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Hydraulic stability of nominal and sacrificial toe berms for mound breakwaters on steep sea bottoms

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ABSTRACT

When mound breakwaters are placed on steep sea bottoms in combination with very shallow waters, the design of the toe berm becomes a relevant issue. Toe berms built close to the water surface on a steep sea bottom must withstand such high wave loads that their design may not be feasible with available quarrystones. In this study, a new design method was developed to reduce the rock size by increasing the toe berm width. The analysis involved specific 2D small-scale tests with toe berms of different rock sizes and widths, placed on a m = 1/10 bottom slope with the water surface close to the toe berm crest. Two new concepts were introduced to better characterize damage to wide toe berms: (1) the most shoreward toe berm area which effectively supports the armor layer, in this study referred to as the primary or "nominal" toe berm and (2) the most seaward toe berm area which serves to protect the nominal toe berm, in this study called the secondary or the "sacrificial" toe berm. Damage to the nominal toe berm was used to describe hydraulic stability of wider toe berms. Given a standard toe berm of three rocks wide (nominal toe berm), an equivalent toe berm with damage similar to the nominal toe berm was defined by increasing the berm width and decreasing the rock size. The reduction in rock size showed an inverse 0.4-power relation with the relative berm width.

Keywords: hydraulic stability; mound breakwater; nominal toe berm; sacrificial toe berm; shallow water; steep sea bottom; toe berm.

Highlights:

a) **Toe berm is a relevant design element of mound breakwaters placed on steep sea bottoms combined with very shallow waters.**
b) **Toe berm damage is critical when the still water level is near the toe berm crest.**
c) **Toe berms may require rocks larger than the size available at the construction site.**
d) **Rock size for the toe berm can be reduced by increasing the toe berm width.**

1. Introduction

The design of rubble mound breakwaters usually focuses on the main armor layer. When concrete armor units are used, it is common to construct a rock toe berm of three to four rocks wide to provide support for the armor layer (see CIRIA/CUR/CEFMEF, 2007). Toe berm stability depends mainly on design wave storm characteristics, water depth and the sea bottom slope existing at the construction site. Toe berms in very shallow waters behave in manner a completely different from those built in non-breaking conditions (see Hovestad, 2005). On gentle sea bottoms, it is common to design deep submerged rock toe berms. However, on rocky coastlines with steep sea bottoms, coastal structures may require emerged toe berms with heavy rocks; toe berm hydraulic stability may be even more critical than armor stability. Herrera and Medina (2015) conducted laboratory tests with a steep bottom slope (m = 1/10) and concluded that most damage occurs when the still water level (SWL) is near the crest of the toe berm. In these conditions, for certain wave storms, the required nominal diameter (D_{n50}) may be so large that it is not possible to design standard toe berms with rocks from available quarries. In these cases, Besley and Benechere (2009) and Herrera and Medina (2015) recommended moving the toe position to deeper or shallower waters where it is feasible to construct the toe berm with rock sizes available at the construction site. Nevertheless, if the toe position cannot be moved due to environmental, economic or operational requirements, this design change is not possible. Other design changes for toe berms are given in the literature; authors such as Burcharth and Liu (1995) or Van Gent and Van der Werf (2014) proposed using concrete units for the toe berm, while USACE (2006) suggested excavating trenches, drilling piles or anchoring bolts to the sea bottom to support the toe stones on rocky coastlines.

The most popular formulas to predict damage to rock toe berms were obtained from small-scale tests with different toe berm geometries. However, toe berm widths (B_t) and thicknesses (t_t) were not usually introduced as explicative parameters of the observed toe berm damage. Eq. (1) is equivalent to the formula given by Gerding (1993), which is based on laboratory tests with a bottom slope m = 1/20, two toe berm widths ($B_t = 3D_{n50}$

and 12D_{n50}), two toe berm thicknesses (t_t = 2.3D_{n50} and 8.8D_{n50}), and different water depths at the toe (7.5$D_{n50} \leq h_s \leq 29.4 D_{n50}$).

$$N_{od} = \frac{1}{\left(0.24\left(\frac{h_t}{D_{n50}}\right)+1.6\right)^{1/0.15}}\left(N_s\right)^{1/0.15} \tag{1}$$

in which N_{od} is the damage number, $N_s = H_{st}/(\Delta D_{n50})$ is the stability number, $\Delta = (\rho_r - \rho_w)/\rho_w$ is the relative submerged mass density of rocks, ρ_r is the mass density of rocks, ρ_w is the mass density of sea water, H_{st} is the significant wave height at the toe of the structure, and h_t is the water depth above the toe berm.

Eq. (2) is equivalent to the formula proposed by Van der Meer (1998), based on the data given by Gerding (1993), but using the dimensionless parameter h_t/h_s.

$$N_{od} = \frac{1}{\left(6.2\left(\frac{h_t}{h_s}\right)^{2.7}+2.0\right)^{1/0.15}}\left(N_s\right)^{1/0.15} \tag{2}$$

Ebbens (2009) and Baart et al. (2010) proposed Eq. (3) to estimate the toe berm damage from laboratory tests with three bottom slopes (m = 1/20, 1/50 and 1/10), two toe berm widths (B_t = 3.7D_{n50} and 5.3D_{n50}), two toe berm thicknesses (t_t = 2.2D_{n50} and 3.2D_{n50}), and different water depths at the toe (2.7$D_{n50} \leq h_s \leq 18 D_{n50}$).

$$N_{\%} = 0.038\left(\xi_{0p}{}^{*}\right)^{3/2}\left(N_s\right)^3 \tag{3}$$

in which $N_{\%}$ is the percentage of damage, $\xi_{0p}{}^{*} = m/(H_{st}/L_{0p})^{1/2}$ is the surf similarity parameter where m is the bottom slope, and $L_{0p} = gT_p{}^2/2\pi$ is the deep water wave length corresponding to the peak period, T_p.

Eq. (4) is equivalent to the formula proposed by Muttray (2013), based on experiments conducted by different authors, including the data given by Gerding (1993) and Ebbens (2009).

$$N_{od} = \left(0.58 - 0.17\frac{h_t}{H_{st}}\right)^3 (N_s)^3 \tag{4}$$

Van Gent and Van der Werf (2014) obtained Eq. (5) from laboratory tests with a bottom slope m = 1/30, two toe berm widths ($B_t = 3D_{n50}$ and $9D_{n50}$), two toe berm thicknesses ($t_t = 2D_{n50}$ and $4D_{n50}$), and different water depths ($8.6D_{n50} < h_s < 27.4D_{n50}$ and $7D_{n50} \leq h_t \leq 25D_{n50}$). Eq. (5) explicitly considers the influence of B_t and t_t on toe berm stability.

$$N_{od} = 0.032\left(\frac{t_t}{H_{st}}\right)\left(\frac{B_t}{H_{st}}\right)^{0.3}\left(\frac{\hat{u}_\delta}{\sqrt{gH_{st}}}\right)(N_s)^3 \tag{5}$$

in which $\hat{u}_\delta = \frac{\pi H_{st}}{T_{m-1,0}}\frac{1}{\sinh(kh_t)}$, $k = \frac{2\pi}{L_{m-1,0}} = \frac{2\pi}{\frac{g}{2\pi}T_{m-1,0}{}^2}$, $T_{m-1,0} = \frac{m_{-1}}{m_0}$, m_i is the *i-th* spectral moment given by $m_i = \int_0^\infty S(f) f^i df$, being *S(f)* the wave spectrum.

Van Gent and Van der Werf (2014) also proposed multiplying the design N_{od} value by a factor f_B (see Eq. 13) when $3D_{n50} < B_t \leq 9D_{n50}$, as described in Section 6.

Finally, Eq. (6) proposed by Herrera and Medina (2015) is based on laboratory tests with a steep bottom slope (m = 1/10), one toe berm width ($B_t = 3D_{n50}$), one toe berm thickness ($t_t = 2D_{n50}$), and water depths at the toe berm in the range of $-0.5D_{n50} \leq h_s \leq 5.01D_{n50}$.

$$N_{od} = \left[\left(\frac{(H_{s0}L_{0p})^{1/2}}{\Delta D_{n50}} - 5.5\right)\left(-0.2\frac{h_s}{D_{n50}} + 1.4\right)\exp\left(0.25\frac{h_s}{D_{n50}} - 0.65\right)\right]^{1/0.15} \tag{6}$$

in which H_{s0} is the significant deep water wave height. Herrera and Medina (2015) described the two toe berm damage definitions, N_{od} and $N_{\%}$, used in Eqs. (1) to (6).

$$N_{od} = \frac{N}{B/D_{n50}} \tag{7}$$

in which N is the number of displaced rocks and B is the total width of the wave flume. Herrera and Medina (2015) found that $N_{\%}$ is usually one order of magnitude lower than the damage number N_{od}. Both N_{od} and $N_{\%}$ take into account the total number of rocks displaced from the toe (N). However, N is not suitable to measure the damage to toe berms with different geometries, since a larger N is required to significantly damage larger toe berms. When increasing the toe berm width ($B_t > 3D_{n50}$), rocks situated in the most seaward area do not directly contribute to support the armor, but only to protect the most shoreward area of the toe berm. Since toe berm stability should be considered together with the stability of the main armor layer (see Lamberti, 1994), the most seaward area of the toe structure can be considered as a “sacrificial” toe berm, and the most shoreward area of three nominal diameters wide, as the “nominal” toe berm necessary to support the armor layer (see Fig. 1).

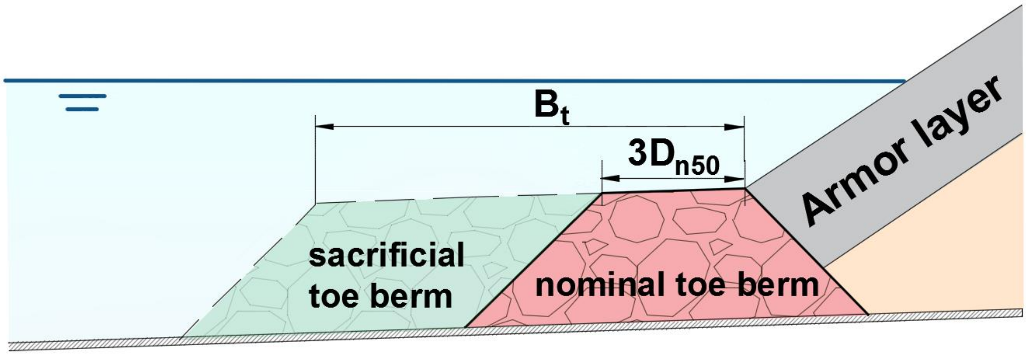

Fig. 1. Sketch of sacrificial and nominal toe berms.

This study analyzes the influence of the nominal diameter (D_{n50}) and the toe berm width ($B_t = nD_{n50}$) on the hydraulic stability of the nominal toe berm, where n is the number of rock rows placed on the upper layer of the toe berm. To this end, 2D physical tests were conducted using small-scale models of breakwaters with double-layer randomly-placed cube armors and rock toe berms, placed on a steep bottom slope (m = 1/10). Different pairs of (D_{n50}, B_t) were tested with the SWL close to the crest of the toe berms. The required rock size given by Eq. (6) for a nominal toe berm ($B_t = 3D_{n50}$) was modified to account for wider toe berms (n > 3) based on damage measurements of the nominal toe berm. In this paper, the experimental setup is described in Section 2. Tests with different toe berm sizes and widths are analyzed in Section 3. Section 4 describes a design method based on a new equation with its confidence intervals, providing an integrated graph to design rock toe berms. A practical application is given in Section 5. Formulas given in the literature are compared in Section 6. Finally, conclusions are drawn in Section 7.

2. Physical model tests

2D physical model tests were conducted in the wind and wave test facility (30m x 1.2m x 1.2m) of the Laboratory of Ports and Coasts at the *Universitat Politècnica de València*

(LPC-UPV) with a piston-type wavemaker and a steep sea bottom (m = 1/10). Fig. 2 shows a longitudinal cross section of the LPC-UPV wave flume with the location of the wave gauges used in this study.

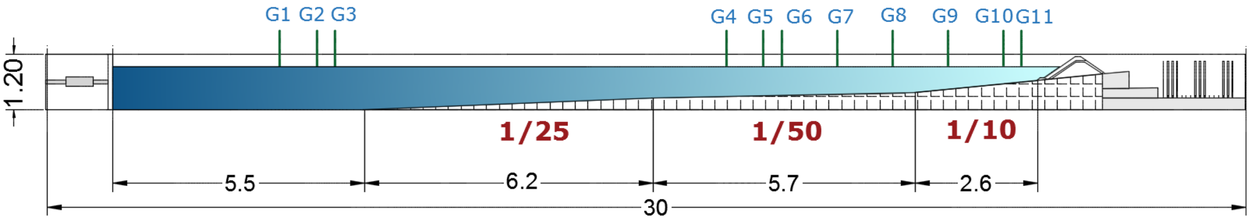

Fig. 2. Longitudinal cross section of the LPC-UPV wave flume (dimensions in meters).

The test model depicted in Fig. 3 corresponds to a conventional $cot\alpha$ = H/V = 3/2 non-overtopped mound breakwater, protected with a double-layer, randomly-placed cube armor with nominal diameter D_n(cm) = 3.97 and weight W(g) = 141.5. The cube armor was built on a filter layer with D_{n50}(cm) = 1.78 and D_{n85}/D_{n15} = 1.35. The granulometric characteristics of the core material were D_{n50}(cm) = 0.68 and D_{n85}/D_{n15} = 1.64.

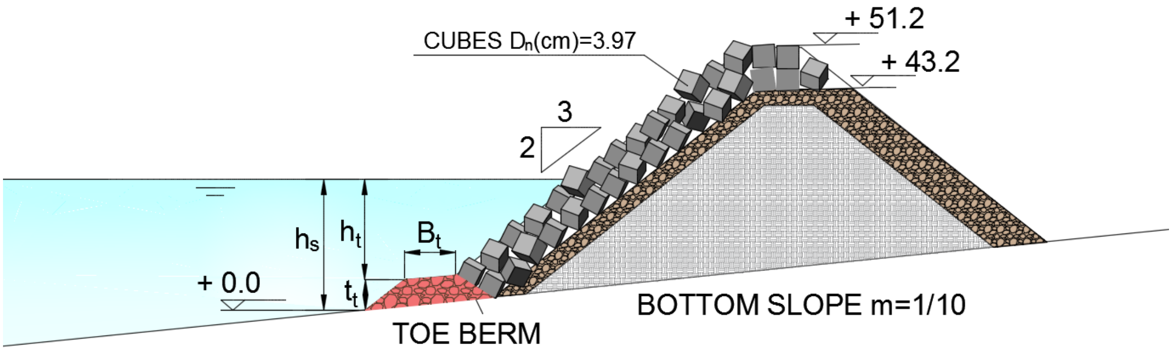

Fig. 3. Configuration of the cube armored model (dimensions in centimeters).

Toe berms were tested with three rock sizes, D_{n50}(cm) = 3.04, 3.99 and 5.12, with a mass density ρ_r(g/cm^3) = 2.70. Three toe berm widths (n = 3, 5 and 12) were applied with D_{n50}(cm) = 3.04 and 3.99; the nominal toe berm was considered as the most shoreward area of the berm with a width of three times the rock nominal diameter ($B_t = 3D_{n50}$). The nominal toe berm was placed first; later, the sacrificial toe berm was placed, using rocks painted in a different color to be easily distinguished. Only the nominal toe berm (n = 3) was tested with D_{n50} (cm) = 5.12. In all cases, the toe berm thickness was fixed at $t_t = 2D_{n50}$, and the water depth was h_{ss}(cm) = 8, measured at the toe of the nominal toe berm ($n = 3$) for all configurations (see Fig. 4). With h_{ss}(cm) = 8, the SWL was very close to the crest of the toe berms ($1.5 \leq h_{ss}/D_{n50} \leq 2.6$). Note that $h_s = h_{ss}$ only when $B_t = 3D_{n50}$.

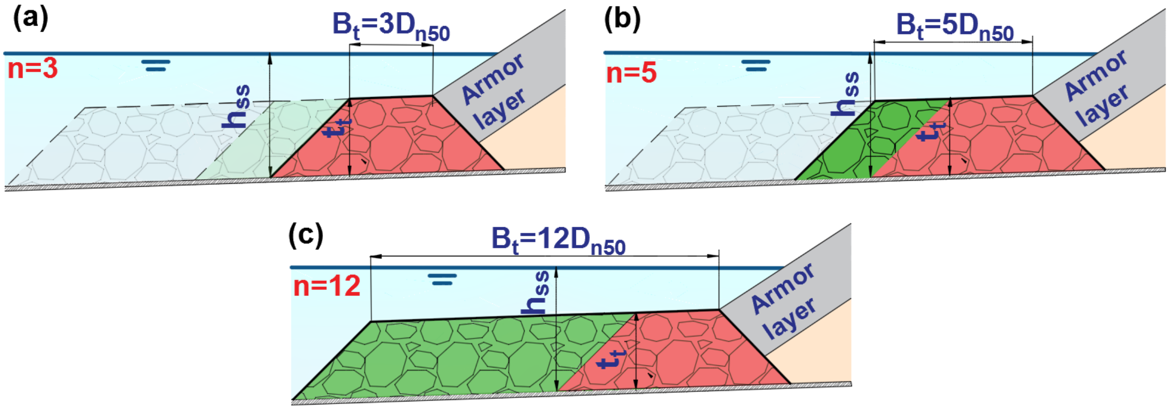

Fig. 4. Configuration of tested toe berms: (a) $B_t = 3D_{n50}$, (b) $B_t = 5D_{n50}$ and (c) $B_t = 12D_{n50}$.

Random wave runs of 500 waves were generated following JONSWAP (Υ= 3.3) spectrum, and the AWACS Active Absorption System was activated to avoid multi-reflections. Test series were conducted following the methodology described by Herrera and Medina (2015). Five different peak periods were considered, T_p(s) = 1.20, 1.50, 1.80, 2.20 and 2.40; for each T_p, values of significant wave height at the wave generating zone (H_{sg})

were increased from no damage to wave breaking in front of the wavemaker. H_{sg} was increased in steps of 2cm in the range of 8≤ H_{sg} (cm) ≤20. The toe berm was rebuilt after each test series defined by the water depth at the toe of the nominal toe berm (h_{ss}), the rock size (D_{n50}), and the toe berm width ($B_t = nD_{n50}$).

Two damage parameters were measured after each test: (1) N_{od}, corresponding to the total damage of the toe berm of width $B_t = nD_{n50}$ ($n \geq 3$); and (2) N_{od}* corresponding only to the damage of the nominal toe berm. Fig. 5 shows a model with D_{n50} (cm) = 3.99 and $B_t = 5D_{n50}$; blue rocks correspond to the nominal toe berm and brown rocks correspond to the sacrificial toe berm. Table 1 summarizes the test conditions and the range of parameters used in this study.

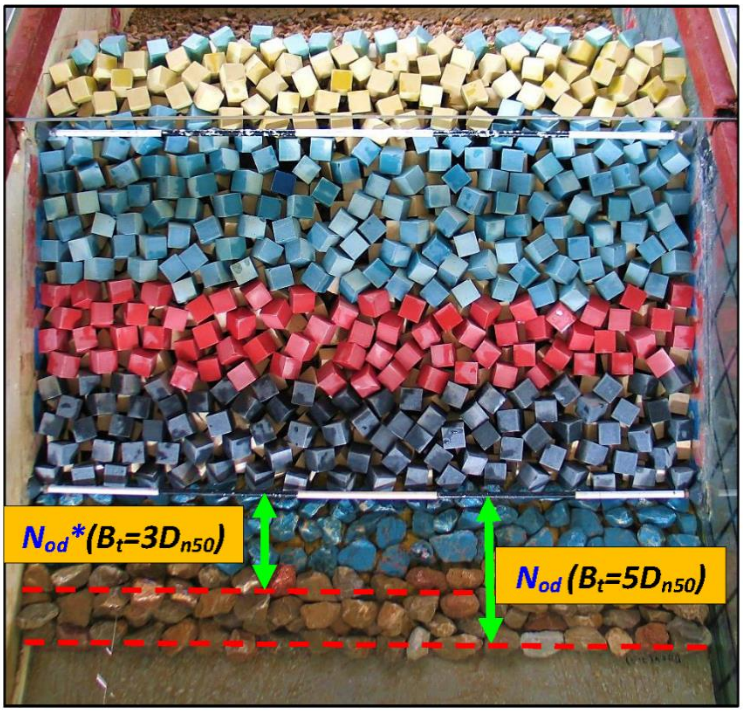

Fig. 5. Nominal (blue rocks) and sacrificial (brown rocks) toe berms with $B_t = 5D_{n50}$.

Table 1. Test conditions.

Parameter	Symbol	Value
Slope angle (-)	$cot\alpha$	3/2
Bottom slope (-)	m	1/10
Cube armor size (cm)	D_n	3.97
Rock filter size (cm)	D_{n50}	1.78
Rock core size (cm)	D_{n50}	0.68
Rock toe size (cm)	D_{n50}	3.04, 3.99 and 5.12
Rock toe density (g/cm^3)	ρ_r	2.7
Relative toe width (-)	B_t/D_{n50}	3-12
Relative toe thickness (-)	t_t/D_{n50}	2
Relative water depth at toe berm (-)	h_s/D_{n50}	1.5-3.5
Relative water depth at the nominal toe berm (-)	h_{ss}/D_{n50}	1.5-2.6
Relative significant wave height at generating zone (-)	H_{sg}/h_{ss}	1.0-2.5
Wave steepness at generating zone ($s_{gp}=2\pi H_{sg}/gT_p^2$) (-)	s_{gp}	0.01-0.07
Stability number at generating zone ($N_s=H_{sg}/\Delta D_{n50}$) (-)	N_s	1.0-3.8
Damage level of the nominal toe berm (-)	N_{od}*	<4.8
Total damage level (-)	N_{od}	<11.1

Water surface elevation was measured using eleven capacitive wave gauges. One group of wave gauges (G1, G2 and G3) was placed near the wavemaker while ten wave gauges (G4 to G11) were placed along the wave flume (see Fig. 2). The LASA-V method described by Figueres and Medina (2004) was used to estimate incident and reflected waves at the generating zone (wave gauges G1, G2 and G3).

3. Data Analysis

3.1. Wave analysis

Using the water surface elevation, waves were characterized with a time and frequency domain analysis. When dealing with waves breaking on a m = 1/10 bottom slope combined with shallow waters, it is not easy to obtain reliable incident wave characteristics. The deep water wave conditions are the most reliable reference in these cases.

In this study, waves were characterized in deep water conditions following the methodology described in Herrera and Medina (2015). The average of the highest one-third incident waves ($H_{i,1/3}$) measured at G1, G2 and G3 was used to estimate the deep water significant wave height (H_{s0}) using the shoaling coefficients given by Goda (2000). Fig. 6 shows the measured $H_{1/3,i}$ versus the deep water significant wave height (H_{s0}) estimated using the methodology given in Goda (2000).

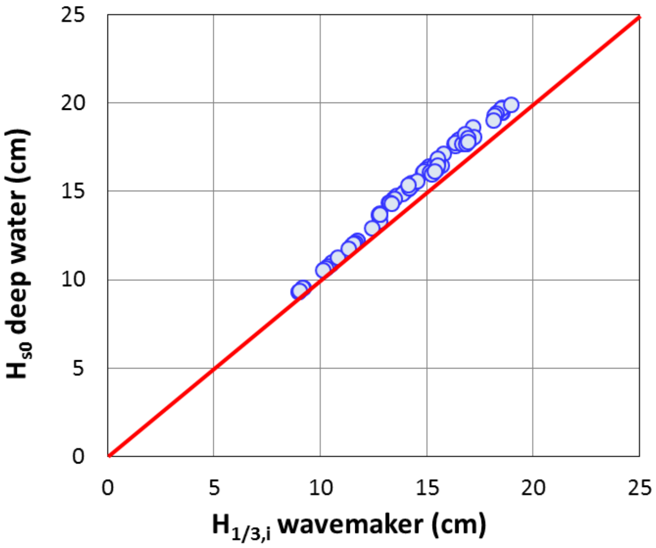

Fig. 6. Measured $H_{1/3,i}$ at the wave generating zone versus deep water significant wave height, H_{s0}.

The deep water significant wave height (H_{s0}) and the deep water wave length obtained from the peak period ($L_{0p} = gT_p^2/2\pi$) were used to characterize the toe berm damage. According to Herrera and Medina (2015), $(H_{s0}\, L_{0p})^{1/2}$ seems to be the best explicative variable to represent toe berm damage in very shallow waters combined with steep sea bottoms.

3.2. Damage analysis

Toe berm stability was analyzed using the total toe berm damage (N_{od}), along with the nominal toe berm damage (N_{od}*). After each test, the total number of rocks displaced from the toe berm (N) were counted and the damage parameter, N_{od}, was determined using Eq. (7). N_{od} corresponded to the damage to both the sacrificial and nominal toe berms. The damage parameter, N_{od}*, was also determined using Eq. (7) but considering only the number of rocks displaced from the nominal toe berm (Fig. 7). Both damage parameters considered the cumulative damage of each test series.

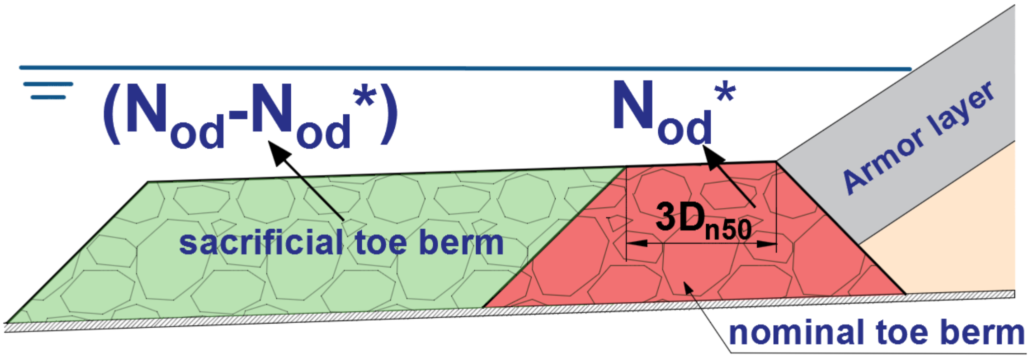

Fig. 7. Total toe berm damage (N_{od}) and nominal toe berm damage (N_{od}*).

Figs. 8 and 9 show total and nominal toe berm damage corresponding to D_{n50} (cm) = 3.04, 3.99 and 5.12 and different toe berm widths (n = 3, 5 and 12). Only the maximum cumulative damage obtained after test runs characterized by T_p is plotted here.

3.2.1. Total toe berm damage (N_{od})

Fig. 8 shows the measured N_{od} as a function of the variable $(H_{s0}\ L_{0p})^{1/2}$ for the seven tested models. N_{od} increased almost linearly with the variable $(H_{s0}\ L_{0p})^{1/2}$ in all cases. Given *n*, N_{od} was larger when reducing D_{n50}. Given D_{n50}, N_{od} increased when increasing *n*. Smaller rock sizes and wider toe berms led to larger values of total toe berm damage (N_{od}).

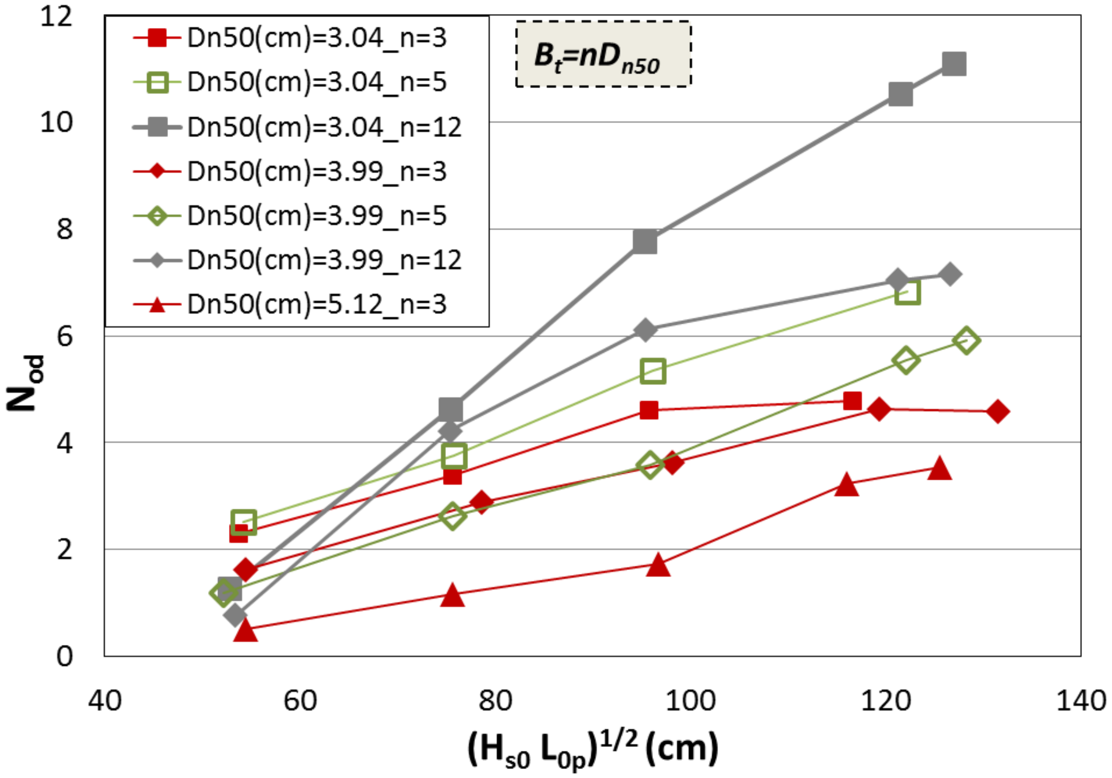

Fig. 8. Total toe berm damage (N_{od}) as a function of toe berm width ($B_t = nD_{n50}$) and rock size (D_{n50}).

3.2.2. Nominal toe berm damage (N_{od}*)

Fig. 9 shows the measured nominal toe berm damage (N_{od}*) as a function of the variable $(H_{s0}\ L_{0p})^{1/2}$. Given a toe berm width ($B_t = nD_{n50}$), N_{od}* was larger when reducing D_{n50}.

Given a rock size (D_{n50}), N_{od}* increased when reducing the toe berm width (n). Thus, larger rock sizes as well as wider toe berms led to less nominal toe berm damage (N_{od}*).

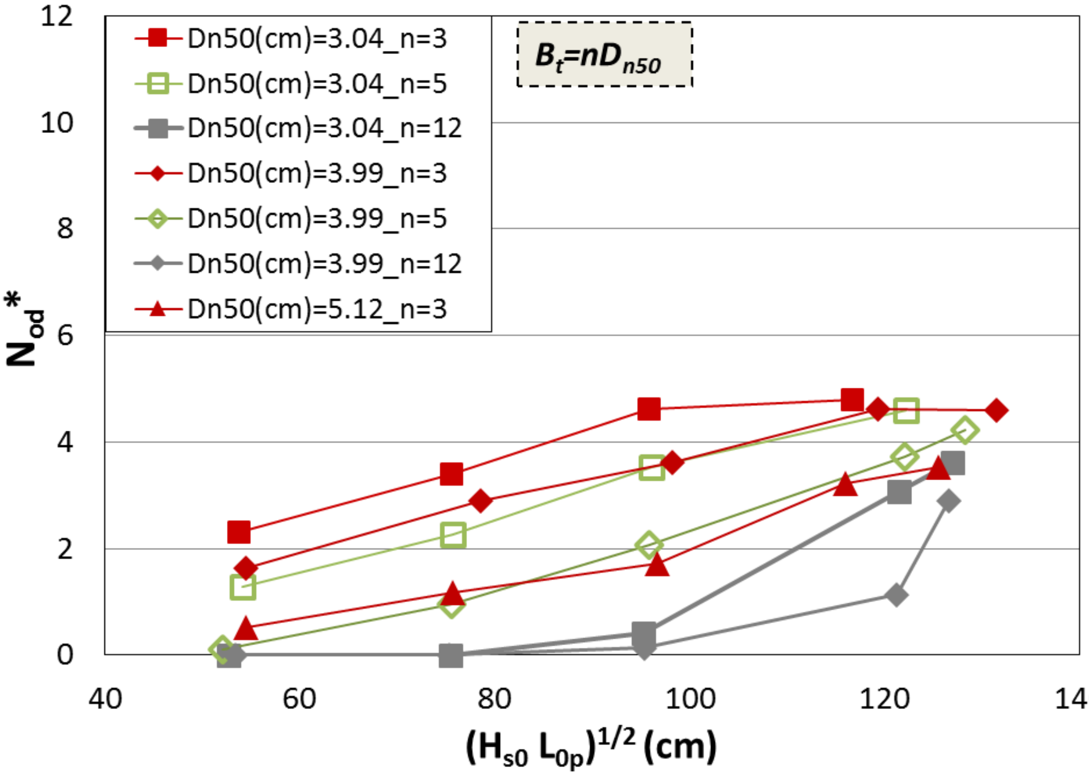

Fig. 9. Nominal toe berm damage (N_{od}*) as a function of toe berm width ($B_t = nD_{n50}$) and rock size (D_{n50}).

3.2.3. Comparison of total and nominal toe berm damage measurements

Fig. 10 compares measured total toe berm damage (N_{od}) and nominal toe berm damage (N_{od}*). The wider the toe berm, the lower the N_{od}* but the higher the N_{od}. Given a rock size (D_{n50}), a wider toe berm would reduce N_{od}*, although the N_{od} would increase. Thus, the total toe berm damage (N_{od}) is not a good estimator of the hydraulic stability of toe berms when comparing different berm widths ($B_t > 3D_{n50}$); the N_{od}* corresponding to the damage to the nominal toe berm, which is actually supporting the armor layer, is the toe berm damage which should be taken into account when analyzing breakwater hydraulic stability. Damage observed on the sacrificial toe berm is not relevant when analyzing the hydraulic performance of mound breakwaters.

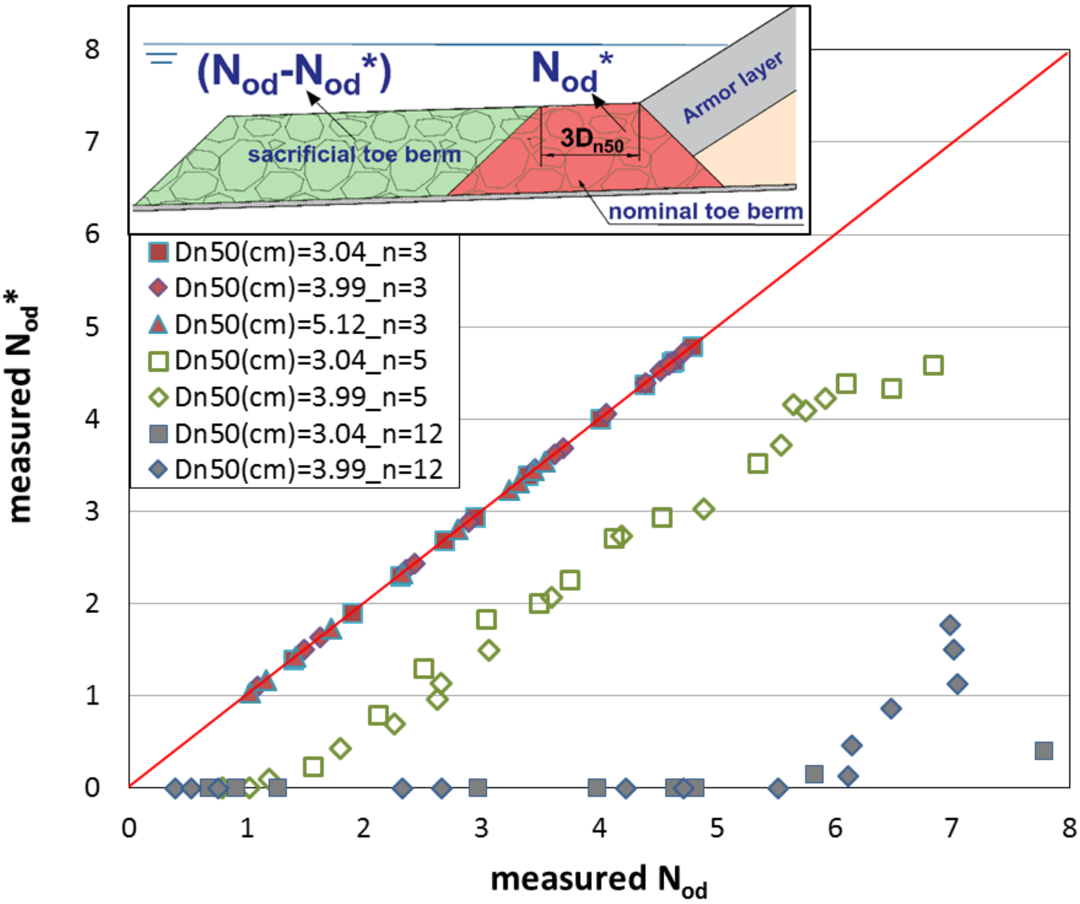

Fig. 10. Comparison of measured total toe berm damage (N_{od}) and measured nominal toe berm damage (N_{od}*).

Hereafter, only the damage to the nominal toe berm, N_{od}*, is considered. Eq. (6), proposed by Herrera and Medina (2015), is extended here to design toe berms with $3D_{n50} \leq B_t \leq 12D_{n50}$ and $t_t = 2D_{n50}$, placed on steep sea bottoms (m = 1/10) when the SWL is close to the crest of the toe berm ($1.5 \leq h_{ss}/D_{n50} \leq 2.6$), $0.02 \leq s_{0p} \leq 0.07$ and $0.4 \leq h_{ss}/H_{s0} \leq 1.0$.

4. New design method for toe berms in shallow water and m =1/10

Fig. 9 shows that several tests with different D_{n50} and n provided similar values of N_{od}* for specific wave conditions, $(H_{s0} L_{0p})^{1/2}$. Under the same wave conditions (H_{s0}, T_p), the toe berm with D_{n50}(cm) = 3.99 and n = 5 provided almost the same N_{od}* as the toe berm with D_{n50}(cm) =5.12 and n=3. Analogously, the toe berm with D_{n50}(cm) = 3.04 and n = 5 gave values of N_{od}* similar to those of the toe berm with D_{n50}(cm) = 3.99 and n = 3. These findings suggest that the rock size can be reduced by increasing the toe berm width. It is possible to keep N_{od}* constant by reducing D_{n50} and increasing n, or *vice versa*. The relationship between D_{n50} and N_{od}* can be described by Eq. (8), using as a reference the nominal toe berm (n = 3) with rock size $D_{n50} = D_{n50,3}$.

$$\frac{D_{n50,n}}{D_{n50,3}} = \left(\frac{3}{n}\right)^k \tag{8}$$

where $D_{n50,3}$ is the nominal diameter of rocks for the nominal toe berm ($n = 3$), $D_{n50,n}$ is the nominal diameter of rocks for wider toe berms ($3 < n \leq 12$), and k is a positive parameter to be calibrated using the test results described above (k = 0.4). Eq. (8) indicates that given a nominal toe berm with n = 3 and $D_{n50} = D_{n50,3}$, an equivalent toe berm can be defined with higher n ($n > 3$) and lower D_{n50} ($D_{n50,n} < D_{n50,3}$) to provide similar N_{od}*.

Because Eq. (6) is valid to design toe berms using rocks with n = 3, the estimated N_{od} given by Eq. (6) corresponds to the nominal toe berm damage (N_{od}*), and Eqs. (6) and (8) can be combined as follows:

$$N_{od}* = \left(\frac{(H_{s0}L_{0p})^{1/2}}{\Delta D_{n50,n}\left(\frac{n}{3}\right)^k} - 5.5 \right) \tag{9}$$

$$x\left[\left(-0.2\frac{h_{ss}}{D_{n50,n}\left(\frac{n}{3}\right)^k}+1.4\right)\exp\left(0.25\frac{h_{ss}}{D_{n50,n}\left(\frac{n}{3}\right)^k}-0.65\right)\right]^{1/0.15}$$

The best agreement between the measured $N_{od}*$ and the estimated $N_{od}*$ given by Eq. (9) was found for k = 0.4. The goodness of fit between measured and calculated values and the 90% confidence interval are described below.

Eq. (9) extends the application range of Eq. (6) to deal with wider toe berms. Eq. (9) with k = 0.4 provides the required rock size for toe berms with $3D_{n50} \leq B_t \leq 12D_{n50}$ and $t_t=2D_{n50}$, placed on a m = 1/10 sea bottom with $1.5 \leq h_{ss}/D_{n50} \leq 2.6$, $0.02 \leq s_{0p} \leq 0.07$, $0.4 \leq h_{ss}/H_{s0} \leq 1.0$, using the damage parameter $N_{od}*$. When designing with $N_{od}*$, common values for acceptable damage may be directly used. In this study, the criterion proposed by Herrera and Medina (2015) was considered: no significant movement of toe berm rocks ($N_{od}* < 0.5$), significant rock movements ($N_{od}* = 1.0$), moderate damage but toe berm still providing support to the armor ($N_{od}* = 2.0$), and toe berm failure ($N_{od}* = 4.0$).

4.1. Confidence intervals

Assuming a Gaussian error distribution, the 90% confidence interval for the toe berm damage estimation given by Eq. (9) is:

$$N_{od}*\Big|_{5\%}^{95\%} = N_{od}* \pm 0.83 \tag{10}$$

Fig. 11 compares measured $N_{od}*$ and that estimated given by Eq. (9) with the 90% confidence interval given by Eq. (10). The few outliers for small $N_{od}*$ shown in Fig. 11 are on the safe side, estimated $N_{od}* >$ measured $N_{od}*$.

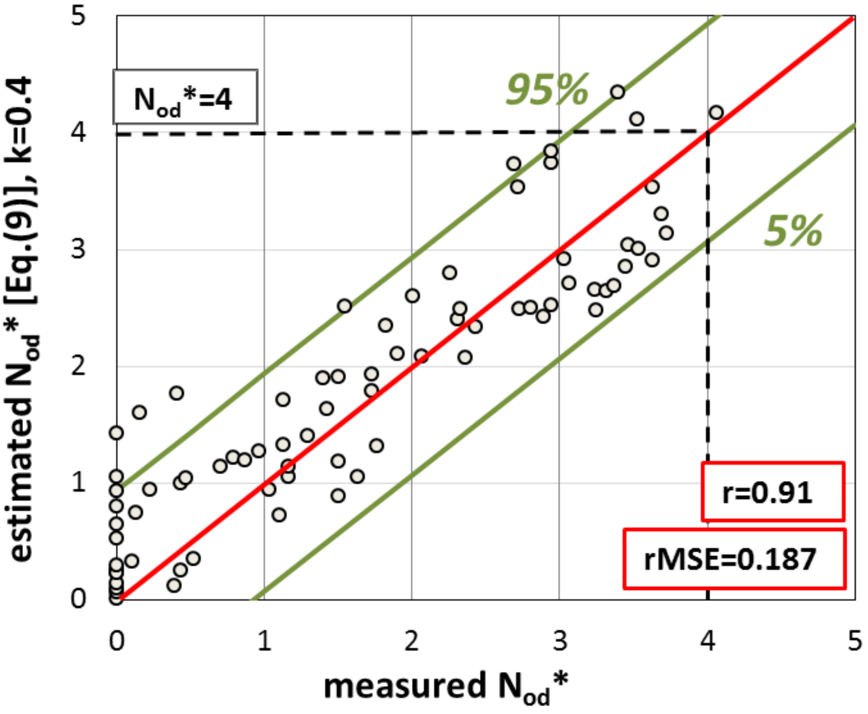

Fig. 11. 90% confidence interval of estimated $N_{od}*$ given by Eq. (9) with k = 0.4.

In order to measure the goodness of fit between the $N_{od}*$ measured in the tests and that estimated by Eq. (9), the relative mean squared error (*rMSE*) and the correlation coefficient (*r*) were calculated:

$$rMSE = \frac{MSE}{VAR} = \frac{\frac{1}{N_t}\sum_{i=1}^{N_t}(t_i - e_i)^2}{\frac{1}{N_t}\sum_{i=1}^{N_t}(t_i - \bar{t})^2} \tag{11}$$

$$r = \frac{\sum_{i=1}^{N_t}(t_i - \bar{t})(e_i - \bar{e})}{\sqrt{\sum_{i=1}^{N_t}(t_i - \bar{t})^2 \sum_{i=1}^{N_t}(e_i - \bar{e})^2}} \tag{12}$$

in which *MSE* is the mean squared error, N_t is the number of observations, t_i is the target value, e_i is the estimated value, *VAR* is the variance of target values, and $\bar{t}$ and $\bar{e}$ are the average of target and estimated values, respectively. $0 \leq rMSE \leq 1$ estimates the proportion of variance in the observed values not explained by Eq. (9); the lower the *rMSE,* the better the predictions. $0 \leq r \leq 1$ measures the degree of correlation between measured and estimated values of $N_{od}*$; the higher the *r*, the better the predictions. Eq. (9) with $k = 0.4$ provided $rMSE = 0.187$ and $r = 0.91$.

4.2. Design approach for equivalent toe berms

Given an acceptable level of damage (e.g. $N_{od}* = 0.5$ or 1.0), Eq. (6) is used first to calculate the rock size for a nominal toe berm, $D_{n50,3}$, and Eq. (9) can be used later to define wider toe berms ($3 < n \leq 12$) with smaller rocks ($D_{n50,n}$).

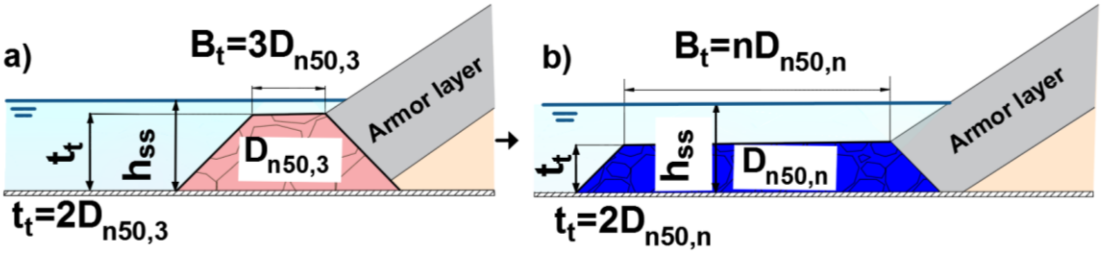

Fig. 12. (a) Nominal toe berm ($n = 3$) and (b) equivalent wider toe berm ($3 < n \leq 12$).

A practical application of this process is given by the design graph shown in Fig. 13, which is valid for $N_{od}* = 0.5$ and 1.0. Fig. 13a shows the nominal diameter of rocks for a nominal toe berm ($D_{n50,3}$), estimated with Eq. (6), as a function of the deep water wave conditions, $(H_{s0}\ L_{0p})^{1/2}$, for $h_{ss}/D_{n50,3}$ = 1.5, 2.0 and 2.5. Fig. 13b shows the relation between nominal diameters ($D_{n50,3}$ and $D_{n50,n}$) as a function of the toe berm width ($3 \leq n \leq 12$). $D_{n50,n}$ can be selected by the designer considering the rock sizes available at the construction site. The ranges of application of Eq. (6) in Fig. 13 are $0.02 < s_{0p} < 0.07$, $-0.15 < h_{ss}/H_{s0} < 1.5$, and $-0.5 \leq h_{ss}/D_{n50,3} \leq 5.01$.

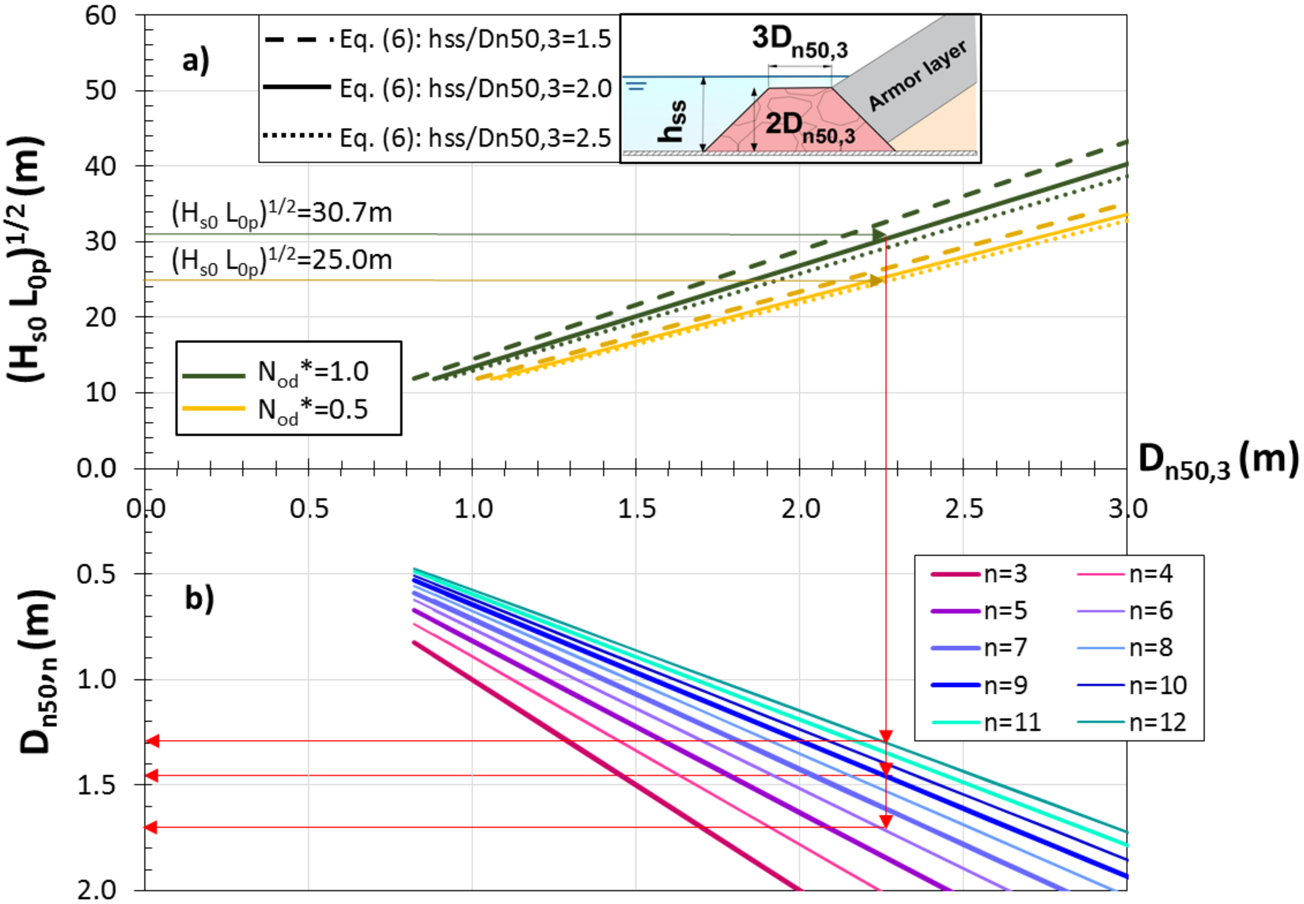

Fig. 13. (a) $D_{n50,3}$ estimated with Eq. (6) and (b) $D_{n50,n}$ as a function of $D_{n50,3}$ and the toe berm width (n).

Red arrows in Fig. 13 indicate the relationship considered in the example given below.

5. Application example

In this section, an example is given to design a rock toe berm placed on a $m = 1/10$ sea bottom combined with a SWL close to the crest of the toe berm ($h_{ss} \approx 2D_{n50}$); the recommended design value of $N_{od}* = 1$, given by Herrera and Medina (2015) for Eq. (6), is considered first. According to Eq. (6) and given a typical design storm for the Alboran Sea (H_{s0}(m) = 5, T_p(s) = 11, $(H_{s0}\,L_{0p})^{1/2}$ = 30.7 m and water depth at the nominal toe berm h_{ss}(m) = 4.5), the required rock size for a nominal toe berm ($B_t = 3D_{n50}$ and $t_t = 2D_{n50}$) is $D_{n50,3}$(m) = 2.23, which corresponds to 30-tonne rocks if the mass density is ρ_r(g/cm^3) = 2.70. In order to reduce the size of the required rocks (if not available at quarry), Eq. (9) with k = 0.4 is applied. When considering a double toe berm width (n = 6), the required rock size is reduced to $D_{n50,n}$(m) = 1.7 (13-tonne rocks). If only 6-tonne rocks are available at the construction site, a wider toe berm with n = 12 is required. Fig. 14 depicts the rock weight (W) in tonnes depending on the toe berm width (n). Rocks with W(t) = 30, 13, 8.0 and 5.7 (D_{n50} (m) = 2.23, 1.7, 1.44 and 1.28) could be used when considering toe berm widths having n = 3, 6, 9 and 12, respectively.

If $N_{od}*$ = 0.5 rather than $N_{od}*$ = 1.0 were considered as the design condition, the toe berms described above would withstand a design storm, $(H_{s0}\,L_{0p})^{1/2}$ = 25.0 m, which corresponds to a weaker design storm: H_{s0}(m) = 4 and T_p(s) = 10).

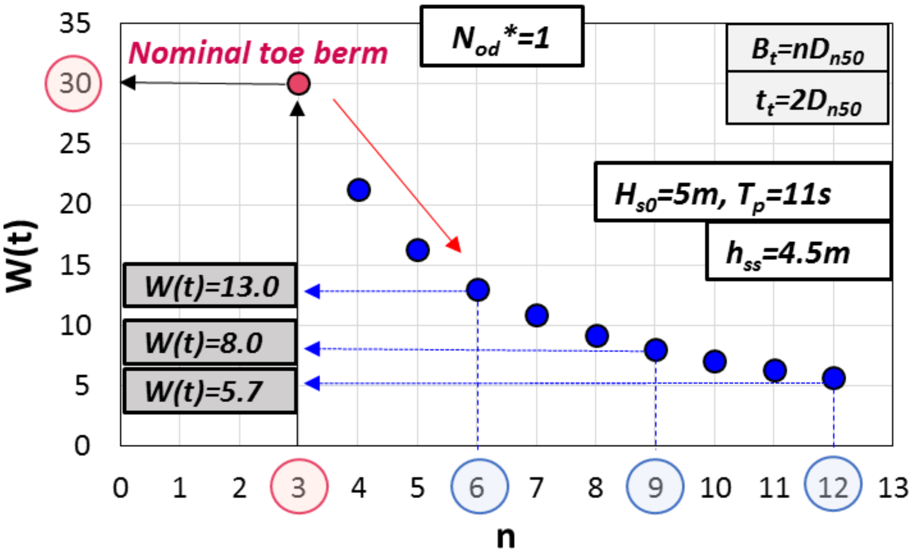

Fig. 14. Rock weight (W) depending on the toe berm width ($B_t = nD_{n50}$).

6. Comparison with existing formulas

As mentioned in Section 1, Van Gent and Van der Werf (2014) specifically introduced the toe berm width (B_t) as an explicative parameter of toe berm damage (N_{od}). For a certain amount of acceptable damage, the required rock size according to the study by these experts is given by Eq. (5), which was obtained from laboratory tests with a m = 1/30 bottom slope, and no severe depth-limited wave breaking. Two toe berm widths were considered ($B_t = 3D_{n50}$ and $9D_{n50}$), and only the total toe berm damage (N_{od}) was measured after each test. In order to consider that N_{od} increases with the toe berm width, these authors proposed multiplying the design N_{od} value by a factor f_B when $3D_{n50} < B_t \leq 9D_{n50}$.

$$f_B = \left(\frac{B_t}{3D_{n50}}\right)^{1/2} = \left(\frac{n}{3}\right)^{1/2} \tag{13}$$

Thus, when $B_t = 3D_{n50}$, Eq. (5) is directly applicable with $N_s = H_{st}/(\Delta D_{n50})$ and $D_{n50}=D_{n50,3}$. If $3D_{n50} < B_t = nD_{n50} \leq 9D_{n50}$ and $t_t = 2D_{n50}$, Eq. (5) may be rewritten as follows:

$$D_{n50,n} = 0.32\left(\frac{H_{st}}{\Delta\left(N_{od} f_B\right)^{1/3}}\right)\left(\frac{2D_{n50,n}}{H_{st}}\right)^{1/3}\left(\frac{nD_{n50,n}}{H_{st}}\right)^{0.1}\left(\frac{\hat{u}_\delta}{\sqrt{gH_{st}}}\right)^{1/3} \tag{14}$$

Eq. (14) is equivalent to Eq. (5) for n = 3. Manipulating Eqs. (5) and (14) and replacing f_B by the expression given by Eq. (13), the relation between the required nominal diameters for the design of a toe berm with $3 < n \leq 9$ and a nominal toe berm with n = 3, can be calculated using Eq. (15).

$$\frac{D_{n50,n}}{D_{n50,3}} = \left(\frac{3}{n}\right)^{1/6} \cdot \left(\frac{D_{n50,n}}{D_{n50,3}}\right)^{1/3} \cdot \left(\frac{nD_{n50,n}}{3D_{n50,3}}\right)^{0.1} = \left(\frac{3}{n}\right)^{\frac{2}{17}} \tag{15}$$

$D_{n50,n}/D_{n50,3}$ follows the potential relationship given by Eq. (8) but with the shape parameter k = 2/17 instead of k=0.4 used in Eq. (9). Fig. 15 shows the N_{od}* measured in this study and that estimated by Eq. (9) when using k = 2/17 rather than k = 0.4. Eqs. (5) and (14) given by Van Gent and Van der Werf (2014) are valid for toe structures placed on a m=1/30 bottom slope, with an armor slope cotα =2.0, $3D_{n50} \leq B_t \leq 9D_{n50}$, $2D_{n50} \leq t_t \leq 4D_{n50}$, $7D_{n50} < h_t \leq 25D_{n50}$, $1.2 \leq h_s/H_{st} \leq 4.5$ and $0.012 \leq s_{0p} \leq 0.042$. Eqs. (5) and (14) are beyond the range of variables tested in this study; this explains the poor agreement between the N_{od}* measured in this study and that estimated by Eq. (9) when using k = 2/17 (instead of k=0.4). Further research is required to test the range of variables not included in Van Gent and Van der Werf (2014) or in the present study (e.g. $3.5D_{n50}<h_s<8.6D_{n50}$ and $1/30<m<1/10$).

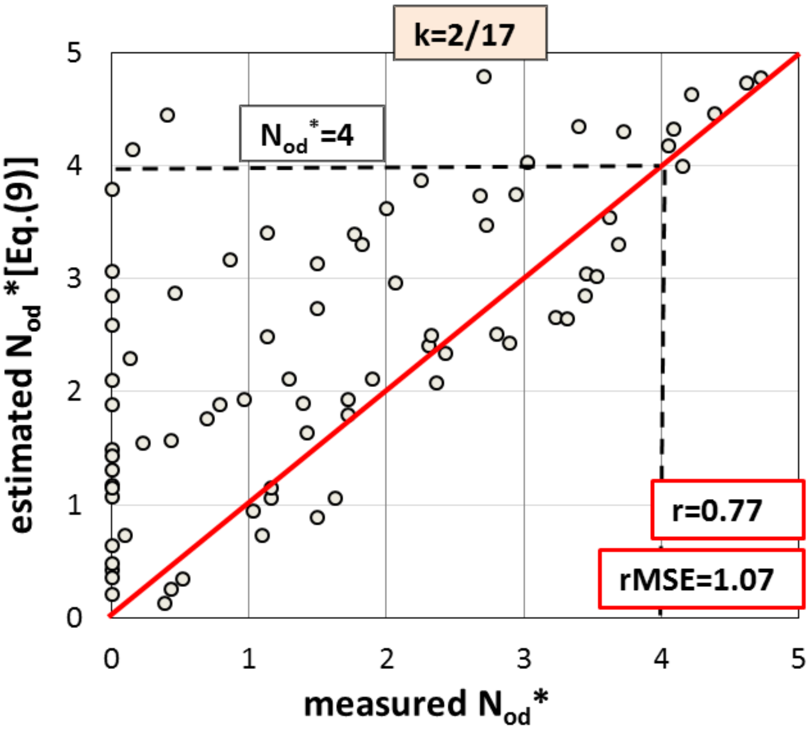

Fig. 15. Comparison of the N_{od}* measured in tests and that given by Eq. (9) using k = 2/17 rather then k = 0.4.

Thus, the parameter *k* depends on the test conditions. The divergence between $k = 2/17$ and 0.4 highlights the distinct performance of the toe berm when dealing with plunging waves breaking on a steep sea bottom ($m = 1/10$) combined with very shallow waters (as seen in the case of this study), or when dealing with gentler sea bottoms ($m = 1/30$) and no severe depth-limited wave breaking (as seen in Van Gent and Van der Werf, 2014). These two cases indicate that rock size and toe berm width should be considered together when designing a rock toe berm.

7. Summary and Conclusions

Although the hydraulic stability tests of toe berms reported in the literature consider different bottom slopes, ($1/50 \leq m \leq 1/10$), toe berm widths ($3D_{n50} \leq B_t \leq 9D_{n50}$) and toe berm thicknesses ($2D_{n50} \leq t_t \leq 8.8D_{n50}$), toe berm geometry is usually not taken as an explicative parameter of the toe berm damage. Only Van Gent and Van der Werf (2014) explicitly considered the influence of toe berm width ($B_t = nD_{n50}$) on toe berm stability. When considering wide toe berms ($n > 3$), common toe berm damage values ($0.5 \leq N_{od} \leq 4.0$) cannot be directly applied since more rock displacements are required to significantly damage wider toe berms.

This study proposes two new concepts to better characterize the hydraulic stability of wide toe berms ($3 < n \leq 12$): nominal and sacrificial toe berms. Two areas were distinguished for the toe berm: (1) the most shoreward area of the toe berm (nominal toe berm, $n=3$) which supports the armor layer and (2) the most seaward area (sacrificial toe berm) which protects the nominal toe berm. New physical tests were carried out at LPC-UPV with toe berms of different rock sizes (D_{n50}(cm) = 3.04, 3.99 and 5.12) and toe berm widths (n = 3, 5 and 12). Tests were conducted with a m = 1/10 bottom slope and a SWL close to the top of the berm ($1.5 \leq h_{ss}/D_{n50} \leq 2.6$). The toe berm damage was measured after each test considering: (1) the total toe berm damage (N_{od}), and (2) the damage to the nominal toe berm ($N_{od}*$). For wider toe berms ($n > 3$), $N_{od}*$ turned out to be a better descriptor of toe berm damage; $N_{od}*$ decreased when increasing the toe berm width (n). When using $N_{od}*$, recommended design values of conventional toe berm damage can be directly used ($0 \leq N_{od}* \leq 4$).

Given an acceptable level of damage to the nominal toe berm ($N_{od}*$) as a design condition, it is possible to significantly reduce the rock size (D_{n50}) by increasing the toe berm width (n) according to Eq. (8). For steep sea bottoms (m = 1/10) and shallow waters, this reduction in rock size showed an inverse 0.4-power relationship with the relative toe berm width. Using the formula given by Van Gent and Van der Werf (2014) with a gentle bottom slope m = 1/30 and a toe berm thickness $t_t = 2D_{n50}$, the reduction in rock size also followed Eq. (8) but showed an inverse 2/17-power relationship with the toe berm width. Thus, the shape parameter k = 2/17 and 0.4 given in Eq. (8) depends on the water depth and the sea bottom slope existing at the construction site, and it determines the breaking type and the wave impact affecting the toe berm.

To design toe berms placed on m = 1/10 bottom slopes, the rock size reduction may be especially important when the wave conditions are so adverse that it is not possible to find the required rock sizes at the construction site. Thus, the proposed method can be used in these cases for the design of rock toe berms within the ranges m=1/10, $3D_{n50} \leq B_t \leq 12D_{n50}$, $t_t = 2D_{n50}$, $1.5 \leq h_{ss}/D_{n50} \leq 2.6$, $0.02 \leq s_{0p} \leq 0.07$, $0.4 \leq h_{ss}/H_{s0} \leq 1.0$ and $0 \leq N_{od}* \leq 4$. The validity is limited to water depths close to the crest of the toe berm. Further research is required to examine the transition area from shallow waters with m=1/10 analyzed in this study, and the deeper waters and milder bottom slope tested by Van Gent and Van der Werf (2014). Also the effect of other slope angles and toe thicknesses should be investigated.

In shallow waters combined with steep sea bottoms (m=1/10), when using sacrificial toe berms, it is convenient to regularly monitor the toe berm. After severe storms, the sacrificial toe berm may be partially washed away and additional dumping of rocks at the toe may be necessary to continue providing full support to the armor layer.

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