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UNIT CUBE CAPACITANCE CALCULATION BY MEANS OF FINITE ELEMENT ANALYSIS.

César-S. Cañas-Peñuelas[1], S. Catalan-Izquierdo[1], Jose-M. Bueno-Barrachina[1], Francisco Cavallé-Sesé[1]

[1] Institute of Electrical Technology
24, Av. Juan de la Cierva, Parque Tecnológico de Valencia
46980, Paterna, Valencia (Spain)
Phone number:+0034 961 366670; Fax number:+0034 961 366680
e-mail: cesar.canas@ite.es; saturnino.catalan@ite.es; jmanuel.bueno@ite.es; fcavalle@ite.upv.es

Abstract.

The study in electric field distribution in the space by different charged geometries, allows to improve the designs of the devices used in electrical engineering. In this work the capacities of a charged unit cube by two different methods have been calculated by means of technical of finite element analysis. Firstly infinite elements in the air that surrounds the charged unit cube were considered, and secondly the conditions of contour of the Trefftz method were used. The proposed models has been achieved to calculate the capacitance of a unit cube based on the size of the element.

Key words

Capacitance, Edge-effect, Unit cube, Surface charge density, Trefftz Method, Transfinite Elements Method.

1. Introduction

Since the decade of the fifties [1;2], the estimation the electrostatic capacitance is widely studied. The interest in this type of problems has been growing throughout the years, due to the development of the aerospace industry.

Nowadays different iterative algorithms are used to calculate the electrostatic capacitance of any model, thanks to the performance that offer the computers. These algorithms are designed to obtain the superficial charge distributions, the charge density and the electrostatic capacitance. The disadvantage of these algorithms is that they are designed for a few specific cases, so that if the model changes, the algorithm must be reconstructed almost in his totality.

In contrast to the disadvantage of the custom iterative algorithms, there are commercial softwares based on the finite elements method (FEM), which integrate generic algorithms applicable to most popular problems of electrostatics, independently of the geometry. This increases the flexibility, the costs and the times of development are minimized.

In this work we have used the commercial software ANSYS [3], since this commercial software has a specific module to solve problems of electrostatics.

In this paper a method based on FEM is proposed to study the case of a unit cube, since the electrostatic capacitance of this regular polyhedron has been solved witch accuracy.

In order to calculate the electrostatic capacitance of a unit cube, a method based on transfinite elements boundary condition [4-7], and another based on the Trefftz method [8-14], have been used. In both methods, a three-dimensional (3D) model has been generated that allows to modify the geometric variables and the size of the elements, with the target to obtain a family of curves that permits to validate this model and to optimize future models.

2. Transfinite Elements Method

This method is based in three volumes generation. Fig 1 shows a section of the used model, where A, B and C represent the three generated volumes. The first of them, is the volume to calculate its electrostatic capacitance, in this case a unit cube. In the volume mesh there has not been changed the spacing ratio (see Fig 2). The second volume generated surrounds the first one, in this case an spherical volume with radius *a* which is centered in the coordinates origin of the first one is used. The material of this volume is the vacuum with relative dielectric permittivity equal a 1. The third volume which has spherical form with radius *b* surround the second one and receives the infinity boundary condition.

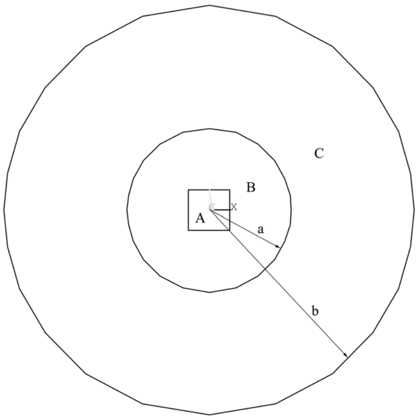

Fig 1. Section of 3D model utilized in Transfinite Elements Method.

The electrostatic capacitance calculated by this method depends on the element size, the mesh divisions of the first volume [6;7;15;16], the distance of the third volume [4] with respect to the coordinates origin and the mesh size of third volume [5].

Table I shows the variables of simulation and the obtained results, where SE_A, SE_B and SE_C are the maximum element sizes that compose the volumes A, B and C respectively. These results are comparable with the theoretical results of the unit cube capacitance in the vacuum ($0.66067 \cdot 4 \cdot \pi \cdot \varepsilon_0$), previously calculated by other authors [7;17-20].

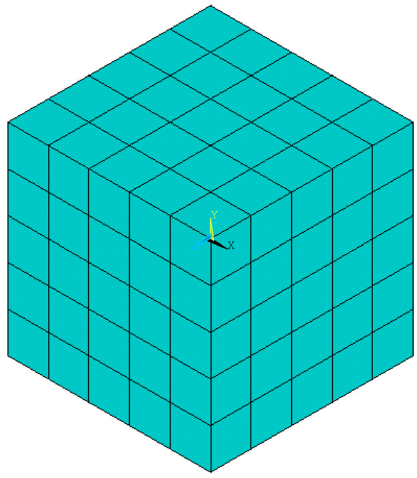

Fig 2. Mesh of unit cube witch uniform spacing ratio.

3. Transfinite Elements Method

In this method, three volumes are also created, but with some differences with the previous method.

The first volume, as in the previous case, is designed to calculate its electrostatic capacitance, in this case a unit cube centred on the coordinates origin. In the mesh of this volume the spacing ratio has been changed, to improve the unit cube edge [6] (see Fig 3).

Table I: Modelling results which Transfinite Elements.

a (m)	*b (m)*	*SE_A (m)*	*SE_B (m)*	*SE_C (m)*	*Capacitance (F)*	*Capacitance ($4 \cdot \pi \cdot \varepsilon_0$)*
3	4.00	0.167	2.09	1.00	7.5900E-11	0.6821552
		0.111	1.40		7.5600E-11	0.6794589
		0.0833	1.05		7.5500E-11	0.6785602
		0.0667	0.838		7.5400E-11	0.6776614
		0.0556	0.698		7.5400E-11	0.6776614
		0.0476	0.598		7.5400E-11	0.6776614
		0.0417	0.524		7.5400E-11	0.6776614
		0.0370	0.465		7.5300E-11	0.6767626
		0.0333	0.419		7.5300E-11	0.6767626
		0.0303	0.381		7.5300E-11	0.6767626
5	5.50	0.20	1.20	0.50	7.7937E-11	0.7004628
		0.10	1.20		7.7200E-11	0.6938390
		0.05	1.20		7.7020E-11	0.6922212
		0.05	0.40		7.6900E-11	0.6911427
		0.05	0.20		7.6897E-11	0.6911158
6	6.50	0.201	0.80	0.50	7.7980E-11	0.7008493
7	7.50	0.20	0.80	0.50	7.7820E-11	0.6994113
8	8.50	0.20	0.80	0.50	7.7579E-11	0.6972453
9	9.50	0.20	0.80	0.50	7.7571E-11	0.6971734
10	10.5	0.20	0.80	0.50	7.7445E-11	0.6960409
14	14.5	0.20	0.80	0.50	7.6976E-11	0.6918258
	1.50			1.00	7.5813E-11	0.6813733
	16.0			2.00	7.4750E-11	0.6718195
	17.0			3.00	7.4590E-11	0.6703815
	18.0			4.00	7.4570E-11	0.6702017
	24.0			10.00	7.4657E-11	0.6709837
	64.0			50.00	7.8000E-11	0.7010290
	114.0			100.00	8.0600E-11	0.7243967

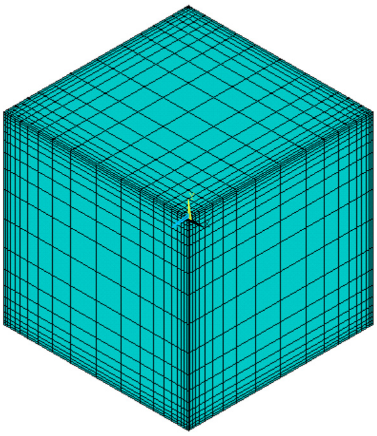

Fig 3. Mesh of unit cube witch non-uniform spacing ratio.

The second volume is an cubic volume with side L_2 which surrounds the first one and is also centred on the origin of coordinates. At the exterior surface of this volume infinite boundary condition is imposed. This boundary condition forces the perpendicularity of the electric field lines to it. It should be noted the importance of the value of L_2, since if the distance between the first volume and the external surface of the second one is small, the deformation of the electric field lines increases because of the restricted infinite boundary condition.

The third volume allows to generate a Trefftz domain [10;12], where the Trefftz equations are applied, with the Trefftz method boundary conditions [11;14]. This volume, is situated between the external surface of the first volume and the second one. In this case it is used a spherical volume of radius *a* and centred on the coordinates origin (see Fig 4).

Table II shows the obtained results and the simulation variables, where SR_L_1 is the mesh spacing ratio of the lines that compose the volume A, and *SE_A*. *SE_B* and *SE_C* are the maximum element size of the elements that compose the volumes A, B and C respectively. It can be seen in this table that the values of the capacitance obtained with the method Trefftz are similar with those of the theoretical unit cube capacitance in the vacuum ($0.66067 \cdot 4 \cdot \pi \cdot \varepsilon_0$).

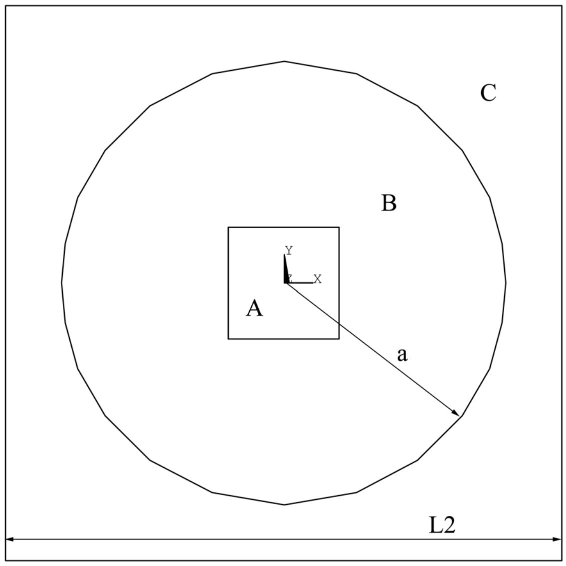

Fig 4. Section of 3D model utilized in Trefftz Method.

4. Model validation

In this paper the two proposed models were validated comparing the simulation results with the theoretical ones. Comparisons between the superficial charge density obtained in the diagonal and in the edge of the models, were made with the results obtained by other authors in order to further validate the proposed models [21;22]

The unit cube edge charge density is shown in Fig 5. The edge distribution is rescaled with the values at centre of the edge of the cube, *Y* is the distance from the centre of the edge of the cube.

Table II: Modelling results witch Trefftz Elements.

SR_L_1 (%)	L_2 (m)	*a* (m)	*SE_A* (m)	*SEC_B* (m)	*SE_C* (m)	*Capacitance* (F)	*Capacitance* ($4 \cdot \pi \cdot \varepsilon_0$)
0.00	5.00	2.00	0.1	0.50	3.14	7.3756E-11	0.6628859
5.00	5.00	2.00	0.05	0.50	3.14	7.3536E-11	0.6609086
10.00	5.00	2.00	0.05	0.50	3.14	7.3530E-11	0.6608547
			0.1	0.0455		7.3617E-11	0.6616366
			0.05	1.00		7.3577E-11	0.6612771
			0.05	2.50		7.3775E-11	0.6630566
11.00	5.00	2.00	0.05	0.50	3.14	7.3529E-11	0.6608457
12.00	5.00	2.00	0.05	0.50	3.14	7.3528E-11	0.6608367
15.00	2.00	0.75	0.05	0.133	1.18	7.3537E-11	0.6609176
				0.133	0.589	7.3562E-11	0.6611423
	5.00	2.00		0.625	3.14	7.3536E-11	0.6609086
				0.333	3.14	7.3525E-11	0.6608097

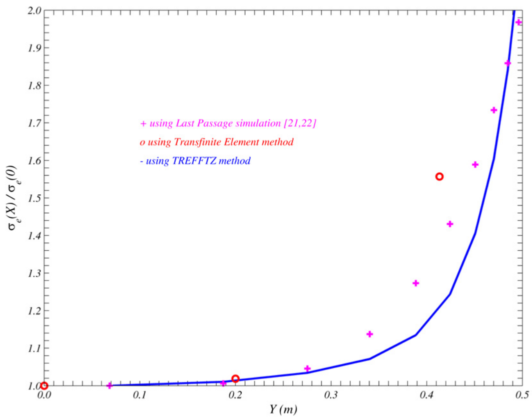

Fig 5. The edge distribution of a unit cube.

The validation of the edge distribution, has been realised using the equation (1), described in [21;22]. This equation is used in the last-passage method and is valid as boundary of the containing singularities. For any edge of a conducting surface, the charge distribution $\sigma(x,\delta)$ on a curve parallel to the edge. but separated from it by distance δ . with δ small.

$$\sigma(x,\delta) = \delta^{\pi/\alpha-1} \cdot \sigma_e(x) \quad (1)$$

The unit cube superficial charge density distribution in the diagonal of one of the faces, obtained with two presented models is show in Fig 6. The diagonal is rescaled witch the values at geometric centre of the cube face, r is the distance from the geometric centre of the face at corner of the cube.

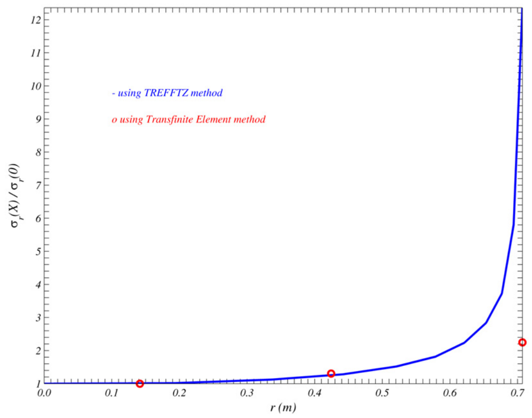

Fig 6. The diagonal distribution of a unit cube face.

5. Conclusions

In this paper it has been demonstrated the validity of FEM to calculate the unit cube capacitance. The superficial charge density distributions have also been calculated in the edges and in the diagonal of one of the faces of the unit cube.

The model also includes the volume that surrounds the conductor and take into account the corner and the edge effect of this.

The proposed models based on FEM are not only valid to calculate the capacitance of the unit cube and to obtain the surface charge distributions, as well as the electric field distribution in the volume that surrounds the conductor, but is also valid for any geometric figure thanks to the flexibility of the FEM, which could be a helpful tool for calculation of complex systems which are the most popular problems in the aerospace industry.

References

[1] C.J.Bouwkamp. A simple method of calculating electrostatic capacity, Physica 24 (1958) 538-542.
[2] D.K.Reitan, T.J.Higgins. Calculation of the Electrical Capacitance of a Cube, J. Appl. Phys. 22 (1951) 223-226.
[3] Release 9.0 Documentation for ANSYS, © 2005 SAS IP, Inc.
[4] D.Halverson, G.Friedman, I.Tsukerman. Local approximation matching for open boundary problems, IEEE Trans. on Mag. 40 (2004) 2152-2154.
[5] J.F.Lee, Z.J.Cendes. Transfinite elements: A highly efficient procedure for modeling open field problems, J. Appl. Phys. 61 (1987) 3913-3915.
[6] F.H.Read. Improved Extrapolation Technique in the Boundary Element Method to Find the Capacitances of the Unit Square and Cube, J. Comput. Phys. 133 (1997) 1-5.
[7] F.H.Read. Capacitances and singularities of the unit triangle, square, tetrahedron and cube, Compel. 23 (2004) 572-578.
[8] M.Gyimesi, I.Tsukerman, D.Lavers, T.Pawlak, D.Ostergaard. Hybrid finite element-Trefftz method for open boundary analysis, IEEE Trans. on Mag. 32 (1996) 671-674.
[9] M.Gyimesi, J.S.Wang, D.Ostergaard. Hybrid p-element and Trefftz method for capacitance computation, IEEE Trans. on Mag. 37 (2001) 3680-3683.
[10] I.Herrera. Trefftz Method: A General Theory, Numer. Meth. Part. D. E. 16 (2000) 561-580.
[11] Ch.Hochard. A Trefftz approach to computational mechanics, Int. J. Numer. Meth. Eng. 56 (2003) 2367-2386.
[12] Zi-Cai Li, Tzon-Tzer Lu, Hung-Tsai Huang, Alexander H.-D.Cheng. Trefftz, collocation, and other boundary methods - A comparison, Numer. Meth. Part. D. E. 23 (1996) 93-144.
[13] A.P.Zielinski, O.C.Zienkiewicz. Generalized finite element analysis with T-complete boundary solution functions, Int. J. Numer. Meth. Eng. 21 (1985) 509-528.
[14] A.P.Zielinski, I.Herrera. Trefftz method: Fitting boundary conditions, Int. J. Numer. Meth. Eng. 24 (2005) 871-891.
[15] E.W.Bai, K.E.Lonngren. On the capacitance of a cube, Comput. Electr. Eng. 28 (2002) 317-321.
[16] S.Ghosh, A.Chakrabarty. Estimation of capacitance of different conducting bodies by the method of rectangular subareas, J. Electrostat. 66 (2008) 142-146.
[17] C.S.Brown. Capacity of the regular polyhedra, Comput. Math. Appl. 20 (1990) 43-56.
[18] J.A.Given, J.B.Hubbard, J.F.Douglas. A first-passage algorithm for the hydrodynamic friction and diffusion-limited reaction rate of macromolecules, J. Chem. Phys. 106 (1997) 3761-3771.
[19] E.Goto, Y.Shi, N.Yoshida. Extrapolated surface charge method for capacity calculation of polygons and polyhedra, J. Comput. Phys. 100 (1992) 105-115.
[20] C.O.Hwang, M.Mascagni. Electrical capacitance of the unit cube, J. Appl. Phys. 95 (2004) 3798-3802.
[21] J.A.Given, C.O.Hwang. Edge distribution method for solving elliptic boundary value problems with boundary singularities, Phys. Rev. E. 68 (2003) 046128-6.
[22] C.O.Hwang, T.Won. Edge charge singularity of conductors, J. Korean. Phys. Soc. 45 (2004) S551-S553.