Intelligent Approach to Inventory Control in Logistics under Uncertainty Conditions

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ABSTRACT

The article presents a proposal for a combined application of fuzzy logic and genetic algorithms to control the procurement process in the enterprise. The approach presented in this paper draws particular attention to the impact of external random factors in the form of demand and lead time uncertainty. The model uses time-variable membership function parameters in a dynamic fashion to describe the modelled output fuzzy (sets) values. An additional element is the use of genetic algorithms for optimisation of fuzzy rule base in the proposed method. The approach presented in this paper was veryfied according to four criteria based on a computer simulation performed on the basis of the actual data from an enterprise.

1. OVERVIEW OF INVENTORY MANAGEMENT ISSUES

As a result of the on-going globalisation and mass consumption, the demand on the goods market is characterised by intense dynamics and a certain level of uncertainty, especially in large agglomerations and urban areas. The logistical processes that occur there as part of supply networks focus primarily on the flow of the streams of material goods, but also take into account the flows of necessary information and financial resources. The volatility of these processes and certain level of uncertainty cause all sorts of inventory to amass at various levels of the logistic network in order to ensure the continuity of production and the uninterrupted availability of the finished products to customers. The goods amassed in the nodal points of the logistic network act as buffers that mitigate the differences in customer demand for the products. In practice, despite the use of modern systems, such as JIT (Just In Time), ERP (Enterprise Resource Planning), MRP (Material resource Planning), it is not possible to entirely eliminate the inventory. In fact, economic processes are stochastic in nature (which results from both the operating environment of these processes and the impact of their surroundings), so it is possible to identify them only to a certain extent, with a greater or smaller error (Wolski, 2010). Due to the impact of random factors on the nodal elements of the supply network (manufacturing plants, distribution centres, warehouses, etc.) through the volatility of demand for semi-finished products or finished products, lead time changeability, vendors' limited capabilities, etc., the optimal policy for the supply and inventory control logistics is of utmost importance to the effectiveness of the entire logistic network.

As a result of the above-mentioned factors and the ever-increasing competition among entities, logistics companies are often forced to keep a high inventory level in order to maintain the desired service level. This behaviour makes it possible to dynamically respond

to unexpected changes in the demand or other external factors but it generates increased costs at the same time. These are, in particular, associated with the carrying the inventory, leasing additional storage space and freezing the limited financial resources in the inventory. On the other hand, the inventory level that is too low in relation to the stock-keeping units characterised by an unusual demand pattern which are essential for the enterprise can lead to the occurrence of external costs caused by lost resources. They can be expressed as cash but also as a customer loss, lowering the reputation of the enterprise or a loss of its competitiveness. This situation is also conducive to the formation of additional transport costs associated with the implementation of unplanned deliveries.

2. OVERVIEW OF INVENTORY CONTROL SOLUTIONS

Due to the impact of the aforementioned factors, the optimal inventory control is a complex decision-making process that requires analysis of multiple criteria and parameters, which in practice are usually non-deterministic in nature. The result is that the basic decisions about how much merchandise should be purchased and at what point in time in order to minimise the stocking and stock-carrying costs and meet the established level of customer service are made in conditions of uncertainty. The subject literature, both domestic and international, provides numerous rich sources on the topic of inventory management. The most popular classical methods for determining inventory levels include, first and foremost, the Economic Order Quantity (EOQ) model, the Re-Order Point (ROP) models and Re-Order Cycle (ROC) models (Krzyżaniak, Cyplik, 2007). However, the applicability of these methods is quite limited as it often requires the adoption of limitations on the stationarity of demand or the known and fixed lead time. The extensions of these methods take into account certain variability with regard to the demand or the lead time by introducing an additional parameter in the form of a safety stock, which aims to cover the unexpected changes in the demand (Grzybowska, 2010), (Niziński, Żurek, 2011), (Krawczyk, 2011). In addition to the above-mentioned methods, one may also encounter other control models, such as: the reorder point model using fixed reorder cycles or the combined re-order point and fixed re-order cycle model (Wolski, 2010). Few papers indicate the problem of inventory control in the conditions of demand discontinuity. When dealing with this issue, authors often present methods created by Wagner-Within and Silver-Meal. Compared to the domestic literature, the list of international publications on the subject of inventory control is definitely more extensive and takes into account a greater number of determinants and characteristics of the task being considered (Axaster, 2006), (Lang, 2009), (Nahmias 2010). An important element raised in foreign publications is the simultaneous inclusion of several products in the control models, which is much closer to the reality (Frank, 2009), (Li, Cheng, Wang, 2007), (Maity, 2007), (Maity, 2009). Due to the difficulty of simultaneously taking into account many parameter variables in the analytical models, more and more papers suggest identifying uncertainty through the introduction of a fuzzy environment. Some articles (Mandal, Roy, 2006), (Roy, 2007), (Taleizadeh, 2009), (Hsieh, 2002), (Maiti, 2006), present an approach that assumes that demand, lead time, stock-carrying costs, customer service, etc. are fuzzy

values. Due to the great complexity and elaborateness of the problem, researchers have been increasingly proposing the use of genetic algorithms to find optimal solutions to the issue (Taleizadeh, 2013), (Khanlarpour, 2013), (Gupta K, 2015). Despite this, in most cases the suggested methods do not take into account the impact of several random factors on the control system at the same time. Therefore, it seems reasonable to develop models and methods for solving the problems of procurement logistics with the use of artificial intelligence techniques. These include, in particular, fuzzy reasoning models supported by the use of genetic algorithms as a synergic element used to further enhance the quality of the solution.

3 THE USE OF FUZZY LOGIC AND GENETIC ALGORITHMS TO SOLVE THE PROBLEM OF INVENTORY CONTROL IN CONDITIONS OF DEMAND AND LEAD TIME UNCERTAINTY

As mentioned previously, the impact of many external determinants on the procurement logistics subsystem leads to the situation where taking the right decision in this respect requires methods and tools with the ability to specify events characterised by uncertainty, information inaccuracy and adaptation to the changing system parameters. Hence, the theory of fuzzy sets and fuzzy reasoning systems is suitable for the wide range of application in the field of inventory control and management in logistics. An additional element that supports and complements the functioning of fuzzy system in the proposed control method will be the use of genetic algorithm. Its aim will be to optimise the knowledge base contained in the fuzzy rules by optimal selection of weights for these rules.

3.1. Fuzzy Logic

Fuzzy logic is an example of a multi-valued logic. Closely related to the theory of fuzzy sets, it was introduced by L. Zadeh. In contrast to the classical logic, the fuzzy logic theory assumes that there may be an infinite number of intermediate values between the false state and the true state. This means that an element of a given set may belong to this set only to a certain degree. This reasoning leads to the formulation of the definition of fuzzy set. According to it, the fuzzy collection A in a non-empty space X is a set of pairs:

$$A = \{(x, \mu_A(x)) : x \epsilon X\} \qquad (1)$$

    where:

$$\mu_A : X \to [0.1]$$

is known as the membership function of fuzzy set A. This representation assigns a degree of fuzzy set membership of each element $x \epsilon X$ to the fuzzy set A. One can distinguish the following cases:

    $\mu_A(x) = 1$ in the case of full membership of an element x to the fuzzy set A ($x \epsilon A$).
    $\mu_A(x) = 0$ in the case of no membership of the element x to the fuzzy set A ($x \notin A$).
    $0 < \mu_A(x) < 1$ in the case of a partial membership of an element x to the fuzzy set A

Similarly to the classical approach, fuzzy sets make it possible to perform a series of operations in the form of a sum, product, etc.

Another important concept necessary to describe fuzzy systems is the linguistic variable, i.e. the input or output quantity in the fuzzy system that is estimated using linguistic values (*high demand, long lead time, etc.*).

3.2. Genetic Algorithms

Genetic algorithms are algorithms designed to search for optimal solutions to artificial human-created optimisation problems. Their functioning is based on the mechanisms of natural selection and the process of heredity. They combine the evolutionary principle of survival of the fittest individuals (solutions of decision problem). When considering a set of solutions to the decision problem, one can compare it to the population of organisms. Each solution (individual) can be attributed its own characteristics of adaptation to certain set conditions (criterion function that measures the quality of a certain solution). This allows you to simulate the evolutionary processes by duplicating better solutions in the future "generations" and eliminating those that are not as good at satisfying the optimisation problem criteria. This operation is carried out on the basis of the reproduction of genetic code and more specifically on the possibility of collating partial ideas (similarly to crossover) that come from a variety of solutions, which results in better innovative solutions to the problem. In practice, this requires adoption of a method of transforming a specific solution into a uniquely representative code string, often called a chromosome. In this way, using the so-defined code strings (solution population), one can perform processing, e.g. the crossover and mutation operations, and receive new solutions (Goldberg, 1989).

3.3. Proposed method for solving the problem

The approach proposed in the article involves describing the uncertainty of input and output system parameters through fuzzy sets. Then, on the basis of the knowledge base contained in the rules, the optimal sharpened control parameters are determined. The rule base consists of a set of conditional instructions. The generalised inference rule *modus tollens* can be specified in the following way:

Antecedent	$y \ is \ B'$
Implication	$IF \ x \ is \ A \ THEN \ y \ is \ B$
Conclusion	$x \ is \ A'$

Where: $A, A' \subseteq X \ and \ B, B' \subseteq Y$ are fuzzy sets, and x and y are linguistic variables.

The input data supplied to the fuzzification block are fuzzified, i.e. the degree of their membership to particular fuzzy sets is determined. Then, each rule is run in the inference block and also activation degrees are calculated for the antecedents contained within them. Each rule is assigned a certain weight *w*. In this way, rules with higher weights have a greater impact on the determination of the output variable value. In order to guarantee the required control efficiency, a genetic algorithm was used to optimise the weight values for a fuzzy

system rule set. Therefore, the decision variable is the vector of the rule weights. The optimisation process is performed on the basis of the minimisation of the function being a weighted sum of standardized three sub-criteria: the average inventory level, the number of stock-outs and the number of deliveries for a fixed period of time, on the basis of training data sets. The above-mentioned problem can be represented as follows:

$$F(W) = \varphi_1 \frac{f_1^{**} - f_1(W)}{f_1^* - f_1^{**}} + \varphi_2 \frac{f_2^{**} - f_2(W)}{f_2^* - f_2^{**}} + \varphi_3 \frac{f_3^{**} - f_2(W)}{f_3^* - f_3^{**}} \quad \rightarrow \quad min \qquad (3)$$

$$W \geq 0 \qquad (4)$$

$$\sum_{i=1}^{3} \varphi_3 = 1 \qquad (5)$$

Where:

$F(W)$ – cumulative criterion function

W – vector of weights for a rule set

f_1^* , f_1^{**} – maximum and minimum, respectively, for the function determining the average inventory level

f_2^*, f_2^{**} – maximum and minimum, respectively, for the function determining the number of stock-outs

f_3^{**}, f_3^* – maximum and minimum, respectively, for the function determining the number of required deliveries

$\varphi_1, \varphi_2, \varphi_3$ - weights for partial criteria

The structure of a single chromosome and the applied two-point crossover operator are presented on Fig. 1 and Fig. 2, respectively.
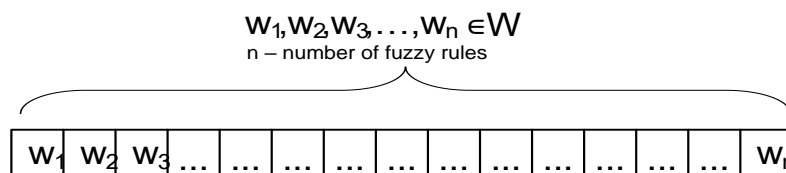

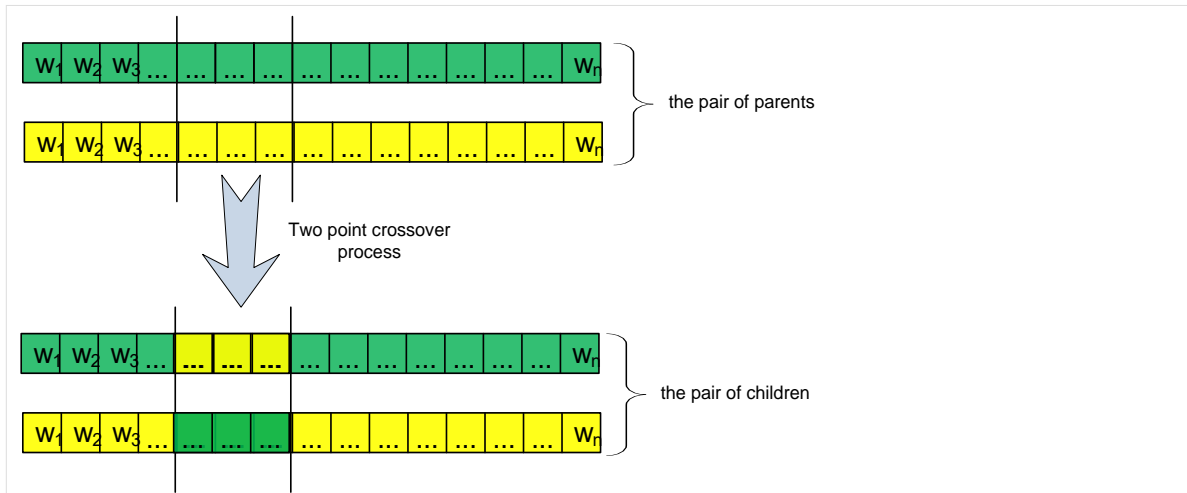**Fig. 1. Sample chromosome structure**

Fig. 2. Sample two-point crossover

The functioning of the reasoning block optimized by the genetic algorithm results in the output system value, which is a sharp value as a consequence of the defuzzification procedure. Fig. 3 shows the general scheme of the entire system of the presented approach.
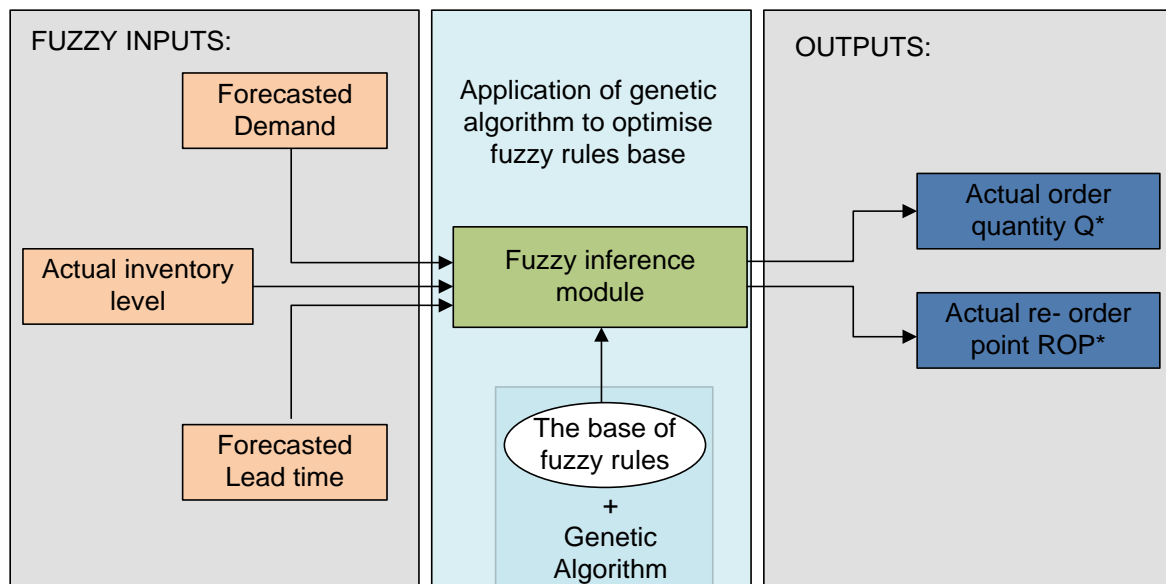

Fig. 3. Scheme of the proposed system

The input to the system comprises three variables, which are the most important in shaping the current inventory level management policy. These include the variables describing the forecast demand, the actual inventory level on a given day and the random lead time. The control parameters in the proposed system are the actual re-order point and the actual order quantity. The first specifies the emergency inventory level at which an order must be placed and the latter designates the item batch size that is appropriate for a certain moment. All variables, both input and output, are defined as linguistic variables determined on a set of linguistic values. For example, one of the input variables can be as follows: $Forecast\ demand = \{small, medium, large\}$. Each estimation of the linguistic variable is assigned an appropriate fuzzy set. Fig. 4 shows the methods of describing the uncertainty of input parameters in the control system.
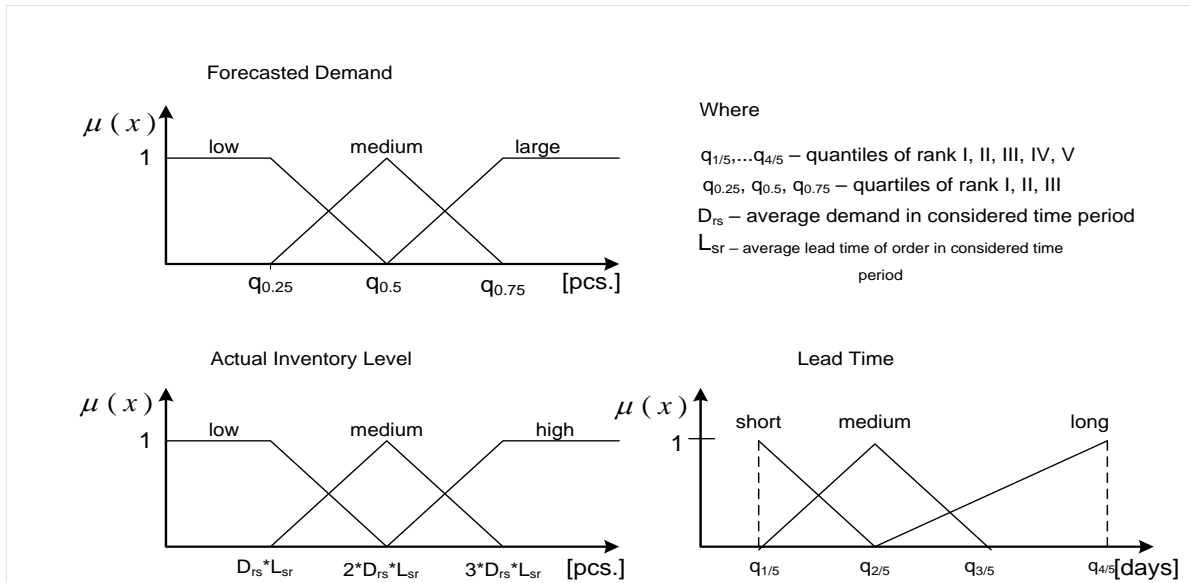
Forecasted Demand

Where

$q_{1/5},...q_{4/5}$ – quantiles of rank I, II, III, IV, V

$q_{0.25}, q_{0.5}, q_{0.75}$ – quartiles of rank I, II, III

D_{rs} – average demand in considered time period

L_{sr} – average lead time of order in considered time period

Actual Inventory Level

Lead Time

Fig. 4. Input system parameters described by means of fuzzy sets

The presented fuzzy sets are described by the proposed triangular and trapezoidal membership functions. The characteristic points on the horizontal axes of the diagrams are determined based on historical observations of a certain variable in a fixed time horizon. Output system parameters were defined in a similar way (Fig. 5).
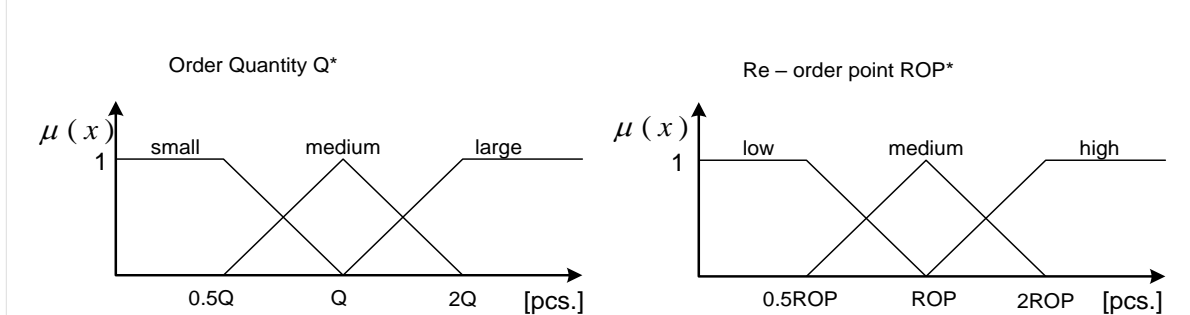
Order Quantity Q*

Re – order point ROP*

Fig. 5. Output system parameters described by means of fuzzy sets

Characteristic points on the horizontal axis are identified on the basis of the following formulas:

$$Q = \sqrt{\frac{2PK_z}{K_u}} \qquad (6)$$

where:

P – estimated demand for the product within a specified time horizon (e.g. a year)

K_z – stocking unit costs

K_u – stock-carrying costs

$$ROP = D_{sr} * L_{sr} + k * \sigma_d * \sqrt{L_{sr}} \qquad (7)$$

where:

D_{sr} – average demand for the article on a given day

L_{sr} – average lead time

σ_d – standard demand deviation

k – adopted safety factor specifying the level of customer service

The approach proposed in this paper is an example of continuous inventory monitoring and control system. The input and output variables are updated for the adopted time interval (e.g. one day). Hence, the identified characteristic points in the fuzzy space for the output variables are only the initial values in the simulation of the entire analysed planning period. Within each successive one-day time interval, the parameters of the membership function describing the outputs from the system are modified based on the identified prediction error (Fig. 6.) Thanks to this, the system has a greater ability to adapt and intelligently identify any unusual situations.
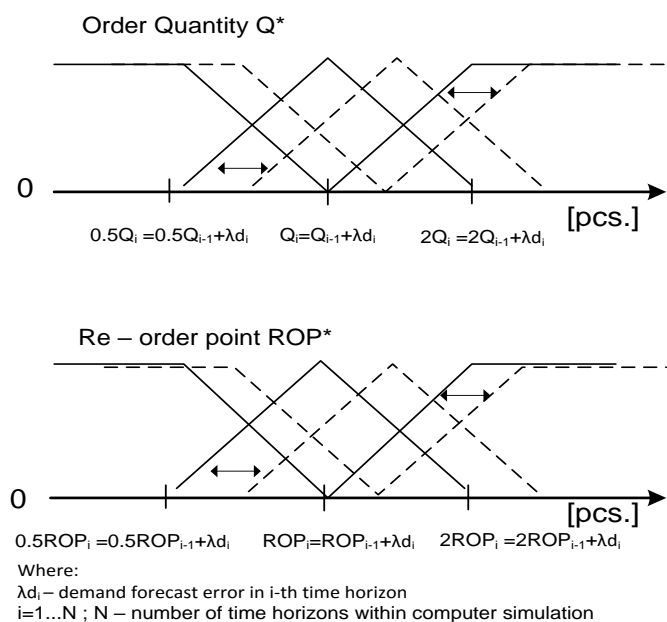

Order Quantity Q*

$0.5Q_i = 0.5Q_{i-1} + \lambda d_i$ $Q_i = Q_{i-1} + \lambda d_i$ $2Q_i = 2Q_{i-1} + \lambda d_i$ [pcs.]

Re – order point ROP*

$0.5ROP_i = 0.5ROP_{i-1} + \lambda d_i$ $ROP_i = ROP_{i-1} + \lambda d_i$ $2ROP_i = 2ROP_{i-1} + \lambda d_i$ [pcs.]

Where:
λd_i – demand forecast error in i-th time horizon
i=1...N ; N – number of time horizons within computer simulation

Fig. 6. The method of updating the membership function parameters for output system variables

4. SAMPLE CALCULATION AND RESULTS

In order to verify the effectiveness of the proposed solution, a computer simulation has been performed on the inventory level of certain product, based on historical demand data from the enterprise within a period of six months. The simulation results were compared with the results obtained from the classical ordering level method and the combined re-order point and fixed reorder cycle model. These methods are broadly presented in (Axsater, 2006). For the set of all possible combination of rules in the reasoning module, optimisation process was performed for weights as shown in the previous section. As a consequence, a set of 27 most useful rules was received together with their assigned optimal weights. The efficiency of the optimisation procedure is shown in Fig. 7.
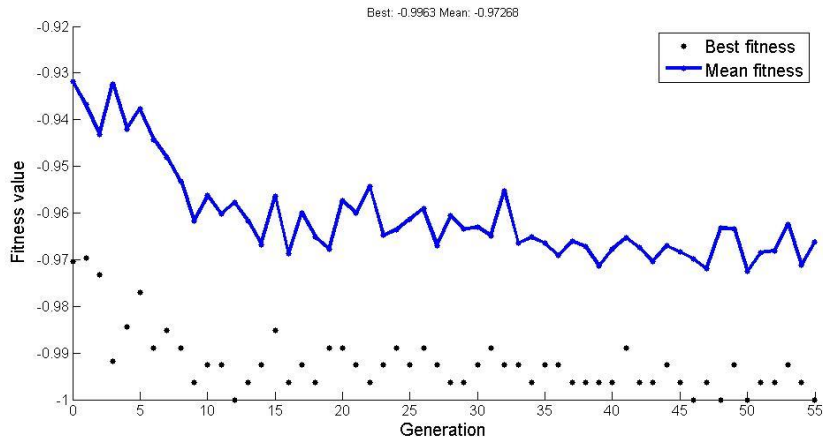
Fig. 7. Efficiency of the genetic algorithm in the procedure of optimisation of rule set weights

The final comparison of the results of the inventory level simulation was made on the basis of the adopted criteria in the form of the total inventory costs for the considered period, the average inventory level, the number of deliveries and the number of the encountered stock-outs. The simulations were carried out for 25 data sets. The final results were averaged. Fig. 8 shows the simulation scheme as per the proposed approach.
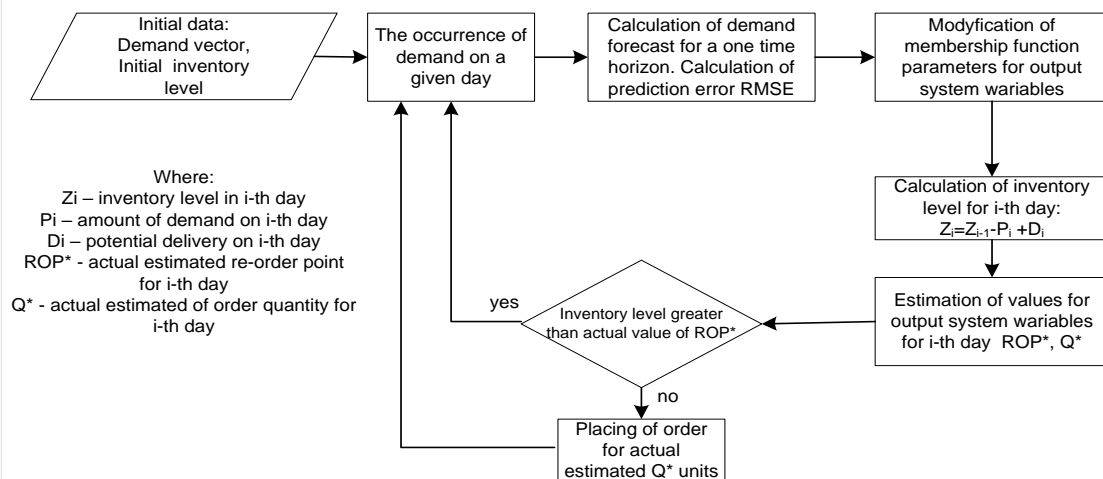

Fig. 8. Simulation scheme as per the proposed fuzzy method

The sequence of all the steps presented in the simulation scheme is performed each time for each day of the entire six-month period. Fig. 9 contains the presentation of the results of the proposed approach in relation to two methods according to selected criteria.
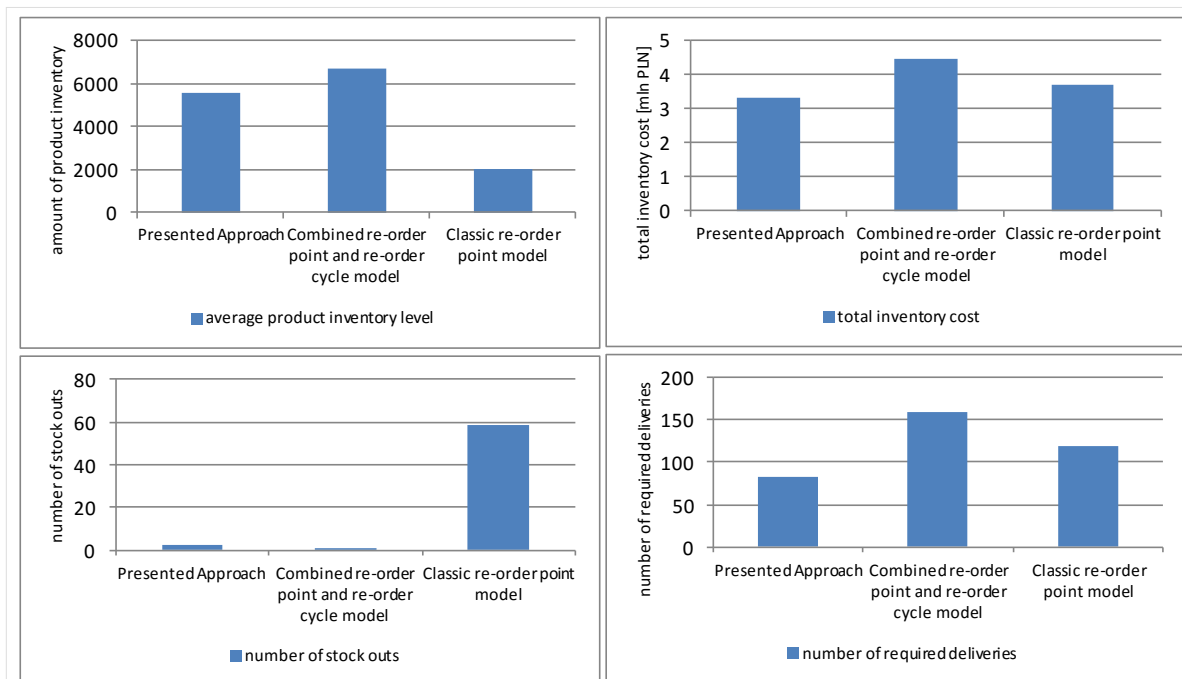
Fig. 9. Simulation results

5 CONCLUSION

The approach presented in this paper and the performed simulations illustrate that the classical method of determining the inventory level are inefficient and ineffective where random factors in the form of a large uncertainty in demand, lead time, etc. impact the inventory control system. On the basis of the received results, the proposed approach brought results that were better by a dozen or so percent compared to the other two classical methods according to the adopted estimation criteria. For the stock-out number criterion, the proposed fuzzy solution turned out to be a little worse than one of the methods being compared. This is dictated by the large number of deliveries for the combined re-order point and re-order cycle method, which is associated with far higher stocking and stock-carrying costs. Further work will be focused on attempts to include an additional uncertainty factor in the form of a limited supply of goods from suppliers and further tests of the method using a greater number of data sets.

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