## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Abstract</td>
<td>i</td>
</tr>
<tr>
<td></td>
<td>Resumen</td>
<td>iii</td>
</tr>
<tr>
<td></td>
<td>Resum</td>
<td>v</td>
</tr>
<tr>
<td></td>
<td>Acknowledgements</td>
<td>vii</td>
</tr>
<tr>
<td></td>
<td>Contents</td>
<td>ix</td>
</tr>
<tr>
<td>1</td>
<td>INTRODUCTION</td>
<td>15</td>
</tr>
<tr>
<td>1.1</td>
<td>Motivation and background</td>
<td>15</td>
</tr>
<tr>
<td>1.2</td>
<td>Objectives</td>
<td>16</td>
</tr>
<tr>
<td>1.3</td>
<td>Outline</td>
<td>17</td>
</tr>
<tr>
<td>2</td>
<td>CURVE SQUEAL</td>
<td>19</td>
</tr>
<tr>
<td>2.1</td>
<td>Introduction</td>
<td>19</td>
</tr>
<tr>
<td>2.2</td>
<td>Curving behaviour</td>
<td>22</td>
</tr>
<tr>
<td>2.2.1</td>
<td>Insertion of a free wheelset in a curve</td>
<td>24</td>
</tr>
<tr>
<td>2.2.2</td>
<td>Insertion of a bogie in a curve</td>
<td>26</td>
</tr>
<tr>
<td>2.3</td>
<td>Frictional excitation mechanisms</td>
<td>27</td>
</tr>
<tr>
<td>2.3.1</td>
<td>Negative friction slope model</td>
<td>28</td>
</tr>
<tr>
<td>2.3.2</td>
<td>Constant friction mechanism</td>
<td>30</td>
</tr>
<tr>
<td>2.4</td>
<td>Other types of excitation</td>
<td>32</td>
</tr>
<tr>
<td>2.4.1</td>
<td>Excitation by roughness</td>
<td>32</td>
</tr>
<tr>
<td>2.4.2</td>
<td>Excitation by discrete irregularities</td>
<td>33</td>
</tr>
<tr>
<td>2.5</td>
<td>Conclusions</td>
<td>34</td>
</tr>
<tr>
<td>3</td>
<td>REVIEW OF THE MODELLING OF THE VEHICLE/TRACK DYNAMIC INTERACTION</td>
<td>35</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>35</td>
</tr>
<tr>
<td>3.2</td>
<td>Wheelset models</td>
<td>38</td>
</tr>
</tbody>
</table>
3.3 Track models ................................................................. 39
3.4 Contact models ............................................................ 41
  3.4.1 Normal contact problem ............................................ 42
  3.4.2 Tangential contact models ........................................ 45
  3.4.3 Transient contact conditions during squeal .................. 49
3.5 Train/track dynamic interaction models .......................... 50
  3.5.1 Frequency-domain models ....................................... 50
  3.5.2 Time-domain models .............................................. 55
3.6 Conclusions ................................................................. 57

4 WHEEL/RAIL ROLLING CONTACT MODELLING .................... 59
4.1 Introduction ................................................................. 59
4.2 Wheel/rail rolling contact model ................................... 61
  4.2.1 Elastic model ......................................................... 63
  4.2.2 Kinematic model ..................................................... 66
4.3 Numerical algorithm ..................................................... 71
  4.3.1 Algorithm for solving the normal contact problem ......... 72
  4.3.2 Algorithm for solving the tangential contact problem .... 75
4.4 Numerical issues ............................................................ 82
  4.4.1 Numerical errors due to spatial/temporal discretisations ... 82
  4.4.2 Numerical errors due to a falling friction coefficient ....... 86
4.5 Regularisation of Coulomb’s law on a steady-state tangential contact problem ........................................ 90
  4.5.1 Introduction of the regularisation of Coulomb’s law .......... 90
  4.5.2 Regularisation on a steady-state tangential contact problem 93
4.6 Study of falling friction effect on rolling contact parameters 95
  4.6.1 Introduction ............................................................. 95
  4.6.2 First analysis through a 2D approach .......................... 97
  4.6.3 Parameters of the rolling contact model ...................... 102
5 VEHICLE/TRACK DYNAMIC INTERACTION MODEL IN THE TIME DOMAIN

5.1 Introduction
5.2 Generation of the train/track interaction model through substructuring techniques
5.3 Flexible and rotating wheelset model
5.4 Cyclic and flexible track model based on the Moving Element Method
5.4.1 Formulation of the 1D Moving Element Method
5.4.2 3D Moving Element Method
5.4.3 Rail support models
5.4.4 Pseudo-static deformation of the track based on MEM
5.5 Method for the temporal solving of the train/track interaction
5.5.1 Modal approach for reducing the dimension of the problem
5.5.2 Static modal correction
5.5.3 Method for decoupling the system after a modal approach
5.5.4 Simpson and Magnus integrators for solving the modal system
5.5.5 Magnus expansion for periodic interaction forces
5.6 Calculation of interaction forces
5.6.1 Calculation of wheel/rail contact forces
5.6.2 Calculation of forces in the rail supports
5.7 Conclusions

6 APPLICATIONS OF THE WHEELSET/TRACK INTERACTION MODEL

6.1 Introduction
6.2 Contributions of the 3D MEM track model in the high-frequency domain for a single wheelset
6.2.1 Introduction
6.2.2 Vehicle model ................................................................. 168
6.2.3 Track model ................................................................. 171
6.2.4 Wheel/rail contact model ............................................. 172
6.2.5 Results ............................................................................. 173
6.2.6 Discussion ................................................................. 177

6.3 Linear stability analysis and non-linear time-domain simulation for a single
wheelset negotiating a curve ................................................... 178
6.3.1 Introduction ................................................................. 178
6.3.2 Wheel and track models ............................................. 179
6.3.3 Wheel/rail contact model ............................................. 182
6.3.4 Solution procedure ..................................................... 183
6.3.5 Results ............................................................................. 185
6.3.6 Discussion ................................................................. 190

6.4 Investigation of stick/slip oscillations in curving conditions for constant
friction ..................................................................................... 191
6.4.1 Introduction ................................................................. 191
6.4.2 Wheelset, track and contact models ......................... 191
6.4.3 Results ............................................................................. 193
6.4.4 Discussion ................................................................. 206

6.5 Conclusions ................................................................. 207

7 CONCLUSIONS AND FUTURE WORK .............................................. 211
7.1 Conclusions ................................................................. 211
7.2 Future work ................................................................. 217

APPENDIX A: Influence coefficients for the elastic half-space 219
APPENDIX B: Hertzian model for normal contact 223
APPENDIX C: Published articles linked to the Thesis 229
References 257