Measuring track vertical stiffness through dynamic monitoring

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This paper proposes a methodology for the evaluation of the track condition by means of the measurement of the track stiffness. This magnitude is calculated from vertical acceleration data measured at the axle box of trains during their normal operation. From the corresponding vertical acceleration spectra, the dominant vibration frequencies for each track stretch are identified and the combined stiffness is then determined. The stiffness without the contribution of the rail is then calculated. The results obtained for a high-speed ballasted track in several track stretches are within the range $120–130\,\text{kN/mm}$, a result consistent with direct stiffness measurements taken during previous studies. Therefore, the proposed methodology may be used to obtain a first insight to the track condition by means of a continuous measurement of the track combined stiffness. This offers an alternative to traditional stationary stiffness measuring devices and might be a useful complement to dedicated continuous monitoring vehicles.

1. Introduction

In order to justify their economic cost and ensure their profitability, railway services must be fast, comfortable and safe. One of the key aspects to achieve these requirements is the degree of maintenance of the track and vehicles. Poor or insufficient maintenance will reduce the performance of the service and may even compromise its safety, whereas an excessive control of the track may increase costs beyond a reasonable level and reduce the track capacity. This is particularly important in those track stretches more prone to degradation, such as transition zones between ballasted tracks and slab tracks (Insa et al., 2014), and other sections with sudden changes of stiffness which increase dynamic loads (López, 2001). This context points out the need for a thorough and cost-effective monitoring of the track condition so as to plan maintenance operations. With this in mind, several different methods and devices have been developed and tested for track monitoring, ranging from simple visual inspection to sophisticated inspection cars equipped with sensors.

Stationary methods usually rely on devices which are placed at certain points of the track where measurements are taken to...
assess the condition of such points. Examples of this are the falling weight deflectometer (FWD) extensively used in the UK (Burrow et al., 2007) and impact hammers combined with accelerometers (Kaewunruen and Remennikov, 2007). The main issue with these methods is that they are stationary and usually require the track to be occupied, hence affecting the regular railway service.

As for inspection cars, it is worth mentioning the Mauzin cars used by the French national railway company (Société nationale des chemins de fer français, SNCF) since the 1960s, which allow measuring several track geometric parameters at up to 200 km/h (Alias, 1984). The data thus gathered are processed with a specific computer application in order to assess the evolution of the track and design the required maintenance operations (Meier-Hirmer, 2007). A similar device called Séneca is used by the Spanish railway operator (Renfe-Adif) to inspect the condition of the track, overhead line and signalling systems of high-speed lines at up to 350 km/h (Via Libre, 2008).

The Swedish rail administration (Banverket) uses a device called a rolling stiffness measuring vehicle (RSMV) which, as its names suggests, measures the track stiffness as a way of identifying those sections where maintenance is required (Berggren, 2009; With and Bodare, 2009). Other more complex methods have been implemented to evaluate the condition of different track components separately, as well as the ground below, whether continuously along the track or at located points (Breul et al., 2008). Among them are the ground-penetrating radar (GPR), which is fast and non-destructive (Fateh, 2005), and the light dynamic penetrometer, which allows soil resistance to be measured (Zhou, 1997).

These vehicles allow continuous monitoring of the track, without interrupting the normal service. They are, however, rather expensive and complex devices which are not always affordable or might be insufficient to inspect a large network. For this reason, a complementary approach is to equip regular trains with sensors that provide a first insight of the track condition while operating, hence using dedicated inspection cars only when a problem has been already hinted at by such measurements.

The most promising parameter to control is the combined track stiffness, as it is strongly related to the track behaviour in relation to dynamic loads and notably affects the process of wear and degradation. This relation has been extensively studied, both experimentally (Cámara et al., 2012; Cuelar et al., 2012) and through computer models and theoretical studies (Alves et al., 2010; Chebli et al., 2008; Rhayma et al., 2011).

Within this context, the present paper aims to calculate the combined track stiffness (both with and without the contribution of the rail stiffness) by means of acceleration data gathered from accelerometers placed on a train axle box. In this way, a first insight to the track condition may be achieved, hence providing a reliable tool for preliminary identification of track defects.

2. Materials and methods

In this section the method for data gathering and processing is explained, and the mathematical tools used to calculate the combined track stiffness are thoroughly described.

2.1 Monitoring methodology

As already explained, the aim in this paper is to obtain the combined track stiffness by means of measuring and processing acceleration data measured in the train axle box. Therefore, monitoring such accelerations is one of the key aspects of this study. In order to do so, uniaxial piezoelectric accelerometers were placed in the axe box of a train (Figure 1) operating along one of the Spanish high-speed lines.

![Figure 1. Sensors placed in the train axle box](a)
Four sensors were used to measure both vertical and lateral accelerations in both sides of the axle box. The main characteristics of these sensors are shown in Table 1.

Measurements were taken during two different days with a sampling frequency of 12,000 Hz, to assess the influence of such parameters and avoid losing any relevant information. All data gathered were stored in a laptop in real time and conveniently processed afterwards. Vertical acceleration spectra were obtained by means of the fast Fourier transform (FFT) for each track stretch of either 50 or 100 m. Particular attention was paid to singular track features such as tunnels, bridges, crossings, transitions and so on, which are most prone to degradation.

2.2 Axle natural frequency

In order to evaluate the data gathered, the track–vehicle interaction was modelled by means of a simple one mass (quarter car) model (Melis, 2008) as shown in Figure 2. In this model, the mass \( m \) represents the unsprung mass of the train; that is, the axle and wheel. This mass moves along the track, and the parameters \( k \) and \( c \) represent the stiffness and damping of the track–vehicle interaction, respectively. This simplified model was chosen because the sprung mass (i.e. the train carriage), although being much bigger than the unsprung mass, presents a much smaller vibration frequency owing to the low stiffness and high damping of the train dampers. Therefore, the effect of the sprung mass on the dynamic load the train applies to the track is negligible, and the natural frequency of the system can be assimilated to that of a single oscillating mass.

The mass vertical oscillation owing to its interaction with the track is given by the following differential equation

\[
m \frac{d^2 y}{dt^2} + c \frac{dy}{dt} + ky = 0
\]

where \( y \) is the mass vertical displacement and \( t \) is time. If damping is neglected, which is a common assumption \((c=0)\), the analytical solution for Equation 1 is

\[
y = A \sin \left( \sqrt{\frac{k}{m}} t \right) + B \cos \left( \sqrt{\frac{k}{m}} t \right)
\]

where \( A \) and \( B \) are constants which depend on the system initial condition. Assuming such a condition to be at rest with initial position \( y_0 \), Equation 2 becomes

\[
y = y_0 \cos \left( \sqrt{\frac{k}{m}} t \right)
\]

This corresponds to a sinusoidal harmonic movement whose natural frequency is

\[
f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}
\]

The characteristics of the train monitored (including the mass and stiffness of the unsprung mass) were provided by the train manufacturer. Therefore, assuming a free vibration state, it is possible to obtain from the model ruled by Equation 3 the results shown in Figure 3. From the vibration spectrum (Figure 3(b)) the natural frequency of the train axle was found to be about 58.5 Hz. Therefore, when analysing the acceleration spectrum of the data recorded, the natural frequency of the train–track system would appear to be in a bandwidth around 58.5 Hz. For the purpose of this study, the range selected for the Fourier analysis was between 55 and 65 Hz.

Bearing this in mind, for each track stretch the corresponding vertical acceleration spectrum was studied to identify the most dominant frequency within the aforementioned bandwidth. Once found, this frequency value was used to calculate the track combined stiffness \( k \) by means of Equation 4.

It is worth noting that, despite being a rather simple model, the quarter car model yields a vertical movement for the vibrating mass in good agreement with that provided by more complex models such as the two, three or four masses, as well

<table>
<thead>
<tr>
<th>Sensor</th>
<th>Sensitivity: mV/g</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical left acceleration</td>
<td>50</td>
</tr>
<tr>
<td>Lateral left acceleration</td>
<td>102.2</td>
</tr>
<tr>
<td>Lateral right acceleration</td>
<td>103.2</td>
</tr>
<tr>
<td>Vertical right acceleration</td>
<td>50</td>
</tr>
</tbody>
</table>

Table 1. Sensors used and their sensitivity

Figure 2. Quarter car model
as far more sophisticated methods of analysis such as Simpack, Adams or UM software (Melis, 2008).

It is also important to note that the mass \( m \) in Equation 4 is not only the unsprung mass of the train (i.e. wheel and axle). A certain contribution from the track itself should be taken into account, namely the rail and sleeper. This is of particular importance as the value of the vibratory mass has a direct influence on the calculated track stiffness, as shown in Figure 4.

The results obtained from this study suggest that, for a ballasted track, the mass of the rail between two consecutive sleepers and about 30% of the sleeper mass should be added to the global ‘vibrating mass’ \( m \) used in Equation 4. This is further validated when comparing the combined track stiffness calculated by this methodology with that shown by Cuellar et al. (2012) and Tijera et al. (2012).

### 2.3 Track stiffness without rail contribution

When studying the track vertical stiffness it is important to evaluate whether such magnitude comprises the contribution of the rail bending stiffness. For example, consider a wheel load of \( Q = 90 \) kN and a vertical displacement at the rail head of 1 mm. Then the combined stiffness according to Equation 5 is 90 kN/mm. However, the load transmitted by the rail foot to the sleeper is usually only about 30% of the wheel load for ballasted tracks; hence the track vertical stiffness under the rail is only about 27 kN/mm.

Bearing this in mind, once the combined track stiffness \( k \) is known as explained before, it is possible to calculate the track vertical deformation \( z \) (at the wheel–rail contact) as

\[
z = \frac{Q}{k}
\]

This relation is only valid under the assumption of small deformations. \( Q \) represents the dynamic load, which was calculated by multiplying the static load (i.e. the train weight per wheel, provided by the train manufacturer) by a factor of 1.5 to account for the all the dynamic overloads. This factor was obtained from Cámara et al. (2012).

This value of deformation \( z \) is used to calculate the ‘without rail’ stiffness; that is, the track stiffness without considering the rail contribution. The purpose of this is to allow calculation of the platform stiffness by means of Equation 6, which assumes the combined stiffness to be the sum of different ‘springs’ connected in series

\[
\frac{1}{k_g} = \frac{1}{k_b} + \frac{1}{k_p} + \frac{1}{k_f}
\]

where \( k_g \) is the combined ‘without’ stiffness, \( k_b \) is the ballast layer stiffness, \( k_p \) is the platform stiffness and \( k_f \) is the pad and fastening system stiffness. As the particular stiffness of the pad
and fastening systems is usually provided by the manufacturer, and the ballast stiffness is relatively well known, it is possible to obtain the platform stiffness. It is worth noting that platform stiffness refers to the combined stiffness of the layers under the ballast: that is, the subgrade and embankment.

The vertical analysis of the track is usually carried out by means of the Zimmermann–Timoshenko theory (1926). However, this method has several disadvantages, one of them being that the track is not a beam of constant stiffness supported by a Winkler material. Another drawback is that it is rather complicated to measure in situ the Ballast coefficient needed to calculate bending moments and deformations.

The track is a relatively simple structure which consists of two parallel rails held by elastic supports. As the distance between sleepers (supports) may be different along the track, and the vertical stiffness of each support may also be different, the stiffness of each track stretch may present noticeable variations (Melis, 2013). However, if a constant distance between sleepers is assumed, both the vertical stiffness under the rail and the stiffness of each rail length are constant. Therefore, the general method can be greatly simplified.

In this particular case, the methodology developed by Unold (1925) and completed by Dischinger (1942) and Lorente de Nó (1980) is used, as explained by Melis (2008). This method assumes the rail to be a continuous beam supported by discrete and equidistant elastic foundations (i.e. the sleepers) (Figure 5) whose elasticity is ruled by a parameter $\delta$, which is the inverse of the stiffness without considering the rail contribution (i.e. the ‘without rail’ stiffness).

From this parameter, the following relation is defined

$$\delta = \lambda \frac{L^3}{EI}$$

where $L$ is the distance between consecutive sleepers, $EI$ is the rail stiffness ($E$ being Young’s modulus and $I$ the inertia) and $\lambda$ is a non-dimensional parameter called ‘constant of yielding’.

![Figure 5. Train load (one wheel) over a sleeper](image)

![Figure 6. Vertical acceleration spectrum. Ballasted track, embankment 17](image)

![Figure 7. Vertical acceleration spectrum. Ballasted track, embankment 22](image)

![Figure 8. Vertical acceleration spectrum. Ballasted track, embankment 32](image)
It is assumed that $\lambda$ is known. Then the following constants $p$ and $q$ are calculated

8. $p = \sqrt{\frac{1 + 48\lambda}{3}}$

9. $q = \sqrt{\frac{4}{3} + 2p}$

And from these constants, the following parameters are calculated

10. $a = \frac{2(p - 1)}{1 + p + q}$

11. $b = \frac{1 + p - q}{1 + p + q}$

These parameters allow calculation of all the reaction forces in the sleepers and the bending moments at any point of the rail depending on whether the wheel (load) is over a sleeper or between sleepers. In the first case, the reaction under the sleeper is

12. $R_{\lambda o} = \frac{1 + 3p}{3pq} Q$

Figure 9. Vertical acceleration spectrum. Ballasted track, embankment 44

Figure 10. Vertical acceleration spectrum. Ballasted track, embankment 59

Figure 11. Vertical acceleration spectrum. Ballasted track, embankment 66

Figure 12. Vertical acceleration spectrum. Ballasted track, embankment 80
and the corresponding vertical deformation is

\[ z = \delta R_{Ao} \]

However, \( z \) is already known from Equation 5. Therefore, it is possible to calculate \( \lambda \) so that the result of Equation 13 is equal to that obtained from Equation 5. Once \( \lambda \) is known, \( \delta \) is easily calculated from Equation 7 and therefore the ‘without rail’ stiffness is obtained.

3. Results and discussion
As mentioned earlier, the main purpose of this paper is to present a methodology to calculate the combined track stiffness (with and without the rail contribution) by means of vertical acceleration data measured in a train axle box. Afterwards, acceleration spectra were calculated for different track stretches of either 50 or 100 m. Figures 6–13 show some examples of the acceleration spectra obtained.

From these figures it is clear that, despite the logical stiffness variations along the track owing to different factors, a dominant frequency may be identified within the 55–65 Hz range previously defined. This is, as explained before, assumed to correspond to the natural frequency of the train unsprung masses, which for the cases shown in Figures 6–13 is always around 60 Hz with little variation and very close to the value of 58.5 Hz previously calculated.

Once this frequency is known, the combined stiffness for each track stretch may be calculated from its respective dominant frequency following the aforementioned methodology. The results for the track stretches represented in Figures 6–13 are shown in Table 2.

In order to assess the performance of this methodology, the results obtained are then compared with stiffness values from two different sources. The first corresponds to the stiffness measurements carried out by Tijera et al. (2012) in different high-speed lines in Spain whose structure and characteristics are equal to the line studied in this article. The stiffness was obtained by measuring the wheel load and rail deflection. The other data set was obtained by Cuellar et al. (2012) from a 1:1 scale track model of the same typology (ballasted track) common in the Spanish high-speed network. Loads equivalent to that of actual trains passing at a speed between 300 and 360 km/h were applied to the model, and the resulting measured stiffness was also validated with data measured

<table>
<thead>
<tr>
<th>Track stretch</th>
<th>Natural frequency: Hz</th>
<th>Track stiffness: kN/mm</th>
<th>Deformation: mm</th>
<th>Without rail stiffness: kN/mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Embankment 17</td>
<td>58</td>
<td>118.79</td>
<td>0.7</td>
<td>46.48</td>
</tr>
<tr>
<td>Embankment 22</td>
<td>60</td>
<td>127.1</td>
<td>0.65</td>
<td>51.29</td>
</tr>
<tr>
<td>Embankment 32</td>
<td>60</td>
<td>127.1</td>
<td>0.65</td>
<td>51.29</td>
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<tr>
<td>Embankment 44</td>
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<td>0.65</td>
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</tr>
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<td>Embankment 59</td>
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<td>Embankment 66</td>
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<tr>
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<tr>
<td>Embankment 155</td>
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<td>0.65</td>
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</tr>
<tr>
<td>Embankment 173</td>
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<td>51.29</td>
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<tr>
<td>Embankment 192</td>
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<td>0.70</td>
<td>46.48</td>
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<tr>
<td>Embankment 255</td>
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<td>127.1</td>
<td>0.65</td>
<td>51.29</td>
</tr>
</tbody>
</table>

Table 2. Calculated track stiffness (with and without rail)
The authors wish to thank Renfe Operadora for granting permission to monitor their trains and for their kind support during the monitoring campaign.

Acknowledgements

The methodology proposed allows measuring the track stiffness as a main condition parameter by means of acceleration data measured in the trains’ axle box. This kind of data is usually available from dedicated monitoring vehicles, although it could be also recorded from normal trains during their daily operation. In this way, a first insight to the track condition may be taken continuously, without requiring stationary stiffness measuring devices. More complex monitoring equipment would only be deployed when a defect was spotted, hence allowing for a more efficient use of this kind of machinery.

The methodology is rather straightforward and provides a good approximation to the track combined stiffness when compared to data measured by previous studies. The results obtained for a high-speed ballasted track are between 120 and 130 kN/mm, which is consistent with previous direct measurements.

Therefore, the method proposed might become an easy and cost-effective way of carrying out a preliminary monitoring of the track condition which complements the use of other more accurate and complex devices. To achieve this, the next step of research would be to study the correlation between the variations of measured combined stiffness and the specific maintenance needs of the line.

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