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Additional Information

# Development and validation of a radial turbine efficiency and mass flow model at design and off-design conditions

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## Abstract

Turbine performance at extreme off-design conditions is growing in importance for properly computing turbocharged reciprocating internal combustion engines behaviour during urban driving conditions at current and future homologation cycles. In these cases, the turbine operates at very low flow rates and power outputs and at very high blade to jet speed ratios during transitory periods due to turbocharger wheel inertia and the high pulsation level of engine exhaust flow. This paper presents a physically based method that is able to extrapolate radial turbines reduced mass flow and adiabatic efficiency in blade speed ratio, turbine rotational speed and stator vanes position. The model uses a very narrow range of experimental data from turbine maps to fit the necessary coefficients. By using a special experimental turbocharger gas stand, experimental data have been obtained for extremely low turbine power outputs for the sake of model validation. Even if the data used for fitting only covers the turbine normal operation zone, the extrapolation model provides very good agreement with the experiments at very high blade speed ratio points; producing also good results when extrapolating in rotational speed and stator vanes position.

*Keywords:* Turbocharger, Adiabatic efficiency extrapolation, Mean-line

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## Acronyms

VGT	Variable geometry turbine
FGT	Fixed geometry turbine
ICE	Internal combustion engine

## Nomenclature

$A$	Area ( $m^2$ )
$a$	Rotor discharge coefficient (-)
$b$	Reduced mass flow fitting coefficient (-)
$c$	Reduced mass flow fitting coefficient (-)
$C_D$	Discharge coefficient (-)
$c_{ss}$	Isentropic jet velocity ( $ms^{-1}$ )
$c_p$	Specific heat capacity at constant pressure ( $JK^{-1}$ )
$D$	Diameter ( $m$ )
$d$	Reduced mass flow fitting coefficient (-)
$K$	Efficiency equation coefficient (-)
$L$	Length ( $m$ )
$\dot{m}$	Mass flow rate ( $kg s^{-1}$ )
$\dot{m}_{red}$	Reduced mass flow rate ( $kg K^{1/2} s^{-1} bar^{-1}$ )
$n$	Rotational speed ( $rpm$ )
$n_{red}$	Reduced rotational speed ( $rpm K^{-1/2}$ )
$n_b$	Number of blades (-)
$p$	Pressure ( $Pa$ )
$R$	Perfect gas constant ( $J kg^{-1} K^{-1}$ )
$r$	Rotor radius ( $m$ )
$sd$	Standard deviation (-)

$sp$	Spacing between stator blades ( $m$ )
$t$	Blades and/or channel width ( $m$ )
$T$	Temperature ( $K$ )
$\dot{Q}$	Heat flux ( $W$ )
$u$	Blade tip speed ( $ms^{-1}$ )
$v$	Absolute velocity ( $ms^{-1}$ )
$VGT$	VGT position (%)
$\dot{W}$	Power ( $W$ )
$w$	Relative velocity ( $ms^{-1}$ )

#### **Greek symbols**

$\alpha$	Absolute velocity angle ( $rad$ )
$\beta$	Relative velocity angle ( $rad$ )
$\gamma$	Specific heat capacities ratio ( $-$ )
$\delta$	Angle between consecutive stator blades ( $rad$ )
$\eta$	Efficiency ( $-$ )
$\theta$	Tangential velocity component ( $ms^{-1}$ )
$\Pi$	Pressure ratio ( $-$ )
$\rho$	Density ( $kgm^{-3}$ )
$\sigma$	Blade to jet speed ratio ( $-$ )
$\varphi$	Angle of the stator vanes ( $rad$ )

#### **Subscripts and superscripts**

0	Turbine inlet station
1	Stator inlet station
2	Stator outlet station
2'	Stator throat station
2a	Stator vanes axis of rotation station
3	Rotor inlet station
4	Rotor outlet station
<i>geom</i>	Refers to geometry
<i>metal</i>	Refers to metal angle

<i>Neq</i>	Refers to equivalent nozzle
<i>red</i>	Refers to reduced variables
<i>s</i>	Isentropic conditions and Stator
<i>t</i>	Total conditions
<i>T</i>	Refers to turbine
<i>TE</i>	Distance between stator blades axis of rotation and trailing edge
<i>th</i>	Refers to throat
<i>ts</i>	Total to static
-	Average value

## 1. Introduction

In the past years an important increment of interest in improving the prediction of transient and partial load conditions of turbocharged reciprocating internal combustion engines (ICE) has appeared. Due to the strict emission regulations engine manufactures focus engine design in operating conditions different from the traditional full load conditions. As it is showed in [1], during engine transient and partial load design conditions for the ICE the turbocharger turbine works at off-design conditions. In these off-design conditions the turbine works at high blade to jet speed ratios ( $\sigma$ ) or low pressure ratios and low power outputs as shown in [2] due to turbocharger wheel inertia and pulsating flow in the exhaust of the ICE. Traditional measurements of turbine maps in gas stands are unable to capture this behaviour [3]. Only a narrow range turbine map is provided by manufacturers as a standard practice. Turbine maps are necessary when using 1D or 0D modelling tools to predict the whole engine behaviour. In 1D modelling approach the one-dimensional unsteady non-homentropic mass, momentum and energy conservation laws (Euler equations) are solved. Specific source terms are used to simulate the friction and heat exchange in the pipes. The 1D simulation codes make possible the calculation of gas dynamics engine behaviour at low computational costs. Some engine components are modelled

20 with a 0D approach, using specific lumped parameter models or performance  
21 maps. That is the case of cylinders, injectors, valves, compressors and turbines  
22 which are coupled to the 1D computational domain as it is described in [4].  
23 For that reason, turbocharged ICE designers must rely on map extrapolation  
24 tools when predicting engine performance outside of turbine design operative  
25 conditions [5]. It is typical in pulsating flow conditions, requiring different mod-  
26 elling approaches similar to the proposed in [6], where meanline one-dimensional  
27 models are discussed and in [7], where non-adiabatic pressure loss boundary con-  
28 dition is discussed. One-dimensional tools are also used in design process for  
29 fast evaluation of turbine capabilities as in [8]. In [9] a Taylor series expansion  
30 is used to develop a model able of predicting mass flow parameter of radial  
31 turbines.

32 CFD models for turbine design have been developed in [10]. This approach is  
33 useful when turbine CAD files are ready. However, this information is not always  
34 available for automotive engines simulation. Full three-dimensional simulations  
35 can reproduce turbocharger behaviour only at a very high computational cost  
36 [11] what means that these simulations are only performed at few operating  
37 points [12]. Therefore, for whole driving cycle simulations, 1D or 0D approaches  
38 must be used to keep low computational costs and an adequate precision.

39 In the last years, several proposals have appeared in the literature regarding  
40 this topic. Some of them are based on pure theoretical approaches [13] but  
41 use parameters in losses models that have not been proved to be general on  
42 a wide range of turbine sizes or VGT (variable geometry turbine) positions.  
43 The same can be said from [6] for adiabatic and from [7] for non-adiabatic  
44 turbines modelling. Other models are based on physical considerations but  
45 use empirical parameters for fitting stator outlet flow angles, without a clear  
46 correlation against physical values of average flow angles [14]. Furthermore, the  
47 model proposed in [14] relies on tangent functions, which are mathematically  
48 unstable during fitting procedures using numerical methods. Moreover, the  
49 model shown in [14] was only validated for blade to jet speed ratio extrapolation,  
50 not for turbocharger speed or VGT position extrapolation. Some models in the

51 literature are based on the characterisation of the different losses of the turbine  
52 [15] such as passage losses or tip clearance losses [16] but no general procedure  
53 for coefficients fitting or a comprehensive model validation at highly off-design  
54 conditions have been reported yet. Other models are purely empirical and use  
55 the information of the map to fit coefficients as has been done in [17] for SI and  
56 DI engines control. A similar approach has been used in [18] for automotive  
57 engines simulation, also with a control oriented objective. A review of the  
58 advantages and disadvantages of each kind of model has been performed in [19].

59 As any extrapolation tool, the extrapolation models have always faced an  
60 important validation problem. The validity of the modelling can be checked for  
61 the measured conditions but not in the outside area, where it is really interesting  
62 to use the model. To overcome this problem, special tests have been designed in  
63 a gas stand to measure a turbine outside of its design range, at extremely high  
64  $\sigma$  [20]. This new approach provides wider  $\sigma$  range than using a closed circuit in  
65 the compressor to extended the operational range of the turbine [21].

66 In the present paper, a model for turbine characteristics extrapolation is  
67 presented. The developed model is suitable to be used in 1D and 0D control  
68 oriented simulation codes. The model is capable of extrapolating to non mea-  
69 sured VGT positions and reduced speeds. Furthermore, it has been developed  
70 based on a database of measured turbochargers and it has been validated in a  
71 high  $\sigma$  range by using experimental results from special gas stand tests [20].

72 The main differences between the model proposed in this paper and literature  
73 models lies in the enhanced extrapolation capabilities, the generality and the  
74 low quantity of input data needed. The proposed model is able to extrapolate  
75 to non-measured VGT position maps, to non-measured reduced speeds and to  
76 non-measured blade to jet speed ratios both mass flow parameter and adiabatic  
77 efficiency. Any radial turbine can be extrapolated with the model if a standard  
78 map or a number of measured operative conditions (higher than 7) are available  
79 as an input. Therefore, the model proposed in this paper can be generally used  
80 for extrapolating different radial turbines only requiring basic geometry data.

81 The paper has been divided in five main sections. In section 2, a brief

82 description of the experimental study is exposed. In sections 3 and 4, the  
83 extrapolation model is exposed in detail, covering both the reduced mass flow  
84 extrapolation and the adiabatic efficiency extrapolation. Finally, in section 5,  
85 the model is also validated using the special experimental results at high  $\sigma$ . In  
86 section 6, work conclusions are listed.

## 87 **2. Experimental study**

88 Extremely high blade to jet speed ratio experimental data were obtained in  
89 order to check the map extrapolation model. The experimental arrangement  
90 used to obtain these data is summarised in the following subsection.

### 91 *2.1. Experimental setup*

92 A standard turbocharger gas stand was modified so very low turbine output  
93 powers were measurable. The modifications were necessary since in normal con-  
94 ditions, the turbine operating point is controlled by means of the turbocharger  
95 compressor, which acts as a brake [21]. In standard turbocharger gas stands,  
96 as the rig described in [22], both the compressor inlet and outlet lines are con-  
97 nected to the ambient, and a back-pressure valve is placed inside the latter so  
98 the compressor flow rate can be restricted. The turbine is fed with pressurised  
99 air at its inlet, whereas its outlet is connected to the ambient. As the operating  
100 range of a radial compressor is quite limited for a given rotational speed when  
101 used in a standard gas stand, only narrow turbine maps can normally be mea-  
102 sured. On the one hand, the maximum power consumed by the compressor, for  
103 a given speed, is produced at its maximum flow rate, which is limited due to  
104 flow choking in any zone of the inducer or diffuser inlet. On the other hand, the  
105 minimum power consumed by the compressor is limited by surge phenomena  
106 at low flow rates. The turbine produces lower power outputs when rising its  $\sigma$ ,  
107 so the maximum value that can be measured is limited by the minimum power  
108 consumed by the compressor and the friction losses.

109 In the experimental campaign designed for high  $\sigma$  testing, the gas stand  
110 was modified so the turbocharger compressor was used as a kind of centrifugal



111 turbine. This way, the turbocharger turbine power output could be lowered and  
112 its  $\sigma$  risen at constant turbo speed and in steady-state conditions, as part of  
113 the mechanical power losses could be overcome by the compressor. Pressurised  
114 air was fed to the turbocharger compressor at its inlet and expanded inside it,  
115 thus producing power instead of consuming it. A detailed description of the  
116 experimental campaign and its physical basis can be found in [20].

117 The turbocharger measured during this experimental campaign has a VGT.  
118 It is sized for a 2 litres turbocharged reciprocating, compression-ignition ICE.  
119 As the adiabatic efficiency of the turbine was the objective to be obtained, it was  
120 measured in almost-adiabatic conditions: the turbine and oil inlet temperatures  
121 were kept as close as possible to the compressor outlet temperature during the  
122 experiments, so the internal heat fluxes were minimised. In spite of this, residual  
123 heat transfer phenomena have an important effect on gas stand measurements  
124 as shown in [23]. When appearing, these effects have been taken into account  
125 using the correlations proposed in [24]. The ducts and the turbocharger were  
126 also thermally insulated in order to reduce the external heat transfer.

127 Residual heat transfer effects were anyway measured due to oil temperature  
128 increment caused by friction losses and considered to get pure adiabatic effi-  
129 ciency as described in [25]. The turbine flow rate, inlet and outlet pressures and  
130 inlet and outlet adiabatic temperatures were obtained experimentally, as well  
131 as the turbocharger speed, to calculate the turbine efficiency at high  $\sigma$  Table 1  
132 shows details about instrumentation errors. A summary of the results is shown  
133 in sections 4 and 5 of present paper. More details about these results can be  
134 found in [20]. The details about uncertainty calculation in experimental results  
135 and in measured turbine efficiency were already discussed in reference [26].

## 136 *2.2. Experimental outputs description*

137 The experiments provided the necessary data to calculate turbine map pa-  
138 rameters: reduced speed, reduced mass flow, pressure ratio, adiabatic efficiency  
139 and  $\sigma$ . As it is usual in turbocharger practice several lines of constant reduced  
140 speed have been measured.

141 The turbine reduced speed  $n_{red}$  was computed using equation (1).

$$n_{red} = \frac{n}{\sqrt{T_{0t}}} \quad (1)$$

Table 1: Accuracies of measurement sensors

Variable	Sensor type	Accuracy
Gas pressure	Piezoresistive	$\pm 2500$ Pa
Gas and metal temperature	K-type thermocouple	$\pm 2.2$ K
Gas mass flow	V-cone	$\pm 0.5$ %
Oil pressure	Piezoresistive	$\pm 2500$ Pa
Oil temperature	RTD	$\pm 0.15$ K
Oil mass flow	Coriolis	$\pm 0.1$ %

142 where  $n$  is the turbocharger speed and  $T_{0t}$  is the turbine total inlet temperature,  
 143 according to stations numbering shown in Figure 1a. The turbine reduced flow  
 144 rate  $\dot{m}_{red}$  was computed using equation (2).

$$\dot{m}_{red} = \frac{\dot{m} \cdot \sqrt{T_{0t}}}{p_{0t}} \quad (2)$$

145 where  $\dot{m}$  is the turbine mass flow rate and  $p_{0t}$  is the turbine inlet total pressure.  
 146 The turbine total to static pressure ratio  $\Pi_{0,4(ts)}$  is computed using equation  
 147 (3).

$$\Pi_{0,4(ts)} = \frac{p_{0t}}{p_4} \quad (3)$$

148 where  $p_4$  is the turbine outlet pressure. The turbine adiabatic efficiency  $\eta_{ts}$  is  
 149 computed as the ratio of the turbine power output  $\dot{W}_T$  and the turbine isentropic  
 150 power output  $\dot{W}_{T,s}$  as shown in equation (4).

$$\eta_{ts} = \frac{\dot{W}_T}{\dot{W}_{T,s}} = \frac{\dot{m}_T \cdot \bar{c}_p \cdot (T_{0t} - T_{4t}) - \dot{Q}_T}{\left( \dot{m}_T \cdot \bar{c}_p \cdot T_{0t} - \dot{Q}_T \right) \cdot \left[ 1 - \left( \frac{p_4}{p_{0t}} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad (4)$$

151 where  $\bar{c}_p$  is the average specific heat capacity at constant pressure of air,  $T_{4t}$  is  
 152 the turbine outlet total temperature,  $\gamma$  is the specific heat capacities ratio of  
 153 air and  $\dot{Q}_T$  is the residual heat transfer inside the turbine in almost adiabatic  
 154 conditions as seen in [23] and [25].  $\dot{Q}_T$  comes from the oil outlet temperature  
 155 increment. Oil outlet temperature is higher than oil inlet due to friction losses.  
 156 The higher oil outlet temperature generates residual heat fluxes travelling from  
 157 oil bearings housing to turbine housing; as oil inlet has been kept equal to turbine  
 158 inlet in almost adiabatic tests. Therefore, these have to be subtracted to get  
 159 adiabatic turbine power from temperature difference measurement through the  
 160 turbine [25]. Finally, the blade to jet speed ratio,  $\sigma$ , is calculated using equation  
 161 (5).

$$\sigma = \frac{2 \cdot \pi \cdot n \cdot r_3}{\sqrt{2 \cdot \bar{c}_p \cdot T_{0t} \cdot \left[ 1 - \left( \frac{p_4}{p_{0t}} \right)^{\frac{\gamma-1}{\gamma}} \right]}} \quad (5)$$

162 where  $r_3$  is the maximum turbine wheel radius Figure 1a. The experimental  
 163 data will be used to validate the extrapolation model in sections 4 and 5 of the  
 164 paper.

### 165 3. Reduced mass flow extrapolation procedure

#### 166 3.1. Equivalent nozzle model development and main hypotheses

167 The extrapolation procedure is based on modelling the turbine as a single  
 168 equivalent nozzle. In that way, an equation of the throat area of that equivalent  
 169 nozzle must be deduced and it must depend only on easy measurable geometry  
 170 of the turbine and on the information available in a standard map. In Figure 1a  
 171 this procedure is sketched, where it is shown that the equivalent nozzle covers  
 172 from station 0 to 4 of the radial turbine. Continuity equation can be applied to  
 173 stator, rotor and equivalent nozzle, as shown in equation (6). The velocity of the  
 174 equivalent nozzle in that equation can be obtained comparing the enthalpy drop  
 175 of the stages (stator and rotor) with the enthalpy drop of the equivalent nozzle  
 176 [14]. Figure 1b shows the ratio of pressure drop in the rotor to total pressure

177 drop in the VGT. From this comparison equation (7) can be obtained, where  
 178 the velocities  $v_{2'}$  and  $w_4$  have been introduced as function of mass flow using  
 179 equation (6). For the derivation of the equation (7) it has been assumed that  
 180  $p_3 = p_{2'}$  and that the flow is incompressible ( $\rho_3 = \rho_{2'}$ ) due to the small size of  
 181 the vaneless space. Solving for the mass flow and comparing with equation (6)  
 182 again it is possible to find the expression for the equivalent nozzle area shown in  
 183 equation (8) [14]. At that point it is necessary to transform this equation into  
 184 an equation where only the variables available in turbine maps appear, as well  
 185 as some global geometrical definitions.

$$\dot{m}_T = A_{2'} \rho_{2'} v_{2'} = A_4 \rho_4 w_4 = A_{Neq} \rho_4 v_{Neq} \quad (6)$$

$$v_{Neq}^2 = \left( \frac{\dot{m}_T}{A_{2'} \rho_{2'}} \right)^2 + u_3^2 - u_4^2 + \left( \frac{\dot{m}_T}{A_4 \rho_4} \right)^2 - w_3^2 \quad (7)$$

$$A_{Neq} = A_4 \sqrt{\frac{1 + \left( \frac{u_4}{v_{Neq}} \right)^2 - \left( \frac{u_3}{v_{Neq}} \right)^2 + \left( \frac{w_3}{v_{Neq}} \right)^2}{\left( \frac{A_4}{A_{2'}} \right)^2 \left( \frac{\rho_4}{\rho_{2'}} \right)^2 + 1}} \quad (8)$$

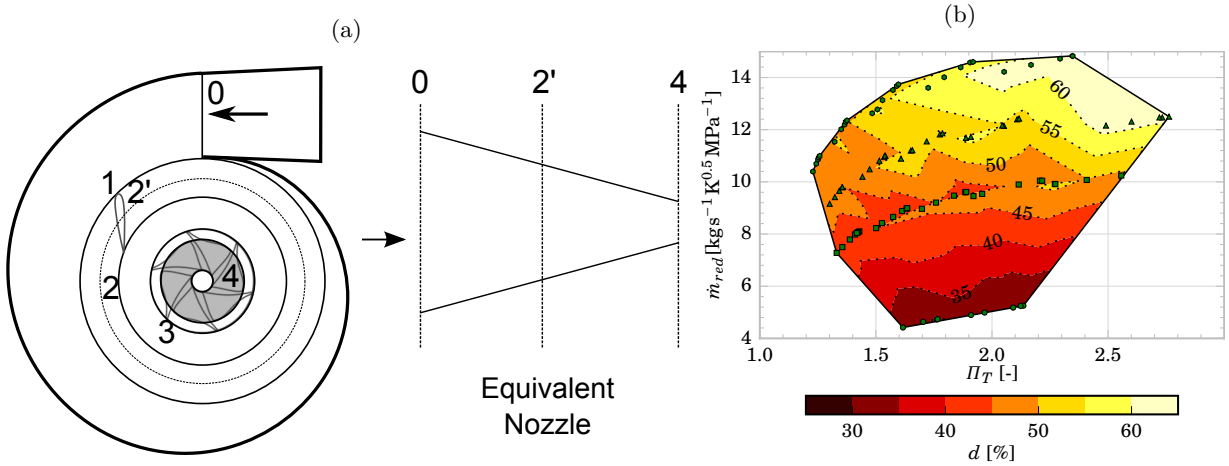


Figure 1: (a) Reduction of a radial turbine to an equivalent nozzle and stations distribution.  
 (b) Measured ratio of pressure drop in the rotor to total pressure drop in the turbine ('d') in  
 the VGT map [27] of T#1A

186 The model must have the capability to compute the geometrical throat sec-  
 187 tion of the stator vanes ( $A_{2'}^{geom}$ ) based on VGT position and geometry of the  
 188 turbine. Equation (9) shows how the throat length of the stator ( $l_{th2'}$ ) can  
 189 be obtained, making use of Figure 2a and of equation (10). In order to get  
 190 the necessary  $\varphi_2^{metal}$  value for equation (9) several VGTs, corresponding to Ta-  
 191 ble 2 have been characterised. Figure 1b shows the mass flow map of T#1A  
 192 detailing 4 VGT positions. If detailed vanes geometry were available a more  
 193 precise geometrical characterisation would be possible, as the one described in  
 194 [28]. Figure 2b shows empirically obtained relations between stator blades av-  
 195 erage angle ( $\varphi_2^{metal}$ ) and VGT position for the three tested turbines. In the  
 196 case that this information was not possible to be obtained, the average values  
 197 of slope and y-intercept of the mean line shown in Figure 2b can be used. The  
 198 angle of the stator vanes can then be calculated using equation (11), obtained  
 199 from Figure 2b for any VGT position percentage. It must be taken into account  
 200 that the relations shown in Figure 2b come from cold conditions and variations  
 201 at hot operational conditions, due to metal thermal expansion and loading in  
 202 the blades mechanism, must be expected. These thermal expansions introduce  
 203 further uncertainty in calculating geometrical throat length ( $l_{th2'}$ ).

$$l_{th2'} = 2r_2 \cdot \sin(\delta/2) \cdot \cos(\varphi_2^{metal}) \quad (9)$$

$$r_2 = \sqrt{(r_{2a} - L_{TE} \cdot \cos(\varphi_2^{metal}))^2 + (\sin(\varphi_2^{metal}) \cdot L_{TE})^2} \quad (10)$$

$$\varphi_2^{metal} \text{ (deg)} = -0.004 \cdot VGT + 79.36 \quad (11)$$

204 Finally  $A_{2'}^{geom}$  can be calculated as shown in equation (12)

$$A_{2'}^{geom} = l_{th2'} \cdot t_2 \cdot n_2 \quad (12)$$

Table 2: Tested VGTs data

	T#1A	T#1B	T#2
Turbine wheel diameter [mm]	40	39	38
Compressor wheel diameter [mm]	48.5	48	46
Water-cooling	yes	yes	no
Engine displacement [l]	2.0	2.0	1.6

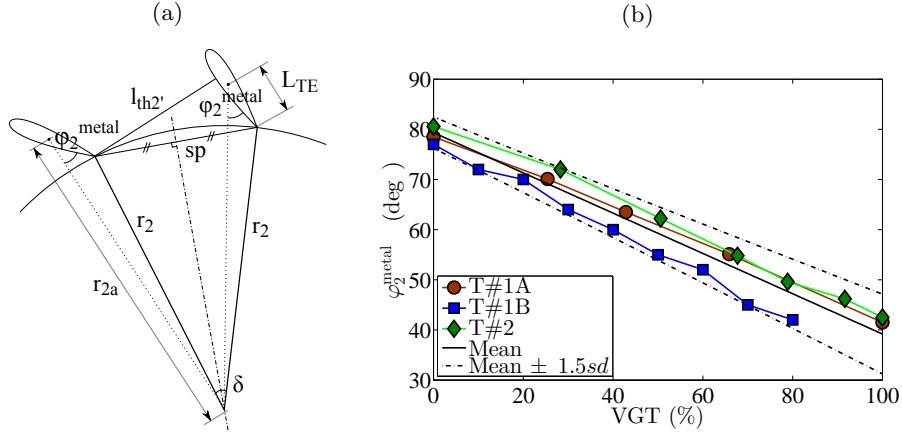


Figure 2: (a) Geometrical relations of a VGT stator vanes (b) Linear relations between stator vanes angle and VGT opening

205 Continuing with the simplifications of equation (8) and knowing the diame-  
 206 ters of turbine rotor it is possible to group the terms of  $u_3$  and  $u_4$ . After that,  
 207 using the total to static turbine adiabatic efficiency definition (equation (9))  
 208 it is possible to introduce the isentropic velocity  $c_{ss}$  to replace the equivalent  
 209 nozzle velocity ( $v_{Neq}$ ). An additional approximation can be made in this point  
 210 following the dimensional analysis of equation (13).

$$\frac{v_{Neq}^2}{c_{ss}^2} = \eta_{ts} + \frac{v_4^2}{c_{ss}^2} = O[10^{-1}] + O[10^{-4}] \Rightarrow \frac{v_{Neq}^2}{c_{ss}^2} \approx \eta_{ts} \quad (13)$$

211 With this approximation it is now possible to introduce the blade to jet

212 speed ratio, defined in equation (5), in the equivalent nozzle area expression.  
 213 At this point the equivalent nozzle area can be approximated by equation (14),  
 214 where the term  $(w_3/c_{ss})^2$  is lumped in 'b' fitting coefficient.

$$A_{Neq} = A_4 \sqrt{\frac{1 + \frac{\sigma^2}{\eta_{ts}} \left[ \left( \frac{D_4}{D_3} \right)^2 - 1 \right] + \frac{b}{\eta_{ts}}}{\left( \frac{A_4}{A_{2'}} \right)^2 \left( \frac{\rho_4}{\rho_{2'}} \right)^2 + 1}} \quad (14)$$

215 There is an inherent problem with equation (14). At certain conditions;  
 216 mainly at high reduced speeds and low pressure ratios; the term inside of the  
 217 square root may become negative, specially at very low values of  $\eta_{ts}$ . To avoid  
 218 possible instabilities during extrapolations a constant average value of efficiency  
 219 is assumed ( $\eta_{ts} = \bar{\eta}_{ts} = 0.8$ ), relying on 'b' coefficient to provide sufficient  
 220 numerical flexibility to the model.

221 Finally, it is necessary to find an expression for the density ratio shown in  
 222 equation (14). Authors have stated that the hypotheses taken in [14] are not  
 223 consistent enough for non-measured VGT positions extrapolation. So, they have  
 224 been revised and changed here with the objective of finding an expression for  
 225 the density ratio by using the equation of ideal gases and expressing the density  
 226 ratio as a product of pressure ratio ( $p_4/p_3$ ) and temperature ratio ( $T_3/T_4$ ). The  
 227 temperature ratio can be estimated using adiabatic efficiency of the rotor. How-  
 228 ever, rotor adiabatic efficiency is not available in turbine maps so an additional  
 229 assumption is made in this point and for this single purpose: i.e. the efficiency  
 230 of the rotor is equal to turbine total to static efficiency. This strong hypothesis  
 231 is made for the single purpose of getting a temperature ratio and it implies the  
 232 following thermodynamic assumptions:

- 233 • The polytropic index is equal in the rotor and in the stator.
- 234 • Absolute kinetic term at turbine inlet and relative kinetic term at rotor  
 235 inlet are neglected.
- 236 • Turbine inlet velocity is equal to the outlet velocity ( $v_1 = v_4$ ).

237 Taking into account the previous assumption it is possible to express the  
 238 area of the equivalent nozzle using equation (15).

$$A_{Neq} = \frac{a \cdot A_4^{geom} \cdot \sqrt{1 + \frac{\sigma^2 \cdot \left[ \left( \frac{D_4}{D_3} \right)^2 - 1 \right] + b}{\bar{\eta}_{ts}}}}{\sqrt{1 + \left( c \cdot \frac{A_4^{geom}}{A_{2'}^{geom}} \right)^2 \cdot \frac{\left( \frac{1}{\Pi_{2',4}} \right)^2}{\left( 1 - \eta_{ts} \cdot \left( 1 - \left( \frac{1}{\Pi_{2',4}} \right)^{\frac{\gamma-1}{\gamma}} \right) \right)^2}}} \quad (15)$$

239 In this equation a new constant ('d') has been introduced (equation (16))  
 240 through the term  $\Pi_{2',4}$  (equation (17)).  $\Pi_{2',4}$  represents pressure ratio in the  
 241 VGT rotor and 'd' term has been shown in Figure 1b on the VGT map of T#1A.  
 242 This means another last assumption: the stator pressure drop to total pressure  
 243 drop ratio is constant for a VGT position. It is easy to obtain equation (17)  
 244 from that assumption introducing the 'd' fitting coefficient.

$$d = \frac{p_{2'} - p_4}{p_{0t} - p_4} \quad (16)$$

$$\Pi_{2',4} = 1 + d [\Pi_{0,4}^{ts} - 1] \quad (17)$$

### 245 3.2. Model coefficients analysis and calibration

246 Summarizing, both coefficients 'b' and 'd' of equation (15) have physical  
 247 meaning, as they come from theoretical considerations, even with simplifying  
 248 hypotheses. The validity of introduced hypotheses for mass flow parameter  
 249 extrapolation in radial VGTs will be further checked in sections 3.3 and 5 de-  
 250 scribing experimental validation of the model. Therefore their values must have  
 251 a coherent order of magnitude. The analysis of all coefficients in equation (15)  
 252 is detailed below:

- 253 • Coefficient 'a': it represents the discharge coefficient of the rotor (equation  
 254 (18)). As the rotor outlet geometrical area ( $A_4^{geom}$ ) is independent of VGT  
 255 position 'a' can be considered constant for a given turbocharger. The



256 order of magnitude of this coefficient must be between 0 and 1 based on  
 257 the definition of discharge coefficient.

$$A_4 = A_4^{geom} \cdot a \quad (18)$$

258 • Coefficient 'b': it comes from velocity triangles [14]. It is a difficult pa-  
 259 rameter to be calculated experimentally but its order of magnitude can  
 260 be estimated as shown in equation (19). It can also be expected that  
 261 more open VGT positions must give higher 'b' values than closer VGT  
 262 positions; due to the higher radial velocities at rotor inlet, for the same  
 263 pressure ratio and peripheral speed values.

$$\begin{aligned} b &= \left(\frac{v_0}{c_{ss}}\right)^2 + \left(\frac{w_3}{c_{ss}}\right)^2 = \frac{A_{Neq}}{A_0} \cdot \left(\frac{1}{\Pi_T}\right)^{(1/\gamma)} + O[10^{-1}] = \\ &= O[10^{-1}] + O[10^{-1}] \rightarrow O[10^{-1}] \leq b \leq O[10^0] \end{aligned} \quad (19)$$

264 • Coefficient 'c': it represents the quotient between rotor discharge coeffi-  
 265 cient (coefficient 'a') and the stator discharge coefficient ( $C_{Ds}$  as shown in  
 266 equation (20)), which in case of VGT should be dependant on stator vanes  
 267 position (equation (9)). For more open VGT positions  $A_{2'}^{geom}$  is high so  
 268 'c' is expected to be low. In addition 'c' must fulfil always inequation (21)  
 269 since in spite of the effect of throat variation in the stator, effective sec-  
 270 tion, introduced through the discharge coefficient ( $C_{Ds}$ ), must be always  
 271 below 1

$$A_{2'} = A_{2'}^{geom} \cdot C_{Ds} \quad (20)$$

$$C_{Ds} = \frac{a}{c} \leq 1 \quad (21)$$

272 • Coefficient 'd' (equation (16)): from experimental measurements of pres-  
 273 sure in the space between stator and rotor in a VGT, it is possible to  
 274 estimate the order of magnitude of the coefficient 'd' [27] and to confirm

275 that the assumption of constant value for a given VGT position is coher-  
 276 ent for small radial VGTs. Figure 1b shows that the coefficient increases  
 277 with VGT opening with a variation in the range of [0.35, 0.6].

278 Once the equivalent nozzle area (equation (15)) is known, the reduced mass  
 279 flow can be calculated using the expression of flow through an orifice with isen-  
 280 tropic expansion, equation (22).

$$\dot{m}_{red} = A_{Neq} \sqrt{\frac{\gamma}{R}} \left( \frac{1}{\Pi_{0,4}^{(ts)}} \right)^{\frac{1}{\gamma}} \sqrt{\frac{2}{\gamma-1} \left[ 1 - \left( \frac{1}{\Pi_{0,4}^{(ts)}} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad (22)$$

281 As it has been already explained, the extrapolation model discussed above is  
 282 evolved from a previous model described in [14]. That model had the problem of  
 283 being unable to extrapolate to non-measured VGT positions. Their hypotheses  
 284 have been revised and improved here. Nevertheless, in order to get the new  
 285 extrapolation capabilities the following procedure is also needed.

286 The main approach to develop the possibility of extrapolation to VGT posi-  
 287 tions is based on the analysis of the physical meaning of the fitting coefficients  
 288 of the model. These coefficients are constant for a given VGT position, i.e. they  
 289 are independent of turbine speed and of pressure ratio.

Table 3: Coefficient 'a' values

	Average	T#1A	T#1B	T#2
$a \pm sd$	$0.40 \pm 0.07$	0.41	0.47	0.33

290 Using the turbochargers listed in Table 2, it is possible to provide a first  
 291 approach for the behaviour of the fitting coefficients that give good precision  
 292 in terms of reduced mass flow prediction compared to experiments. So fitting  
 293 the coefficients of equation (15) for several VGTs it is possible to survey their  
 294 behaviour with VGT position. The fitting is performed using the available data  
 295 of the map in which the reduced mass flow is known. Previously described  
 296 physical trends have been imposed in the fitting procedure, for instance the 'b'

297 coefficient of more open VGT position is imposed to be higher or equal than for  
298 the closer position. Significance test have been performed to check the necessity  
299 of the different coefficient proving each of them to be statistically significant  
300 with a p-value lower than 0.05.

301 In Figure 3 the obtained results for the fitting coefficients that vary with  
302 VGT position ('b', 'c' and 'd') are shown for the turbochargers listed in Ta-  
303 ble 2. This results were used to detect the trends in the coefficients. For the  
304 coefficient 'b' an increasing trend is obtained, according to the expected be-  
305 haviour. Figure 3a shows that the values of this coefficient are close to a linear  
306 behaviour of positive slope. For 'c' coefficient the trend with VGT position must  
307 be decreasing; this has been confirmed in Figure 3c, obtained from the fitting of  
308 several VGTs and several positions after imposing such a negative trend. Again  
309 'c' trend is close to linear behaviour but now with a negative slope. Coefficient  
310 'd' behaviour is in accordance with experimental results from Figure 1b and  
311 shows similar positive trend than constant 'b', as it is shown in Figure 3e. In  
312 that way, coefficients dependence on VGT position has been obtained.

### 313 *3.3. Model validation*

314 Using the previous information it is possible to provide an imposed trend for  
315 the global map fitting. Only one fitting will be now needed for each turbocharger  
316 in which 'a' will be constant and a linear trend with VGT position for the other  
317 three coefficients will be imposed. In that way, seven coefficients must be fitted  
318 using a non-linear fitting procedure for each VGT. The values of the discharge  
319 coefficient of the rotor ('a'), for the three turbochargers, are summarised in Table  
320 3. It can be observed that for T#2 rotor discharge coefficient ('a') is lower than  
321 for T#1A and T#1B, probably due to lower wheel diameter. As initial values  
322 for the fitting, average values from the dependence study have been used and  
323 the upper and lower bounds are 3 times standard deviation. Furthermore it  
324 is necessary to add the condition of equation (21) to make 'c' coefficient to be  
325 fully physical, as the discharge coefficient of the stator cannot be higher than  
326 unity. In that way the results of Figure 3b, Figure 3d and Figure 3f have been

327 obtained for the three coefficients and the three VGTs. The linear fitting of the  
 328 'b', 'c' and 'd' constants is shown for the three VGTs as well as the average value  
 329 and the boundaries for the fitting of new turbochargers. Lower boundary of 'b'  
 330 coefficient is zero instead of using standard deviation in order to avoid negative  
 331 values, which are not physically possible. Lower boundary of 'c' is calculated to  
 332 avoid stator discharge coefficient values higher than one (equation (21)).

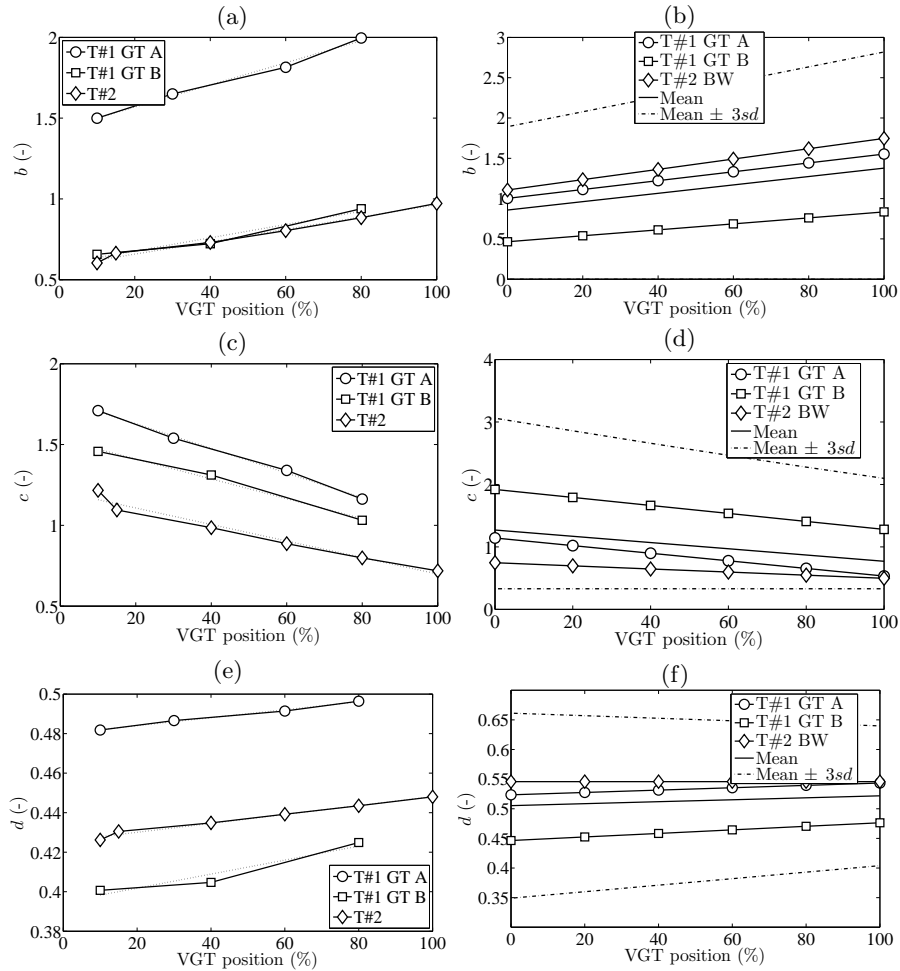


Figure 3: Reduced mass flow coefficients dependence with VGT position (a, c and e) and final fitted coefficients (b, d and f)

333 In that way, for extrapolating a new turbine reduced mass flow map using

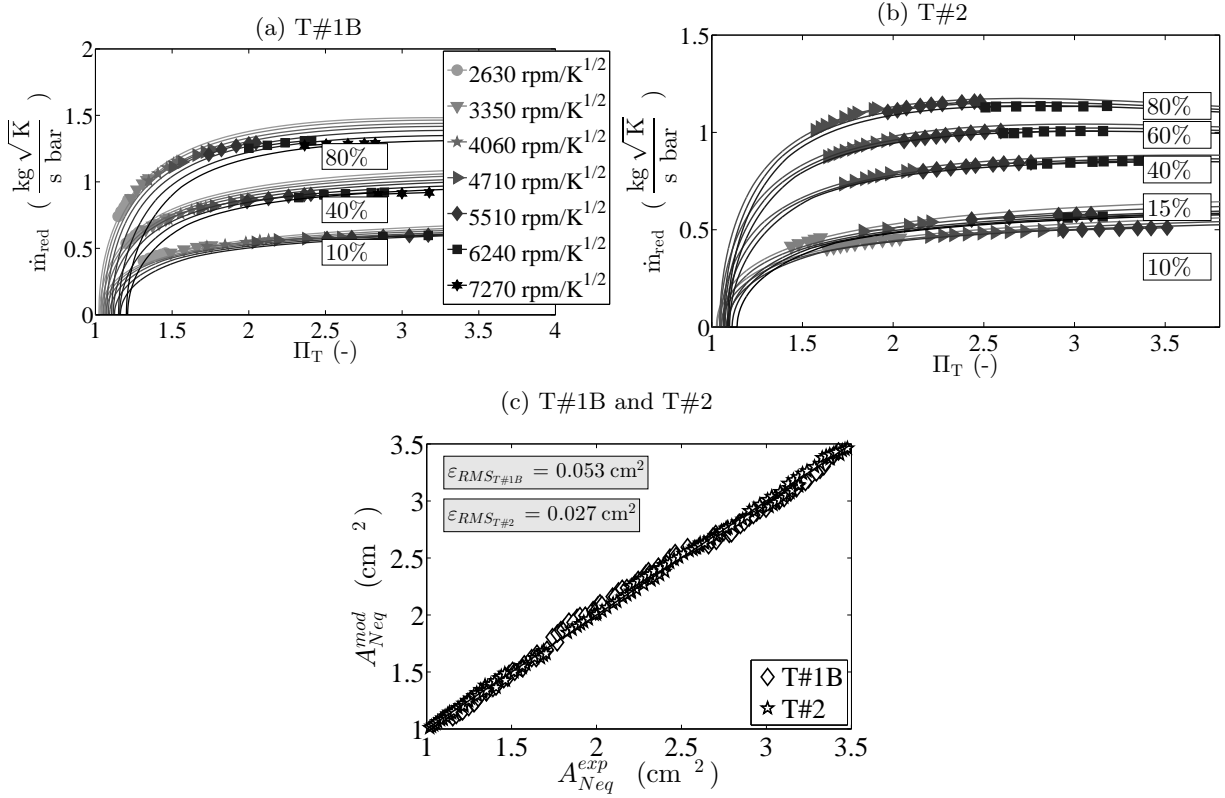


Figure 4: Fitted equivalent area and reduced mass flow for T#1B (a) and for T#2 (b) and measured versus modelled  $A_{Neq}$  (c)

334 this approach a first fitting of seven coefficients must be done using the data  
 335 from the map. After that fitting the reduced mass flow for any VGT position  
 336 and pressure ratio or reduced speed can be calculated.

337 The values of the fitted coefficients correspond to the mass flow parameter  
 338 fittings shown in Figure 4 where dots correspond to measured reduced mass flow  
 339 and lines to interpolated and extrapolated values for different VGT positions  
 340 of T#1B and T#2. Figure 4 shows that the errors in modelling reduced mass  
 341 flow are small enough to consider valid the above explained procedure. Figure  
 342 4 also shows the calculated values of  $A_{Neq}$  that range between 1/3 and 1/10 of  
 343  $A_0^{geom}$  (refers to station 0 in Figure 1a), depending on VGT position, mass flow  
 344 and turbo speed. In case of T#2, Figure 4b, the values of the most open VGT

345 position (80%) have not been used for model fitting but extrapolated.

## 346 4. Adiabatic efficiency extrapolation model

### 347 4.1. Model development and main hypotheses

348 In order to improve efficiency fitting model it is necessary to extrapolate  
349 both in rotational speeds and in VGT position. As the model constants devel-  
350 oped in [14] are dependant on both variables new revision and analysis must be  
351 performed. The procedure to develop the model is based on the analysis of the  
352 available data of the turbines used to refine the models.

353 As described in [14] the efficiency extrapolation is based on using the Euler  
354 equation of turbomachinery and assuming constant meridional component ve-  
355 locities. In that way in equation (23), which represents the definition of total  
356 to static adiabatic efficiency, it is possible to express the numerator in terms  
357 of velocities using Euler equation and turbine enthalpy drop as shown equation  
358 (24). The tangential velocities in that equation can be expressed in terms of  
359 meridional velocity using equation (25) and equation (26). Using them now in  
360 equation (23) and taking into account the isentropic evolution in the denomina-  
361 tor it is possible to obtain equation (27). From these assumptions it is possible  
362 to obtain equation (28) for constant tip speed maps using the definition of  $\sigma$ . It  
363 is worth noting the dependence of equation (28) on  $A_{Neq}$  term that must be cal-  
364 culated using equation (15). From equation (28) the tangent of rotor inlet angle  
365 ( $\alpha_3$ ) is a numerically unstable term. However, a transformation can be used  
366 to convert rotor inlet angle into stator outlet angle as shown in equation (29),  
367 using mass flow and angular momentum conservation equations [1]. Coefficient  
368  $z_3^{geom}$  is a geometrical coefficient that can be obtained theoretically from Figure  
369 2a, as shown in equation (30), where  $l_{th2'}$  can be obtained from equation (9) and  
370  $\varphi_2^{metal}$  for equation (11) respectively and 'c' coefficient is defined in equation  
371 (21). In that way the dependence with VGT position has been also introduced  
372 here. The coefficient 'c' has been fitted previously along with the rest of the  
373 reduced mass flow coefficient using the available data of the map. From all the

374 previous steps, equation (31) can be written for the extrapolation, where some  
375 terms have been lumped into  $K_i$  coefficients for simplification (equations (32),  
376 (33) and (34)). In equation (31),  $K_2^*$  coefficient, shown in equation (34), plays  
377 an important role as a fitting constant ('z') has been added multiplying the sine  
378 function. The rest of the parts of efficiency equation (31) are related to physical  
379 values from turbine geometry or from the map as it has been already described  
380 in [14].  $\varphi_2^{metal}$  and  $\beta_4^{metal}$  assumes that there are negligible deviation angles in  
381 expansion stages, what is usually accepted at high pressure ratios [1].

$$\eta_{ts} = \frac{T_{0t} - T_{4t}}{T_{0t} - T_{4ts}} \quad (23)$$

$$\dot{W} = \dot{m}c_p (T_{0t} - T_{4t}) = \dot{m} (u_3 v_{\theta 3} - u_4 v_{\theta 4}) \quad (24)$$

$$v_{\theta 3} = v_0 \tan \alpha_3 \quad (25)$$

$$v_{\theta 4} = u_3 \left( \frac{r_4}{r_3} \right) - v_0 \tan \beta_4 \quad (26)$$

$$\eta_{ts} = \frac{u_3 v_0 \tan \alpha_3 - \left[ u_3 \left( \frac{r_4}{r_3} \right) - v_0 \tan \beta_4 \right] u_3 \left( \frac{r_4}{r_3} \right)}{c_p T_{0t} \left( 1 - \left( \frac{1}{\Pi_{0,4}^{(ts)}} \right)^{\frac{\gamma-1}{\gamma}} \right)} \quad (27)$$

$$\begin{aligned} \eta_{ts} &= \quad (28) \\ &= -2 \left( \frac{r_4}{r_3} \right)^2 \sigma^2 + 2 \frac{A_{Neq}}{A_0^{geom}} \left( \tan \alpha_3 + \frac{r_4}{r_3} \tan \beta_4 \right) \left[ \frac{1}{\Pi_{0,4}^{(ts)}} \right]^{\frac{1}{\gamma}} \sigma \end{aligned}$$

$$\tan \alpha_3 = z_3^{geom} \sin \varphi_2^{metal} \quad (29)$$

$$z_3^{geom} = c \cdot \frac{r_2 \cdot 2\pi \cdot t_3}{t_2 \cdot l_{th2'} \cdot n_b} \cdot \frac{A_0^{geom}}{A_3^{geom}} \quad (30)$$

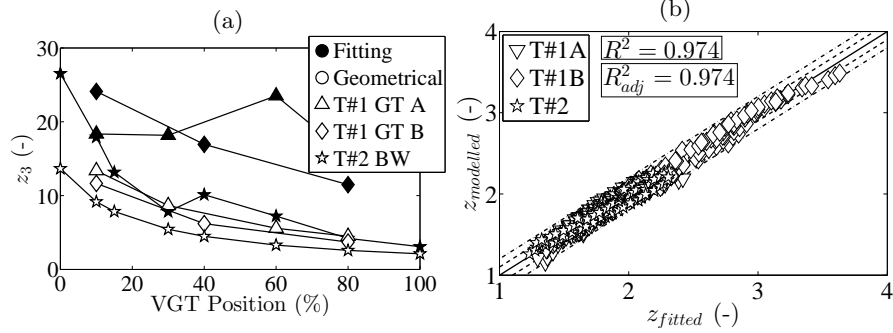


Figure 5: Difference between fitted and geometrical  $z_3^{geom}$  coefficient (a) and modelled 'z' versus fitted 'z' (b)

$$\eta_{ts} = -K_1\sigma^2 + K_2^* \left(1 - \frac{K_3}{\sigma^2}\right)^{\frac{1}{\gamma-1}} \cdot \sigma \quad (31)$$

$$K_1 = 2 \left(\frac{r_4}{r_3}\right)^2 \quad (32)$$

$$K_3 = \frac{u_3^2}{2c_p} \quad (33)$$

$$\begin{aligned} K_2^* &= \\ &= 2 \frac{A_{Neq}}{A_0^{geom}} \left( z \cdot z_3^{geom} \sin(\varphi_2^{metal}) + \sqrt{\frac{K_1}{2}} \tan(\beta_4^{metal}) \right) \end{aligned} \quad (34)$$

382 The reason for using a fitting coefficient 'z' lies in the various simplifica-  
383 tions made to obtain equation (31) that make impossible a good fitting of the  
384 efficiency, specially if off-design conditions must be covered. For example, as-  
385 suming radial velocity equal to the axial one, assuming incompressible flow be-  
386 tween stator and rotor stages and negligible flow deviation in stator and rotor  
387 blades. Indeed, comparing the  $z_3^{geom}$  coefficient using equation (30) and fitting  
388 it to match experimentally obtained adiabatic efficiency gives different results  
389 as shown in Figure 5a.



390 *4.2. Model calibration and validation*

391 There is a fitting 'z' coefficient for each point of turbine map. If this coeffi-  
392 cient is calculated and plotted against blade to speed ratio and reduced speed,  
393 different planar surfaces appear one for each VGT position. From these surfaces  
394 the most simple approach is to use linear decreasing trend for blade to speed  
395 ratio, being the slope and the y-intercept values linearly dependant on reduced  
396 speed. As the normal vector of the surfaces is similar the VGT dependence can  
397 be added directly in the independent term of the surface equation. This is done  
398 considering that the 'z' coefficient tends to increase until the maximum efficiency  
399 VGT position is reached and decrease later on. Therefore a parabolic trend with  
400 VGT position is proposed since maximum efficiency is reached around 60% of  
401 VGT opening. All these considerations lead to equation (35) for the 'z' coef-  
402 ficient where  $n_{red}$  is in ( $rpm/K^{1/2}$ ) and VGT is in (%). In this equation, six  
403 constants must be fitted for a given turbocharger using the data of the whole  
404 map (all available positions). In Figure 5b the level of correlation between fit-  
405 ted 'z' values and modelled 'z' values using equation (35) is shown. It can be  
406 observed that modelled values correlate with the fitted ones as proved by the  $R^2$   
407 and  $R_{adj}^2$  values. Significance test have been performed to check the necessity  
408 of the different coefficient proving each of them to be statistically significant  
409 with a p-value lower than 0.05. The standard deviation of the six coefficients  
410 are shown in the third column of Table 4. It is advised to use three times the  
411 standard deviation for new calibrations boundaries.

$$z = -(a' \cdot n_{red} + b') \cdot \sigma + (c' \cdot n_{red} + d' \cdot VGT^2 + e' \cdot VGT + f') \quad (35)$$

412 In Figure 6 a flowchart of the procedure used to extrapolate mass flow and  
413 efficiency is presented. Following the flowchart, the extrapolation procedure  
414 starts with the input of the available map data. Using this information the  
415 equivalent nozzle area can be solved from equation (22) and used in a non-linear  
416 fitting procedure in order to calibrate the coefficients 'a', 'b', 'c' and 'd'. For  
417 that purpose the boundaries of the different coefficients for the fitting procedure

418 are taken from Table 3 and Figures 3b, 3d, 3f. Using the fitted 'c' value and  
419 the information from the map the efficiency fitting coefficients are calibrated.  
420 After this step all the necessary coefficients are calibrated so the system of  
421 equations composed of equation (15) and equation (31) can be solved with an  
422 iterative procedure to obtain the extrapolated values of equivalent nozzle area  
423 and efficiency. Finally, the extrapolated mass flow parameter is obtained from  
424 equation (22) substituting the extrapolated equivalent nozzle area.

425 After fitting both the reduced mass flow coefficients and the efficiency co-  
426 efficients using the data of the map, the model can be used for extrapolations.  
427 As the efficiency appears in the equivalent nozzle area (equation (15)) and the  
428 equivalent nozzle area appears in the efficiency expression (equation (31)) both  
429 equations must be solved at the same time. It can be done using an itera-  
430 tive procedure as defined in [29]. In Figure 7 the modelled efficiency is plotted  
431 against the measured one for the whole map of each turbocharger and the root  
432 mean square error is shown. In Figure 7 the solid red line indicates perfect fit,  
433 the dashed line indicates 2.5 efficiency points deviation and the dash and dotted  
434 line indicates 5 efficiency points deviation. In Figure 8 the model is extrapo-  
435 lating in blade to jet speed ratio using all the available experimental data to fit  
436 the coefficients of the model. Exception is made for the 80% VGT position in  
437 T#2 that has been fully extrapolated (Figure 8f) while the rest of the cases were  
438 interpolated between measured points. Good agreement between experimental  
439 data (dots) and extrapolated results (lines) can be observed as deduced from  
440 Figure 7.

441 Figure 8f shows that for  $4874 \text{ rpm}/K^{1/2}$  the measured efficiency is equal to  
442 the corresponding to  $5814 \text{ rpm}/K^{1/2}$  what makes no sense. The model extrap-  
443 olation is showing a more coherent behaviour and probably correcting measure-  
444 ments errors.

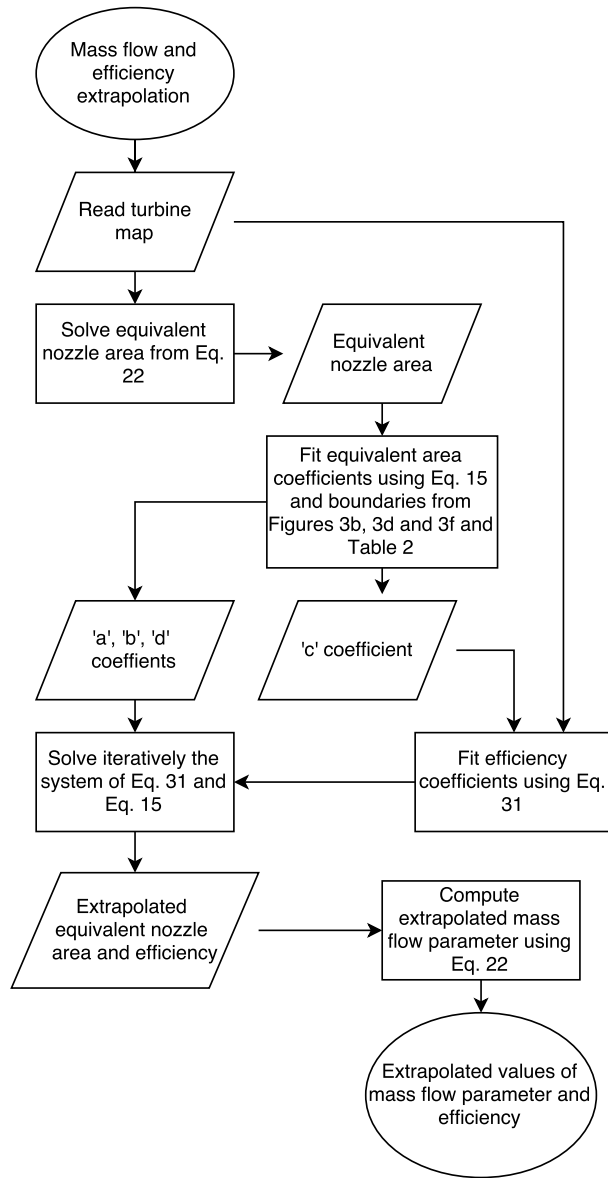


Figure 6: Procedure for mass flow parameter and efficiency extrapolation

Table 4: Efficiency coefficients average and standard deviation

Coefficient	Average	$sd$
$a'$	$1.47 \cdot 10^{-4}$	$3.58 \cdot 10^{-4}$
$b'$	3.33	$3.34 \cdot 10^{-1}$
$c'$	$3.59 \cdot 10^{-4}$	$3.21 \cdot 10^{-4}$
$d'$	$-7.67 \cdot 10^{-5}$	$9.05 \cdot 10^{-5}$
$e'$	$2.73 \cdot 10^{-2}$	$9.45 \cdot 10^{-3}$
$f'$	1.68	$2.38 \cdot 10^{-1}$

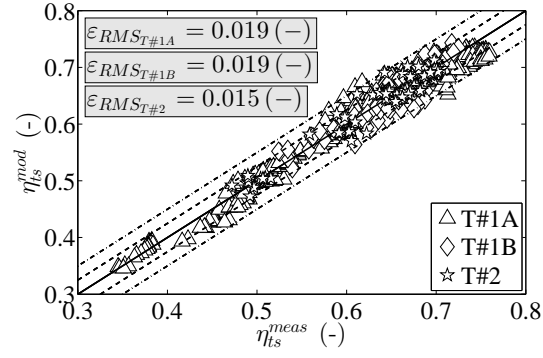


Figure 7: Root mean square error of efficiency fitting for the different turbochargers

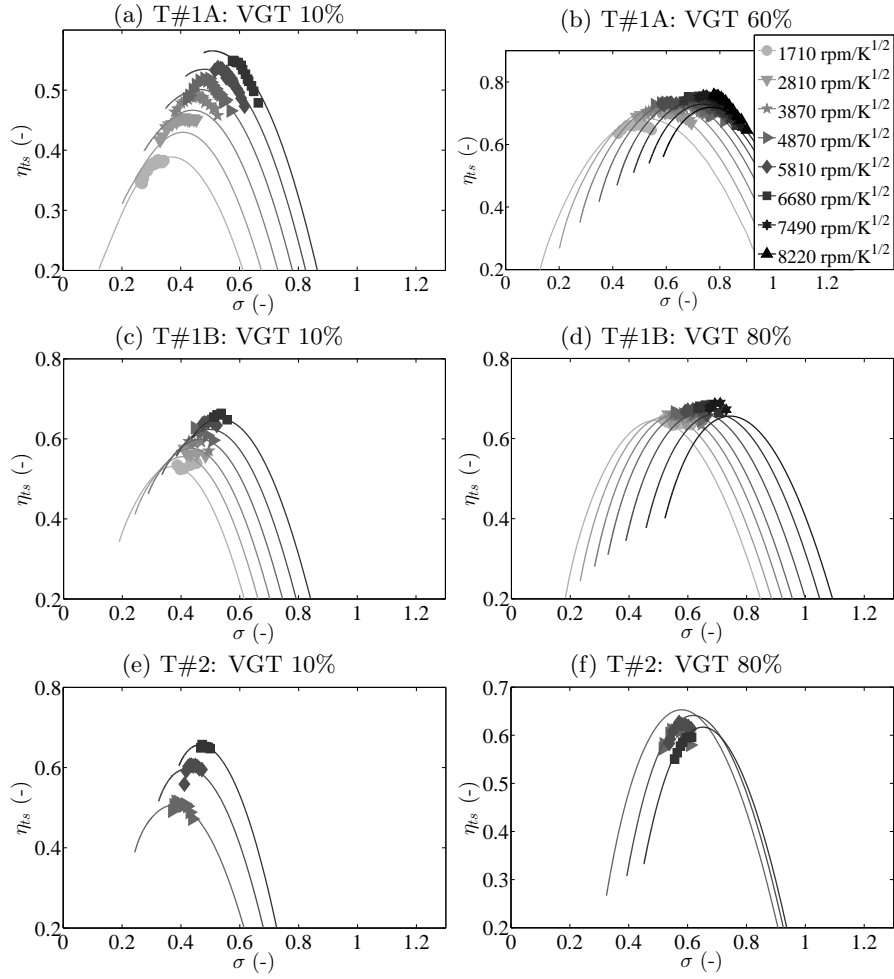


Figure 8: Efficiency extrapolation in blade to jet speed ratio for different VGT positions of T#1A, T#1B and T#2 (dots correspond to experimental data and solid lines to model results)

445 **5. Discussion of model results at high blade to jet speed ratio extrap-**  
 446 **olation**

447 Figure 9 shows the model results against the experimental data for T#1A  
 448 corresponding to the extrapolation results for reduced mass flow rate. Figure 9  
 449 shows the experimental data as points and the model results as solid lines. In  
 450 Figure 9, the model is fitted using only five points per speed line, the ones of

451 maximum pressure ratio (minimum  $\sigma$ ) corresponding to the full points in Figure  
 452 9. Indeed, neither the maximum nor the minimum rotational speeds for each  
 453 VGT have been used during the fitting process, so Figures 9a and 9f are only  
 454 extrapolated results. In this way the model has to extrapolate in both  $\sigma$  and  
 455 turbine reduced speed. Therefore,  $1706 \text{ rpm}/K^{1/2}$  and  $6679 \text{ rpm}/K^{1/2}$  have  
 456 been fully extrapolated.

457 The model shows very good agreement with the experimental data when  
 458 extrapolating at both lower and higher reduced speeds. At extremely low ex-  
 459 pansion ratios (high  $\sigma$ ), the error starts to grow: it occurs at points where the  
 460 adiabatic efficiency of the turbine is lower than zero and it consumes power  
 461 instead of producing it, so very off-design flow patterns should take place in the  
 462 turbine at those conditions [20]. Biggest errors are shown at VGT 80% and  
 463 VGT 10% at reduced speed of  $4874 \text{ rpm}/\sqrt{K}$  (figure 9d)

464 Figure 10 shows the model extrapolation results for efficiency against blade  
 465 to jet speed ratio. Filled point correspond to the data used for model fitting  
 466 and empty points correspond to data used for blind checking. The mean square  
 467 error of the whole extrapolation in this case is  $\varepsilon_{RMS} = 0.02 (-)$ . Only the  
 468 experimental data corresponding to central reduced speeds and low  $\sigma$  were used  
 469 for model fitting (bold points in Figure 10). In Figure 10d VGT 80 % position  
 470 was fully extrapolated by the model (all experimental points are empty). In this  
 471 case the model is able to reproduce with good precision an entire VGT position,  
 472 up to very high  $\sigma$ . The differences between the model and the experimental  
 473 data are maximum at the reduced speed of  $4874 \text{ rpm}/\sqrt{K}$  and a VGT position  
 474 of 60 %. This might be explained by the experimental error, caused by the very  
 475 low enthalpy drops measured in the turbine at high  $\sigma$  [20]. They are affected  
 476 by the residual heat transfer effects since at high  $\sigma$  uncertainty is introduced,  
 477 even when measuring in almost adiabatic conditions and when compensating  
 478 these effects. In general turbine adiabatic efficiency experimentally obtained  
 479 is affected by the relatively high combined uncertainty of the turbine enthalpy  
 480 drop, residual heat transfer and isentropic power at very low speeds. At other  
 481 speeds, the worst results have an error of 5% to 7% points of efficiency. However,

482 the general quality of the prediction is high and, the model is able to produce  
 483 good extrapolations very far from the points to which it was fitted.

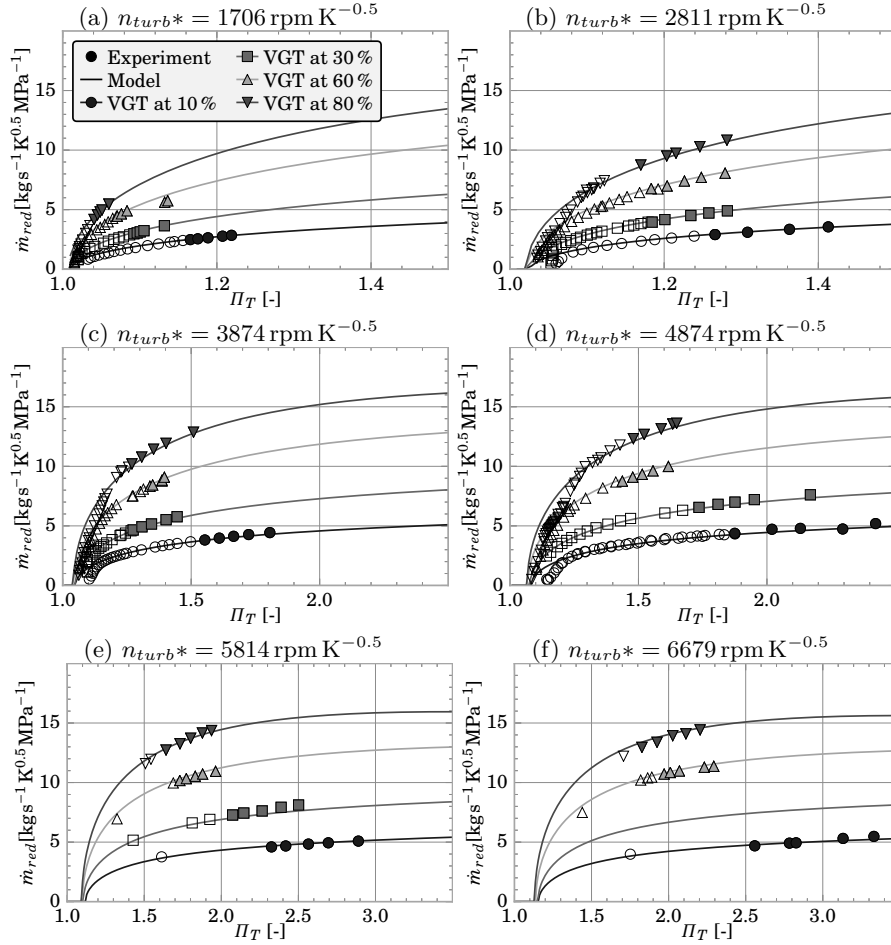


Figure 9: Reduced mass flow rate extrapolation results where extreme rotational speeds were fully extrapolated (dots correspond to experimental data and solid lines to model results)

## 484 6. Conclusions

485 In this paper, a method for extrapolating radial turbine performance in terms  
 486 of VGT position, rotational speed and blade speed ratio is presented. In order  
 487 to fully validate the model, a special turbocharger gas stand has been also used,  
 488 which provides means for measuring at very high blade to jet speed ratio.

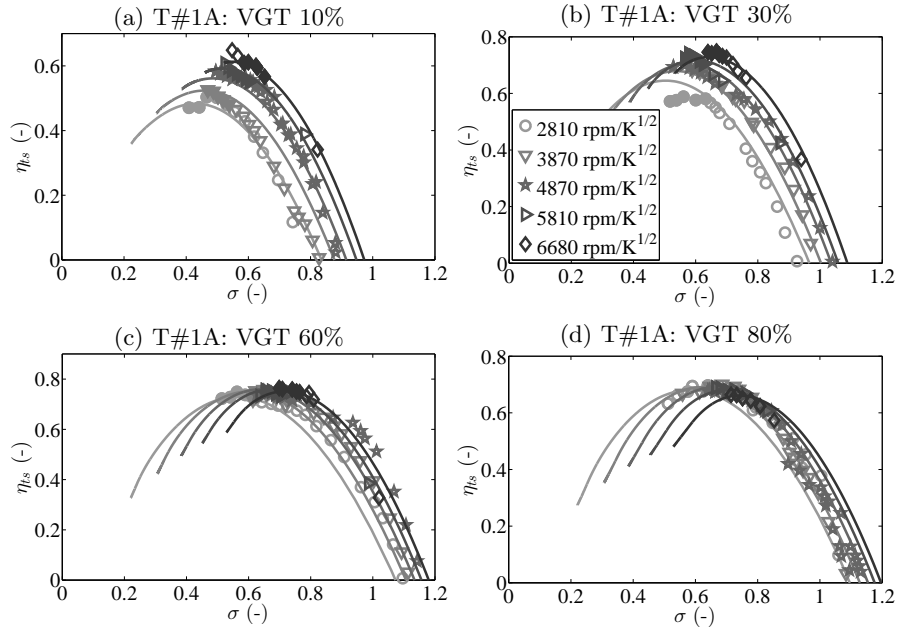


Figure 10: Efficiency extrapolation results for T#1A checking model capabilities at high blade to jet speed ratio (dots correspond to experimental data and solid lines to model results)

489 The method uses thirteen calibration coefficients which are fitted using a  
 490 limited set of available turbine map data. Seven coefficients are fitted indepen-  
 491 dently for the reduced mass flow and six for the efficiency. The reduced mass  
 492 flow calibration coefficients must be fitted in the first place since one of them,  
 493 the quotient between the rotor and the stator discharge coefficients (coefficient  
 494 'c'), is used in the efficiency model equation.

495 The reduced mass flow and the efficiency variables are interrelated since the  
 496 reduced mass flow appears in the efficiency equation and the efficiency appears  
 497 in the mass flow equation. As both variables appear implicitly, after fitting  
 498 the calibration coefficients, a system with the reduced mass flow and the effi-  
 499 ciency equations must be solved, using an iterative procedure, for extrapolation  
 500 purposes.

501 The model shows good agreement with the experimental data even when  
 502 it is calibrated with a very limited set of data. Both mass flow and efficiency



503 can be extrapolated beyond typical turbine map measured range in whatever  
504 variable, i.e. VGT position, reduced speed and blade to jet speed ratio.

505 One of the main advantages of the model is that it can be used easily for  
506 any radial VGT or FGT (fixed geometry turbine), as the geometrical parame-  
507 ters are easy to measure and the initial conditions and boundaries for the fitting  
508 procedure have been stated in the paper. A good extrapolation is expected for  
509 any energy conversion system using radial turbines since model self calibration  
510 coefficients do not depend on turbine size and model is based on reduced or  
511 non-dimensional parameters for representing turbines performance. It is only  
512 necessary to have a standard turbine map measured in almost adiabatic condi-  
513 tions (or adiabaticized) and with at least two VGT positions to fit the necessary  
514 coefficients for reduced mass flow and efficiency extrapolation. Nevertheless, the  
515 higher the number of available VGT positions the better for the quality of the  
516 extrapolation model.

517 The accuracy of the extrapolation model regarding mass flow parameter is  
518 high in the intermediate expansion ratio range. At low expansion ratio the model  
519 is still able to reproduce the behaviour of most of the experimental points, even  
520 with negative turbine adiabatic efficiencies. The accuracy of the efficiency model  
521 is high in the intermediate  $\sigma$  range. At high  $\sigma$  the model curves cross most of  
522 the experimental points, mainly at intermediate turbine reduced speeds. Good  
523 results for both variables are also obtained when extrapolating turbine reduced  
524 speeds. Finally, the prediction of full VGT position maps is also accurate in the  
525 whole expansion ratio range.

526 The main limitation of this approach is that the model has to be calibrated  
527 for each turbine using turbocharger manufacturers map data or some operative  
528 points tested a priori to get model maximum predictability. Since there are 13  
529 coefficients (7 for mass flow parameter and 6 for efficiency), at least 7 turbine  
530 operative points must be available for model self-fitting procedure. The higher  
531 the number of points the better the fitting results. If no data are available the  
532 predictions of the model will be less accurate but can still be possible by using  
533 average values of the proposed model coefficients.

534 One possible future work is to use a CFD approach in a similar way to the  
535 proposed in [30] in order to double check the validity of the proposed methods.  
536 Furthermore, available experimental off-design measurements can be used for  
537 validation.

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### 543 **References**

- 544 [1] N. Watson, M. Janota, Turbocharging the Internal Combustion Engine,  
545 Macmillan Publishers, Ltd., 1982.
- 546 [2] H. Moustapha, M. Zelesky, N. Baines, D. Japikse, Axial and radial turbines,  
547 Concepts NREC, Vermont, 2003.
- 548 [3] H. Hiereth, K. Drexl, P. Prenninger, Charging the internal combustion  
549 engine, Springer, 2007.
- 550 [4] M. Barratta, E. Spessa, Numerical Simulation Techniques for the Pre-  
551 diction of Fluid-Dynamics, Combustion and Performance in IC Engines  
552 Fuelled by CNG, Computational Simulations and Applications, InTech,  
553 Dr.Zhu Jianping (Ed.), 2011. doi:10.5772/25081.
- 554 [5] G. Martin, P. H. C. Caillol, V. Talon, Implementing turbomachinery  
555 physics into data map-based turbocharger models, SAE technical paper  
556 2009-01-0310.
- 557 [6] M. Chiong, S. Rajoo, A. Romagnoli, A. Costall, R. Martinez-  
558 Botas, Integration of meanline and one-dimensional methods

- 559 for prediction of pulsating performance of a turbocharger tur-  
560 bine, *Energy Conversion and Management* 81 (2014) 270 – 281.  
561 doi:<http://dx.doi.org/10.1016/j.enconman.2014.01.043>.
- 562 [7] M. Chiong, S. Rajoo, A. Romagnoli, A. Costall, R. Martinez-Botas, Non-  
563 adiabatic pressure loss boundary condition for modelling turbocharger tur-  
564 bine pulsating flow, *Energy Conversion and Management* 93 (2015) 267 –  
565 281. doi:<http://dx.doi.org/10.1016/j.enconman.2014.12.058>.
- 566 [8] J. Macek, Z. Zak, O. Vitek, Physical model of a twin-scroll turbine with un-  
567 steady flow, SAE Technical Paper 2015-01-1718doi:10.4271/2015-01-1718.
- 568 [9] X. Fang, Q. Dai, Modeling of turbine mass flow rate performances using  
569 the taylor expansion, *Applied Thermal Engineering* 30 (13) (2010) 1824 –  
570 1831. doi:<http://dx.doi.org/10.1016/j.applthermaleng.2010.04.016>.
- 571 [10] M. Gugau, H. Roclawski, On the design and matching of turbocharger  
572 single scroll turbines for pass car gasoline engines, *J. Eng. Gas Turbines*  
573 *Power* 136 (12). doi:10.1115/1.4027710.
- 574 [11] D. Palfreyman, R. Martinez-Botas, The pulsating flow field in a mixed flow  
575 turbocharger turbine: an experimental and computational study, *Proceed-*  
576 *ings of the ASME Turbo Expo 2004* (2004) 697–708.
- 577 [12] M. Padzillah, S. Rajoo, R. Martinez-Botas, Influence of speed and fre-  
578 quency towards the automotive turbocharger turbine performance under  
579 pulsating flow conditions, *Energy Conversion and Management* 80 (2014)  
580 416 – 428. doi:<http://dx.doi.org/10.1016/j.enconman.2014.01.047>.
- 581 [13] A. Romagnoli, R. Martinez-Botas, Performance prediction of a noz-  
582 zled and nozzleless mixed-flow turbine in steady conditions, *Inter-*  
583 *national Journal of Mechanical Sciences* 53 (8) (2011) 557 – 574.  
584 doi:<http://dx.doi.org/10.1016/j.ijmecsci.2011.05.003>.
- 585 [14] F. Payri, J. R. Serrano, P. Fajardo, M. A. Reyes-Belmonte, R. Gozalbo-  
586 Belles, A physically based methodology to extrapolate performance maps

- 587 of radial turbines, *Energy Conversion and Management* 55 (0) (2012) 149  
588 – 163.
- 589 [15] N. Baines, A meanline prediction method for radial turbine efficiency, In:  
590 6th International conference on turbocharging and air management sys-  
591 tems. Proc. IMechE C554-6 (1998) 315–325.
- 592 [16] R. Dambach, H. Hodson, Tip leakage flow: a comparison between small  
593 axial and radial turbines, *IMechE Sym S* 767.
- 594 [17] L. Eriksson, Modeling and control of turbocharged SI and DI engines, *Oil  
595 Gas Sci Technology-Revue de l'IFP* 62(4) (2007) 523–538.
- 596 [18] M. Canova, Development and validation of a control-oriented library for the  
597 simulation of automotive engines, *Int J Engine Res* 5(3) (2004) 219–228.
- 598 [19] S. Zhu, K. Deng, S. Liu, Modeling and extrapolating mass flow charac-  
599 teristics of a radial turbocharger turbine, *Energy* 87 (2015) 628 – 637.  
600 doi:<http://dx.doi.org/10.1016/j.energy.2015.05.032>.
- 601 [20] J. R. Serrano, A. O. Tiseira, L. M. García-Cuevas, L. B. Inhestern, H. Tar-  
602 toussi, Radial turbine performance measurement under extreme off-design  
603 conditions, Submitted to *Energy*.
- 604 [21] J. Galindo, J. R. Serrano, C. Guardiola, C. Cervelló, Surge limit definition  
605 in a specific test bench for the characterization of automotive turbocharg-  
606 ers, *Experimental Thermal and Fluid Science* 30 (5) (2006) 449 – 462.  
607 doi:<http://dx.doi.org/10.1016/j.expthermflusci.2005.06.002>.
- 608 [22] R. Bontempo, M. Cardone, M. Manna, G. Vorraro, Steady and un-  
609 steady experimental analysis of a turbocharger for automotive ap-  
610 plications, *Energy Conversion and Management* 99 (2015) 72 – 80.  
611 doi:<http://dx.doi.org/10.1016/j.enconman.2015.04.025>.
- 612 [23] J. R. Serrano, P. Olmeda, F. J. Arnau, A. Dombrovsky, L. Smith, Analysis  
613 and methodology to characterize heat transfer phenomena in automotive

- 614 turbochargers, ASME. J. Eng. Gas Turbines Power 137(2) (2014) 021901–  
615 021901–11. doi:<http://dx.doi.org/10.1115/1.4028261>.
- 616 [24] J. R. Serrano, P. Olmeda, F. J. Arnau, M. A. Reyes-Belmonte, H. Tartoussi,  
617 A study on the internal convection in small turbochargers. proposal of heat  
618 transfer convective coefficients, Applied Thermal Engineering 89 (2015) 587  
619 – 599. doi:<http://dx.doi.org/10.1016/j.applthermaleng.2015.06.053>.
- 620 [25] J. R. Serrano, P. Olmeda, A. Páez, F. Vidal, An experimental procedure to  
621 determine heat transfer properties of turbochargers, Measurement Science  
622 and Technology 21 (3) (2010) 035109.
- 623 [26] P. Olmeda, A. Tiseira, V. Dolz, L. Garca-Cuevas, Uncertainties in power  
624 computations in a turbocharger test bench, Measurement 59 (2015) 363 –  
625 371. doi:<http://dx.doi.org/10.1016/j.measurement.2014.09.055>.
- 626 [27] L. García-Cuevas, Experiments and modelling of automotive turbocharg-  
627 ers under unsteady conditions, Ph.D. thesis, Universitat Politècnica de  
628 València (2014).
- 629 [28] F. S. Ahmed, S. Laghrouche, A. Mehmood, M. E. Bagdouri, Estimation  
630 of exhaust gas aerodynamic force on the variable geometry turbocharger  
631 actuator: 1D flow model approach, Energy Conversion and Management 84  
632 (2014) 436 – 447. doi:<http://dx.doi.org/10.1016/j.enconman.2014.03.080>.
- 633 [29] M. Reyes-Belmonte, Contribution to the experimental characterization and  
634 1-D modelling of turbochargers for IC engines, Ph.D. thesis, Universitat  
635 Politècnica de València (2013).
- 636 [30] G. Besagni, R. Mereu, P. Chiesa, F. Inzoli, An Integrated Lumped  
637 Parameter-CFD approach for off-design ejector performance evalua-  
638 tion, Energy Conversion and Management 105 (2015) 697 – 715.  
639 doi:<http://dx.doi.org/10.1016/j.enconman.2015.08.029>.