

Electric Levitation Using ϵ -Near-Zero Metamaterials

Francisco J. Rodríguez-Fortuño,^{1,2} Ashkan Vakil,¹ and Nader Engheta^{1,*}

¹*Department of Electrical and Systems Engineering, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA*

²*Nanophotonics Technology Center, Universitat Politècnica de València, 46022 Valencia, Spain*

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The ability to manufacture metamaterials with exotic electromagnetic properties has potential for surprising new applications. Here we report how a specific type of metamaterial—one whose permittivity is near zero—exerts a repulsive force on an electric dipole source, resulting in levitation of the dipole. The phenomenon relies on the expulsion of the time-varying electric field from the metamaterial interior, resembling the perfect diamagnetic expulsion of magnetostatic fields. Leveraging this concept, we study some realistic requirements for the levitation or repulsion of a polarized particle radiating at any frequency, from microwave to optics.

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Levitation of objects with action at a distance has always been intriguing to humans. Several ways to achieve this [1], such as aerodynamic, acoustic, or electromagnetic methods, including radiation pressure [2], stable potential wells [3], and quantum Casimir-Lifshitz forces [4], exist. A fascinating approach for levitation is based on diamagnetic repulsion, in which materials with permeability $|\mu| < 1$ are repelled away from a magnetic field. A striking example of perfect diamagnetism ($\mu = 0$) is the levitation of magnets over superconductors based on the Meissner effect—the complete expulsion of the magnetic field by a superconductor [5]. With the advent of metamaterials—designed structures with electromagnetic properties that may not be found in nature—we ask whether a material may be conceived exhibiting similar field expulsion, but involving the electric field. We show how a special subcategory of metamaterials, called epsilon-near-zero materials [6–9], exhibits such an electric classic analog to perfect diamagnetism, exerting a repulsion on nearby sources. Repulsive forces using anisotropic and chiral metamaterials have been investigated [10–16], but our proposal uses a different mechanism based on field expulsion. The possibility of switching on and off the repulsion of particles away from a surface in micromechanical devices can lead to new functionalities [4] such as ultralow friction (superlubricity) [17] or elimination of unwanted adhesion. At small scales, capillary, electrostatic, and van der Waals forces can dominate over the elastic restoring forces of structures, causing parts to unwantedly adhere to the substrate during fabrication or functioning [18–20]. Overcoming this is an important challenge in nanotechnology. Strong repulsive forces of atoms near surface resonances (where $\epsilon = -1$) are well-known [21–23], but their resonant nature makes them sensitive to losses. In contrast, our proposal, which works near the regime $\epsilon = 0$, is more robust to losses and material dispersion, enabling broader band levitation of electric sources.

Figure 1 illustrates the basis of our proposal for electric levitation in the vicinity of an ϵ -near-zero (ENZ) metamaterial. Figure 1(a) displays a magnet over a perfect diamagnetic substrate such as a superconductor, demonstrating, in a simple form, the well-known physics of the Meissner effect. The superconductor expels the magnetic flux density \mathbf{B} from its interior. This phenomenon is occasionally associated with $\mu = 0$ [24]—one would expect that a material with $\mu = 0$ expels the magnetic

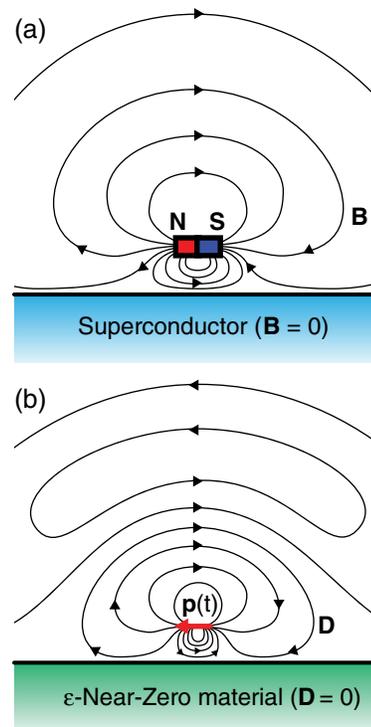


FIG. 1 (color online). ENZ analog to perfect diamagnetism. Analogy between (a) the Meissner effect and (b) the levitation of an electric radiating dipole source over an ϵ -near-zero (ENZ) substrate.

fields away from its surface (since the normal component of the magnetic flux density \mathbf{B} should be zero outside the material near its surface). This will exert a magnetostatic gradient force on the magnet and levitate the magnet to a stable point. Inspired by this notion, we envision an analogous classical scenario, shown in Fig. 1(b). Can we have an electric dual of perfect diamagnetism involving metamaterials? In other words, would an ENZ material, in which $\epsilon = 0$, exhibit a similar behavior, albeit classical, for time-varying electric fields? In an ENZ substrate, the displacement current \mathbf{D} generated by a polarized particle will be expelled in an analogous (but classic) fashion, and thus may result in levitation of the particle. On the surface, the apparent simplicity of this analogy may be misleading: one could argue that the same analog to perfect diamagnetism could be achieved with the use of a classic conductor, inside which the electric field is zero. However, in such a case, free charges accumulate on the surface of a conductor, creating an outside electric field normal to the surface of the conductor, in turn forming a different field profile than that of Fig. 1(b). Such field profile results in attraction rather than levitation. In contrast, an ENZ material does not accumulate free charges inside its bulk or at its boundary, and thus, the normal component of the electric displacement \mathbf{D} is conserved at the interface; however, inside the ENZ material, electric displacement is zero, forcing the external electric field to be parallel to the outside surface of an ENZ material, in complete analogy to the magnetic flux \mathbf{B} on a perfect diamagnet with $\mu = 0$. Since ENZ materials cannot exist at zero frequencies, the proposed mechanism works for a source oscillating at an angular frequency ω , as Fig. 1(b) depicts, implying that the electric source is radiating energy, but the static analogy is still valid for the near fields and for forces at electrically small distances, where the quasistatic approximation applies. Importantly, the effect is still present for values of permittivity close to, but not exactly zero, indicating a relatively broader bandwidth effect.

To understand the physics behind our proposal, we consider an idealized and simple scenario by considering a point electric dipole oscillating monochromatically with an angular frequency ω at a height h above a homogeneous semi-infinite substrate with permittivity $\epsilon_{\text{subs}} = \epsilon_0(\epsilon' + i\epsilon'')$ as depicted in Fig. 2(a). A full rigorous solution to the Maxwell equations can be obtained for this problem by applying the well-known Sommerfeld integral [25–28], from which the time-averaged force acting on the dipole can be calculated. The full electro-dynamical calculation is provided in the Supplemental Material [29], but it is convenient to have at our disposal a simple analytical expression. The force obtained is only strong for heights much smaller than the wavelength, suggesting that this phenomenon is purely a near-field effect. Applying the “quasistatic” (i.e., near-field) approximation to the integral

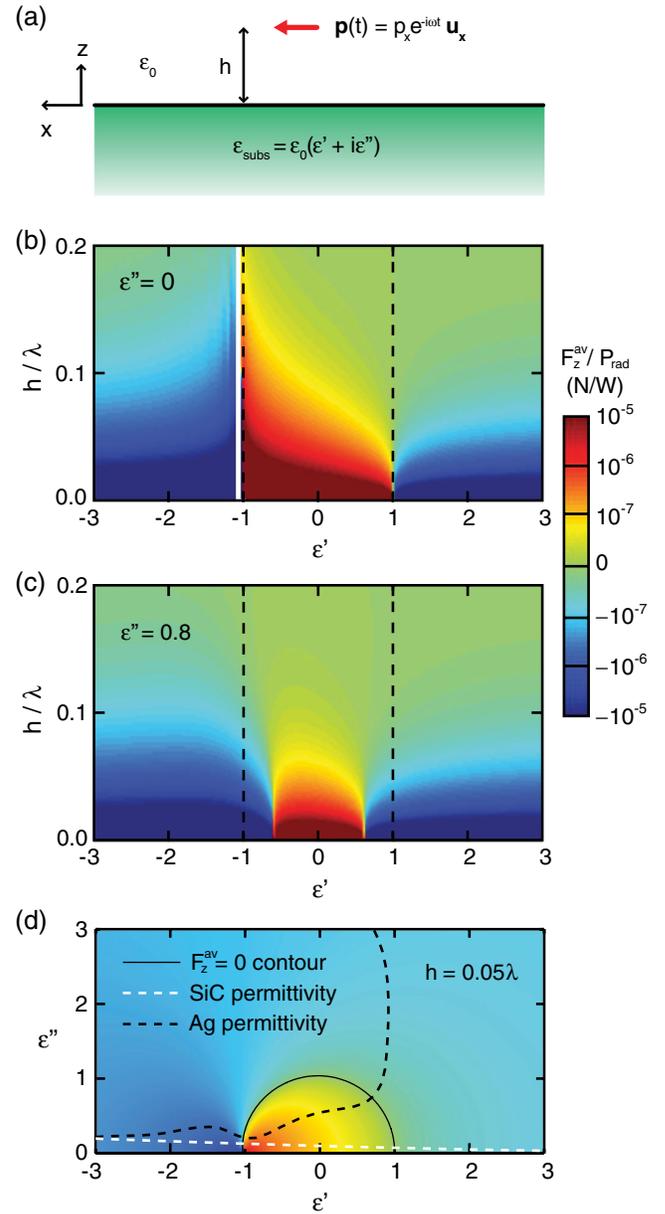


FIG. 2 (color online). Repulsive force dependence on height and substrate permittivity. (a) Geometry of the problem. (b,c,d) Plot of the time-averaged force acting on a horizontal electric point dipole per unit radiated power of the dipole, as a function of (b,c) the height over the substrate (vertical axis) and the real part of the permittivity of the substrate (horizontal axis), and as a function of (d) the real and imaginary part of the permittivity of the substrate for a fixed height $h = 0.05\lambda$. The imaginary part of the permittivity of the substrate is set to zero (lossless) in (b) and set to a very high value $\epsilon'' = 0.8$ in (c).

for the force and considering the dominant term at low heights (see Supplemental Material [29] for more details) keeps all the relevant physics of the problem unchanged, so we obtain the following simple approximate expression for the vertical time-averaged force:

$$F_z^{\text{ave}} \approx -\sigma \frac{9}{512\pi^4 c} \times \text{Re} \left[\frac{\epsilon_{\text{subs}} - \epsilon_0}{\epsilon_{\text{subs}} + \epsilon_0} \right] P_{\text{rad}} \left(\frac{h}{\lambda} \right)^{-4} \quad (1)$$

where $\sigma = 1$ and 2 for a horizontal and vertical dipole, respectively, and $P_{\text{rad}} = (4\pi^3 c / 3\epsilon_0 \lambda^4) |\mathbf{p}|^2$ is the radiated power of the dipole if it were placed in unbounded free space. The quasistatic approximation employed is equivalent to the image theory for static charges over a substrate [30]. An oscillating charge q polarizes the surface, inducing an image charge $q' = -Sq$, which is instantaneously correlated with itself with a complex amplitude given by the image coefficient $S = (\epsilon_{\text{subs}} - \epsilon_0) / (\epsilon_{\text{subs}} + \epsilon_0)$. When $\text{Re}(S) < 0$, as is the case for an ENZ substrate, the dipole and its image are in a like-charge configuration, resulting in a repulsive force. This phenomenon is known in the case of excited atoms in front of a surface [21–23]. Interestingly, the force is independent of the oscillating frequency of the dipole for a fixed dipole moment \mathbf{p} and absolute height h , or, equivalently, for a fixed P_{rad} and height to wavelength (h/λ) ratio. This is exactly true also in the fully retarded case.

To interpret the result, Figs. 2(b) and 2(c) depict the force per unit radiated power acting on a horizontal electric dipole (HED) as a function of (h/λ) and ϵ_{subs} . The analogous results for a vertical dipole, provided in the Supplemental Material [29], show a similar behavior. Any reorientation of the dipole, expected due to the existence of a torque, will be a superposition of the horizontal and vertical scenarios.

Let us consider the lossless case first [Fig. 2(b)]. The color in the figure represents the magnitude of the vertical time-averaged force acting on the dipole per unit radiated power; negative (blue) values represent attraction toward the substrate, while positive (red) values represent repulsion away from the substrate. When the substrate is a conventional dielectric ($\epsilon' > 1$), there is an attractive force acting on the dipole when it is close to the surface: this is well-known and it is interpretable as the force on the dipole due to its image. When the permittivity of the substrate is equal to that of air ($\epsilon' = 1$), the dipole is radiating in free space and there is no force acting on the dipole. The interesting effect is observed when we further reduce the permittivity, in the region $-1 < \epsilon' < 1$, where a repulsive force exists on the dipole. In particular, when the substrate permittivity is near zero ($\epsilon' \approx 0$), the electric displacement field radiated by the dipole cannot penetrate into the substrate, leading to the field distribution shown in Fig. 1(b), exhibiting an analogous ENZ-based Meissner-like effect. This can be understood as a force pushing the system away from the high-energy configuration that exists when the emitter is very close to the substrate, associated with very high values of the electric field, “squashed” between dipole and substrate. Similar interpretations can be made for other kinds of sources even if they are not adequately modeled with a point dipole. It is worth noting that a diverging resonant force exists when ϵ' is close to -1 as shown in Figs. 2(b)–2(d).

Such resonant force is a result of surface plasmon resonances at the boundary between air and substrate [16] and is well-known in the context of atoms near surfaces supporting surface plasmons [21–23]. Ideally, this resonance can result in an arbitrarily large force acting on the dipole, but as is typical with resonant phenomena, it is limited in practice by the losses. However, the repulsion mechanism at the ENZ condition ($\epsilon' = 0$) does not rely on the existence of a resonant mode but on a property of the substrate at a particular frequency, leaving us to expect that such repulsion may be more robust to losses. That is indeed the case, as shown in Fig. 2(c), which shows the force when the substrate has very high losses ($\epsilon'' = 0.8$, high compared to practical ENZ materials, e.g., SiC is a natural ENZ material [31] at around 29 THz where it has $\epsilon'' \approx 0.1$). We have deliberately chosen an exaggerated imaginary value of $\epsilon'' = 0.8$ to illustrate our point. The repulsive force around $\epsilon' = -1$ based on the surface plasmon resonance has disappeared completely, but the repulsion in the region around $\epsilon' = 0$ is still strong despite the high imaginary part, showing that our approach is robust to loss. This can be clearly seen in Fig. 2(d), where the repulsive force at a constant height is plotted as a function of the real and imaginary parts of the substrate permittivity, together with experimental permittivity values of Ag [32] and SiC [31]. Since Eq. (1) is frequency independent [aside from the $(h/\lambda)^{-4}$ term for a fixed P_{rad}], we can consider the locus of all points in the (ϵ', ϵ'') space that result in a repulsive force, from which we can obtain the associated frequency bandwidth of repulsion for any dispersive ENZ substrate as $\epsilon(\omega)$ moves through this region [see the dashed lines in Fig. 2(d)]. For the Lorentz model of SiC given in Ref. [31], a relatively wide repulsive force fractional bandwidth of 6% around the 29 THz ENZ frequency is obtained.

Figure 3 shows the time-averaged force on the electric dipole normalized to the radiated power in unbounded free

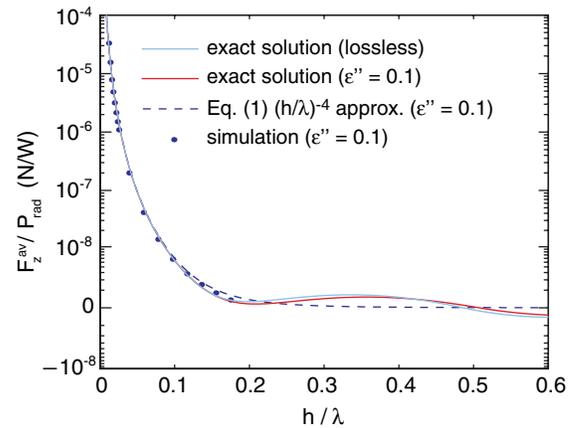


FIG. 3 (color online). Repulsive force dependence on height. Plot of the normalized time-averaged force acting on a horizontal electric point dipole over an ENZ substrate as a function of the normalized height using independent calculation methods: exact solution, near-field approximation, and numerical modeling.

space, as a function of (h/λ) when the substrate is ENZ ($\epsilon' = 0$). We show three independent verifications of the effect: (i) the exact electrodynamic solution given by Sommerfeld's integral, (ii) the near-field approximation given by Eq. (1), and (iii) the numerical calculations of the force obtained from numerical simulations using Maxwell's stress tensor [33]. All these independent results agree almost perfectly for low heights $(h/\lambda) < 0.2$.

So far, we have only referred to the repulsive force acting on a point electric dipole, which is a good approximation to an electrically small emitter or polarizable particle, but in order to achieve levitation, the repulsive force should be compared to the emitter's weight. We can use the simple expression of the force given by Eq. (1) to explore different scenarios. At microwave frequencies, electrically small sources and distances are relatively easier to work with; therefore, we can exploit the relatively big repulsion forces at very low heights by considering electrically small antennas placed very close to an ENZ substrate. Let us assume a dipole cylindrical antenna of length $\lambda/4$ and diameter $\lambda/100$ made of copper: this assumption is only used to calculate the weight of the antenna and the dipole moment. Substituting the values into Eq. (1), we obtain that to have repulsive force greater than the weight of the antenna, the current supplied to the antenna must be $I(\text{A}) > (5.56 \times 10^4 \text{ A s}^{-3/2}) [f(\text{GHz})]^{-3/2} (h/\lambda)^2$ (details provided in the Supplemental Material [29]). For example, at 1 GHz, a current of 6 A is sufficient to levitate the described antenna a distance $\lambda/100$ (i.e., 3 mm) from the ENZ substrate. The calculations presented here are only a rough estimate of the achievable levitating phenomena. An experimental setup should use an electrically small antenna and should take into account the effect of the ENZ substrate in the input impedance of the antenna. Considering the possibility of an electric breakdown or discharges resulting from molecular ionization in intense electric fields, one may probably require high vacuum to levitate heavy weights, a limitation not present in the more common magnetic levitation techniques.

At optical frequencies, the particles have comparatively very low mass, so smaller forces can be considered. An illuminated fluorescent particle, radiating at the ENZ frequency of the substrate, or an illuminated polarizable particle could be used as electric emitters. At such nanoscale sizes, the successful repulsion from a surface requires overcoming the effects of thermal and quantum fluctuations which give rise to van der Waals and Casimir forces [26,34,35]. In principle, the proposed field-expulsion repulsive force can overcome these forces, if a sufficiently high input pump power is used. However, the pump laser itself [36] will also exert a force in the emitter. Included in the Supplemental Material [29] (Fig. S4) is a self-consistent calculation of a 200 nm diameter polarizable spherical particle, resonant at a wavelength of $1.8 \mu\text{m}$, located over an ENZ substrate, and illuminated by a pump plane wave.

The calculation includes the dipole and pump laser forces (which depend linearly on the input power) and the Casimir force acting on the particle. A repulsive force greater than the particle weight and the Casimir attraction is obtained for a relatively low input pump power density of 100 mW/mm^2 , demonstrating the feasibility of our proposal. We studied scenarios at smaller wavelengths and distances, where the Casimir forces become stronger, but overcoming them requires much larger incident powers. Also, in practice, unwanted electrostatic forces arising from different phenomena such as the accumulation of static charges in the dielectrics involved might exist, and although some techniques such as conductive coating and grounding can be used to reduce them, it is difficult to avoid them entirely [18,19]. The presence of such electrostatic forces should be experimentally accounted for [4,37,38].

Finally, it is worth mentioning that the nonresonant repulsive force presented in this Letter is not localized, in contrast with gradient forces. As such similar to some previously proposed levitation scenarios [39], the applications of our proposal are different from those of gradient forces used in optical tweezers, for which the force is localized to the focus of the tweezers.

In conclusion, we have proposed a method for repelling electromagnetic sources away from an ENZ surface. The mechanism is very robust to both realistic losses and to frequency detuning from the ENZ frequency (and thus robust to material dispersion), rare traits in metamaterial applications. ENZ behavior can be found in natural materials at certain frequencies or can be designed for specific frequencies using the notion of metamaterials. The effect may have useful potential applications, and we have shown realistic requirements for levitation. Further studies can be pursued on this analogy to the Meissner effect, inspired by the concepts from superconductor levitation, such as the imitation of flux-pinning effects which take place in a Type-II superconductor [40], by introducing air channels on ENZ media.

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*Corresponding author.
engheta@ee.upenn.edu

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