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Vehicle Routing and Scheduling Problem for a multi-period, multi-perishable product system with time window: A Case study

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Abstract: The well-known Vehicle Routing Problem (VRP) is to find proper sequence of routes in order to minimize transportation costs. In this paper, a mixed-integer programming model is presented for a food distributer company and the model outputs are to determine the optimal routes and amount of pickup and delivery. In the objective function, the costs of transportation, holding, tardiness and earliness are considered simultaneously. The proposed model with respect to real conditions is multi-period and has two different time periods: one for dispatching vehicles to customers and suppliers and the other for receiving customers' orders. Time window and split pickup and delivery are considered for perishable products. The proposed model is nonlinear and will be linearized using exact techniques. At the end, model is solved using GAMS and the sensitivity analysis is performed. The results indicate that the trend of changes in holding and transportation costs in compared to tardiness and earliness costs are closed together and are not so sensitive to demand changes.

Key words: Vehicle routing problem, Time window, Split pick-up and delivery, Scheduling.

1. Introduction

Transportation, earliness, tardiness and holding costs are very important in a service company. In order to increase profit, there should be a balance between these costs in such a way that total cost is minimized.

The well-known Vehicle Routing Problem (VRP) is to find proper sequence of routes in order to minimize transportation cost. The aim of this problem is to find the shortest route and minimizing the number of vehicles in use. By adding different constraints to VRP, various kinds of this problem are extended. In VRP with time windows (VRPTW), demand of customer must be supplied in the allowed time interval. In VRP with split delivery (SDVRP), a customer may be served by more than one vehicle. When both time window and split delivery are considered, the problem is called SDVRPTW. Vehicle routing and scheduling problem (VRSP) is an extension of VRP which has additional time constraints such as time window, total available time etc. In vehicle routing problem, the amount of distributed products and transportation scheduling are usually considered separately. Considering time window causes to include real world conditions in the model. Moreover, Vehicle Routing Problem with Soft Time Window (VRPSTW) has studied significantly less than VRPTW (Duygu Tas *et al.,* 2014). However, there is still a large gap between real needs of industry and what has been done theoretically (Andersson *et al.,* 2010).

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In this paper, a food distributer (Amirpakhsh Company) that delivers and receives perishable products in different areas is considered as a real case. The objective of this paper is modeling of routing and scheduling problem simultaneously in order to minimize total cost.

The solution determines optimized vehicle routes, distribution system and pickup and delivery scheduling. Also, the amount of product that remains at the end of each period is calculated. The other contribution of this paper is considering two kinds of time periods. One of them is related to receiving orders by visitors and the other is related to sending products and vehicle movement in pickup and delivery process. The paper is organized as follows:

In the coming section, the literature of VRP and its extensions are briefly reviewed. Section 3 includes problem definition and mixed-integer linear programming model. Sensitivity analysis and concluding remarks are discussed in sections 4 and 5 respectively.

2. Literature review

Yang and Yu (2011) used improved Ant Colony Optimization (IACO) to solve PVRPTW and showed that IACO can generate very good solutions. Baldacci *et al.* (2012) reviewed mathematical modeling, relaxation and recent exact approaches for CVRP and VRPTW. They also compared computational performance of exact algorithm for both of them. Cetinkaya *et al.* (2013) introduced a new kind of VRP that called two-phase VRP with arc time window (TS-VRP-ATWs). They used a heuristic approach based on Memetic Algorithm (MA) to solve the mixed-integer programming formulation. The computational results show that MIP formulation produce optimum answer up to 50 nodes and the proposed MA can generate satisfactory solution in a reasonable time. Belfiore and Yoshizaki (2013) proposed Scatter Search (SS) for Fleet Size and Mixed Vehicle Routing Problem with Time Windows and Split Delivery (FSMVRPTWSD). The results show that proposed algorithm is comparable with the best results of the literature.

Cacchiani *et al.* (2012) proposed a hybrid optimization algorithm for PVRP and showed the efficiency of proposed algorithm to produce high quality outputs. Rahimi-Vahed (2015) studied the determining optimum size of fleets for multi-depot VRP, PVRP and multi depot PVRP. They considered vehicle capacity constraint, route duration and budget and proposed a new Modular Heuristic Algorithm (MHA) to solve it. Computational results showed that MHA generated high quality solutions for these three problems in a reasonable time.

Zhang *et al.,* (2014) introduced a new model that called environmental VRP (EVRP) and proposed Artificial Bee Colony (ABC) algorithm to solve it. Its objective is reduction of adverse effects of vehicles on environment. The environmental effects are measured by emission amount of carbon dioxide. Numerical results show that hybrid ABC on average acts 5% better than ABC.

Diego *et al.* (2014) studied multi-trip VRP in which vehicles return to depot after the end of service in order to reload. They proposed hybrid Genetic Algorithm as a solution approach.

Shahin Moghadam *et al.* (2014) considered cross-dock, customer and suppliers for VRSP. They presented a nonlinear mixed integer programming model and used SA and a hybrid algorithm based on Ant Colony System and SA to solve it. Computation results show the best performance of hybrid algorithm.

Hasani-Goodarzi and Tavakkoli-Moghaddam (2012) presented a MILP model of SDVRP with capacity constraint for multi-products cross-dock. The results show optimum vehicle routes and number of required vehicles. Silva *et al.,* (2015) investigated VRP with split delivery (SDVRP) and used multi-start iteration local search based approach to solve it. Computational results show that proposed approach could improve solution comparing some other heuristics. Ai-min *et al.* (2009) presented SDVRP model that includes soft time window, fixed cost of vehicles and vehicle full-load coefficient and solved in using Simulate Annealing algorithm.

In VRP scheduling problem, perishable goods, split delivery and time window have not been considered simultaneously. Considering this research gap, periodic VRP with split pick-up and delivery, scheduling and time window constraints are formulated in this paper.

3. Problem definition

Amirpakhsh Company is an Iranian food distributer firm that carries out delivery and pick-up perishable products. In order to holding and transportation of these products, it needs suitable equipment such as refrigerated vehicles. Usually this kind of vehicle is more expensive and consumes more fuel than regular ones. Furthermore, perishable good should be delivered in a desirable time interval by refrigerated vehicles. So in the proposed model, a maximum available time constraint is considered.

The company distributes different kinds of products that only dairy products are considered in this paper. In the proposed model, total cost includes transportation, holding, earliness and tardiness costs. Because of split delivery and pickup, the demand of each node can be met by more than one vehicle. Due to the importance of meeting demand in proper time, tardiness costs considered only in delivery process. There are two kinds of time periods in the model that are called t and t'. Index t indicates time period of delivery to customers while t' is related to time period of receiving customers' orders. In other words, a part of customer's demand in period t' can be met by delivering products in period t. The difference between t and t' indicates earliness/tardiness time. The aim of model is to determine the optimal sequence of routes and the best amount of delivery and pick-up.

3.1. Model assumptions

The model assumptions are as follows:

- Problem is considered multi-period with short term horizon.
- At the beginning of the planning period, vehicles are in depot and after completion of service they will return to depot again.
- Shortage is allowed and considered as lost sale.
- The demand of a customer can be met by more than one vehicle.
- Vehicles are heterogeneous and equipped to refrigerator.
- Outdoor temperature is considered fixed.
- Traffic volume and accidents do not effect on distribution.

3.2. Proposed model

In this section, the proposed model is described. Indices, parameters and variables are as follows:

Indices

i , i' : Index of supplier (i , $i' \in I$)

j , j' : Index of customer (j , $j' \in J$)

r: Index of product ($r \in R$)

m: Index of vehicle in delivery process ($m \in M$)

m': Index of vehicle in pickup process ($m' \in M'$)

O: Depot

t: Index of time period for dispatching vehicle to customers and suppliers ($t \in T$)

t': Index of time period for receiving customer order ($t' \in T'$)

Parameters

Co_m: Fixed cost of using vehicle m in delivery process

$Co'_{m'}$: Fixed cost of using vehicle m' in pick-up process

$C_{jj'}^{m}$: Transportation cost of vehicle m from customer j to customer j'

$C'^{m'}_{ii'}$: Transportation cost of vehicle m' from supplier i to supplier i'

$T_{jj'}$: Travelling time from customer j to customer j'

$T'_{ii'}$: Travelling time from supplier i to supplier i'

TP :Total available time in each period

$d_{jr}^{t'}$: Demand of customer j for product r in period t'

b_{ir}^{t}: Production capacity of supplier i for product r in period t

Q_m: Capacity of vehicle m in delivery process

$Q'_{m'}$: Capacity of vehicle m' in pickup process

B_r: Volume of product r

W_r: Earliness and tardiness penalty for product r

W'_r: Holding cost of product r at depot

S: Soft time window

h: Hard time window

M: Big number

Variables

$$x_{jj'}^{tm}\begin{cases}1 & if\ vehicle\ m\ moves\ from\ customer\ j\ to\ j'\ in\ period\ t\\ 0 & O.W.\end{cases}$$

$$x'^{tm'}_{ii'}\begin{cases}1 & if\ vehicle\ m'moves\ from\ supplier\ i\ to\ i'in\ pickup\ process\ in\ period\ t\\ 0 & O.W.\end{cases}$$

$Z_{jr}^{tt'm}$: Amount of product r delivered by vehicle m in period t to meet period t' demand of customer j

$a_{ir}^{tm'}$: Amount of product r collected from supplier i by vehicle m' in period t

U_r^t: Amount of product r remain at the end of period t

The proposed model is mixed integer non-linear programming and is shown by equations 1 to 30.

$$\begin{aligned}Min &\sum_{j\in(o,J)}\sum_{\substack{j'\in(o,J)\\ j'\neq j}}\sum_{m\in M}\sum_{t\in T}C_{jj'}^{m}\,x_{jj'}^{mt}\\ &+\sum_{j'\in J}\sum_{m\in M}\sum_{t\in T}Co_m x_{0j'}^{mt}\\ &+\sum_{j\in J}\sum_{m\in M}\sum_{t'\in T}\sum_{t=t'-h}^{t'+h}\sum_{r\in R}W_r Z_{jr}^{mtt'}max(|t-t'|-s\,,0)\\ &+\sum_{r\in R}\sum_{t\in T}W'_r U_r^t\\ &+\sum_{i\in(o,I)}\sum_{\substack{i'\in(o,I)\\ i'\neq i}}\sum_{m'\in M'}\sum_{t\in T}C'^{m'}_{ii'}\,x'^{m't}_{ii'}\\ &+\sum_{i\in I}\sum_{m'\in M'}\sum_{t\in T}Co'_{m'}\,x'^{m't}_{0i}\end{aligned}\qquad(1)$$

Subject to

$$\sum_{\substack{j'\in(o,J)\\ j'\neq j}}x_{jj'}^{mt}\leq 1\quad;\ \forall\ m\in M\,,t\in T\,.j\in(o,J)\qquad(2)$$

$$\sum_{\substack{i'\in(o,I)\\ i'\neq i}}x'^{m't}_{ii'}\leq 1\ \ ;\ \forall\ m'\in M'\,,t\in T\,,i\in(0,I)\qquad(3)$$

$$\sum_{\substack{j\in(0,J)\\ j\neq j'}}x_{jj'}^{mt}=\sum_{j\in(o,J)}x_{j'j}^{mt}\ \ ;\ \ \forall\,m\in M\,,t\in T\,,j'\in J\qquad(4)$$

$$\sum_{\substack{i\in(0,I)\\ i\neq i'}}x'^{m't}_{ii'}=\sum_{i\in(o,I)}x'^{m't}_{i'i}\ \ ;\ \ m'\in M',t\in T\,,i'\in I\qquad(5)$$

$$\sum_{t=t'-h}^{t'+h}\sum_{m\in M}Z_{jr}^{mtt'}=d_{jr}^{t'}\quad\forall\ j\in J\,,r\in R\ \,,t'\in T\qquad(6)$$

$$\sum_{j\in J}\sum_{r\in R}\sum_{t'=t-h}^{t+h}B_r Z_{jr}^{mtt'}\leq Q_m\ \ ;\ \ \forall\ m\in M\ \,,t\in T\qquad(7)$$

$$\sum_{m'\in M'}a_{ir}^{m't}\leq b_{ir}^{t}\quad;\quad\forall\ i\in I\,,r\in R\,,t\in T\qquad(8)$$

$$\sum_{i\in I}\sum_{r\in R}B_r a_{ir}^{m't}\leq Q'_{m'}\quad;\quad\forall\,m'\in M',t\in T\qquad(9)$$

$$\sum_{i\in I}\sum_{t\in T}\sum_{m'\in M'}a_{ir}^{m't}\geq\sum_{j\in J}\sum_{t\in T}\sum_{t'=t-h}^{t+h}\sum_{m\in M}Z_{jr}^{mtt'}\ ;\forall\ r\in R\quad(10)$$

$$U_r^t=\sum_{i\in I}\sum_{m'\in M'}a_{ir}^{m't}-\sum_{j\in J}\sum_{m\in M}\sum_{t'=t-h}^{t+h}Z_{jr}^{mtt'}+U_r^{t-1}\quad;\ \forall\ r\in R\,,t\in T\qquad(11)$$

$$\sum_{j\in J}\sum_{\substack{j'\in(o,J)\\ j'\neq j}}x_{jj'}^{mt}\leq M\sum_{j\in J}x_{oj}^{mt}\ \ ;\ \ \forall\ m\in M\ \,,t\in T\qquad(12)$$

$$\sum_{i\in I}\sum_{\substack{i'\in(o,I)\\ i'\neq i}}x'^{m't}_{ii'}\leq M\sum_{i\in I}x'^{m't}_{oi}\ \ ;\forall\ m'\in M',t\in T\qquad(13)$$

$$\sum_{j\in(o,J)}\sum_{\substack{j'\in(o,J)\\ j'\neq j}}T_{jj'}x_{jj'}^{mt}\leq TP\ \ ;\forall\ m\in M\ \,,t\in T\qquad(14)$$

$$\sum_{i\in(o,I)}\sum_{\substack{i'\in(o,I)\\ i'\neq i}}T'_{ii'}x'^{m't}_{ii'}\leq TP\ ;\forall\ m'\in M'\ \,,t\in T\qquad(15)$$

$$\sum_{r\in R}\sum_{t'=t-h}^{t+h}Z_{jr}^{mtt'}\leq M\sum_{\substack{j'\in(o,J)\\ j'\neq j}}x_{jj'}^{mt}\,;\forall\,j\in J\,,m\in M\,,t\in T\qquad(16)$$

$$\sum_{r\in R}a_{ir}^{m't}\leq M\sum_{\substack{i'\in(o,I)\\ i'\neq i}}x'^{m't}_{ii'}\ ;\ \forall\ i\in I\,,m'\in M',t\in T\qquad(17)$$

$$\sum_{\substack{j\in(o,J)\\ j\neq j'}}x_{jj'}^{mt}\leq\sum_{r\in R}\sum_{t'=t-h}^{t+h}Z_{jr}^{mtt'}\,;\forall\,j'\in J\,,m\in M\,,t\in T\qquad(18)$$

$$\sum_{\substack{i\in(o,I)\\ i\neq i'}} x'^{m't}_{ii'} \leq \sum_{r\in R} a^{m't}_{i'r} \;;\; \forall\, i' \in I\,, m' \in M', t \in T \tag{19}$$

$$\sum_{j\in S}\sum_{\substack{j'\in S\\ j'\neq j}} x^{tm}_{jj'} \leq |S|\,;\;\; \forall\;\; S \subset J\,, |S| > 1\,, |S| \neq |J| - 1,\quad \forall\, t \in T, m \in M \tag{20}$$

$$\sum_{j\in S}\sum_{\substack{j'\in S\\ j'\neq j}} x'^{m't}_{ii'} \leq |S'|\;;\; \forall\, S' \subset J\,, |S'| > 1\,, |S'| \neq |J| - 1,\quad \forall\;\; t \in T, m' \in M' \tag{21}$$

$$x^{tm}_{jj'}\;,\; x'^{m't}_{ii'} \in \{0,1\} \quad;\quad Z^{mtt'}_{jr}\;, a^{m't}_{ir},\; U^{t}_{r} \;\geq 0 \tag{22}$$

The objective of the problem is to minimize operation and transportation costs in pick-up and delivery process, earliness and tardiness costs in delivery process and holding cost. If a vehicle is arrived to a node in the allowed time interval, earliness and tardiness costs is equaled to zero. Otherwise, if it is arrived to a node between hard and soft time window, earliness and tardiness costs are imposed in the model. Two last phrase of objective is related to operation and transportation costs in pick-up process that are the same as delivery process.

Constraints (2) and (3) prove that a vehicle in each period can service to only one node. Constraint (4) implies the consecutive motion of vehicles in delivery process. Constraint (5) is the same as constraint (4) but it is for pick-up process. Constraint (6) proves that total amount of product r that is carried to customer j by vehicle m must be equal to the customers' demands in period t'. Constraint (7) is related to limited capacity of vehicles in delivery process. Constraint (8) shows that amount of product r that is collected from supplier i in each period is not more than production power of supplier i. Constraint (9) is similar to constraint (7) but it is for pick-up process. Constraint (10) shows that total amount of input product r to the depot, must not fewer than total amount of output product r, over planning horizon. Constraint (11) is related to the amount of remained products in the depot at the end of period t. Constraint (12) implies that at first, vehicles are in the depot and start their motion from there. Constraint (13) is the same as constraint (12) but it is for pick-up process. Total available time of vehicles, in delivery process is presented in constraint (14). Constraint (15) is similar to constraint (14) but it is for pick-up process. Constraint (16) states that if there is no route to customer j, no product will be delivered to that customer. Constraint (17) is the same as constraint (16) but it is for pick-up process. Constraint (18) guarantees that if a vehicle has to deliver products to a node, it can meet that node. Constraint (19) is similar to constraint (18) but it is for pick-up process. Constraints (20) and (21) are related to elimination of sub tours. Binary and positive variables are shown in constraint (22).

$$\sum_{j\in J}\sum_{m\in M}\sum_{t'\in T}\sum_{t=t'-h}^{t'+h}\sum_{r\in R} \mathrm{W_r Z^{mtt'}_{jr}} \max(|t-t'| - s\,, 0) \tag{23}$$

As observed, phrase (23) is nonlinear. So as, it is linearized as followed:

At first, phrase (23) is converted to phrase (25) by using equation (24)

$$\max(|\,t-t'\,|-s,0)=NM_{tt'} \tag{24}$$

Then following constraints are added:

$$\sum_{j\in J}\sum_{m\in M}\sum_{t'\in T}\sum_{t=t'-h}^{t'+h}\sum_{r\in R} \mathrm{W_r Z^{mtt'}_{jr}}\, NM_{tt'} \tag{25}$$

$$NM_{tt'} \geq 0 \tag{26}$$

$$-NM_{tt'} - s \leq t - t' \leq NM_{tt'} + s \tag{27}$$

Phrase (25) is still nonlinear. With respect to $t-t' \leq 2$ it is obvious that if $a=1$, then variable NM is binary. By using equation $Z^{mtt'}_{jr} NM_{tt'} = K_{mjrtt'}$, phrase (25) is linearized as followed:

$$\sum_{j\in J}\sum_{m\in M}\sum_{t'\in T}\sum_{t=t'-h}^{t'+h}\sum_{r\in R} \mathrm{W_r}\, K_{mjrtt'} \tag{28}$$

Finally, in order to complete linearization process, following constraints must be added to phrase (28).

$$\mathrm{Z^{mtt'}_{jr}} - M\,(1 - NM_{tt'}) \leq K_{mjrtt'} \leq \mathrm{Z^{mtt'}_{jr}} + M\,(1 - NM_{tt'}) \tag{29}$$

$$\leq M \times NM_{tt'} \tag{30}$$

$$NM_{tt'} \in \{0,1\} \tag{31}$$

4. Numerical example

The proposed model is solved using Gurobi solver in GAMS on a laptop with CPU=Core i3-370M, processor 2.40 GHz and 4G RAM. Model parameters are as follows:

There are nine customer nodes, four supplier nodes, five vehicles in delivery process and three vehicles in pick-up process. Four different products are considered. The number of time periods that demands of customers are received to the depot is 3 and the number of time periods in order to service at the customers, and suppliers are 5. Parameters are as followed:

Co_1=70, Co_2=70, Co_3=140, Co_4=140, Co_5=140

$Co_{1'}$=200, $Co_{2'}$=200, $Co_{3'}$=200

Q_1=200, Q_2=200, Q_3=400, Q_4=400, Q_5=400

$Q_{1'}$ = 700, $Q_{2'}$ = 700, $Q_{3'}$ = 700

B_1=0.5, B_2=1 ,B_3=1, B_4=1

W_1= 5, W_2= 5, W_3= 5, $W_{\_4}$= 5

$W_{1'}$=3, $W_{2'}$=3, $W_{3'}$=3, $W_{4'}$=3

S=1, h=3, T=8

Other parameters are shown in Tables 1 to 6. To illustrate, node 0 indicates depot and nodes 1 to 9 indicate customers in Table 1. Also, the last number in Table 2 indicates that traveling from customer 9 to 8 by vehicle 5 costs 2 hundred dollars.

Table 1. Travelling time from customer j to j' (in hour).

$T_{jj'}$	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	1	2	3	4	3	2	1	3	3
1	1	0	1	2	3	3	2	2	3	3
2	2	1	0	1	2	2	3	3	4	4
3	3	2	1	0	1	2	3	4	4	5
4	4	3	2	1	0	1	2	3	3	4
5	3	3	2	2	1	0	1	2	2	3
6	2	2	3	3	2	1	0	1	1	2
7	1	2	3	4	3	2	1	0	1	1
8	3	3	4	4	3	2	1	1	0	1
9	3	3	4	5	4	3	2	1	1	0

Table 2. Transportation cost of vehicle m from customer j to j' (in hundred dollars).

$C^m_{jj'}$	**1**	**2**	**3**	**4**	**5**
0.1	1	1	2	2	2
0.2	2	2	4	4	4
0.3	3	3	6	6	6
0.4	4	4	8	8	8
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
9.7	2	2	4	4	4
9.8	1	1	2	2	2

Table 3. Production capacity of supplier i for product r in period t.

b^t_{ir}	**1**	**2**	**3**	**4**	**5**
1.1	130	0	18	100	0
1.2	130	8	100	100	0
1.3	50	0	8	100	100
1.4	55	10	5	100	0
2.1	60	0	6	100	0
2.2	110	10	2	100	80
2.3	0	41	150	100	20
2.4	10	0	38	0	50
3.1	100	0	247	0	0
3.2	110	6	5	0	70
3.3	0	0	100	0	0
3.4	200	0	100	0	20
4.1	71	1	0	200	0
4.2	90	6	0	200	0
4.3	0	15	306	0	0
4.4	100	0	0	140	0

Table 4. Travelling time from supplier i to i' (in hour).

$T'_{ii'}$	**0**	**1**	**2**	**3**	**4**
0	**0**	**4**	**3**	**2**	**1**
1	**4**	**0**	**1**	**2**	**3**
2	**3**	**1**	**0**	**1**	**2**
3	**2**	**2**	**1**	**0**	**1**
4	**1**	**3**	**2**	**1**	**0**

Table 5. Demand of customer j for product r in period t'.

$d^{t'}_{jr}$	**1**	**2**	**3**
1.1	25	14	1
1.2	13	11	2
1.3	22	12	3
1.4	24	15	4
2.1	23	12	5
2.2	22	11	6
2.3	21	11	7
2.4	18	10	8
. . .	. . .	. . .	. . .
9.1	21	11	8
9.2	25	51	0
9.3	21	15	0
9.4	18	11	2

Table 6. Transportation cost of vehicle m' from supplier i to i' (in hundred dollars).

$C'^{m'}_{ii'}$	**1**	**2**	**3**
0.1	4	4	4
0.2	3	3	3
0.3	2	2	2
0.4	1	1	1
1.2	1	1	1
1.0	4	2	2
1.3	2	2	2
1.4	3	3	3
2.1	1	1	1
2.0	3	2	2
2.3	1	1	1
2.4	2	2	2
3.0	2	2	2
3.1	2	2	2
3.2	1	1	1
3.4	1	1	1
4.0	1	1	1
4.1	3	3	3
4.2	2	2	2
4.3	1	1	1

The results of the model show that transportation, holding, earliness and tardiness costs are 41, 38 and 21 percent of total cost. If the products were not perishable, constraints 14 and 15 had been removed, fixed and transportation costs would be decreased as well. Therefore, objective function would decrease about 10 percent.

5. Results and Sensitivity Analysis

Due to importance of customers' demands, the results of demand changing are investigated on the transportation, holding, earliness and tardiness costs. The results of changes and their trends are shown in Figure 1. As observed, increase in demand of customers leads to up trend of all costs. As well, Changes' trends in holding and transportation costs are closed to each other.

As shown, amounts of earliness and tardiness costs for changes, less than 60% in demand is close to the other costs. However, increase more than 60% in demand, causes to exponential increase in earliness and tardiness costs. In this situation, products that are in the depot must be delivered to the customers; so, holding cost is reduced. However, transportation cost is increased because of the increase in the number of transportation. Nonetheless, earliness and tardiness costs have the greatest increase.

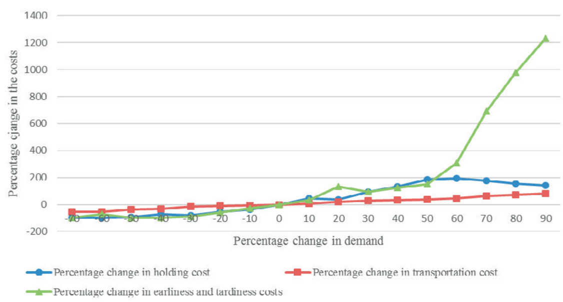

Figure 1. Changes' trend in costs that are created by changes in demand.

In Figure 2, customers' demand and the capacity of vehicles are increased simultaneously. As shown in Figure 2, holding cost is very sensitive to these changes while transportation cost is not so sensitive. Also, increase in Earliness and tardiness costs fairly shows a linear trend.

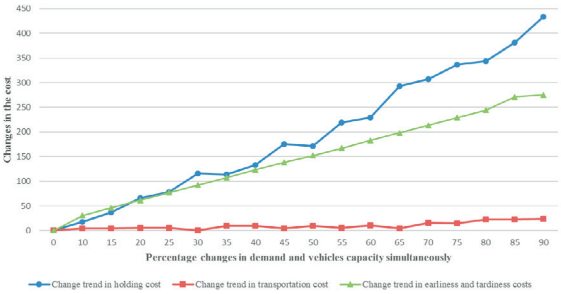

Figure 2. Changes trend in costs that are created by changes in demand and capacity of vehicles simultaneously.

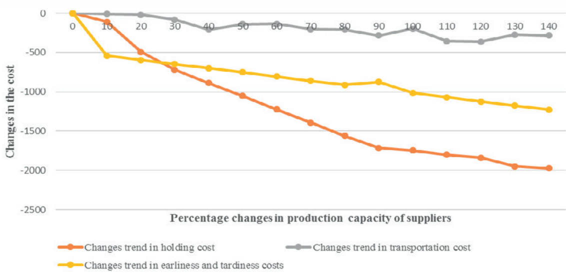

Figure 3. Changes trend in costs that are created by changes in production capacity of suppliers.

As shown in Figure 3, total cost decreases due to the increase in suppliers' production capacity. Transportation cost has the least decrease and holding cost has the most decrease.

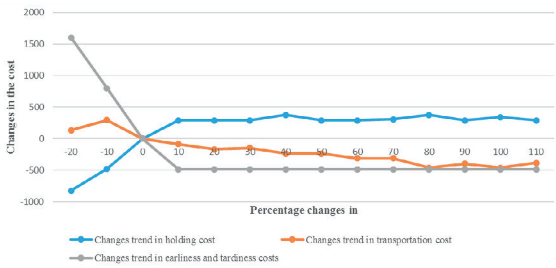

Figure 4. Changes trend in costs that are created by changes in capacity of vehicles in delivery process.

As shown in Figure 4, if capacity of delivery vehicles increases, holding cost will increase and transportation, earliness and tardiness costs will decrease.

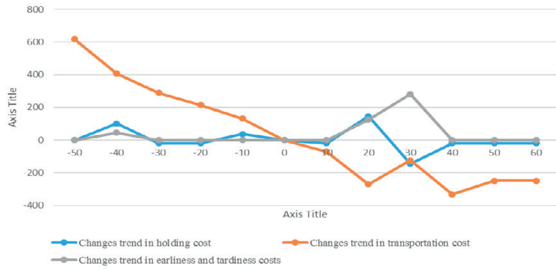

Figure 5. Changes trend in costs that are created by changes in capacity of vehicles in pick-up process.

Figure 5 shows that increase in capacity of pick-up vehicles leads to decrease in transportation cost. However, holding, earliness and tardiness costs are not so sensitive to this change.

6. Conclusion

The vehicle routing problem (VRP) is a practical problem that is implemented a lot in real-world conditions. So as, many different kinds of VRP are created with respect to adding real condition constraints. In this paper transportation, holding, earliness and tardiness costs are considered simultaneously and sensitivity analysis is used. The results indicate that the trend of changes in holding and transportation costs in compared to tardiness and earliness costs are closed together and are not so sensitive to demand changes. However, tardiness and earliness costs are increased exponentially. Therefore, it can be concluded that total costs are increased exponentially by increase more than 60% in customers' demands.

Sensitivity analysis helps managers to make better decisions and understand how to invest and develop their company with the lowest cost. Also they can focus on cost items that are more important and try to decrease them in order to increase the profit.

Although there is a lot of work in VRP, even so, the workspace in this field is wide. For future study, adding real-world application constraints, increase in dimension of problem and solve it with meta-heuristic algorithms are proposed. As well as, even using stochastic and fuzzy data, considering traffic and accident effects can be proposed.

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