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Additional Information

1 **DEVELOPING A NEW TOOL BASED ON A QUANTILE REGRESSION**
2 **MIXED-TGC MODEL FOR OPTIMIZING GILTHEAD SEA BREEM**
3 **(*Sparus aurata* L) FARM MANAGEMENT**

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15
16 **Abstract**

17 In this work, a seasonal quantile regression growth model for the gilthead sea bream (*Sparus aurata* L)
18 based on an aggregation of the quantile *TGC* models with exponent 1/3 and 2/3, named the “*Quantile*
19 *TGC-Mixed Model*”, is presented. This model generalizes the proposal of Mayer *et al.* (2012) in the sense
20 that the new model is able to describe the evolution of weight distribution throughout an entire production
21 cycle, which could be a powerful tool for fish farm management. The information provided by the model
22 simulations enables us to estimate total fish production and final fish size distribution, and helps to design
23 and simulate production and sales plan strategies considering the market price of different fish sizes, in
24 order to increase economic profits. The most interesting alternative in the studied case results in sending
25 all production when 0.25 quantile fish reach 600 g, although on each fish farm it would be necessary to
26 evaluate optimum strategy depending on its own quantile regression model, the production cost and the
27 market price.

28 **Keywords:** Modelling fish growth, quantile regression, growth in marine cages, Thermal Growth
29 Coefficient (*TGC*), fish farm management.

30 **1.- Introduction**

31 Optimization of fish farm management is necessary to maintain and increase production
32 profitability and ensure the sustainability of aquaculture. Aspects related with feeding
33 management are very important, but are usually studied in terms of nutrient levels,
34 ingredients and feeding rates, among other factors. Nevertheless, fish stock aspects such
35 as seasonal growth, density, optimum harvest size and weight dispersion, among others,
36 are largely unknown in Mediterranean marine species, although some studies have been
37 carried out (Gasca-Leyva, E. Hernández, J.M., Veliov, V.M., 2008; Araneda, M.E.;
38 Hernández, J.M., Gasca-Leyva, E., 2011, 2011; Araneda, M.E., Hernández, J.M.,
39 Gasca-Leyva, E., Vela, M.A., 2013; Dominguez-May, R., Hernández J.M., Gasca-
40 Leyva, E., 2011; Sánchez-Zazueta, E.; Hernández, J.M.; Martínez-Cordero, F.J., 2013).

41 In recent years, production of gilthead sea bream (*Sparus aurata* L) and sea bass
42 (*Dicentrarchus labrax* L) has increased and consequently the sale price has declined,
43 making it necessary to adjust production costs (feed, fingerling, labour, etc.) and
44 increase income. An alternative to improve income could be the added value of new
45 products (fillet, pre-cooking, etc.), but also through optimization of the production
46 process and stock management, for example by optimizing feed ingredients (Martínez-
47 Llorens, S., Tomas-Vidal, A., Jover, M., 2012), food rations (León, C. J., Hernández, J.
48 M., Gasca-Leyva, E., 2001), optimum stocking (Seginer & Halachmi, 2008) or
49 harvesting size in RAS (Seginer & Ben-Asher, 2011). In current offshore marine
50 systems, classification of fish by size is not actually possible, as at the end of the

51 production cycle a variability of sizes is obtained and the final income depends on the
52 percentage of fish size at harvesting, as the sale price depends on fish weight.

53 To estimate and optimize several management aspects, it is essential to have good
54 growth models adapted to each species and area of production. A large number of
55 papers in recent years (Baer, A., Schultz, C., Traulsen, I., Krieter, J., 2011; Dumas A.,
56 France, J., Bureau, D., 2007, 2010; Dumas & France, 2008; Libralato & Solidoro, 2008;
57 Mayer, P., Estruch, V.D., Blasco, J., Jover M., 2008, Mayer, P., Estruch, V.D., Martí,
58 P., Jover M., 2009; Moses, M. E., Hou, C., Woodruff, W. H., West, G. B., Nekola, J. C.,
59 Zuo, W., Brown, J. H., 2008; Seginer & Halachmi, 2008) aim to describe and predict
60 the growth of fish with different objectives. Almost all of them have in common that
61 only the dynamic of the average value of the time-dependent weight is described by
62 means of simple or multiple regression models, but weight dispersion is considered by
63 few authors; for example, Mayer *et al.* (2009) in gilthead sea bream with quantile
64 regression growth models, Hurtado-Herrera, M., Dominguez-May, R., Gasca-Leyva, E.,
65 (2013), in tilapia and Araneda *et al.* (2013) in white shrimp using characteristic growth
66 curves for different fish sizes.

67 The Thermal-unit Growth Coefficient (*TGC*) model was reported by Mayer *et al.* (2008,
68 2009) and Mayer, P., Estruch, V. D., Jover, M., (2012) in gilthead sea bream. When
69 determining the production conditions, the *TGC* model becomes an interesting
70 management tool for describing growth in marine farms in the western Mediterranean.
71 Mayer *et al.* (2012) establish two periods of growth for gilthead sea bream, using a
72 simple regression mixed model for the mean of the weight based on two *TGC* models
73 corresponding to two different exponents.

74 Except in recirculating systems, water temperature varies throughout the year, and for
75 this reason, following previous works (Ursin, 1963; Akamine, 1993; Moreau, 1987,
76 Fontoura & Agostinho, 1996; Hernández *et al.*, 2003; León, C.J., Hernández, J.M.,
77 León-Santana, M., 2006; Seginer & Halachmi; 2008; Dumas & France, 2008), Mayer *et*
78 *al.* (2012), included a sinusoidal temperature curve in the growth models to simulate the
79 seasonal *TGC* growth.

80 As mentioned above, in most of the studies that explore weight dynamics using
81 mathematical models a simple description of the evolution of the mean weight at a
82 given time interval is considered, which is acceptable as a reasonable exercise of
83 simplification. However, in aquaculture the starting point is an initial population of fish
84 provided by the hatchery whose weight follows a statistical distribution, which can be
85 estimated from representative initial samples. It is undisputable that in-depth knowledge
86 of various sizes in a batch at the end of the cycle would facilitate management in the
87 aquaculture farm and knowledge of sizes could be obtained from the statistical
88 distribution of the weight. Therefore, it seems reasonable to describe the evolved body
89 weight distribution. Thus, in the event of achieving a good description of the changes in
90 weight distribution versus time, a complete statistical description of the weight would
91 be available at any time, and not only a simple average value.

92 Quantile regression (Koenker & Bassett, 1978) helps estimate the evolution of the
93 growth data distribution and is very suitable for analysing data in contexts characterized
94 by heteroscedasticity, such as reference charts in medicine, survival analysis, financial
95 and economics research or environment modelling (Yu, K., Lu & Z., Stander, J., 2003;
96 Vaz, S., Martin, C. S., Eastwood, P. D., Ernande, B., Carpentier, A., Meaden, G. J., &
97 Coppin, F., 2008). Linear quantile regression estimates multiple rates of change,
98 providing more complete information about the relationships between variables than

99 that obtained from linear least square regression (Cade & Noon, 2003). Quantile
100 regression has proved to be a powerful tool for the detection of different growth patterns
101 caused by environmental conditions and the characteristics of the fish population
102 provided by the hatcheries (Mayer *et al.* 2009).

103 The aim of the current paper consists of developing a quantile regression approach to
104 the gilthead sea bream growth in commercial production conditions, based on previous
105 work presented in Mayer *et al.* (2012). Quantile regression is a radically different and
106 alternative approach compared with the least-squares regression approach. So, a new
107 study of *TGC* models is required, with different powers and focused on suitability of the
108 mixed model from the quantile regression. Using a simulation model based on the *TGC*-
109 Mixed model, which considers the different stages throughout the growth period and the
110 local sea water temperature curve, a dynamic management tool for fish farms will be
111 test to improve fish stock growth estimates, optimizing production and maximizing
112 profits considering the market sale price of fish.

113 **2.- Material and Methods**

114 **2.1. Mathematical models**

115 The continuous and linearized growth model used by Mayer *et al.* (2012) was
116 considered:

$$117 \quad W^b(t) = W_0^b + TGC_b \cdot ST(t_0, t). \quad (1)$$

118 where $ST(t_0, t)$ (sometimes written ST for simplicity) represents the accumulated
119 effective temperature (°C) in the time interval $[t_0, t]$ (days), $ST(t_0, t_0)=0$, W_0 (g) is the

120 initial weight (when $t=t_0$) and b is a dimensionless parameter which takes the values b
 121 $=1/3$ and $b=2/3$.

122 The function that provides the water temperature at each time, $T(t)$ ($^{\circ}\text{C}$), is equal to the
 123 derivative with respect to time t of $ST(t_0, t)$, i.e. $dST(t_0, t)/dt=T(t)$. Considering for each
 124 day, i , $i=1, 2, \dots, n$, the mean of the daily temperature, T_i , an immediate discrete-time
 125 version of (1), is obtained (2)

$$126 \quad W_n^b = W_0^b + TGC_b \cdot \sum_{i=1}^n T_i, \quad n = 1, 2, \dots \quad (2)$$

127 If $b=1/3$, then we have the designated TGC model (Cho, 1992). Note that the function
 128 $T(t)$ can take different expressions depending on environmental conditions (Akamine,
 129 1993).

130 The equation (1) could be expressed in an equivalent integral form

$$131 \quad W^b(t) = W_0^b + k \cdot b \cdot \int_{t_0}^t T(t) \cdot dt \quad (3)$$

132 i.e.

$$133 \quad W(t) = \left(W_0^b + k \cdot b \cdot \int_0^t T(t) dt \right)^{\frac{1}{b}} \quad (4)$$

134 In the case of marine farms in Mediterranean fixed locations, the time-dependent water
 135 temperature develops according to regular annual periods of 365 days modelled by the
 136 function $T(t)$ ($^{\circ}\text{C}$) (Mayer *et al.*, 2012)

$$137 \quad T(t) = T_m + T_D \cdot \sin\left(\frac{2\pi}{365} \cdot (t - \alpha)\right) \quad (5)$$

138 , where $t \geq 0$, and T_m (°C) is the average annual temperature, T_D (°C) is the amplitude
 139 and α is a tuning parameter. The fitted values for the parameters of the temperature
 140 function $T(t)$, described in (5), corresponding to the sea area where the studied marine
 141 farm is located, are $T_m=18.8525$, $T_D=-6.6997$ and $\alpha=312.4609$ and, in the case of
 142 gilthead sea bream, it is more appropriate to use the effective temperature, $T(t)-12$,
 143 instead of $T(t)$ (Mayer *et al.*, 2012). So, the weight at the day t , $W(t)$ (g), is obtained

$$144 \quad W(t) = \left(W_0^b + TGC_b \cdot \left((T_m - 12) \cdot (t - t_0) - T_D \frac{365}{2\pi} \left(\cos\left(\frac{2\pi(t - \alpha)}{365}\right) - \cos\left(\frac{2\pi(t_0 - \alpha)}{365}\right) \right) \right) \right)^{\frac{1}{b}} \quad (6)$$

145 From the basic model described by the equation (6), three quantile regression models
 146 were developed to simulate the indeterminate seasonal growth of gilthead sea bream.
 147 Two of them were obtained by fitting the data to equation (2), assuming the values
 148 $b=1/3$ and $b=2/3$ (as in Mayer *et al.*, 2012), and the quantiles 0.025, 0.05, 0.10, 0.20,
 149 0.25, 0.30, 0.40, 0.50, 0.60, 0.70, 0.75, 0.80, 0.90, 0.95 and, 0.975. The third model was
 150 built by aggregation of the two models mentioned before, establishing two stages of
 151 growth for each quantile.

152 **2.2. Data description, statistical analysis and design of the models**

153 Models have been developed considering actual data on final weight and its
 154 corresponding actual values of accumulated effective temperature, starting at the
 155 beginning of the cycle from several samples corresponding to 20 batches of farmed
 156 gilthead sea bream in real conditions of growth (Mayer *et al.* 2008, 2009, 2012). More
 157 specifically, the actual weights of 22805 fish were used for the fitting. Sampling and
 158 biometrics were performed at various times in the course of the production cycle in each
 159 batch.

160 With the actual paired data available (sum of effective temperatures and weight), the
 161 quantile regression fitting was performed considering the discrete model (7)-(8)

$$162 \quad W_{f,\tau} = (A_{b,\tau} + TGC_{b,\tau} \cdot ST)^{\frac{1}{b}} \quad (7)$$

163 i.e.

$$164 \quad W_{f,\tau}^b = A_{b,\tau} + TGC_{b,\tau} \cdot ST \quad (8)$$

165 with $b=1/3$ and $b=2/3$. Values for $A_{b,\tau}$ and $TGC_{b,\tau}$ were estimated by means of quantile
 166 regression, using the “*quantreg*” procedure available in package R (Koenker, 2008), for
 167 the quantiles $\tau = 0.025, 0.05, 0.10, 0.20, 0.25, 0.30, 0.40, 0.50, 0.60, 0.70, 0.75, 0.80,$
 168 $0.90, 0.95$ and, 0.975 . According to the generic model (7)-(8) and (2), values $A_{b,\tau}$
 169 correspond to estimations of initial weight raised to the power b , $W_{0,\tau}^b$, $b=1/3, b=2/3$,
 170 which should be approximately equal to the corresponding percentiles obtained by
 171 analysing the weight distribution corresponding to the start of cycle in the case of a
 172 good fit. These two models, $b=1/3, b=2/3$, are integrated into one by computing the
 173 critical points of change in the growth dynamic, in a similar way to that described in
 174 Mayer *et al.* (2012). The non-zero solutions for W in (9) for the different τ quantiles are
 175 theoretical critical values of the weight in which the instantaneous rate of change, in
 176 terms of weight depending on accumulated temperature, is the same for both models.

$$177 \quad \frac{TGC_{1/3,\tau} \cdot W^{2/3}}{1/3} = \frac{TGC_{2/3,\tau} \cdot W^{1/3}}{2/3} \quad (9)$$

178 So, the hypothesis is assumed that in the critical values of the weight a smooth
 179 transition from the dynamic described by the model given by (7) with $b=1/3$ to the
 180 dynamic described by the model with $b=2/3$ occurs.

181 Once the values $W_{0,\tau}$ and $TGC_{b,\tau}$ have been computed to estimate the evolution of the
182 weight distribution of gilthead sea bream, two simulation models were considered for
183 each quantile from equation (6) with $b=1/3$ and $b=2/3$, and assuming the temperature
184 function, $T(t)$, given in (5). These models were designated the seasonal quantile 1/3-
185 *TGC* model and the seasonal quantile 2/3-*TGC* model, respectively.

186 From the seasonal quantile models 1/3-*TGC* and 2/3-*TGC*, taking into account the
187 critical values of the weight obtained previously, the definitive simulation model,
188 named quantile seasonal *TGC*-Mixed model was built by aggregation.

189 The values $A_{b,\tau} = W_{0,\tau}^b$ and $TGC_{,\tau}$, obtained by means of quantile regression, after
190 linearization (Eq. 8), for models with $b=1/3$ and $b=2/3$, are shown in Table 1. It is
191 necessary to remark that the value $A_{2/3,0.025}=-0.517<0$ is unacceptable because
192 $A_{2/3,0.025}=W_{0,0.025}^{2/3}$ and the result of squaring a number cannot be negative.

193 Table 2 shows the quantile critical weight values of the weight, $W_{c,\tau}$, and the sum of
194 effective temperatures, ST_{τ} , at which the critical weight values are reached.

195 Fig. 1 shows the actual data (black points) and the graph of the fitted quantile models
196 for $\tau=0.025, 0.05, 0.25, 0.50, 0.75, 0.95$ and 0.975 . The dashed line corresponds to the
197 quantile $\tau=0.50$. The *TGC*-1/3 model, the *TGC*-2/3 model and the *TGC*-Mixed model
198 are represented in Fig. 1 a, b and c, respectively. Note that Fig. 1 a shows clearly that
199 the *TGC*-1/3 model tends to overestimate weight as from a certain moment in the
200 growth cycle.

201 To obtain the quantile *TGC*-Mixed model, the quantile *TGC*-1/3 model and the quantile
202 *TGC*-2/3 model are coupled for each quantile, τ , in the critical weight values which are

203 obtained considering that instantaneous growth rates based on the cumulative effective
 204 temperature must be the same for the quantile *TGC*-1/3 models and the quantile *TGC*-
 205 2/3 models. The non-zero critical values of weight are obtained by solving $W_{c,\tau}$ in (9):
 206 $W_{c,\tau} = 1/8 (TGC_{2/3,\tau} / TGC_{1/3,\tau})$ g. So, the *TGC*-1/3 model is considered until the weight
 207 reaches the critical weight. From that moment on, the *TGC*-2/3 model is considered for
 208 estimating the weight, assuming the critical weight as the initial weight. This coupling
 209 of the *TGC*-1/3 and *TGC*-2/3 gives rise to the *TGC*-Mixed model. Fig. 1 b justifies that
 210 the *TGC*-2/3 model explains the growth better than the *TGC*-1/3 model, starting from a
 211 certain point in the growth period, and this property is inherited by the *TGC*-Mixed
 212 model in Fig. 1 c.

213 Valuation of the goodness of fit is not immediate in the case of quantile regression. So,
 214 the overall analysis of the quantile model's goodness of fit was approached from
 215 various angles. On one hand, by computing the coefficient $R^1(\tau)$, $\tau \in]0,1[$ (Koenker &
 216 Machado, 1999), which is a natural analogue of Coefficient of Determination, R^2 . While
 217 R^2 is a global measure of goodness of fit in terms of residual variance, $R^1(\tau)$ is a local
 218 measure of goodness of fit for a particular quantile τ , and measures the relative success
 219 of the model at a specific quantile, in terms of an appropriately weighted sum of
 220 absolute residuals. The value of $R^1(\tau)$, $\tau \in]0,1[$ also lies between 0 and 1, and
 221 considering $R^1(\tau)$ as a function of τ , we can obtain a global measure of goodness of fit
 222 of the quantile regression in the range $\tau \in [\tau_0, \tau_1] \subset]0, 1[$, by means of the average
 223 value of the function $R^1(\tau)$ in the interval $[\tau_0, \tau_1]$, $R^1_{[\tau_0, \tau_1]}$, which is computed as is
 224 shown in (10):

225
$$R^1_{[\tau_0, \tau_1]} = \frac{\int_{\tau_0}^{\tau_1} R^1(\tau) d\tau}{\tau_1 - \tau_0} \quad (10)$$

226 In practice, we only have discrete information from the functions $R^1(\tau), \tau \in [\tau_0, \tau_1]$ for
 227 the three models, which correspond to the specific τ values considered in the quantile
 228 regression fit. So, we compute approximations to the $R^1_{[0.025, 0.975]}$ coefficients for the
 229 three models, approximating the integrals (10) numerically by means of the trapezoids
 230 method.

231 The goodness of fit valuation is completed by comparing the distribution of the initial
 232 weights of fishes provided by the hatchery and those obtained in the last sampling, with
 233 the theoretical distributions deduced from the quantile model. By means of Pearson's
 234 Chi-square test, the discrepancy between the observed and the theoretical distributions
 235 are evaluated, indicating whether the differences between the two distributions, if any,
 236 are due to chance. It is interesting to examine the value of chi-square statistic χ^2 , which
 237 allows us to assess the degree of similarity between the theoretical distribution and the
 238 empirical distribution deduced from the actual data. The fit between the two
 239 distributions is greater if the value of the statistic χ^2 is smaller. In addition, the p -value
 240 lets us assess whether the hypothesis that the two distributions could really be the same
 241 is acceptable.

242 The outcome is that the *TGC-Mixed* model is the one which best fits the actual
 243 evolution of the weight distribution depending on the cumulative effective temperature,
 244 taking into account the different goodness of fit results. To analyse the goodness of fit
 245 in all three models, the $R^1(\tau)$ values for the considered quantiles and the average value

246 for the function R_{τ}^1 , $\tau \in [0.025, 0.975]$, denoted $R_{[0.025, 0.975]}^1$, were computed (Table 3).

247 The global valuation of the goodness of fit given by $R_{[0.025, 0.975]}^1$ is similar for the 2/3
248 and the *TGC-Mixed* models, and clearly worse for the 1/3 *TGC*-model.

249 The assessment of the goodness of fit is completed by comparing the actual weights of
250 the initial samplings and the last actual sampling weights (represented by the quantiles
251 computed from the data) with the theoretical quantiles of the initial and final weights
252 provided by the models, respectively (Table 4 and Table 5). From Table 4, we can
253 accept that the estimated distribution of the initial weight, obtained from the 1/3 *TGC*
254 model quantile regression fitting (which coincides initially with the *TGC Mixed* model),
255 is the same as the initial distribution of weight deduced from the actual data (p -
256 value=0.99). On the other hand, we can reject that the estimated distribution of the
257 initial weight, obtained from the 2/3 *TGC* model quantile regression, is the same as the
258 initial weight distribution deduced from the actual data (p -value<0.0001).

259 The analysis of the weight distribution of the last sample is summarized in Table 5,
260 which shows that we reject the hypothesis that the estimated distribution for the final
261 sample is the same as the weight distribution deduced from the actual data for the three
262 models. However, although that the p -values would be especially interesting as
263 indicators of the level of match between distributions, the Chi-Square value provides a
264 good measure to establish which model better fits the actual distribution. When
265 observing Chi-square values in table 5, we may deduce that the *TGC-Mixed* model is
266 the one that better fits the distribution corresponding to the last sample of weights.
267 Table 5 also shows that the *TGC-Mixed* model is the best fitting central quantiles and
268 that *TGC 1/3* model overestimates the weight with respect to the last sample of actual
269 data for all quantiles, but mainly for the upper quantiles. From the above results, it can

270 be deduced that the *TGC-Mixed* model is the one that best fits the evolution of the
 271 weight distribution depending on the cumulative effective temperature.

272 To simulate the growth of gilthead sea bream, two seasonal quantile regression models
 273 based on equation (6) are established: the seasonal quantile *TGC-1/3* model ($b=1/3$) and
 274 the seasonal quantile *TGC-2/3* model ($b=2/3$). Next, from the former models, *TGC-1/3*
 275 and *TGC-2/3*, we constructed the seasonal quantile *TGC-Mixed* model:

$$276 \quad W_{f,\tau}(t) = \left(W_{0,\tau}^{\frac{1}{3}} + TGC_{1/3,\tau} \cdot ST(t_0, t) \right)^3, \quad \text{if } W_{f,\tau}(t) < W_{c,\tau} \quad (11)$$

$$277 \quad W_{f,\tau}(t) = \left(W_{0,\tau}^{\frac{2}{3}} + TGC_{2/3,\tau} \cdot ST(t_0, t) \right)^{\frac{3}{2}}, \quad \text{if } W_{0,\tau}(t) \geq W_{c,\tau} \quad (12)$$

278 For each quantile, τ , to estimate final weights greater than the critical weight, $W_{c,\tau}$, we
 279 consider the model curve corresponding to the *TGC-1/3* model until it reaches the
 280 critical weight (Table 2), and following that moment, considering the critical weight as
 281 the initial weight and resetting the initial time in the cumulative temperature function
 282 ST , the final weight will be estimated using the curve corresponding to the *TGC-2/3*
 283 model.

284 Therefore, for each quantile τ , up to a final weight less than $W_{c,\tau}$, the *TGC-Mixed* model
 285 coincides with the *TGC-1/3* model. In the case of an initial weight greater than or equal
 286 to $W_{c,\tau}$ g, the *TGC-Mixed* model coincides with the *TGC-2/3* model. The *TGC-Mixed*
 287 model leads to a continuous curve representing the final weight for the considered
 288 quantiles of gilthead sea bream. Moreover, the curves are also differentiable at all times
 289 because the *TGC-Mixed* model is constructed so that when the weight is exactly $W_{c,\tau}$ g,
 290 the derivatives of the functions that define the models *TGC-1/3* and *TGC-2/3* coincide.

291 Thus, the transition from the quantile *TGC*-1/3 model to the quantile *TGC*-2/3 model
292 occurs smoothly, without sharp points.

293 The quantile regression fit for the *TGC*-1/3 model and *TGC*-2/3 model provides two
294 values for each quantile: the initial weight and the value of the *TGC*. Therefore, when
295 considering the different quantiles, each model provides an empirical distribution of the
296 initial weight (Table 4). The fit of the data to the *TGC*-1/3 Model provides a theoretical
297 well fitted distribution of the initial weights of the fish supplied by the hatchery, i.e. the
298 theoretical distribution practically coincides with that deduced from the analysis of the
299 samples corresponding to the beginning of the cycle. Moreover, the initial weight
300 distribution deduced from the quantile 2/3-Model is not consistent with the actual initial
301 weights. From an inferential approach, it could be interesting to prove that the *TGC*
302 values corresponding to the different quantiles are significantly different, from the
303 statistical point of view. If the *TGC* corresponding to different quantiles is not different,
304 the growth only depends on the initial weight. But this is not the case. Using the
305 ANOVA test proposed in Koenker & Bassett (1982) we studied the behaviour of the
306 quartiles ($\tau=0.25, 0.50$ and 0.75) and found that the differences for the *TGC*
307 corresponding to the quartiles are statistically significant for the *TGC*-1/3 model (p -
308 value <0.000) and for the *TGC*-2/3 model (p -value <0.000). So, to describe the evolution
309 of the weight distribution over time, we need to know the distribution of the initial
310 weight and the *TGC* values associated with the different quantiles. A smaller Chi-square
311 value in tables 4 and 5 means that the distributions are more similar. The p -value
312 indicates to what extent it would be reasonable to accept the hypothesis that the
313 distributions compared are identical. By and large, it is desirable that the actual and the
314 estimated distributions should coincide at the beginning of the cycle, and moreover that
315 the actual and estimated distributions at the end of the cycle are compatible, i.e. do not

316 differ too greatly. In this sense, the excellent goodness of fit of the initial weights
317 provided by the quantile *TGC*-1/3 Model is a fundamental aspect inherited by the mixed
318 model. On the other hand, the quantile *TGC*-2/3 model provides a better overall fit
319 compared to that obtained from the quantile *TGC*-1/3 model, as the *TGC*-2/3 model fits
320 the weights better at later stages of the cycle, which is not only evident observing the
321 Fig. 1, but also from the values $R_1(\tau)$ and $R_{[0.025,0.975]}^1$ (Table 3). This positive feature of
322 the quantile 2/3-Model is also inherited by the mixed model. Therefore, we can say that
323 the quantile *TGC*-Mixed model captures the best features of the *TGC*- 1/3 and *TGC*-2/3
324 models.

325 Note the great importance of a good fit of the model to the sample distribution at the
326 outset, as the model should explain the generic distribution of the weight of the fishes
327 provided by the hatchery. On the other hand, requiring an excellent fit of the weight
328 distribution obtained from the model to the final sample has relative importance.
329 Obviously, the actual weights at the end of the cycle may be above or below
330 expectations. To validate the model from the point of view of the weight distribution at
331 the end of the cycle, it would be reasonable to see that the results are within what the
332 experience of the marine farm indicates as reasonable margins for the actual production,
333 which is sufficiently justified for the *TGC*-2/3 and the *TGC*-Mixed models (Table 5). In
334 summary, taking into account the results for establishing the goodness of fit for the
335 three models, the *TGC*-Mixed model is the best model for explaining the growth over
336 the entire production cycle.

337 **3.- Results and discussion**

338 In a similar way as in Mayer *et al.* (2012), the development of the quantile *TGC*-Mixed
339 model indicates a range of weights in which a change in the growth dynamic should be

340 considered. In 95% of the cases, in the time period when the sum of effective
341 temperatures is in the interval that goes from 1144 °C to 1642 °C, i.e. when the weights
342 are in the interval from 70.7 to 189.17 g, the growth dynamic changes. Moreover, the
343 change occurs earlier for larger fish than for smaller ones (see Table 2). The results
344 agree with those obtained in Mayer *et al*, 2012, where a weight around 117g is
345 suggested for establishing a point in the change of the dynamics of the evolution of the
346 mean of the weights from the *TGC-1/3* model to the *TGC-2/3* model.

347 We developed a MATLAB® script, which allows us to make simulations introducing a
348 start date of the cycle and an end date as the inputs. As an example of growth simulation
349 using the quantile *TGC-Mixed* model, the evolution of growth distribution for several
350 batches (starting in mid-March, mid-May, mid-July and mid-September, i.e. at days 75,
351 136, 197 and 259, respectively, of a year considering that January 1 is the day 1) is
352 shown (Fig. 2). In all cases, for a better view of the behaviour, the figures represent the
353 period until the curve corresponding to the 0.20 quantile reaches a weight equal to 500
354 g.

355 The curves obtained from simulation of the quantile *TGC-Mixed* model are similar to
356 those used in paediatrics to assess the growth of children, except that in our case we
357 obtain the curves adapted to the starting date of the production cycle, i.e. the starting
358 date of growth.

359 Note that it would be necessary for each fish farm to dispose of its own and
360 characteristic quantile growth curves, which would be obtained on the basis of local
361 temperature and data from historical growth in many batches. A continuous feedback
362 process could be considered for improving the curves by adding information on growth
363 of new batches. For correct use of the quantile model by the fish farm manager, the

364 quantile curves could serve as a reference to evaluate the general evolution in time of
365 the weight distribution of a batch on the farm. For example, growth can be assessed by
366 comparing the relative position of the quantiles of a sample with those deduced from the
367 curves.

368 Knowledge of the weight distribution at any time allows us to obtain approximations of
369 any statistical value, which is an important tool for fish farm managers. To this end, it
370 should be necessary to consider mainly the curves corresponding to core values
371 (quantiles 0.25-0.75), because the lowest and highest curves, corresponding to the
372 lowest and highest quantile values, respectively, represent the extreme variability of
373 production in the plant. In Fig. 3, a simulation of several gilthead sea bream batches was
374 performed, starting in March, May, July and September, considering the different
375 weight intervals when the three quantiles, 0.25, 0.50 and 0.75 reach the weights 400,
376 500 and 600 g, which will allow the evaluation of several alternatives and sale values. It
377 can be observed that there are practically no differences in the final distribution of the
378 final weights for different simulated batches, in each interval of weight and quantile.
379 Nevertheless, there are important differences for the number of days that the quantiles
380 need to reach, for example, 400, 500 or 600 grams (Fig. 4). It can be seen that fish in
381 quantile 0.75 take less time (407, 466 or 531 days respectively, as average of simulated
382 four batches) than those belonging to quantile 0.50 (443, 524 or 602 days), or quantile
383 0.25 (508, 587 or 662 days). If technology for size sieving in marine cages was
384 available, big fish from the 0.75 quartile could be sold first and fish from the 0.25
385 quartile last, optimizing fish management and achieving economic benefits, but this is
386 currently not possible, so fish farm managers have to decide the time for harvesting.
387 Obviously, the production cost is reduced when the growth cycle is shorter, but the total

388 fish biomass sold and sales income will also be lower, so an equilibrium point for
389 optimizing profit must be evaluated.

390 In Table 6, an estimation of final biomass and economic value is shown in each weight
391 interval, which was developed for different alternatives, mean fish weight (400, 500 and
392 600 g) and quantiles (0.25, 0.50 and 0.75), considering the average of four batches. For
393 biomass estimation, we considered 300000 fish per cage and 85% survival, if we have
394 an estimation of the total number of fish, for each day of the cycle, the quantiles curves
395 allows us knowing the biomass corresponding to any quantile, the biomass
396 corresponding to each interval of weights and obviously the total biomass. For
397 economic value, we obtained the sales price of different sea bream sizes from a
398 commercial fish farm in the Mediterranean Sea.

399 Fish production increases with fish weight in each quantile, and also the total sales
400 value, and both are highest for the 0.25 quantile in relation to the 0.50 and 0.75
401 quantiles, as when 0.25 quantile fish reach the fish mean weight (i.e. 400, 500 and 600
402 g), there are many bigger fish (Fig. 3). The higher production figures (174 tons) and
403 value (897 thousand euros) are obtained by considering quantile 0.25 and 600 g, and
404 lower production (87 tons) and value (372 thousand euros) by alternatively considering
405 quantile 0.75 and 400 g. Obviously, the time for growth until the fish reach a higher
406 weight, 662 and 407 days respectively, entails a higher production cost, so a new
407 approach becomes necessary.

408 Income value of sales in relation to days and production volume are presented in Fig. 5.
409 Daily value (€day^{-1}) increases in large fish for all quantiles, being maximum for 0.25
410 quantile, and a similar trend is obtained for total value per unit of production (€Kg^{-1}
411 fish). Nevertheless, daily value per production ($\text{€Kg}^{-1} \text{ fish day}^{-1}$) is opposite, and high

412 values are obtained by considering quantile 0.75 and 400 g, and lower by alternatively
413 considering quantile 0.25 and 600 g.

414 Thus, sale strategies could be designed with the aim of sending fish in the interval 400-
415 500, 500-600 or 600-700 g, when these weights are reached, first by the 0.75 quantile,
416 then by the 0.50 quantile and finally by the 0.25 quartile (Table 7). It seems that the
417 most profitable alternative would be to send 158 tons of fish weighing 600-700 g, with a
418 sale income of 752 thousand euro, but if the market accepts a great variability of fish,
419 from 200 to 800 g, the maximum income would be when the 0.25 quantile reaches 600
420 g (Table 6), because the sent biomass has reached a maximum, 174 tons, and the
421 income would be around 897 thousand euro.

422 Nevertheless, the selected strategy should be applied on each fish farm, taking
423 production cost and the market price into account, because if the value of small or large
424 fish was lower, the economic results could be very different. For example, Seginer &
425 Ben-Asher (2011) reported an increase in sale price related with gilthead sea bream
426 weight and although production cost in a RAS system also rose with fish size, the profit
427 was higher as the harvest size increased.

428 **5.- Conclusions**

429 The quantile regression *TGC*-Mixed model specifically developed for the plant provides
430 a good global representation of the variability of fish growth in the fish farm over the
431 entire production cycle. Thus, the growth model allows simulations of growth,
432 providing the variability of the weight throughout the production cycle and values
433 closer to reality of the total biomass, and its size distribution which is the most
434 important. The information obtained from the growth simulation provided by the model
435 is very powerful because it allows us to design and simulate sales plans taking the sale

436 price into consideration, with a view to optimizing management and economic profits
437 on each fish farm.

438

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441

442 **References**

443 Akamine, T. (1993). A New Standard Formula for Seasonal Growth of Fish in
444 Population Dynamics, *Nippon Suisan Gakkaishi*, 59 (11), pp. 1857-1863.

445 Araneda, M.E.; Hernández, J.M., Gasca-Leyva, E. (2011). Optimal harvesting time of
446 farmed aquatic populations with nonlinear size-heterogeneous growth. *Natural*
447 *Resource Modeling* 24 (4), pp. 477-513.

448 Araneda, M.E., Hernández, J.M., Gasca-Leyva, E., Vela, M.A. (2013). Growth
449 modelling including heterogeneity: Application to the intensive cultura of white shrimp
450 (*P. vannamei*) in freshwater. *Aquacultural Engineering*, 56, pp. 1-12.

451 Baer, A., Schultz, C., Traulsen, I., Krieter, J. (2011). Analising the growth of turbot
452 (*Psetta maxima*) in a commercial recirculation system with the use of three different
453 growth models. *Aquacult. Int.*, 19, pp. 497-511.

454 Cade, B.S., Noon, B.R. (2003). A gentle introduction to quantile regression for
455 ecologists. *Frontiers in Ecology and the Environment*, 1, pp.412-420.

456 Cho, C.Y. (1992). Feeding systems for rainbow trout and other salmonids with
457 reference to current estimates of energy and protein requirements, *Aquaculture*, 100, pp.
458 107-123.

459 Dominguez-May, R., Hernández J.M., Gasca-Leyva, E. (2011). Effect of ration and size
460 heterogeneity on harvest time: Tilapia culture in Yucatan, Mexico. *Aquaculture*
461 *Economics and Management*, 15 (4), pp. 278-301.

462 Dumas A., France, J., Bureau, D. (2007). Evidence of three growth stanzas in rainbow
463 trout (*Oncorhynchus mykiss*) across life stages and adaptation of the thermal-unit
464 growth coefficient, *Aquaculture*, 267, pp. 139-246

465 Dumas, A. and France, J. (2008). Modelling the ontogeny of ectotherms exhibiting
466 indeterminate growth, *Journal of Theoretical Biology*, 254, pp. 76-81.

467 Dumas, A., France, J., Bureau, D. (2010). Modelling growth and body composition in
468 fish nutrition: where have we been and where are we going?, *Review Article*
469 *Aquaculture Research*, 41, pp. 161-181

470 Fontoura, N.F. and Agostinho, A.A. (1996). Growth with seasonally varying
471 temperatures: an expansion of the von Bertalanffy growth model. *Journal of Fish*
472 *Biology* 48(4), pp. 569–584.

473 Gasca-Leyva, E. Hernández, J.M., Veliov, V.M. (2008). Optimal harvesting time in a
474 size-heterogeneous population. *Ecological Modelling* 210, pp. 161-168.

475 Hurtado-Herrera, M., Dominguez-May, R., Gasca-Leyva, E. (2013). Efecto de la
476 estructura de tallas bajo un modelo dinámico de población utilizando curvas
477 características. *Abstraction and Application* 9, pp 11-18.

- 478 Koenker R. and Bassett, G. (1978). Regresión cuantiles. *Econometrica*, 46, pp. 33–50
- 479 Koenker, R. and Bassett, G. (1982). Robust tests for heteroscedasticity based on
480 regression quantiles. *Econometrica* 50, pp. 43–61.
- 481 Koenker, R. and Machado, J. A. F. (1999). Goodness of fit and related inference
482 processes for quantile regression. *Journal of the American Statistical Association*, 94
483 (448), pp. 1296–1310.
- 484 Koenker, R. (2008). quantreg: Quantile Regression. *R package version 4.24*.
485 <http://www.r-project.org>
- 486 León, C. J., Hernández, J. M., Gasca-Leyva, E. (2001). Cost optimization and input
487 substitution in the production of gilthead seabream. *Aquaculture Economics and*
488 *Management* 5, pp. 147-171.
- 489 León, C.J., Hernández, J.M., León-Santana, M. (2006). The effects of water temperature
490 in aquaculture management. *Applied Economics*, 38(18), pp. 2159–2168.
- 491 Libralato, S. and Solidoro, C. (2008). A bioenergetic growth model for comparing
492 *Sparus aurata*'s feeding experiments. *Ecological Modelling*, 214(2-4), pp. 325-337.
- 493 Martínez-Llorens, S., Tomas-Vidal, A., Jover, M. (2012). A new tool for determining
494 the optimum fish meal and vegetable meals in diets for maximizing the economic
495 profitability of gilthead sea bream (*Sparus aurata*, L.) feeding. *Aquaculture Research*,
496 43, pp. 1697-1709.
- 497 Mayer, P., Estruch, V.D., Blasco, J., Jover M. (2008). Predicting growth of gilthead sea
498 bream (*Sparus aurata*) in marine farms under real productions conditions using
499 temperature and time-dependent models. *Aquaculture Research*, 39, pp. 1046-1052.

500 Mayer, P., Estruch, V.D., Martí, P., Jover M. (2009). Use of quantile regression and
501 discriminant analysis to describe growth patterns in farmed gilthead sea bream (*Sparus*
502 *aurata*). *Aquaculture* 292, pp. 30-36.

503 Mayer, P., Estruch, V. D., Jover, M. (2012). A two-stage growth model for gilthead sea
504 bream (*Sparus aurata*) based on the thermal growth coefficient. *Aquaculture*, 358-359,
505 pp. 6–13.

506 Moreau, J. (1987). Mathematical and biological expression of growth in fishes: Recent
507 trends and further developments. *The Age and Growth of Fish*, edited by Robert C.
508 Summerfelt and Gordon E. Hall. The Iowa State University Press, pp. 81-113.

509 Moses, M. E., Hou, C., Woodruff, W. H., West, G. B., Nekola, J. C., Zuo, W., Brown, J.
510 H. (2008). Revisiting a model of ontogenetic growth: estimating model parameters from
511 theory and data. *The American Naturalist*, 171(5), 632–45.

512 Sánchez-Zazueta, E.; Hernández, J.M.; Martínez-Cordero, F.J. (2013). Stocking density
513 and date decisions in semi-intensive shrimp *Litopenaeus vannamei* (Boone, 1931)
514 farming: a bioeconomic approach. *Aquaculture Research*, 44, pp. 574-587.

515 Seginer, I, and Halachmi, I. (2008). Optimal stocking in intensive aquaculture under
516 sinusoidal temperature, price and marketing conditions. *Aquacultural Engineering* 39(2-
517 3), pp. 103-112.

518 Seginer, I, and Ben-Asher, R. (2011). Optimal harvest size in aquaculture, with RAS
519 cultured sea bream (*Sparus aurata*) as an example. *Aquacultural Engineering* 44, pp.
520 55-64.

521 Ursin, E. (1963). On the incorporation of temperature in the von Bertalanffy growth
522 equation. *Meddelelser Fra Danmarks Fisheri-Og Havundersøgelser*, 4(1), 1–16.

523 Vaz, S., Martin, C. S., Eastwood, P. D., Ernande, B., Carpentier, A., Meaden, G. J.,
524 Coppin, F. (2008). Modelling species distributions using regression quantiles. *Journal*
525 *of Applied Ecology*, 45(1), pp. 204–217.

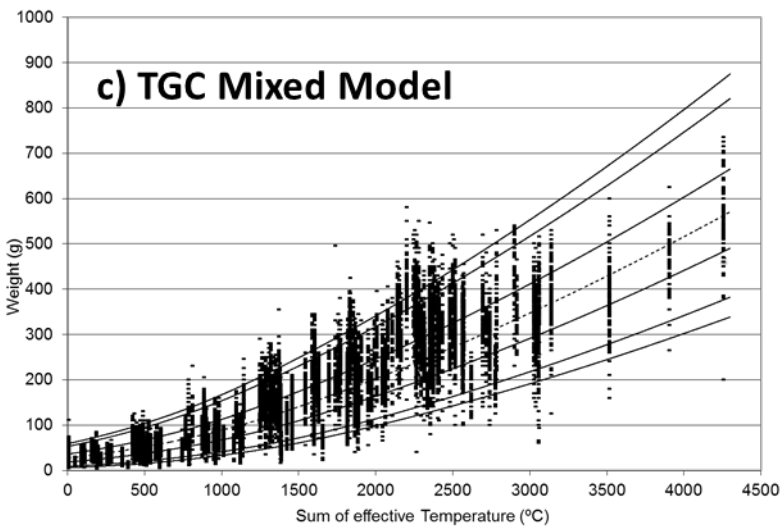
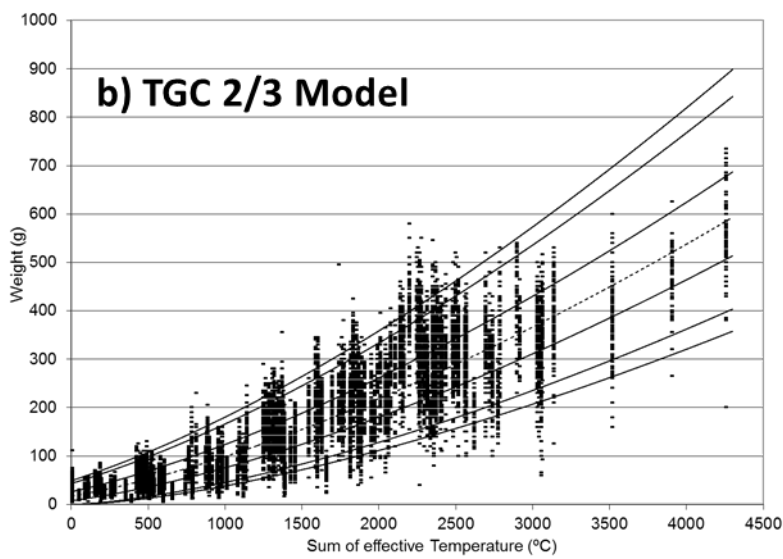
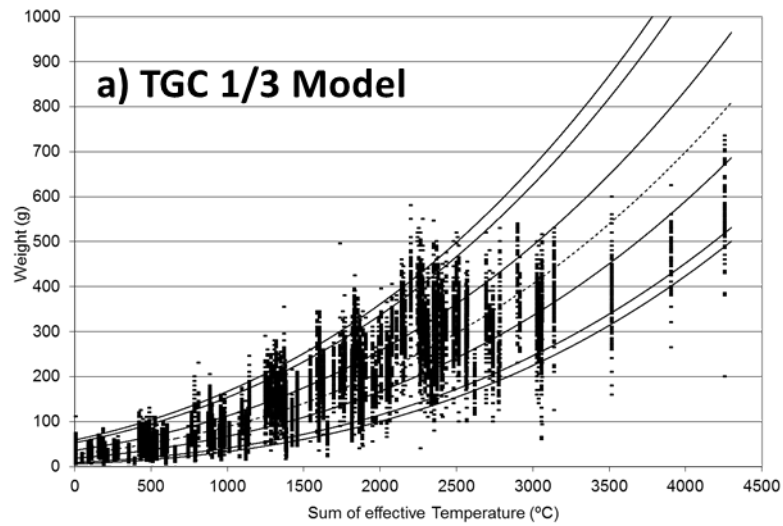
526 Yu, K., Lu, Z., Stander, J. (2003). Quantile regression: applications and current research
527 areas. *Journal of the Royal Statistical Society: Series D (The Statistician)*, 52(3), pp.
528 331–350.

529

530

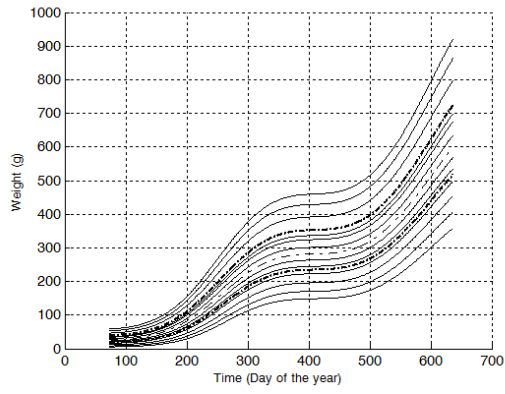
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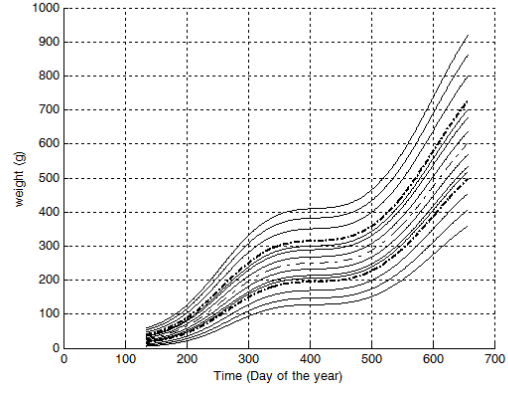


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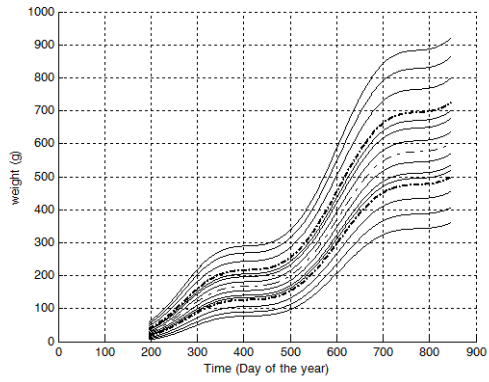
Figure 1. Growth curves of sea bream considering three quantile models: TGC-1/3, TGC-2/3 and Mixed model (dashed curves corresponding to 0.50 quantile and, from bottom to top, the curves corresponding to the quantiles 0.025, 0.05, 0.25, 0.50, 0.75, 0.95 and 0.975).



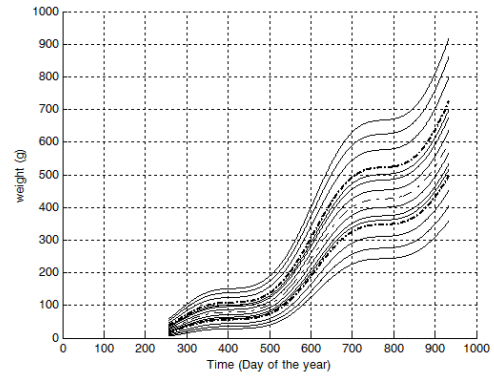
a) March Batch



b) May Batch



c) July Batch



d) September Batch

Figure 2. Simulation of sea bream growth considering four batches: March, May, July and September (Dashed curves corresponding to 0.5 quantile and, from bottom to top, the curves corresponding to the quantiles 0.025, 0.05, 0.1, 0.2, 0.25, 0.30, 0.40, 0.50, 0.60, 0.70, 0.75, 0.80, 0.9, 0.95 and 0.975, bold dashed lines correspond to quantiles 0.20 and 0.80).

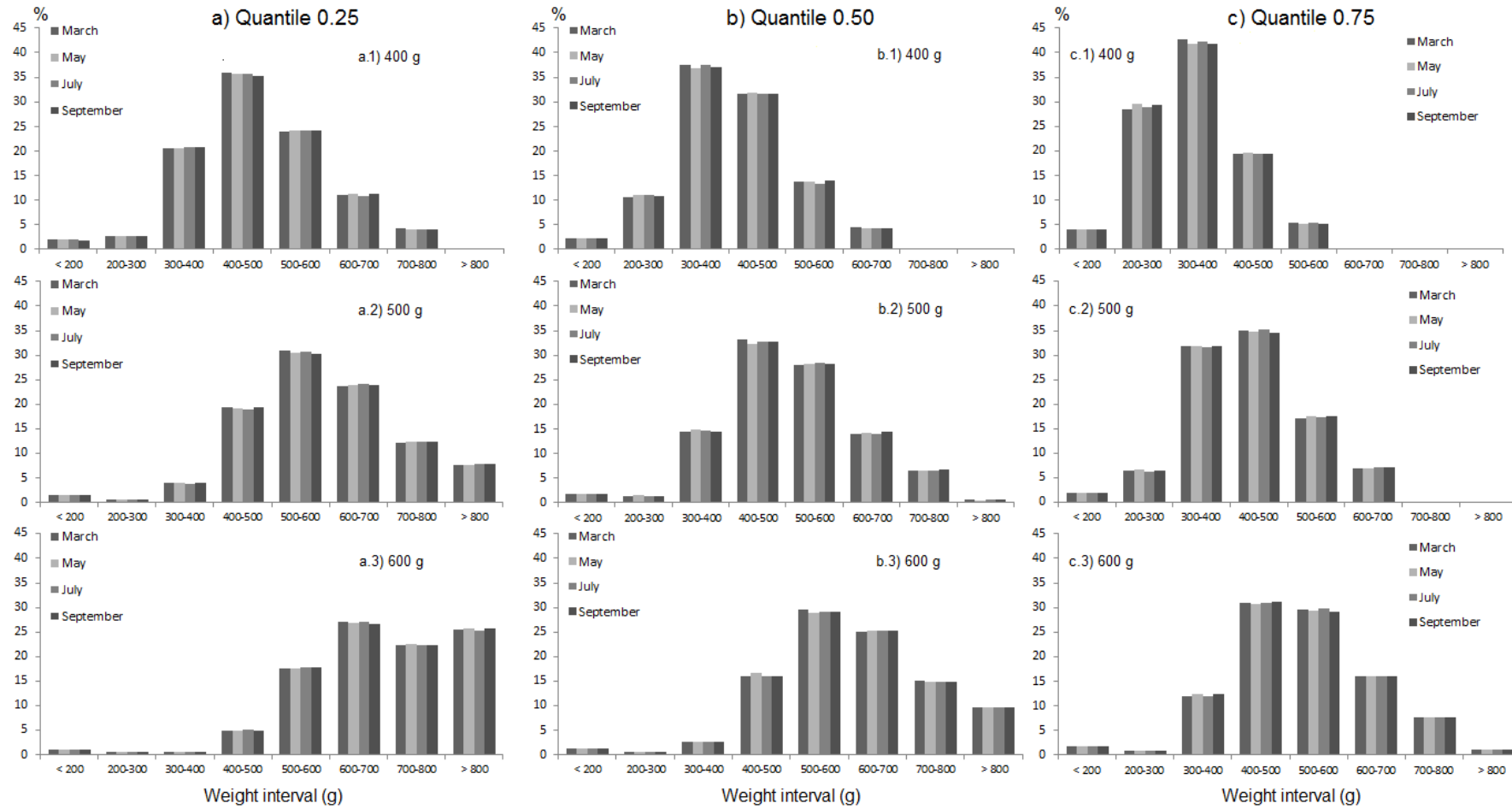
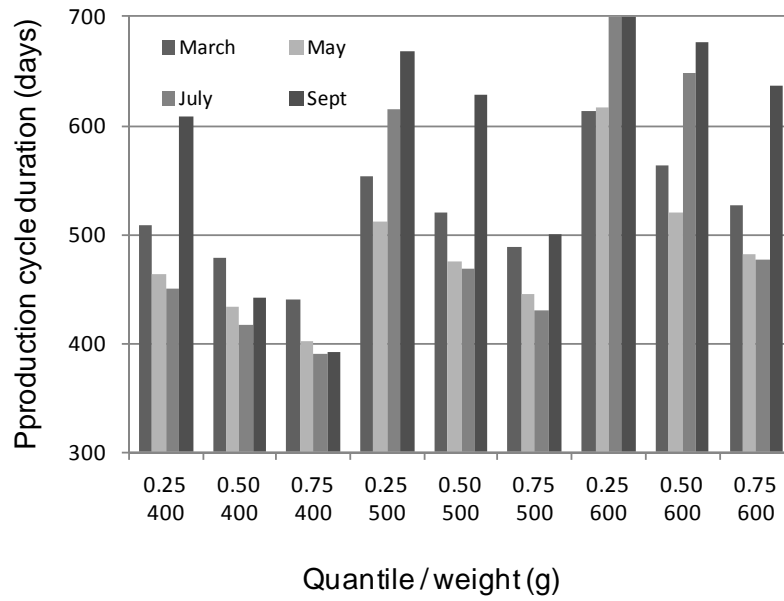


Figure 3. Simulation of percentage of gilthead sea bream in each weight interval when the quantiles (0.25, 0.50 and 0.75) reach the weights 400, 500 and 600 g

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Figure 4. Duration of production cycle (days) of gilthead sea bream computed so that the quantiles 0.25, 0.50 and 0.75 reach the weights 400, 500 and 600 g

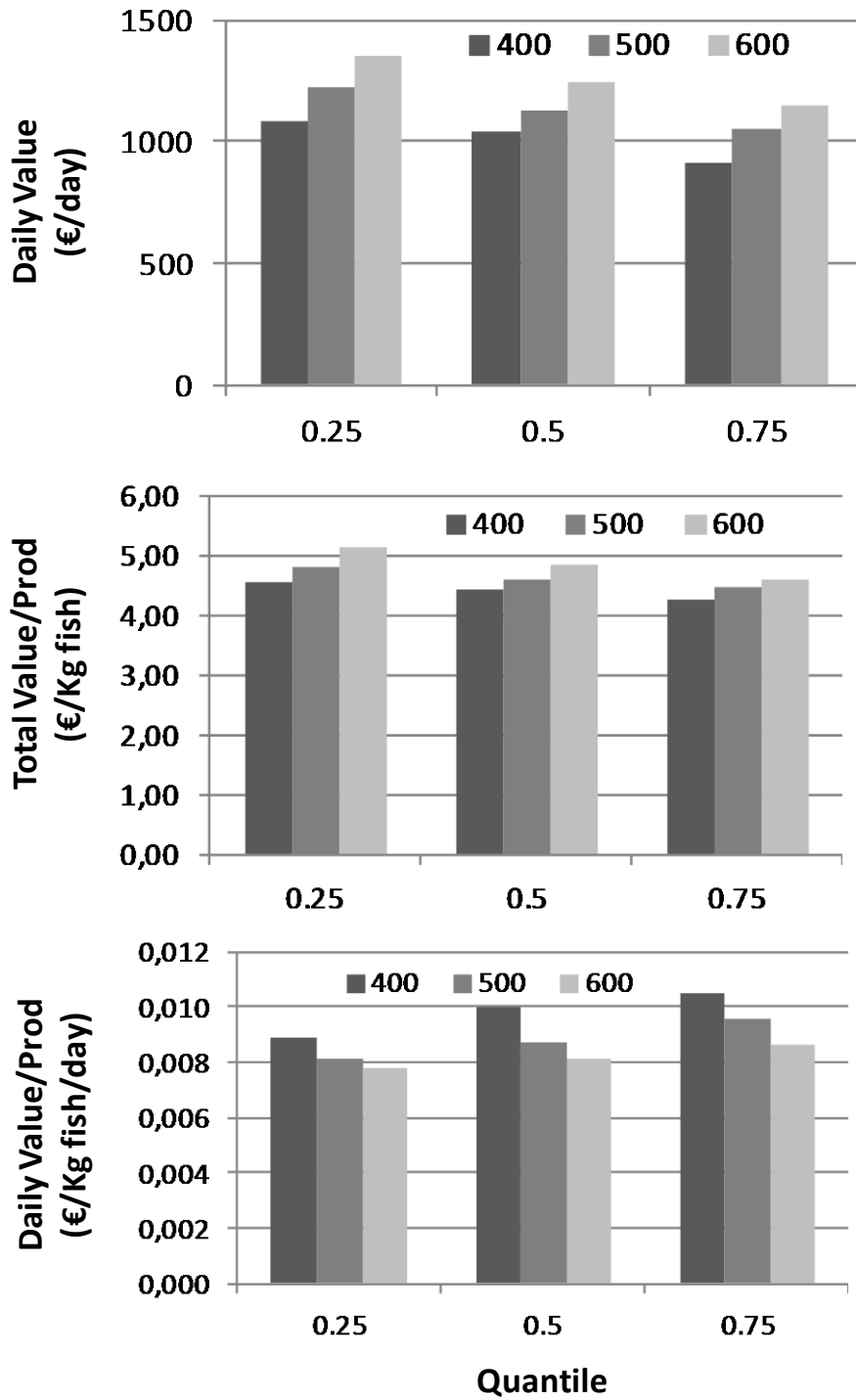


Figure 5. Economic values of sea bream sales in each weight interval for the quantiles 0.25, 0.50 and 0.75, and weights 400, 500 and 600 g

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Table 1. Estimated values of $A_{b,\tau}$ and $TGC_{b,\tau}$ for the two quantile models TGC-1/3 and TGC-2/3

Quantile (τ)	$A_{1/3,\tau}$	$TGC_{1/3,\tau}$	$A_{2/3,\tau}$	$TGC_{2/3,\tau}$
0.025	1.783	0.001432	-0.517	0.011841
0.05	2.047	0.001407	0.291	0.012687
0.10	2.289	0.001415	1.443	0.013387
0.20	2.540	0.001433	3.309	0.013769
0.25	2.627	0.001441	3.886	0.014014
0.30	2.720	0.001443	4.569	0.014167
0.40	2.856	0.001459	5.608	0.014553
0.50	2.994	0.001471	6.488	0.014910
0.60	3.110	0.001494	7.368	0.015288
0.70	3.250	0.001510	8.277	0.015755
0.75	3.302	0.001530	8.776	0.016067
0.80	3.391	0.001540	9.420	0.016303
0.90	3.598	0.001570	11.104	0.017163
0.95	3.758	0.001600	12.341	0.017878
0.975	3.893	0.001615	13.391	0.018542

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Table 2. Sum of effective temperature and sea bream weight at the critical point

Quantile (τ)	ST_τ	$W_{c,\tau}$
0.025	1642	70.7
0.05	1749	91.6
0.10	1725	105.9
0.20	1580	110.9
0.25	1551	115.0
0.30	1517	118.3
0.40	1461	124.1
0.50	1410	130.2
0.60	1343	133.9
0.70	1303	142.0
0.75	1274	144.8
0.80	1235	148.3
0.90	1190	163.3
0.95	1143	174.4
0.975	1144	189.2

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Table 3.-Values of goodness coefficient $R^1(\tau)$ and of $R^1_{[0.025-0.975]}$ for the three quantile models:
TGC-1/3, TGC-2/3 and TGC-Mixed Model

Quantile (τ)	$R^1(\tau)$		
	1/3-Model	2/3-Model	Mixed-Model
0.025	0.500	0.521	0.412
0.05	0.552	0.539	0.462
0.10	0.677	0.617	0.546
0.20	0.689	0.651	0.611
0.25	0.615	0.622	0.581
0.30	0.667	0.631	0.645
0.40	0.636	0.608	0.659
0.50	0.575	0.728	0.600
0.60	0.592	0.618	0.690
0.70	0.578	0.622	0.686
0.75	0.588	0.636	0.630
0.80	0.559	0.615	0.674
0.90	0.529	0.578	0.638
0.95	0.614	0.643	0.651
0.975	0.624	0.649	0.657
$R^1_{[0.025,0.975]}$	0.576	0.627	0.627

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Table 4.-Values of the actual quantiles and estimated quantiles for the initial weight

Quantiles (τ)	Initial weight $W_{0,\tau}$			
	Actual data ($n=1133$)	1/3-Model	Mixed-Model	2/3-Model
0.025	9.00	5.67	5.67	-
0.05	11.00	8.58	8.58	0.16
0.10	14.00	12.00	12.00	1.73
0.20	17.00	16.38	16.38	6.02
0.25	18.00	18.14	18.14	7.66
0.30	20.00	20.12	20.12	9.76
0.40	23.80	23.30	23.30	13.28
0.50	26.00	26.85	26.85	16.53
0.60	28.00	30.09	30.09	19.99
0.70	35.00	34.33	34.33	23.81
0.75	37.00	36.00	36.00	26.00
0.80	40.00	39.00	39.00	28.91
0.90	48.00	46.56	46.56	37.00
0.95	52.00	53.07	53.07	43.36
0.975	58.00	59.00	59.00	49.00
Chi-Square		3.33	3.33	919
d.f.		14	14	13
p -value		0.99	0.99	<0.0001*

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Table 5.-Values of the actual quantiles and estimated quantiles for the final weight considering three models: *TGC-1/3*, *TGC-2/3* and *TGC-Mixed* model

Quantiles	Final weight (last sample)			
	Actual (<i>n</i> =74)	1/3-Model	2/3-Model	Mixed-Model
0.025	384.1	487.2	351.4	332.3
0.05	433.3	517.2	396.5	375.5
0.10	463.0	572.1	445.4	420.9
0.20	500.0	642.4	486.0	462.8
25	511.3	669.5	505.2	481.8
0.30	515.0	693.4	521.2	497.7
0.40	526.0	742.6	553.9	529.6
0.50	540.0	789.8	583.7	561.1
0.60	560.0	846.1	615.1	593.5
0.70	580.5	903.1	652.4	652.4
0.75	607.5	942.2	676.3	654.5
0.80	620.0	980.5	698.1	677.7
0.90	678.5	1082.7	770.3	747.7
0.95	701.8	1175.9	829.8	807.6
0.975	716.8	1243.8	885.0	861.3
Chi ²		1342	104	90
d.f.		14	14	14
<i>P</i> -value		<0.0001	<0.0001	<0.0001

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Table 6.- Simulation of biomass and value of gilthead sea bream (considering a mean batch of 300000 fish and a survival of 85%, and selling price from a commercial fish farm in Mediterranean Sea) by weight interval, average production duration (days) from the beginning of the cycle until the curves $W_{f0.25}$, $W_{f0.50}$ and $W_{f0.75}$ corresponding to when 0.25, 0.50 and 0.75 quartiles reach 400, 500 and 600 g, respectively.

Weight Interval	Mean Weight	Sale Price	W 0.25 = 400 g	508 days	W 0.25 = 500 g	587 days	W 0.25 = 600 g	662 days	
(g)	(g)	(€)	%	Production	Value	%	Production	Value	
< 200	150	1,9	1,9	717	1341	1,5	574	1073	
200-300	250	3,7	2,7	1705	6327	0,7	446	1656	
300-400	350	4,4	20,6	18408	81363	4,0	3525	15582	
400-500	450	4,5	35,7	40937	185036	19,3	22089	99844	
500-600	550	4,5	24,1	33800	152777	30,6	42952	194141	
600-700	650	4,8	11,1	18315	87731	23,9	39656	189951	
700-800	750	4,8	4,1	7793	37331	12,4	23667	113366	
> 800	850	6,1	0,0	0	0	7,8	16798	101797	
Total				121.676	551.904		149.707	717.409	
Increment							28.031	165.505	
								25.108	180.127
Weight Interval	Mean Weight	Sale Price	W 0.50 = 400 g	443 days	W 0.50 = 500 g	526 days	W 0.50 = 600 g	598 days	
(g)	(g)	(€)	%	Production	Value	%	Production	Value	
< 200	150	1,9	2,2	842	1574	1,7	650	1216	
200-300	250	3,7	10,9	6933	25721	1,4	893	3311	
300-400	350	4,4	37,3	33268	147044	14,6	13053	57693	
400-500	450	4,5	31,7	36376	164418	32,8	37581	169864	
500-600	550	4,5	13,7	19214	86848	28,3	39621	179085	
600-700	650	4,8	4,3	7086	33941	14,2	23454	112343	
700-800	750	4,8	0,0	0	0	6,6	12623	60462	
> 800	850	6,1	0,0	0	0	0,6	1246	7553	
Total				103.718	459.546		129.119	591.528	
Increment							25.401	131.981	
								25.286	155.839
Weight Interval	Mean Weight	Sale Price	W 0.75 = 400 g	407 days	W 0.75 = 500 g	466 days	W 0.75 = 600 g	531 days	
(g)	(g)	(€)	%	Production	Value	%	Production	Value	
< 200	150	1,9	4,1	1549	2897	2,1	803	1502	
200-300	250	3,7	29,1	18535	68766	6,6	4176	15492	
300-400	350	4,4	42,2	37641	166374	31,8	28382	125446	
400-500	450	4,5	19,4	22290	100752	34,9	40019	180886	
500-600	550	4,5	5,3	7433	33598	17,4	24439	110462	
600-700	650	4,8	0,0	0	0	7,1	11727	56171	
700-800	750	4,8	0,0	0	0	0,2	430	2061	
> 800	850	6,1	0,0	0	0	0,0	0	0	
Total				87.449	372.387		109.975	492.021	
Increment							22.526	119.634	
								22.561	118.482

586

587 Table 7.- Estimation of biomass and sales income of gilthead sea bream, considering three strategies of
 588 interval weight sale (400-500, 500-600 and 600-700 g) by classifying fish and sequential sale of fish from
 589 0.75, 0.50 and 0.25 quartile.

Sale weight (g)	Biomass (tons)	Sales Income (€)	Mean time (Days)	Sales Income /Days (€)
400-500	114	511768	486	1053
500-600	139	628890	560	1123
600-700	158	752158	614	1225

590