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On star compactifications

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Abstract

We study the ordered structure of the collection of star compactifications by n points and the behavior of these compactifications through quotients obtained by identification of additional points.

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1. Introduction.

When we study compactification in a general topology course, we usually only deal with three types of compactifications: (1) the Alexandroff compactification, which is a one-point compactification; (2) the Stone Čech's compactification, and (3) some elementary examples of compactifications by a finite number of points, for instance:

(1) $[0, 1]$ is a compactification of $(0, 1)$ by two points.
(2) $[0, 1] \cup [2, 3]$ is a compactification of $A = (0, 1) \cup (2, 3)$ by four points.
(3) Geometrically, two disjoint circles on $\mathbb{R}^2$ are a compactification by two points, of the set A in the previous example.

These examples of compactifications by a finite number of points are particular cases of star compactifications. Star topologies were defined in [7] and we mention without proofs some results given in [7]; in particular, the necessary and sufficient conditions for such topologies to be compactifications or T_2-compactifications of a non-compact space.

By compactness of a topological space X, we mean that every open covering of X has a finite subcovering, and by compactification of a non-compact topological space X, not necessarily a Hausdorff space, we mean a compact space containing X as a subspace, which is dense in the compactification.

Our intention is to illustrate the definition with simple star compactifications and present some interesting results about the ordered structure of the collection of star compactifications by n points, of a topological space. We obtain the stability of such collection by finite intersections. On the other hand, we present the behavior of the quotients of star compactifications obtained through equivalence relations in which we make some identification between the additional points. Moreover we establish the relationships between the star compactifications of a topological space and other compactifications of it by a finite number of points.

2. Preliminary results.

2.1. **Star topologies and star compactifications.** In this section we present the definition and some known results concerning star topologies and star compactifications, given in [7].

For this purpose, we consider a non-compact topological space (X, τ) and $X_n = X \cup \{\omega_1, ..., \omega_n\}$, where $\omega_1, ..., \omega_n$ are n distinct points not belonging to X, $n \in \mathbb{N}$.

Proposition 2.1. *Let U_i, $i = 1, ..., n$ be open subsets contained in X. Then*

$$\mathcal{B} = \tau \cup \{(U_i \setminus K) \cup \{\omega_i\} \mid K \subseteq X \text{ closed and compact; } i = 1, ..., n\}$$

is a base for a topology μ on X_n.

This topology is called the **star topology** associated to $U_1, ..., U_n$.

Notice that $\mathcal{B}$ is a closed collection under finite intersections. Throughout this paper we denote the star topology over X_n associated to $U_1, ..., U_n$ by $\mu = \langle\langle U_1, ..., U_n \rangle\rangle$.

Proposition 2.2. *(X_n, μ) is a compactification of (X, τ) if and only if*

(1) *$X \setminus \bigcup_{i=1}^{n} U_i$ is compact, and*
(2) *$U_i \nsubseteq K$ for each K closed and compact subset of X, $i = 1, ..., n$.*

Observe that the second condition implies that for each $i = 1, ..., n$, U_i is nonempty. By the definition of μ we observe that (X, τ) is a subspace of (X_n, μ) and $X \in \mu$.

2.2. **The Alexandroff compactification.** Let (X,τ) be a non-compact topological space and $X_1 = X \cup \{\omega\}$, where ω is a point not belonging to X.

Theorem 2.3. *If* $\eta = \tau \cup \{A \cup \{\omega\} \mid A \in \tau \text{ and } X \setminus A \text{ is compact}\}$, *then* (X_1, η) *is a compactification of* (X,τ) *by one point.*

This compactification is called the **Alexandroff compactification** of (X,τ) . It is the finest compactification of (X,τ) obtained by adding one point, and it is called "the" compactification by one point in the class of Hausdorff spaces, because it is the only Hausdorff compactification by one point when (X,τ) is a Hausdorff and locally compact space. See [6].

3. Ordered structure of the star compactifications by n points.

We present some results regarding the inclusion order relation in the collection of star compactifications by n points of a non-compact topological space, which enable us to conclude that this collection is stable under finite intersections. We exhibit the smallest element of the mentioned collection. This element can be seen as a generalization of the Alexandroff compactification by n points, with $n > 1$. On the other hand, we study the relationship between the open sets that generate two star topologies when one of them is finer than the other.

Let (X,τ) be a non-compact topological space and $W = \{\omega_1, \omega_2, \omega_3, ...\}$ be a set of different elements not belonging to X.
We denote $X_0 = X$, $X_n = X_{n-1} \cup \{\omega_n\}$, $n \geq 1$.

It is known that all compactifications of (X,τ) by n points are seen, up to homeomorphisms, as X_n with a convenient topology, in such way that X is considered a subspace of the compactification.

We denote

$$\mathcal{E}_n = \{\mu \in Top(X_n) \mid (X_n, \mu) \text{ is a star compactification of } (X,\tau)\} \,;$$

the collection of star compactifications of (X,τ) by n points. Observe that for each $\mu \in \mathcal{E}_n$, we have $\tau \subset \mu$, i.e., $X \in \mu$.
Notice that the inclusion order relation defined in $\mathcal{E}_n$ coincides with the order usually defined between compactifications (see [4]).

Proposition 3.1. *Let* $\Omega = \langle\langle U_1, ..., U_n \rangle\rangle$, *where* $U_i = X$ *for each* i. *If* $\mu = \langle\langle V_1, ..., V_n \rangle\rangle$ *is an arbitrary star topology on* X_n, *then* $\Omega \subseteq \mu$.

Proof. Let K be a closed and compact subset of (X,τ). As $V_i \subseteq X$ for each $i = 1, ..., n$, then $A = \{\omega_i\} \cup (X \setminus K) = \{\omega_i\} \cup (V_i \setminus K) \cup (X \setminus K) \in \mu$, since $\{\omega_i\} \cup (V_i \setminus K) \in \mu$ and $X \setminus K \in \tau \subseteq \mu$. Therefore $\Omega \subseteq \mu$. □

This proposition asserts that Ω is the smallest element of the set of star topologies on X_n, ordered by inclusion. Since Ω satisfies the mentioned conditions in Proposition 2.2, then Ω is a star compactification of (X,τ) by n points and Ω is the smallest element of $(\mathcal{E}_n, \subseteq)$.

Remark 3.2. For the case $n = 1$, $\Omega = \langle\langle X \rangle\rangle = \mathcal{B}$, where

$$\mathcal{B} = \tau \cup \{\{\omega_1\} \cup (X \setminus K) \mid K \subseteq X \text{ closed and compact}\},$$

we have that $\Omega = \mathcal{B}$ and Ω is the *Alexandroff compactification* of (X, τ). Since the Alexandroff compactification is the finest compactification of (X, τ) by one point, then Ω *is the only star compactification of* (X, τ) *by one point.*

Proposition 3.3. *If* $\mu = \langle\langle V_1, ..., V_n \rangle\rangle$ *is a star compactification of* (X, τ) *where* $V_1 = V_2 = ... = V_n$*, then* $\mu = \Omega$.

Proof. Let $\{\omega_i\} \cup (V_i \setminus K)$ be a basic open set in μ for some $i = 1, ..., n$ and some K closed and compact subset of X. $X \setminus V_i$ is closed and compact since $\bigcup_{j=1}^{n} V_j = V_i$.
Thus

$$\{\omega_i\} \cup (V_i \setminus K) = \{\omega_i\} \cup [X \setminus (K \cup (X \setminus V_i))] \in \Omega,$$

and $\mu \subseteq \Omega$. $\square$

Remark 3.4.

(1) If $\mu = \langle\langle V_1, ..., V_n \rangle\rangle$ is an arbitrary star topology on X_n with $V_1 = V_2 = ... = V_n$, it could happen that $\mu \nsubseteq \Omega$.
Example: Consider $X = (0, 1)$ with the usual topology as subspace of $\mathbb{R}$; $V_1 = ... = V_n = \left(\frac{1}{4}, \frac{1}{2}\right)$. Then $\{\omega_i\} \in \mu$, but $\{\omega_i\} \notin \Omega$.

(2) Proposition 3.3 is a particular case of Theorem 0.8 in [7]: "Let $A_1, ..., A_n$ and $B_1, ..., B_n$ be two n-tuples of open sets in X. The star topologies associated with those two n-tuples are the same if and only if the sets $A_i - B_i$ and $B_i - A_i$ are contained in compact and closed sets for all $i = 1, ..., n$".

Proposition 3.5. *Let* $\mu = \langle\langle U_1, ..., U_n \rangle\rangle$, $\beta = \langle\langle V_1, ..., V_n \rangle\rangle$ *be two star topologies on* X_n*. If* $V_i \subseteq U_i$ *for each* $i = 1, ..., n$*, then* $\mu \subseteq \beta$.

Proof. Let $A = \{\omega_i\} \cup (U_i \setminus K) \in \mu$. Since $V_i \subseteq U_i$ and $U_i \setminus K \in \tau$, then $A = \{\omega_i\} \cup (V_i \setminus K) \cup (U_i \setminus K) \in \beta$. $\square$

The next proposition asserts that the insersection of two star topologies on X_n is of the same kind and it describes the open sets associated with it.

Proposition 3.6. *Let* $\mu = \langle\langle U_1, ..., U_n \rangle\rangle$, $\beta = \langle\langle V_1, ..., V_n \rangle\rangle$ *be two star topologies on* X_n*, then* $\mu \cap \beta = \eta$ *where* $\eta = \langle\langle U_1 \cup V_1, ..., U_n \cup V_n \rangle\rangle$.

Proof. i) By the previous proposition we have that $\eta \subseteq \mu$ because $U_i \subseteq U_i \cup V_i$, for each $i = 1, ..., n$ and for the same reason, $\eta \subseteq \beta$.
ii) For the other inclusion it is enough to see that all basic open neighborhoods of ω_i in $\mu \cap \beta$ are open neighborhoods of ω_i in η, for each $i = 1, ..., n$.

Let M be a basic open neighborhood of ω_i in $\mu \cap \beta$, for $i = 1, ..., n$, that is, $M = \{\omega_i\} \cup (U_i \setminus K_1) \cup A = \{\omega_i\} \cup (V_i \setminus K_2) \cup B$, where K_1 and K_2 are closed and compact subsets of X and $A, B \in \tau$.
Since $(U_i \cup V_i) \setminus (K_1 \cup K_2) \subseteq (U_i \setminus K_1) \cup (V_i \setminus K_2)$ and $(U_i \setminus K_1) \cup A = (V_i \setminus K_2) \cup B$, then

$$\begin{aligned} (U_i \setminus K_1) \cup A &= (U_i \setminus K_1) \cup (V_i \setminus K_2) \cup A \cup B \\ &= [(U_i \cup V_i) \setminus (K_1 \cup K_2)] \cup (U_i \setminus K_1) \cup (V_i \setminus K_2) \cup A \cup B. \end{aligned}$$

So, if we call $C = (U_i \setminus K_1) \cup (V_i \setminus K_2) \cup A \cup B$, then C is an open set of τ and therefore $M = \{\omega_i\} \cup [(U_i \cup V_i) \setminus (K_1 \cup K_2)] \cup C \in \eta$. □

Proposition 3.7. *Let $\mu = \langle\langle U_1, ..., U_n \rangle\rangle$, $\beta = \langle\langle V_1, ..., V_n \rangle\rangle$ be two star topologies on X_n. If $\mu \subseteq \beta$ then there exist M_i, N_i open sets of τ such that $N_i \subseteq M_i$ for each $i = 1, ..., n$ and $\mu = \langle\langle M_1, ..., M_n \rangle\rangle$, $\beta = \langle\langle N_1, ..., N_n \rangle\rangle$.*

Proof. Let $N_i = V_i$, $M_i = U_i \cup V_i$ for each $i = 1, ..., n$ sets of τ. Therefore $\mu = \langle\langle M_1, ..., M_n \rangle\rangle$, $\beta = \langle\langle N_1, ..., N_n \rangle\rangle$ and $N_i \subseteq M_i$ for each $i = 1, ..., n$. □

Observe that this proposition is a weak version of the reciprocal of Proposition 3.5.

The next proposition guarantees that intersection is a closed operation in the collection $\mathcal{E}_n$ of star compactifications of (X, τ) by n points.

Proposition 3.8. *If $\mu = \langle\langle U_1, ..., U_n \rangle\rangle$, $\beta = \langle\langle V_1, ..., V_n \rangle\rangle$ are two star compactifications of (X, τ) by n points, then $\eta = \langle\langle U_1 \cup V_1, ..., U_n \cup V_n \rangle\rangle$ is a star compactification of (X, τ) by n points.*

Proof. i) We have that $U_i \cup V_i \nsubseteq K$ for each closed and compact subset K of X, because $U_i \nsubseteq K$ for each $i = 1, ..., n$.
ii)

$$\begin{aligned} X \setminus \bigcup_{i=1}^{n} (U_i \cup V_i) &= X \setminus \left(\left(\bigcup_{i=1}^{n} U_i \right) \cup \left(\bigcup_{i=1}^{n} V_i \right) \right) \\ &= \left(X \setminus \bigcup_{i=1}^{n} U_i \right) \cap \left(X \setminus \bigcup_{i=1}^{n} V_i \right), \end{aligned}$$

where $X \setminus \bigcup_{i=1}^{n} U_i$ and $X \setminus \bigcup_{i=1}^{n} V_i$ are closed and compact subsets of X, then $X \setminus \bigcup_{i=1}^{n} (U_i \cup V_i)$ is compact. Therefore, η is a star compactification of (X, τ) by n points. □

4. Certain quotients of star compactifications.

We consider the quotients obtained by an equivalence relation $\diamond$ on X_n; $\diamond = \{(x, x) \mid x \in X_n\} \cup R$, where R is an equivalence relation on $\{\omega_1, ..., \omega_n\}$. The star compactifications of these quotients have an interesting behavior.

Definition 4.1. We say that a compactification (Y, μ) of (X, τ) is of A-class if $X \in \mu$.

Observe that star compactifications are of A-class.

The next theorem asserts that a quotient of A-class compactification of (X,τ), obtained through the mentioned equivalence relation $\diamond$, is an A-class compactification of (X,τ).

Theorem 4.2. *Let (X_n,μ) be an A-class compactification of (X,τ) by n points, $n>1$. If we consider the equivalence relation $\diamond$ on X_n defined above, then $(X_n/\diamond,\mu/\diamond)$ is an A-class compactification of (X,τ) by m points, where $m=|\{\omega_1,...,\omega_n\}/R|\leq n$.*

Proof. We have $X\overset{i}{\hookrightarrow}X_n\overset{\theta}{\longrightarrow}X_n/\diamond$, where i is the topological imbedding and θ is the standard quotient map.
i) $(X_n/\diamond,\mu/\diamond)$ is compact because $\theta:(X_n,\mu)\longrightarrow(X_n/\diamond,\mu/\diamond)$ is a surjective continuous map, where (X_n,μ) is a compact space.

ii) To see that $\theta\circ i(X)$ is dense in $X_n/\diamond$ we need to see that $\overline{\theta\circ i(X)}^{\mu/\diamond}=X_n/\diamond$ $=\{[x]\mid x\in X\}\cup\{[\omega_i]\mid i=1,...,n\}$.
If $A=\{[\omega_i]\mid i=1,...,n\}$, then $|A|=|\{\omega_1,...,\omega_n\}/R|=m$.
Supose that $\overline{\theta\circ i(X)}^{\mu/\diamond}\subsetneqq X_n/\diamond$, that is, that there exists $B\in\mathcal{P}(A)\setminus\{\varnothing\}$ such that $B\in\mu/\diamond$, then $\theta^{-1}(B)\in\mathcal{P}(\{\omega_1,...,\omega_n\})\setminus\{\varnothing\}$ and $\theta^{-1}(B)\in\mu$, but this is a contradiction because X is dense in X_n; therefore $\overline{\theta\circ i(X)}^{\mu/\diamond}=X_n/\diamond$ and $\theta\circ i(X)$ is dense in $X_n/\diamond$.

iii) Let us see that $\theta\circ i:X\to X_n/\diamond$ is a homeomorphism between X and $\theta\circ i(X)$, where $\theta\circ i(X)=\theta(X)=\{[x]=\{x\}\mid x\in X\}$:
i is a homeomorphism between X and $i(X)=X$. By the definition of $\diamond$, θ is a bijective map between $i(X)=X$ and $\theta(X)$, then $\theta\circ i$ is a bijective map between X and $\theta\circ i(X)=\theta(X)$. By the continuity of θ we have that $\theta\circ i$ is continuous.
Let us see that $\theta\circ i$ is an open map:
Let A be an open set of τ, we need to see that $\theta\circ i(A)=\theta(A)\in\mu/\diamond$.
$\theta(A)=\{[a]\mid a\in A\}=\{\{a\}\mid a\in A\}$ because $A\subset X$.
$\theta(A)\in\mu/\diamond$ if and only if $\theta^{-1}(\theta(A))\in\mu$. Since $\theta(A)\subset\theta(X)$ and θ is a bijective map between X and $\theta(X)$, then $\theta^{-1}(\theta(A))=A$. Since i is a homeomorphism between X and $i(X)=X$ and $A\in\tau$, then $i(A)=A=B\cap X$, where $B\in\mu$ and since $X\in\mu$ then $A\in\mu$ and $\theta(A)\in\mu/\diamond$. Thus, $\theta\circ i$ is an open map, and then, it is a homeomorphism between X and $\theta\circ i(X)$. Moreover, $\theta(X)\approx X\in\mu/\diamond$ because $X\in\mu$.
Hence $(X_n/\diamond,\mu/\diamond)$ is an A-class compactification of (X,τ) by m points, where $m=|\{\omega_1,...,\omega_n\}/R|\leq n$. □

Corollary 4.3. $(X_n/\diamond,\mu/\diamond)$ *is homeomorphic to an A-class compactification of (X,τ) by m points, seen as (X_m,η) for an appropriate η.*

Consider (X_n,μ) a star compactification of (X,τ) by n points, with $\mu=\langle\langle U_1,...,U_n\rangle\rangle$. Since (X_n,μ) is of A-class, we know that $(X_n/\diamond,\mu/\diamond)$ is a compactification of (X,τ) by m points, where $m=\left|\{\omega_1,...,\omega_n\}/R\right|\leq n$.
Since the quotient map $\theta: X_n\longrightarrow X_n/\diamond$ is bijective in X, we say $\theta(x)=x$ and $\theta(A)=A$ for each $x\in X$ and for each $A\subseteq X$, and then $X_n/\diamond=X\cup V$ with $V=\{\upsilon_1,...,\upsilon_m\}=\{\theta(\omega_i)\mid i=1,...,n\}$, $\upsilon_1=\theta(\omega_1)$, $\upsilon_k=\theta(\omega_i)$ where $i=\min\{j\in\{1,...,n\}\mid \omega_j\notin \upsilon_1\cup...\cup\upsilon_{k-1}\}$ for each $k=2,...,m$.
We denote $I_i=\{j\in\{1,...,n\}\mid\theta(\omega_j)=\upsilon_i\}$ for each $i=1,...,m$ and if $|I_i|=n_i$, then $I_i=\{i_1,...,i_{n_i}\}$.

The next proposition asserts that $(X_n/\diamond,\mu/\diamond)$ is a star compactification of (X,τ).

Proposition 4.4. $\mu/\diamond=\langle\langle M_1,...,M_m\rangle\rangle$ *where* $M_i=\overset{n_i}{\underset{j=1}{\cup}}U_{ij}$ *for each* $i=1,...,m$.

Proof. Let A be a basic open set of $\langle\langle M_1,...,M_m\rangle\rangle$: If $A\in\tau$, since $\theta^{-1}(A)=A$ and $\tau\subseteq\mu$ then $A\in\mu/\diamond$. If $A=\{\upsilon_i\}\cup(M_i\setminus K)$ for some $i=1,...,m$ and some K closed and compact subset of X, then

$$\begin{aligned}\theta^{-1}(A)&=\{\omega_{i_1},...,\omega_{i_{ni}}\}\cup(M_i\setminus K)\\&=\{\omega_{i_1},...,\omega_{i_{ni}}\}\cup\left(\left(\overset{n_i}{\underset{j=1}{\cup}}U_{ij}\right)\setminus K\right)\\&=\{\omega_{i_1},...,\omega_{i_{ni}}\}\cup\left(\overset{n_i}{\underset{j=1}{\cup}}(U_{ij}\setminus K)\right)\\&=\overset{n_i}{\underset{j=1}{\cup}}\left(\left\{\omega_{i_j}\right\}\cup(U_{ij}\setminus K)\right)\in\mu,\end{aligned}$$

then $A\in\mu/\diamond$. Thus $\langle\langle M_1,...,M_m\rangle\rangle\subseteq\mu/\diamond$.

Consider now $A\in\mu/\diamond$, we see that $A\in\langle\langle M_1,...,M_m\rangle\rangle$ by showing that all its points are interior in $\langle\langle M_1,...,M_m\rangle\rangle$.
We know that $\theta^{-1}(A)\in\mu$ and let z be an element of A :
Case 1: If $z\in X$ then $z\in\theta^{-1}(A)$ and since $\mu\mid_X=\tau$, there exists an open set $\theta^{-1}(A)\cap X=B\in\tau$ such that $z\in B\subseteq\theta^{-1}(A)$ then $z\in\theta(B)=B\subseteq A$. Since $B\in\langle\langle M_1,...,M_m\rangle\rangle$ then z is an interior point of A in $\langle\langle M_1,...,M_m\rangle\rangle$.
Case 2: $z=\upsilon_i$ for some $i=1,...,m$.
$\theta^{-1}(\upsilon_i)=\{\omega_{i_1},...,\omega_{i_{ni}}\}\subseteq\theta^{-1}(A)$ and $\theta^{-1}(A)$ is an open set of μ, then for each $j\in I_i$, there exists $\left\{\omega_{i_j}\right\}\cup(U_{ij}\setminus K_j)$ open set of μ and subset of $\theta^{-1}(A)$.
Since $\left(\overset{n_i}{\underset{j=1}{\cup}}U_{ij}\right)\setminus\left(\overset{n_i}{\underset{j=1}{\cup}}K_j\right)\subseteq\overset{n_i}{\underset{j=1}{\cup}}(U_{ij}\setminus K_j)$ we have:

$$\begin{aligned}E&=\overset{n_i}{\underset{j=1}{\cup}}\left[\left\{\omega_{i_j}\right\}\cup(U_{ij}\setminus K_j)\right]\\&=\theta^{-1}(\upsilon_i)\cup\left[\left(\overset{n_i}{\underset{j=1}{\cup}}U_{ij}\right)\setminus\left(\overset{n_i}{\underset{j=1}{\cup}}K_j\right)\right]\cup\overset{n_i}{\underset{j=1}{\cup}}(U_{ij}\setminus K_j)\\&=\theta^{-1}(\upsilon_i)\cup(M_i\ \setminus K)\cup B\subseteq\theta^{-1}(A),\end{aligned}$$

where $K=\overset{n_i}{\underset{j=1}{\cup}}K_j$ is a closed and compact subset of X and

$B=\overset{n_i}{\underset{j=1}{\cup}}(U_{ij}\setminus K_j)$ is an open set of τ.

Thus, $\upsilon_i \in \theta(E) = \{\upsilon_i\} \cup (M_i \setminus K) \cup B \subseteq A$ and since
$\{\upsilon_i\} \cup (M_i \setminus K) \cup B$ is open in $\langle\langle M_1, ..., M_m\rangle\rangle$, then υ_i is an interior point of A in $\langle\langle M_1, ..., M_m\rangle\rangle$.
By cases 1 and 2 we conclude that $A \in \langle\langle M_1, ..., M_m\rangle\rangle$. □

Therefore, the quotient of a star compactification by n points is a star compactification by m points, where $n \geq m$. Consequently and by Remark 3.2, if $\diamond = R \cup \{(x,x) \mid x \in X\}$, where R is the equivalence relation on $\{\omega_1, ..., \omega_n\}$ in which all these points are related, then $(X_n/\diamond, \mu/\diamond)$ is the Alexandroff compactification of (X, τ).

Moreover, by observing the form of the open sets M_i and in view of the Proposition 3.6 we obtain the next corollary.

Corollary 4.5. $\mu/\diamond = \bigcap_{p\in P} \langle\langle U_{p_1}, ..., U_{p_m}\rangle\rangle$ *where* $P = I_1 \times ... \times I_m$ *and* $p = (p_1, ..., p_m)$.

For instance, if $\diamond$ only identifies ω_1 with ω_2 on X_n then, in X_{n-1} we have:

$$\mu/\diamond \;= \langle\langle U_1 \cup U_2, U_3, ..., U_n\rangle\rangle = \langle\langle U_1, U_3, ..., U_n\rangle\rangle \cap \langle\langle U_2, U_3, ..., U_n\rangle\rangle .$$

On the other hand, the star compactifications show an interesting behavior when, from a star compactification of (X, τ) by m points, we obtain a compactification of (X, τ) by n points, with $n \geq m$, so that certain quotient on X_n gives back the original compactification.

In terms of Theorem 4.2 we have the next proposition.

Proposition 4.6. *If* (X_m, μ) *with* $\mu = \langle\langle U_1, ..., U_m\rangle\rangle$ *is a star compactification of* (X, τ) *by* m *points, then* (X_n, β) *with*

$$\beta = \mu \cup \{B \cup A \mid B \subseteq \{\omega_{m+1}, ..., \omega_n\},\ A \in \mu\ ,\ \{\omega_1\} \cup A \in \mu\}$$

is a star compactification of (X, τ) *by* n *points,* $n \geq m$*, such that a certain quotient of* (X_n, β) *is* (X_m, μ).

Proof. Let see that $\beta = \langle\langle U_1, U_2, ..., U_m, U_1, ..., U_1\rangle\rangle$.
By the definition of $\langle\langle U_1, U_2, ..., U_m, U_1, ..., U_1\rangle\rangle$ we observe that
$\mu \subseteq \langle\langle U_1, U_2, ..., U_m, U_1, ..., U_1\rangle\rangle$.
Let C be a basic open of $\langle\langle U_1, U_2, ..., U_m, U_1, ..., U_1\rangle\rangle$:
If $C \in \tau$ or $C = \{\omega_i\} \cup (U_i \setminus K)$ for some $i = 1, ..., m$ and some K closed and compact subset of X, then $C \in \mu \subseteq \beta$.
If $C = \{\omega_i\} \cup (U_1 \setminus K)$ for some $i = m+1, ..., n$ and some closed and compact subset K of X, we have that $A = U_1 \setminus K \in \tau \subseteq \mu$ and $\{\omega_1\} \cup A \in \mu$, thus $C = B \cup A$ with $B = \{\omega_i\} \subseteq \{\omega_{m+1}, ..., \omega_n\}$, then $C \in \beta$.
Therefore $\langle\langle U_1, U_2, ..., U_m, U_1, ..., U_1\rangle\rangle \subseteq \beta$.
On the other hand let C be an element of β. We see that
$C \in \langle\langle U_1, U_2, ..., U_m, U_1, ..., U_1\rangle\rangle$ showing that all of its points are interior points in $\langle\langle U_1, U_2, ..., U_m, U_1, ..., U_1\rangle\rangle$.

Let z be a point of C :
Case 1: If $z \in X_m$, by the definition of β we have that $C \cap X_m \in \mu$, thus $z \in C \cap X_m \subseteq C$ where $C \cap X_m \in \langle\langle U_1, U_2, ..., U_m, U_1, ..., U_1 \rangle\rangle$ and z is an interior point of C in $\langle\langle U_1, U_2, ..., U_m, U_1, ..., U_1 \rangle\rangle$.

Case 2: If $z = \omega_i$, for some $i = m+1, ..., n$, then $C = B \cup A$ where $\omega_i \in B$, $A \in \mu$, $\{\omega_1\} \cup A \in \mu$, thus there exists a closed and compact subset K of X such that $\{\omega_1\} \cup (U_1 \setminus K) \subseteq \{\omega_1\} \cup A$. Since $U_1 \setminus K \subseteq A$, then $\omega_i \in \{\omega_i\} \cup (U_1 \setminus K) \subseteq B \cup A = C$ where $\{\omega_i\} \cup (U_1 \setminus K)$ is open in $\langle\langle U_1, U_2, ..., U_m, U_1, ..., U_1 \rangle\rangle$.
Therefore ω_i is an interior point to C in $\langle\langle U_1, U_2, ..., U_m, U_1, ..., U_1 \rangle\rangle$.
By cases 1 and 2 we conclude that $C \in \langle\langle U_1, U_2, ..., U_m, U_1, ..., U_1 \rangle\rangle$.
Finally, in the presentation of β as $\langle\langle U_1, U_2, ..., U_m, U_1, ..., U_1 \rangle\rangle$ it is clear that the equivalence relation on X_n that identifies $\omega_1, \omega_{m+1}, ..., \omega_n$ produces the star topology μ. □

Remark 4.7. From a star compactification of $\mu = \langle\langle U_1, ..., U_m \rangle\rangle$ on X_m it is quite simple to generate new compactifications by n points, $n > m$, by repeating the U_i as the open sets associated to new points.

For instance, $\beta = \langle\langle U_1, U_2, ..., U_7, U_4, U_3 \rangle\rangle$ is a star compactification of (X, τ) by nine points obtained from the star compactification by seven points $\mu = \langle\langle U_1, ..., U_7 \rangle\rangle$ of (X, τ). Moreover, if $\diamond$ is the equivalence relation on X_9 that identifies ω_4 with ω_8 and ω_3 with ω_9 then $(X_9/\diamond, \beta/\diamond)$ is the compactification (X_7, μ) .

5. Star compactifications vs Magill compactifications.

A Magill compactification refers to the method of compactification by n points of a non-compact topological space, presented by Magill in [3]. In this section we show the relation between Magill compactifications and star compactifications.

5.1. **The Magill compactifications.** Using the notation that we have introduced in this paper, we present the Magill compactification with his results without proofs.

Proposition 5.1. *If X contains n non-empty open subsets G_i, $i = 1, ..., n$; two by two disjoint such that:*

(1) *$H = X \setminus \bigcup_{i=1}^{n} G_i$ is compact and*
(2) *$X \setminus \bigcup_{j \neq i} G_j$ is not compact for each $i = 1, ..., n$,*

then the collection

$$\mathcal{B}^* = \tau \cup \{A \cup \{\omega_i\} \mid A \in \tau, (H \cup G_i) \cap (X \setminus A) \text{ is compact in } X;\ i = 1, ..., n\}$$

is a base for a topology ρ on X_n.

Observe that $H \cup G_i = X \setminus \bigcup_{j \neq i} G_j$ for each $i = 1, ..., n$ because G_i are mutually disjoint. Under these conditions we have the following propositions.

Proposition 5.2. *(X_n, ρ) is a compactification of (X, τ) by n points.*

Proposition 5.3. *If (X, τ) is locally compact and T_2 then (X_n, ρ) is T_2.*

Remark 5.4. Although Magill always considers Hausdorff topological spaces in [3], it is clear that this construction still provides a compactification of X without this assumption.

5.2. **Relation between star compactifications and Magill compactifications.** In this section we obtain that each Magill compactification of X by n points is a star compactification. Thus the collection of Magill compactifications of X by n points coincides with the collection of star compactifications with associated two by two disjoint open sets. In the case of Hausdorff compactifications, when X is Hausdorff and locally compact, we have that the collections of Magill compactifications of X by n points and of star compactifications of X by n points coincide and are all the possible Hausdorff compactifications of X by n points.

Proposition 5.5. *Each star compactification of (X, τ) by n points, with associated two by two disjoint open sets is a Magill compactification of (X, τ) by n points.*

Proof. Let μ be a star compactification $\langle\langle U_1, ..., U_n\rangle\rangle$ of (X, τ) where the sets U_i are two by two disjoint. The sets U_i are open, non-empty subsets of X and $H = X \setminus \bigcup_{i=1}^{n} U_i$ is compact.
Moreover, $X \setminus \bigcup_{j \neq i} U_j$ is not compact because in the contrary case, since $U_i \subseteq X \setminus \bigcup_{j \neq i} U_j$, with $X \setminus \bigcup_{j \neq i} U_j$ a subset of X closed and compact, this contradicts that (X_n, μ) is a compactification of (X, τ).
Therefore, the sets U_i, $i = 1, ..., n$ produce a Magill compactification that we call ρ.
We assert that $\mu = \rho$.
i) Let $(U_i \setminus K) \cup \{\omega_i\}$ be an element of $\mathcal{B}$, base of the star compactification μ, where K is a closed and compact subset of X.
To see that $(U_i \setminus K) \cup \{\omega_i\} \in \mathcal{B}^*$, base of the Magill compactification ρ, we show that $(H \cup U_i) \cap (X \setminus (U_i \setminus K))$ is compact:

$$
\begin{aligned}
&(H \cup U_i) \cap (X \setminus (U_i \setminus K)) \\
&= \left(X \setminus \bigcup_{j \neq i} U_j\right) \cap (X \setminus (U_i \cap (X \setminus K))) \\
&= X \setminus \left(\left(\bigcup_{j \neq i} U_j\right) \cup (U_i \cap (X \setminus K))\right) \\
&= X \setminus \left(\left(\bigcup_{j=1}^{n} U_j\right) \cap \left(\left(\bigcup_{j \neq i} U_j\right) \cup (X \setminus K)\right)\right) \\
&= \left(X \setminus \bigcup_{j=1}^{n} U_j\right) \cup \left(X \setminus \left(\left(\bigcup_{j \neq i} U_j\right) \cup (X \setminus K)\right)\right) \\
&= \left(X \setminus \bigcup_{j=1}^{n} U_j\right) \cup \left(\left(X \setminus \bigcup_{j \neq i} U_j\right) \cap K\right),
\end{aligned}
$$

where $X \setminus \bigcup\limits_{j=1}^{n} U_j$ is compact and so is $\left(X \setminus \bigcup\limits_{j\neq i} U_j\right) \cap K$ because K is compact and $X \setminus \bigcup\limits_{j\neq i} U_j$ is closed. Thus $(H \cup U_i) \cap (X \setminus (U_i \setminus K))$ is compact, with $U_i \setminus K \in \tau$; then $\mathcal{B} \subseteq \mathcal{B}^*$ and $\mu \subseteq \rho$.

ii) Let $A \cup \{\omega_i\}$ be an element of $\mathcal{B}^*$, base of ρ, then $(H \cup U_i) \cap (X \setminus A)$ is compact and closed. Thus $U_i \setminus [(H \cup U_i) \cap (X \setminus A)] \cup \{\omega_i\} \in \mathcal{B}$.

$$\begin{aligned} U_i \setminus [(H \cup U_i) \cap (X \setminus A)] &= [U_i \setminus (H \cup U_i)] \cup [U_i \setminus (X \setminus A)] \\ &= \left[U_i \setminus \left(X \setminus \bigcup_{j\neq i} U_j\right)\right] \cup [U_i \setminus (X \setminus A)] \\ &= \varnothing \cup [U_i \setminus (X \setminus A)] \\ &= A \cap U_i. \end{aligned}$$

Since $A \in \tau$, we have that $A \cup \{\omega_i\} = A \cup [(A \cap U_i) \cup \{\omega_i\}] \in \mu$ and $\rho \subseteq \mu$. □

Proposition 5.6. *Each Magill compactification of (X, τ) by n points is a star compactification of (X, τ) by n points.*

Proof. Let ρ be a Magill compactification of (X, τ) obtained through n open, non-empty subsets of X, G_i, $i = 1, ..., n$ two by two disjoint.
Let μ be the star topology $\langle\langle G_1, ..., G_n\rangle\rangle$. Reasoning as in the previous proposition we obtain that $\rho = \mu$. Thus ρ is a star compactification of X and we have that $G_i \nsubseteq K$, for each K closed and compact subset of X. □

So we have that the collection of Magill compactifications of X by n points: $\mathcal{M}_n$ and the collection of star compactifications of X by n points associated to n two by two disjoint open sets: $\mathcal{E}_{Dn}$ are the same. From this fact and Proposition 4.4 we obtain the following corollary.

Corollary 5.7. *If (X_n, μ) is a Magill compactification of (X, τ) by n points and $\diamond = R \cup \{(x, x) \mid x \in X\}$, where R is an equivalence relation on $\{\omega_1, ..., \omega_n\}$, then $(X_n/\diamond, \mu/\diamond)$ is a Magill compactification of (X, τ) by m points, where $m = |\{\omega_1, ..., \omega_n\}/R| \leq n$.*

Remark 5.8. There exist star compactifications that are not Magill compactifications.
Let X be the subspace $(0, 1)$ of $\mathbb{R}$ with the usual topology, and $X_2 = X \cup \{\omega_1, \omega_2\}$, $\mu = \langle\langle U_1, U_2\rangle\rangle$ where $U_1 = \left(0, \frac{1}{2}\right)$, $U_2 = (0, 1)$.
It is clear that (X_2, μ) is a star compactification of X by two points. If μ is a Magill compactification, then by Proposition 5.6 we have that $\mu = \langle\langle G_1, G_2\rangle\rangle$ where G_1 and G_2 are non-empty, open and disjoint subsets of X, that satisfy the other conditions to be a compactification. But since $\langle\langle G_1, G_2\rangle\rangle$ is a compactification of X by two points, with G_1 and G_2 disjoints, it follows that there are $a, b \in (0, 1)$ such that $G_1 = (0, a)$, $G_2 = (b, 1)$ with $a < b$; without loss of generality we can suppose that $a < \frac{1}{2} < b$ then, by Proposition 3.5 we have that $\langle\langle U_1, U_2\rangle\rangle \subseteq \langle\langle G_1, G_2\rangle\rangle$.

Moreover, $G_2 \cup \{\omega_2\} \in \langle\langle G_1, G_2\rangle\rangle$ but $G_2 \cup \{\omega_2\} \notin \langle\langle U_1, U_2\rangle\rangle$ because for all $A \cup \{\omega_2\} \in \langle\langle U_1, U_2\rangle\rangle$ there exists $c \in (0,1)$ such that $(0,c) \subseteq A$ and clearly $(0,c) \nsubseteq G_2$.
Therefore $\langle\langle U_1, U_2\rangle\rangle \subsetneqq \langle\langle G_1, G_2\rangle\rangle$ and (X_2, μ) is a star compactification of X that is not a Magill compactification.
Furthermore, for each $n > 1$ we have that $\mathcal{M}_n \subsetneqq \mathcal{E}_n$ because (X_n, Ω)[1] is a star compactification of (X, τ) that is not a Magill compactification.

We mention without proof Theorem 0.5 of [7]:
"Let X be a locally compact Hausdorff space. Then any Hausdorff compactification for X is the star topology associated with an $m-$ tuple of open mutually disjoint subsets of X".

We denote:
$\mathcal{HC}_n$ the collection of T_2 compactifications of X by n points,
$\mathcal{HE}_n$ the collection of T_2 star compactifications of X by n points and
$\mathcal{HM}_n$ the collection of T_2 Magill compactifications of X by n points.

Proposition 5.9. *If (X, τ) is T_2, locally compact and non-compact then* $\mathcal{HC}_n = \mathcal{HE}_n = \mathcal{HM}_n$.

Proof. i) Clearly $\mathcal{HE}_n \subseteq \mathcal{HC}_n$. By Theorem 0.5 of [7] we have $\mathcal{HC}_n \subseteq \mathcal{HE}_n$.
ii) By Proposition 5.6, $\mathcal{HM}_n \subseteq \mathcal{HE}_n$. Let μ be an element of $\mathcal{HE}_n \subseteq \mathcal{HC}_n$, by Theorem 0.5 of [7], μ is a T_2 star compactification of X associated with n open mutually disjoint subsets of X. Then $\mu \in \mathcal{HM}_n$ by Proposition 5.5. □

Remark 5.10. Observe that $\mu = \langle\langle U_1, U_2\rangle\rangle$ is not a T_2 compactification of X, by Remark 5.8.

6. Some examples.

In this section we present simple examples of star compactifications, and examples of compactifications of a topological space by a finite number of points that are not of A-class and, therefore, are not star compactifications.

Example 6.1. Consider the topological space $(\mathbb{R}, \tau)$, where

$$\tau = \{(-x, x) : x > 0\} \cup \{\phi, \mathbb{R}\}.$$

To find the star compactifications by two points of this space we need two open sets U_1, U_2 such that $L = \mathbb{R} \setminus (U_1 \cup U_2)$ is compact. Since the only closed and compact set of this space is ϕ we need that $U_1 \cup U_2 = \mathbb{R}$ and this happens if one of them is $\mathbb{R}$. Thus, for this space we have basically three types of star compactifications by two points: $\langle\langle U, \mathbb{R}\rangle\rangle$, $\langle\langle \mathbb{R}, U\rangle\rangle$ and $\langle\langle \mathbb{R}, \mathbb{R}\rangle\rangle$, where U is an open set of the form $(-x, x)$.

On the other hand, there exist infinity star compactifications by two points of type $\langle\langle U, \mathbb{R}\rangle\rangle$, that depend on the open set $U = (-x, x)$ that we consider. All

[1] Considered as defined in Proposition 3.1.

the compactifications of this type are ordered by inclusion, so that each non-empty subcollection of them has its intersection or its union in the same type of star compactifications. That is, we again obtain a star compactification. However, in general this fact is false.

Example 6.2. Let X be the open interval $(0,1)$ of $\mathbb{R}$ with the usual topology of subspace. In this case there exist seven star compactifications by two points, which are classified in three types: (1) the lowest compactification $\langle\langle X, X\rangle\rangle$; (2) four of the type $\langle\langle X, U_2\rangle\rangle$ or $\langle\langle U_1, X\rangle\rangle$ that are obtained accordingly if U_i, for $i=1,2$ is either $(0,a)$ or $(a,1)$ for some a, with $0<a<1$; and (3) two maximal compactifications of the form $\langle\langle U_1, U_2\rangle\rangle$, where $U_1=(0,a)$ and $U_2=(b,1)$ or on the contrary, where $a,b\in(0,1)$ and in this case the order between a and b is irrelevant because the different possibilities produce the same compactification.

The two maximal compactifications can be seen as the compactification $[0,1]$ of $(0,1)$ where the additional points 0 and 1 correspond to ω_1 and ω_2; $\omega_1=0$ if $U_1=(0,a)$, or $\omega_1=1$ if $U_1=(b,1)$.

Example 6.3. Consider $X=(0,1)\cup(2,3)$ as a subspace of $\mathbb{R}$ with the usual topology. In this case the star compactifications by two points are classified in twelve different types. In these twelve, we find three types of maximal compactifications in which X_2 can be represented as a subset of $\mathbb{R}^2$, where the basic neighborhoods of ω_1 and ω_2 are precisely obtained by the subspace topology of $\mathbb{R}^2$, with the usual topology. These can be represented as in Figure 1:

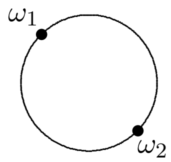
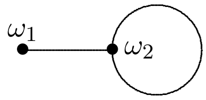
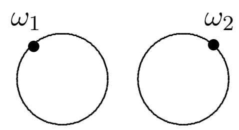
(X_2,μ)	(X_2,ρ)	(X_2,φ)
		
X_2 is the circumference.	X_2 is the union of the segment and the circumference.	X_2 is the union of the two disjoint circumferences.

Figure 1: Maximal compactifications.

Some intermediate compactifications are:

- $\eta=\langle\langle U_1, U_2\rangle\rangle$, where $U_1=(0,1)$ and $U_2=(b,1)\cup(2,3)$, with $0<b<1$. We have that $\eta\subset\varphi$ and $\eta\subset\rho$.
- $\gamma=\langle\langle V_1, V_2\rangle\rangle$, where $V_1=(0,1)\cup(2,a)$ and $V_2=(b,1)\cup(2,3)$, with $2<a<3$ and $0<b<1$. We have that $\gamma\subset\eta$.

- $\alpha = \langle\langle W_1, W_2\rangle\rangle$, where $W_1 = (0,a) \cup (2,3)$ and $W_2 = (b,1) \cup (2,3)$, with $0 < a < 1$ and $0 < b < 1$. Notice that α and γ are incomparable.

We know that every star compactification is of A-class, but there exist compactifications of A-class that are not star compactifications. This is illustrated in the following example.

Example 6.4. Consider the topological space (X,τ) of the real numbers with the usual topology and the simplest possible compactification: (X_n,μ), where $\mu = \tau \cup \{X_n\}$, for $n \geq 1$. We can easily verify that (X_n,μ) is a compactification of A-class of (X,τ) that is not star.

The following propositions provide examples of non A-class compactifications.

Proposition 6.5. *If (X_1,μ) is a compactification of (X,τ) by one point, where $X_1 = X \cup \{\omega\}$, then $\tau \subseteq \mu$.*

Proof. i) $X \in \mu$.
In fact, since (X,τ) is not compact, there exists a covering $\{A_i \mid i \in I\}$ of X by open sets of τ that cannot be reduced to a finite covering.
For each $i \in I$ there exists $B_i \in \mu$ such that $A_i = B_i \cap X$.

$$\begin{aligned} X_1 = \bigcup_{i\in I} B_i \quad &\Longrightarrow \quad X_1 = \bigcup_{k=1}^{n} B_{ik} \quad \text{(because } X_1 \text{ is compact).} \\ &\Longrightarrow \quad X = \bigcup_{k=1}^{n} (B_{ik} \cap X) = \bigcup_{k=1}^{n} A_{ik}. \end{aligned}$$

This is a contradiction and then $X_1 \neq \bigcup_{i\in I} B_i$.
But

$$\begin{aligned} X_1 \supsetneqq \bigcup_{i\in I} B_i \quad &\Longrightarrow \quad \omega \notin B_i, \text{ for all } i \in I \\ &\Longrightarrow \quad B_i \subseteq X, \text{ for all } i \in I \\ &\Longrightarrow \quad A_i = B_i, \text{ for all } i \in I \\ &\Longrightarrow \quad X \in \mu. \end{aligned}$$

$$\begin{aligned} \text{ii) } A \in \tau \quad &\Longrightarrow \quad A = B \cap X \quad \text{for some } B \in \mu \\ &\Longrightarrow \quad A \in \mu. \qquad \text{(because } X \in \mu.) \end{aligned}$$

□

The previous proposition asserts that X is an open set in every compactification of (X,τ) by one point. This is equivalent to saying that each element of τ is an element of μ. Thus, each compactification of (X,τ) by one point is obtained with the original open sets and by adding some additional sets in an appropriate way. However, this fact cannot be generalized to compactifications by more than one point.

Definition 6.6. We say that a topological space (Y,υ) is *hyperconnected* if $\upsilon \setminus \{\varnothing\}$ is a collection closed for finite intersections, that is to say, if each pair of non-empty open sets has non-empty intersection.

Some examples:

(1) (Y, φ), where Y is an infinite set and φ is the cofinite topology, is hyperconnected.
(2) $(\mathbb{R}, \tau)$, where τ is the topology with base $\{(-a, a) \subset \mathbb{R} \mid a > 0\}$, is hyperconnected.
(3) $(\mathbb{R}, \mu)$, where μ is the topology with base $\{(a, +\infty) \subset \mathbb{R} \mid a \in \mathbb{R}\}$, is hyperconnected.
(4) $\mathbb{R}$ with the usual topology is not a hyperconnected space.

The next proposition provides examples of compactifications by more than one point that are not of A-class.

Proposition 6.7. *If (X, τ) is a hyperconnected, non-compact topological space then (X, τ) has a compactification by more than one point in which X is not an open set.*

Proof. Let X_2 be the set $X \cup \{\omega_1, \omega_2\}$

$$\mu = \{A \cup \{\omega_1\} \mid A \in \tau \setminus \{\varnothing\}\} \cup \{\varnothing, X_2\}.$$

It is clear that μ is a topology on X_2 and $X \notin \mu$.
i) (X_2, μ) is compact because if $\{B_i \mid i \in I\}$ is a covering of X_2 by open sets of μ, $X_2 = B_i$, for some $i \in I$, and then this covering can be reduced to a finite one.
ii) $\mu|_X = \tau$.
iii) X is dense in X_2, because on the contrary some $\{\omega_1\}, \{\omega_2\}$ or $\{\omega_1, \omega_2\}$ will be open sets of μ.
Thus, (X_2, μ) is a compactification of (X, τ) by more than one point, in which X is not a open set. □

Remark 6.8. In the previous proposition we can take $Y = X \cup Z$, where $\omega_1 \in Z$, $X \cap Z = \varnothing$ and Z is a finite or infinite set that contains more than one element. Following the proof we obtain that (Y, μ) is a compactification of (X, τ) by more than one point that does not contain X as an open set.

The next fact can be proved in a simple way.

Proposition 6.9. *If (X_n, μ) is a T_1 compactification of (X, τ) then (X_n, μ) is of A-class.*

Question 6.10. If we consider other types of compactifications of (X, τ) by n points, what is the relationship between these types of compactifications and Star (Magill) compactifications?

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