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Atmospheric refraction in geodetic techniques: analysis
of tropospheric effects in GNSS and development of a
3D refractivity model for terrestrial measurements

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“All models are wrong, but some are useful”

George E. P. Box

Abstract

Atmospheric refraction remains one of the main limitations to achieving millimetre-level accuracy in high-precision geodetic measurements. This thesis addresses refraction effects in two different but complementary domains: GNSS-based distance estimation and terrestrial optical observations.

The first part focuses on the quantification and analysis of tropospheric delays at the double-difference (DD) level, within the framework of the Ground-Based Distance Meter (GBDM+) methodology, developed at the Universitat Politècnica de València (UPV). Based on controlled analyses, this thesis shows that although DD processing effectively reduces atmospheric biases, DD residual delays, particularly for baselines longer than 3–4 km or with significant height differences, can still reach several decimetres. These effects must be accounted for to preserve measurement traceability and accuracy. Several empirical models from the literature are evaluated, alongside Precise Point Positioning (PPP)-based estimates obtained from the CSRS-PPP online service. PPP-based estimates provide the additional advantage of uncertainty estimation, allowing its propagation to the final distance, which is particularly relevant for metrological applications.

The second part presents the development and validation of a three-dimensional refractivity model (3D-RM) for terrestrial geodetic measurements. This model combines data from a meteorological sensor network, remote sensing products (ERA5), vertical profiling via the application of the Turbulence Transfer Model (TTM), and terrain information using a Digital Terrain Model (DTM) to enable full-path refraction corrections. Validation with multiple field datasets demonstrates that 3D-RM significantly improves precision and accuracy compared to the conventional station-based corrections, particularly over long distances. A simplified variant (3D-RM 2), which avoids the need to install a sensor at the instrument location, achieves comparable results, increasing the model's flexibility for practical deployment. Although vertical angle corrections could not be fully assessed due to instrumental constraints, observed diurnal patterns suggest the model's potential for future angular applications.

Overall, the thesis advances the understanding of atmospheric effects on geodetic measurements and proposes practical strategies for their modelling and mitigation. Particular emphasis is placed on the role of atmospheric modelling in ensuring traceability and achieving high-precision results in complex, real-world environments.

Resumen

La refracción atmosférica sigue siendo una de las principales limitaciones para alcanzar precisiones del orden de milímetros en mediciones geodésicas de alta precisión. Esta tesis aborda los efectos de la refracción en dos ámbitos diferentes pero complementarios: la estimación de distancias basadas en GNSS y las observaciones ópticas terrestres.

La primera parte se centra en la cuantificación y el análisis de los retardos troposféricos a nivel de dobles diferencias (DD), en el marco de la metodología *Ground-Based Distance Meter* (GBDM+), desarrollada en la Universitat Politècnica de València (UPV). Los análisis realizados muestran que, aunque el procesamiento DD reduce eficazmente los errores atmosféricos, los retardos residuales, especialmente para líneas base superiores a 3–4 km o con diferencias de altura significativas, pueden alcanzar varios decímetros. Estos efectos deben tenerse en cuenta para preservar la trazabilidad y la exactitud de las mediciones. Se evalúan varios modelos empíricos de la literatura, junto con estimaciones basadas en Posicionamiento de Punto Preciso (PPP) obtenidas mediante el servicio en línea CSRS-PPP. Las estimaciones PPP ofrecen, además, la ventaja de la estimación de la incertidumbre, lo que permite su propagación a la distancia, especialmente relevante para aplicaciones metrológicas.

La segunda parte presenta el desarrollo y la validación de un modelo tridimensional de refractividad (3D-RM) para mediciones geodésicas terrestres. El modelo integra datos de una red de sensores meteorológicos, productos de teledetección (ERA5), perfiles verticales generados mediante la aplicación del *Turbulence Transfer Model* (TTM) e información topográfica usando un Modelo Digital de Elevaciones (MDE), lo que permite obtener correcciones de refracción a lo largo de toda la trayectoria. La validación con diversos conjuntos de datos de campo demuestra que el 3D-RM mejora de forma significativa la precisión y la exactitud respecto a las correcciones convencionales basadas en datos en la estación, especialmente en distancias largas. Una variante simplificada (3D-RM 2), que evita tener que instalar un sensor en la posición del instrumento, logra resultados comparables, aumentando la flexibilidad del modelo para su aplicación práctica. Aunque las correcciones de ángulo vertical no se han podido evaluar completamente debido a limitaciones instrumentales, los patrones diurnos observados sugieren el potencial del modelo para futuras aplicaciones angulares.

En conjunto, esta tesis contribuye a una mejor comprensión de los efectos atmosféricos en las mediciones geodésicas y propone estrategias prácticas para su modelización y mitigación. Se hace especial hincapié en el papel de los modelos atmosféricos basados en fundamento físico para garantizar la trazabilidad y alcanzar resultados de alta precisión en entornos reales y complejos.

Resum

La refracció atmosfèrica continua sent una de les principals limitacions per a aconseguir precisions de l'ordre de mil·límetres en mesures geodèsiques d'alta precisió. Esta tesi aborda els efectes de la refracció en dos àmbits diferents però complementaris: l'estimació de distàncies basades en GNSS i les observacions òptiques terrestres.

La primera part se centra en la quantificació i l'anàlisi dels retards troposfèrics a nivell de dobles diferències (DD), en el marc de la metodologia *Ground-Based Distance Meter* (GBDM+), desenvolupada a la Universitat Politècnica de València (UPV). Les anàlisis realitzades mostren que, encara que el processament DD redueix eficaçment els errors atmosfèrics, els retards residuals, especialment per a línies base superiors a 3–4 km o amb diferències d'altura significatives, poden arribar a diversos decímetres. Aquestos efectes han de tindre's en compte per a preservar la traçabilitat i "exactitud de les mesures. S'avaluen diversos models empírics de la literatura, junt amb estimacions basades en Posicionament de Punt Precís (PPP) obtingudes mitjançant el servei en línia CSRS-PPP. Les estimacions PPP ofereixen, a més, l'avantatge de la estimació de la incertesa, permetent la propagació a la distància, especialment rellevant per a aplicacions metrològiques.

La segona part presenta el desenvolupament i la validació d'un model tridimensional de refractivitat (3D-RM) per a mesures geodèsiques terrestres. El model integra dades d'una xarxa de sensors meteorològics, productes de teledetecció (ERA5), perfils verticals generats mitjançant l'aplicació del *Turbulence Transfer Model* (TTM) i informació topogràfica utilitzant un Model Digital d'Elevacions (MDE), el que permet obtindre correccions de refracció al llarg de tota la trajectòria. La validació amb diversos conjunts de dades de camp demostra que el 3D-RM millora de manera significativa la precisió i l'exactitud respecte a les correccions convencionals basades en dades de l'estació, especialment en distàncies llargues. Una variant simplificada (3D-RM 2), que evita haver d'instal·lar un sensor en la posició de l'instrument, aconsegueix resultats comparables, augmentant la flexibilitat del model per a la seua aplicació pràctica. Encara que les correccions d'angle vertical no s'han pogut avaluar completament a causa de limitacions instrumentals, els patrons diürns observats suggereixen el potencial del model per a futures aplicacions angulars.

En conjunt, esta tesi contribuïx a una millor comprensió dels efectes atmosfèrics en les mesures geodèsiques i proposa estratègies pràctiques per a la seua modelització i mitigació. Es fa especial èmfasi en el paper dels models atmosfèrics amb fonament físic per a garantir la traçabilitat i aconseguir resultats d'alta precisió en entorns reals i complexos.

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1. Introduction

1.1 Motivation

High-precision geodetic measurements are crucial in fields such as length metrology, civil engineering, and critical deformation monitoring. The demand for such high precision has grown with applications like the survey and alignment of large-scale scientific installations (e.g. CERN’s geodetic network) and the construction of complex infrastructure projects. These applications require measurement uncertainties at the millimetric or sub-millimetric level, over distances ranging from tens of meters to several kilometres.

Despite of the significant advances in instrumentation and measurement strategies, atmospheric refraction remains a limiting factor for all high-precision geodetic techniques, affecting both satellite-based techniques like GNSS and terrestrial techniques such as total stations (TS), electronic distance meters (EDMs) laser trackers or Terrestrial Laser Scanner (TLS). In both domains, errors due to atmospheric effects can compromise the measurement traceability and prevent instruments from achieving their nominal accuracies, particularly in outdoor, long-range setups.

The study of atmospheric refraction is not new. Researchers have been investigating its effects for decades, with notable efforts in the 1980s [1, 2, 3] to understand the atmospheric mechanisms and their influence on signal propagation. However, many of the ideas explored during that time could not be fully implemented due to limitations in available technology and computational power. Today, advancements in meteorological sensors capable of automatic data recording, the availability of accessible remote sensing products, and increased computational resources provide an opportunity to revisit these foundational ideas and implement them to achieve more effective refractivity correction methods.

This renewed interest in atmospheric modelling is also reflected in the field of metrology, where efforts are being made to improve the measurement traceability and uncertainty assessment of geodetic techniques. For instance, the European GeoMetre project [4] seeks to enhance the traceability of geodetic measurements to the International System of Units (SI) by addressing systematic error sources and developing improved instrumentation and methodologies.

For GNSS-based techniques, atmospheric effects, specifically in the troposphere, introduce signal delays that are difficult to isolate from other sources of error. Although tropospheric errors tend to cancel out in short-baseline double-difference (DD) processing, this assumption breaks down for longer baselines, or when significant height differences exist between receivers. Moreover, GNSS remains problematic for applications requiring strict traceability to the International System of Units (SI), as the measurements depend

on complex models, reference frames, and unobservable parameters. In this context, recent developments like the Ground-Based Distance Meter (GBDM+) [5, 6] aim to fill this gap by treating GNSS as a metrological tool and rigorously propagating uncertainties, including those from atmospheric sources. A deeper understanding and quantification of tropospheric effects is crucial to unlocking the full potential of such techniques.

For terrestrial geodetic instruments, atmospheric refraction can significantly distort both distance and angle measurements, especially in variable or complex environments. The standard correction approach, averaging refractivity values from meteorological sensors located at the endpoints of the measured path, is often inadequate, as it fails to capture vertical and spatial variations in the atmospheric profile along the full line of sight. These limitations are especially pronounced in scenarios involving inaccessible targets or non-homogeneous terrain, where installing sensors at critical points is either impractical or impossible.

To address this limitation, specific instruments designed to compensate the refraction effects are being developed. While historical instruments like the Terrameter LDM-2 paved the way for high-precision distance measurements, their commercial unavailability highlights the ongoing need for innovative solutions. Recently, significant efforts have been made to develop new instruments capable of compensating for refraction effects [7, 8]. However, these new technologies remain under development and are not yet widely available. Consequently, there is still a need for cost-effective and practical solutions for refraction correction that can be implemented efficiently in real-world applications.

This thesis contributes to this broader effort by addressing atmospheric refraction from both GNSS and terrestrial geodetic perspectives. First, it analyses the impact of the troposphere on GNSS in the context of GBDM+ and investigates how different tropospheric models and baseline characteristics affect zero and double-difference processing. Second, it proposes a novel, cost-effective 3D refractivity model (3D-RM) for terrestrial applications, which integrates meteorological sensors, remote sensing products, vertical profiling using the Turbulence Transfer Model (TTM), and terrain data using a Digital Terrain Model (DTM) to compute refractivity corrections.

Beyond these two main lines of work, the thesis is structured around three complementary components:

1. A theoretical study of atmospheric refraction and its influence on geodetic measurements.
2. The development of new modelling approaches and methodologies.
3. Design and execution of controlled field experiments that enable both the validation of these developments and a deeper understanding of atmospheric effects under real-world conditions.

Together, these elements aim to advance both the scientific understanding and the practical mitigation of atmospheric refraction, contributing to the traceability and accuracy of high-precision geodetic measurements.

1.2 Objectives

The main goal of this thesis is to address atmospheric refraction as a limiting factor in high-precision geodetic measurements. This aim is divided into two main objectives:

1. To analyse the impact of tropospheric effects on double-difference (DD) equations, specifically in the context of the Ground-Based Distance Meter (GBDM+) technique.
2. To develop a 3D refractivity model that generates atmospheric corrections for terrestrial techniques in the visible/infrared spectrum by integrating meteorological data, vertical characterization via Turbulence Transfer Model (TTM), and terrain information from Digital Terrain Models (DTMs).

To achieve these objectives, the following sub-objectives are defined:

Tropospheric effects in GBDM+:

- Analyse tropospheric residuals in double-difference (DD) equations under varying baseline lengths and height differences.
- Evaluate the differences in tropospheric delays of both zero differences and double differences using various tropospheric models and estimations.
- Investigate the impact of different tropospheric mapping function at zero and double difference level.

3D refractivity model development:

- Identify and implement a suitable interpolation method for refractivity values.
- Integrate available remote sensing products with the Turbulence Transfer Model (TTM) to enhance vertical atmospheric characterization.
- Develop a cost-effective refractivity correction model that leverages existing sensors and products to achieve improved performance in complex terrains compared to traditional methods.
- Analyse and validate the results of the proposed model through field studies.

1.3 Outline

This document is structured into seven chapters, each addressing different aspects of atmospheric refraction in geodetic measurements, with a focus on both satellite-based and terrestrial techniques. Following this introductory chapter, the thesis is organized as follows.

Chapter 2: Theoretical background

This chapter provides the basic theoretical background required to understand atmospheric effects on signal propagation. It begins with a description of the Earth's atmosphere, including its composition and structure, with special emphasis on the atmospheric boundary layer. Then, the effects of the atmosphere on signal propagation are described, as well as the fundamental concepts related to atmospheric refraction are introduced. Finally different humidity-related parameters are introduced.

Chapter 3: Atmospheric effects in GNSS

Chapter 3 explores the impact of the atmosphere on GNSS signal propagation, with emphasis on the troposphere. The ionospheric effect is briefly introduced for context, including mitigation techniques such as dual-frequency observations. The main focus, however, is on tropospheric delays, both hydrostatic and wet components, and the modelling approaches used to mitigate them.

Chapter 4: Atmospheric effects in terrestrial techniques

This chapter addresses how atmospheric refraction affects terrestrial optical measurements, such as those made with total stations (TSs) and electronic distance meters (EDMs). It is organised into two sections: one examining the influence on distance measurements, and the other on vertical angle observations. Standard correction methods are discussed, alongside their limitations in heterogeneous or complex environments. This chapter motivates the need for a more refined correction model, introduced in Chapter 6.

Chapter 5: GBDM+: principles and tropospheric effect analysis

Chapter 5 presents the GBDM+ methodology, describes its main aspects and explores how the sources of error, particularly those related to the troposphere, are addressed. The focus is placed on analysing how tropospheric delays propagate from zero-difference (ZD) to double-difference (DD) GNSS equations. The chapter evaluates the impact of baseline length, height differences, and model selection (empirical vs. PPP-derived delays), providing a detailed assessment of residual tropospheric effects in the DD domain. These findings are contextualized through case studies in diverse environments, including the CERN geodetic network, the EURO5000 calibration baseline, and the monitoring network

at Cortes de Pallás.

Chapter 6: 3D Refractivity Model for distance and vertical angle correction

This chapter introduces and validates a novel 3D refractivity model (3D-RM) designed to improve correction accuracy in terrestrial geodetic measurements. It begins with the theoretical foundations, including the Turbulence Transfer Model (TTM) and sensible heat flux (H) estimation. The structure and implementation of the 3D-RM are detailed step-by-step, followed by field applications in the Cortes de Pallás area. Case studies demonstrate the model's performance in real conditions, highlighting its advantages over traditional station-based correction methods and its flexibility through a simplified version (3D-RM2). The influence of several model parameters is also assessed. Finally, the application of the model to terrestrial laser scanner (TLS) point clouds is shown as a proof of concept.

Chapter 7: Conclusions and outlook

The final chapter summarizes the main findings of the research, emphasizing the contributions made to the understanding and mitigation of atmospheric effects in high-precision geodetic measurements. In addition, the chapter outlines several directions for future research.

2. Background

This chapter provides the fundamental background necessary to understand the complex physics and processes governing the atmosphere, as well as its interactions with the Earth's surface. It explores the role of the atmosphere in geodetic applications, particularly in relation to signal propagation.

This chapter is structured in three sections. Section 2.1 presents an overview of the Earth's atmosphere, detailing its composition and structure. Section 2.2 examines how the atmosphere influences signal propagation. Finally, Section 2.3 introduces some concepts related to humidity parameters.

2.1 Earth's atmosphere

This section provides an overview of the Earth's atmosphere, focusing on its composition and structure. First, the atmosphere's composition is presented, highlighting the distribution of its primary constituents. Then, the vertical stratification of the atmosphere according to different criteria is explored, providing insight into how these layers affect signal propagation.

2.1.1 Atmosphere composition

The Earth's atmosphere consists of a complex mixture of gases. Table 1 outlines the primary constituents of the atmosphere, their corresponding fractional concentrations by volume, and a brief overview of their distribution across different atmospheric layers.

Nitrogen and oxygen are the dominant constituents of the atmosphere, as shown in Table 1. Typically, atmospheric components are categorized into dry air, water vapour, and aerosols [11]. Alternatively, some authors classify these components into two groups: dry air and trace species [9].

Dry air primarily comprises nitrogen, oxygen, and argon, which constitute the main components of the atmosphere with relatively constant distributions throughout. Water vapour, on the other hand, is highly variable, with concentrations ranging from 10 parts per million (ppm) by volume to as high as 5% by volume [10]. Its spatial and temporal variations depend on local conditions and meteorological factors, making accurate estimations difficult.

Aerosols are small suspended particles, such as pollen and smoke, that also contribute to the composition of the atmosphere.

Table 1: Main atmosphere's constituents (adapted from [9] and [10])

Constituent	Concentration	Distribution
Nitrogen (N_2)	78.08%	Homogeneous
Oxygen (O_2)	20.95%	Homogeneous
Argon (Ar)	0.93%	Homogeneous
Water vapour (H_2O)	0-5%	Highly variable, decreases sharply in Troposphere
Carbon dioxide (CO_2)	380 ppm	Homogeneous
Neon (Ne)	18 ppm	Homogeneous
Helium (He)	5 ppm	Homogeneous
Methane (CH_4)	1.75 ppm	Homogeneous in Troposphere
Krypton (Kr)	1 ppm	Homogeneous
Hydrogen (H_2)	0.5 ppm	Homogeneous
Nitrous oxide (N_2O)	0.3 ppm	Homogeneous in Troposphere
Ozone (O_3)	0-0.1 ppm	Highly variable, decreases sharply in Stratosphere

2.1.2 Atmospheric layers

The Earth's atmosphere is generally described as a series of concentric layers, each with different characteristics. These layers can be defined based on various properties such as chemical composition, temperature profile, or ionization state.

An overview of the atmospheric layers according different criteria is presented in Figure 1.

Altitude (Km)	Composition	Temperature	Ionization
500 —	Heterosphere	Thermosphere	Ionosphere
100 —			
50 —	Homosphere	Mesosphere	
10 —		Stratosphere	Neutral atmosphere
		Troposphere	

Figure 1: Atmospheric layers (adapted from [12])

Chemical composition

Based on chemical composition, the atmosphere can be classified into two different layers [11]:

- **Homosphere.** Extending up to approximately 100 kilometres above the Earth’s surface, the mixing ratios (volume fractions) of the main atmospheric constituents (N_2 , O_2 , Ar , etc.) are relatively uniform and independent of height [10], even though their absolute concentrations decrease with altitude. The upper boundary of this layer is referred to as the turbopause.
- **Heterosphere.** Above 100 kilometres, the atmospheric composition becomes more variable with concentrations of different constituents changing differently with height [10]. Heavier components decrease more rapidly compared to lighter ones, a phenomenon known as molecular diffusion [11].

Temperature distribution

The atmosphere is commonly divided into distinct layers based on the temperature variation with height, which significantly influences the dynamic properties of each layer [9]. Figure 2 shows the vertical temperature profile across the different atmospheric layers.

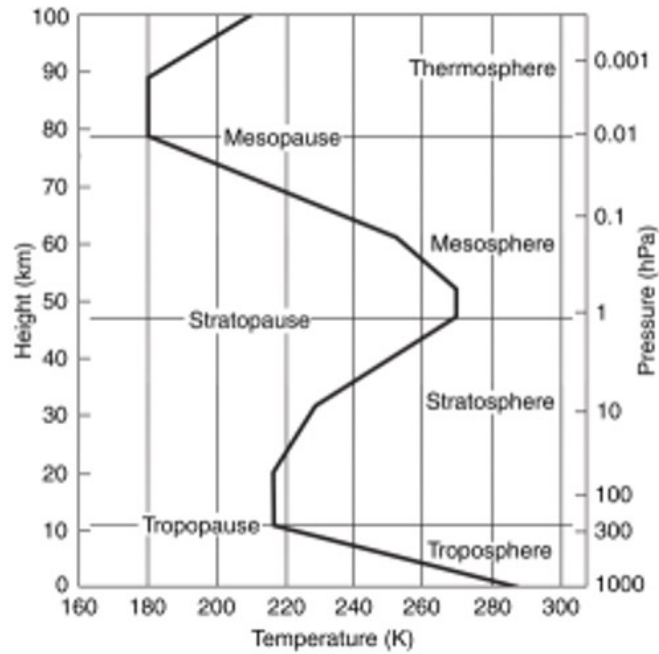


Figure 2: Temperature variation through atmospheric layers (Source: [10])

- Troposphere. The troposphere is the lowest layer of the atmosphere. It is widely accepted that it extends up to 10 kilometres, although some authors indicate a slightly different height above the Earth surface, for example [11] indicates that upper limit is variable and depends on the time and the location and suggest a range from 7-10 km, over the polar regions and to 16-18 km, over the tropical and equatorial regions.

It generally exhibits a consistent temperature decrease with altitude, known as the lapse rate, of approximately 6.5°C per kilometre. Accounting for around 80% of the atmospheric mass, most weather-related phenomena occur in this region [9].

The troposphere can be subdivided in two different layers, the lowest one is known as boundary layer and the higher part is called free atmosphere. The boundary layer, which is the lowest part of the atmosphere, is the region in which terrestrial geodetic measurements are usually taken and there is a strong interaction between the Earth's surface and the atmosphere. Further information about this layer is provided in Section 2.1.3).

The limit between the troposphere and the stratosphere is marked by the tropopause, where the temperature gradient abruptly changes.

- Stratosphere. The stratosphere extends up to approximately 50 kilometres and can be further divided into two regions. The lower part, up to approximately 20

kilometres, is almost isothermal [11] while in the higher portion exhibits a non-constant temperature change with altitude. This layer is characterized by its dryness and high ozone concentration [10]. The stratosphere ends at the stratopause, where the maximum temperature is reached.

- Mesosphere. The mesosphere extends up to 80-85 kilometres. In this region, the temperature decreases again with height with a sharp tendency due to strong absorption of solar radiation [13]. Its minimum temperature occurs at the mesopause, separating the mesosphere from the thermosphere.
- Thermosphere. The thermosphere presents rapid temperature increases due to solar activity, reaching maximum temperatures around 500 kilometers above the Earth's surface.

Ionization state

Another way to categorize the Earth's atmosphere involves considering its ionization state, particularly relevant in satellite geodesy due to its impact on electromagnetic wave propagation. The ionization is an atmospheric phenomenon produced by the energy exchange between gas molecules [11]. Two different layers can be distinguished:

- Neutral atmosphere. The region below approximately 50 kilometres is often referred to as the neutral atmosphere, which lacks ionization, making it a non-dispersive medium for microwaves. This distinction is crucial in spatial geodetic techniques, where mitigating ionospheric effects using dual-frequency signals is ineffective against neutral atmosphere-induced errors.
- Ionosphere. Starting at approximately 50 kilometres above the Earth's surface, the ionosphere is characterized by strong ionization, resulting in the presence of free electrons and positively charged ions [14], with non-uniform distribution. This ionization makes it a dispersive region for microwaves, meaning that the refractive index is frequency-dependent [15]. It can be subdivided into several regions:
 - D layer. The lower region of the ionosphere, reaching up to about 90 kilometres [14], has the lowest electron density concentration and it is most active during the day [11].
 - E layer. Extending from the upper boundary of the D layer up to around 140-150 kilometres, this layer is primarily influenced by solar radiation, with minimal nighttime effects [11].
 - F layer. The F layer is further subdivided into the F1 layer, active only during the day, and the F2 layer, which is characterized by significant electron density variation between day and night, substantially affecting GNSS signal transmission [16]. The F layer reaches from the end of the E layer (140-150 km) to the end of the atmosphere at approximately 1000 km.

It is important to note that the terms “troposphere” and “neutral atmosphere” are often used interchangeably in literature. However, strictly speaking, the term “neutral atmosphere” refers to the non-ionized part of the atmosphere, comprising the troposphere and part of the stratosphere.

2.1.3 Boundary layer

In section 2.1.2, various atmospheric layers were presented based on different division criteria. This section focuses on the lowest part of the atmosphere, known as boundary layer, where terrestrial geodetic measurements typically take place.

The boundary layer can be defined as the atmospheric region influenced by the Earth’s surface parameters, such as ground temperature, soil moisture, and turbulent fluxes [17]. Its thickness is quite variable, ranging from hundreds meters to few kilometres [18].

Some physical processes occurs in the boundary layer, such as turbulence, convection and radiation. These physical processes play a critical role in the propagation of geodetic signals through the atmosphere, influencing refraction and measurement accuracy. Turbulence causes rapid, small-scale variations in temperature and pressure, leading to signal scattering and phase fluctuations. Convection, driven by vertical heat exchange, creates localized temperature differences that can distort signal paths. Radiation impacts surface heating and cooling cycles, shaping diurnal variations in atmospheric density and therefore refractivity. Together, these processes contribute to the variability in signal delay and bending, underscoring the importance of accurately modeling the boundary layer for precise geodetic measurements.

The temperature profile within the boundary layer is not uniform, and several regions can be distinguished, yielding in different temperature gradients. Following the classification proposed in [19], three main regions can be distinguished:

- Lower atmosphere. The first 20-30 meters above ground of the atmosphere are strongly influenced by the Earth’s surface properties. During the day, solar radiation is absorbed by the surface and heats the atmosphere, causing negative vertical temperature gradients. In the evening, as the Earth’s surface cools faster than the atmosphere, strong positive gradients are typically observed. According to [18], the temperature gradient in this region can vary between -0.02 K/m and -0.04 K/m during daytime, while night-time gradients may exceed 0.05 K/m. This value vary depending on local conditions and the presence of surface heterogeneities.
- Intermediate atmosphere. Ranging from 20-30 meters to approximately 100 meters above the ground, the influence of the Earth’s temperature weakens, and the temperature gradient stabilizes in about -0.01 K/m.
- Higher atmosphere. Beginning around 100 meters above ground, the temperature gradient becomes almost independent of the Earth’s surface temperature, with a

commonly accepted value of approximately -0.006 K/m.

2.2 Atmospheric effects on signal propagation

All geodetic techniques based on electromagnetic signals are affected by the refractive properties of the atmosphere. This includes techniques operating in the visible and near infrared range, such as total stations (TS) and electronic distance meters (EDM), as well as those operating in the microwave domain, such as Global Navigation Satellite Systems (GNSS). As the signal travels from transmitter to receiver through the Earth's atmosphere, it is subject to several atmospheric effects that influence both the signal velocity and curvature of the ray path [20], introducing measurement errors that must be properly modelled and corrected for high-accuracy applications. Following [1], the term *geodetic refraction* is commonly used to describe the impact of atmospheric conditions on electromagnetic wave propagation.

The refractive index (n) is defined as the ratio between the speed of light in vacuum (c) and the signal speed in the medium (v):

$$n = \frac{c}{v} \quad (1)$$

The atmospheric delay is directly related to the integral of the refractive index along the signal path (s). For convenience, an average index along the trajectory of length s can be defined as:

$$n = \frac{1}{s} \int_0^s n(x) dx \quad (2)$$

This definition represents the mean refractive index experienced by the signal along its path.

Since the refractive index in the Earth's atmosphere is very close to 1, refractivity (N) is often used in practical applications. The relationship between refractivity and the index of refraction is given by:

$$N = 10^6 \cdot (n - 1) \quad (3)$$

The delay introduced by the atmosphere can be expressed in terms of distances. Let G denote the geometric distance, i.e., the straight line between transmitter and receiver in absence of refraction. The actual propagation path through the atmosphere, denoted S , is curved due to refraction, according to Fermat's principle.

The electrical path length or apparent path length, L_e , is defined as the integral of the refractive index along the true ray path S :

$$L_e = \int_S n(S) dS \quad (4)$$

This represents the effective path that will be measured, affected by the slower propagation speed in the atmosphere than in vacuum and by the bending of the ray.

The excess path length (ΔL) is then defined as the difference between the electrical path and the geometric distance:

$$\Delta L = L_e - G = \int_S n(S) dS - G \quad (5)$$

Using the definition of refractivity in Equation (3), this can be expressed as:

$$\Delta L = 10^{-6} \int_S N(s) ds + (S - G) \quad (6)$$

where the first term accounts for the excess path due to slower signal velocity, while the second term represents the extra distance caused by the curvature of the ray. Both effects must be considered simultaneously in precise geodetic measurements. Some strategies to mitigate refraction effects over distances are discussed in Section 4.1.

Another important parameter when modelling atmospheric refraction effects is the coefficient of refraction (k). This coefficient, dimensionless, represents the degree to which a curved light path deviates from a straight line due to vertical gradients in the refractive index. It is defined as the ratio between the Earth's radius (R) and the effective radius of curvature of the light path (r):

$$k = \frac{R}{r} \quad (7)$$

This coefficient is especially relevant in the modelling of vertical angle corrections, where the curvature of the light path impacts the apparent zenith angles. A commonly adopted mean value is $k = 0.13$, originally proposed by Gauss [1]. However, this approximate value has been shown to be insufficient for representing real-working environments as it varies significantly with atmospheric conditions and height.

As previously noted, the atmosphere not only affects distances, but also curvature. Under the assumption of a stratified atmosphere, Snell's law states [20]:

$$n \cdot \sin \beta = cte \quad (8)$$

where β is the zenith angle. Differentiating this expression leads to:

$$\sin \beta \cdot dn + n \cdot \cos \beta \cdot d\beta = 0 \quad (9)$$

since:

$$dn = (\text{grad } n) \cdot ds = |\text{grad } n| \cdot \cos \beta \cdot ds \quad (10)$$

the curvature of the path is obtained as:

$$\frac{1}{r} = \frac{d\beta}{ds} = -\frac{|\text{grad } n|}{n} \cdot \sin \beta \quad (11)$$

The refractive index gradient ($\text{grad } n$) can be separated into horizontal and vertical components. Because the horizontal component is normally several orders of magnitude smaller, only the vertical component is considered, yielding:

$$\frac{1}{r} = -\frac{1}{n} \frac{dn}{dh} \sin \beta \quad (12)$$

where h is the height.

In terrestrial measurements, the value of n is close to 1 and the zenith angle β is approximately 100 gon, so Equation (12) can be simplified as:

$$\frac{1}{r} = -\frac{dn}{dh} \quad (13)$$

Considering Equation (13), the coefficient of refraction k can be expressed as:

$$k = -R \cdot \frac{dn}{dh} \quad (14)$$

Thus, k provides the link between the vertical gradient of the refractive index and the apparent curvature of light rays. Accurate modelling or estimation of k is essential in precise angular observations, as discussed in detail in Section 4.2.

The index of refraction (n), or refractivity (N), depends on several factors, including atmospheric composition, temperature, pressure, humidity and the signal frequency.

An important physical property of a propagation medium is whether it behaves as dispersive or non-dispersive for certain frequencies. A dispersive medium is one in which the refractive index (n) depends on the signal frequency. This means that signals at different frequencies travel at different speeds, leading to frequency-dependent propagation delays. In contrast, a non-dispersive medium is characterized by a refractive index that remains effectively constant across the relevant frequency band, so all components of the signal are equally delayed.

In the Earth's atmosphere, each layer behaves differently depending on the frequency of the signal:

- The ionosphere presents a strong dispersive behaviour in the microwave domain, particularly in the L-band (1-2 GHz) used by GNSS. This dispersion leads to different delays for signals at different frequencies, so dual-frequency observations can be used to estimate and correct ionospheric delay in GNSS (see Chapter 3).
- The neutral atmosphere is non-dispersive for microwaves, meaning all frequencies are delayed equally. However, it behaves as a dispersive medium in the visible and near infrared range, which is relevant for TS and EDM instruments. In this case, corrections must account for the wavelength-dependence of the refractive index to achieve millimetre or submillimetric level precision (see Chapter 4).

In a dispersive medium, two types of refractive indices are distinguished:

1. The phase refractive index, related with the speed of the signal's carrier wave.
2. The group refractive index, related to the speed of the modulated signal.

As previously noted, the refractive index (n), or refractivity (N), depends not only on the signal's frequency but also on the thermodynamic state of the atmosphere, such as temperature, pressure, humidity, and gas composition (e.g., CO_2 concentration).

In the microwave domain, particularly relevant for GNSS applications, several empirical models have been developed to express refractivity as a function of meteorological parameters. These models, which are based on measurements and physical assumptions about the neutral atmosphere, are presented and discussed in detail in Chapter 3.

For the visible and near infrared range, several formulations have been proposed by different authors to model the relation between refractive index and meteorological parameters. These formulations are discussed in Chapter 4.

2.3 Humidity parameters

This section is focused on humidity parameters, specifically those that are related to the wave propagation. Humidity, which refers to the water vapour content in the air, is a significant factor affecting atmospheric refractivity, especially in the case of microwaves. Understanding these parameters is crucial for accurately modelling atmospheric effects on signal propagation. As introduced in section 2.1.1, the amount of water vapour in air is very variable in both time and location, making its estimation difficult.

Partial pressure of water vapour

The partial pressure of water vapour, denoted as e , represents the contribution of water vapour to the total atmospheric pressure [21]. It is a crucial parameter in determining the refractivity of the atmosphere.

Saturation vapour pressure

The saturation vapour pressure, e_{sat} , is the maximum partial pressure that water vapour pressure can exert in air at a given temperature. When this value is reached, the air cannot hold additional water vapour, and any excess vapour condenses into liquid water. At this point, the air is considered saturated, and the water vapour is in equilibrium with liquid water [22].

Several formulations for calculating the saturation vapour pressure have been proposed in literature over the years. The most commonly used formulations are presented below. For consistency, all equations have been unified so that temperature is expressed in degrees Celsius [C] and saturation vapour pressure (e_{sat}) in kilopascals [kPa].

- Clausius-Clapeyron. This is one of the most extended formulas for calculating the saturation vapour pressure, from which several slightly different values have been derived by diverse authors. The expression is [22]:

$$e_{sat} = e_0 \cdot \exp \left[\frac{L}{R_v} \cdot \left(\frac{1}{T_0} - \frac{1}{T} \right) \right] \quad (15)$$

where L is a latent-heat parameter and R_v is water vapour gas constant. Using nominal values, the numerical form of Equation (15) is:

$$e_{sat} = 0.6113 \cdot \exp \left[5423 \cdot \left(\frac{1}{273.15} - \frac{1}{T + 273.15} \right) \right] \quad (16)$$

- Tetens. The Tetens formula to calculate the saturated water vapour pressure is [23]:

$$e_{sat} = \begin{cases} 0.6105 \cdot \exp \left(\frac{17.269T}{237.3+T} \right), & T \geq 0 \\ 0.6105 \cdot \exp \left(\frac{21.875T}{265.5+T} \right), & T < 0 \end{cases} \quad (17)$$

- Magnus. The formula of Magnus, as given in [13] is:

$$e_{sat} = 0.61078 \cdot \exp \left(\frac{17.1 \cdot T}{235 + T} \right) \quad (18)$$

- Bolton. Bolton proposed the following formula, based on Wexel's results [24]:

$$e_{sat} = 0.6112 \cdot \exp \left(\frac{17.67 \cdot T}{T + 243.5} \right) \quad (19)$$

- Buck. The formula proposed by Buck is [25]:

$$e_{sat} = 0.61121 \cdot \exp \left(\left(18.678 - \frac{T}{234.5} \right) \left(\frac{T}{257.14 + T} \right) \right) \quad (20)$$

Specific humidity

The specific humidity (q) is the mass of water vapour per unit mass of air [21]. It is related to the mixing ratio (w) by:

$$q = \frac{w}{1 + w} \quad (21)$$

Specific humidity remains relatively constant with changes in temperature, making it a useful measure for characterizing the moisture content over time.

Mixing ratio

The mixing ratio (w) is defined as the mass of water vapour contained by a unit mass of dry air [21], that is the ratio of the mass of water vapour (m_v) to the mass of dry air (m_d):

$$w = \frac{m_v}{m_d} \quad (22)$$

Mixing ratio can be expressed as a function of pressure approximately by [11]:

$$w = 0.622 \cdot \frac{e}{P} \quad (23)$$

where e is the partial pressure of water vapour and P is the total pressure of air.

It is similar to the specific humidity but is often used in atmospheric science because it remains constant during adiabatic processes (processes with no heat exchange). It is particularly useful in modelling atmospheric processes because it simplifies the calculation of other humidity-related parameters.

Absolute humidity

Absolute humidity (a) is the total mass of water vapour (m_v) present in a given volume of air (V) [13]. It is expressed in grams of water vapour per cubic meter of air:

$$a = \frac{m_v}{V} \quad (24)$$

Absolute humidity provides a direct measure of the water vapour content but is less commonly used in atmospheric modelling because it varies significantly with temperature and pressure.

Relative humidity

The relative humidity (RH) is a dimensionless measure expressed as a percentage that compares the current amount of water vapour in the air to the maximum amount of water the air can hold at a specific temperature [10]. It can also be expressed as the quotient

between the water vapour partial pressure (e) and the saturated water vapour pressure (e_{sat}), usually expressed as a percentage:

$$RH = \frac{e}{e_{sat}} \cdot 100 \quad (25)$$

Relative humidity is a critical parameter for understanding atmospheric moisture and predicting weather patterns, as it directly influences condensation and evaporation processes.

3. Atmospheric effects in GNSS

Global Navigation Satellite Systems (GNSS) signals, which operate in the microwave frequency range, are affected by the Earth's atmosphere. As these signals travel from satellites to ground-based receivers, they pass through various atmospheric layers that differently influence signal propagation. This chapter introduces the main atmospheric effects caused by the ionosphere and the troposphere and provides an overview of common mitigation strategies.

3.1 Ionosphere

The ionosphere, as introduced in Section 2.1.2, is the upper part of the atmosphere containing a mixture of free electrons and positively charged ions. It affects the propagation of GNSS signals by altering their speed and trajectory. In contrast to the neutral atmosphere, the ionospheric effect is a dispersive medium for the L-band (1-2 GHz), meaning that the induced signal delay depends on frequency.

The ionospheric effect results in a pseudo-range delay that increases the travel time, and a carrier-phase advance (apparent shortening of the wavelength), both of which have equal magnitude but opposite sign [15]. This dual effect has important implications in GNSS signal processing and must be mitigated to achieve high-precision positioning and makes difficult ambiguity resolution.

The ionospheric delay is highly variable and influenced by several environmental factors such as [14]:

- Diurnal and seasonal variations: electron density increases during the day due to the solar radiation and decreases at night due to recombination. Seasons also influence ionization, being strongest near the equinoxes.
- Solar and geomagnetic activity. Solar flares and geomagnetic storms can cause sudden increases in Total Electron Content (TEC), resulting in rapid fluctuations in signal propagation conditions.
- Latitudinal dependence. The equatorial anomaly (typically between $\pm(15^\circ$ to 20° latitude) exhibits the highest TEC levels, due to interactions between electric and magnetic fields, which lift and redistribute ionization to higher altitudes.
- Ionospheric stratification. The ionosphere consists of different layers (D, E, F1, and F2) each contributing differently to the overall ionization (see Section 2.1.2).

From a modelling perspective, the ionospheric delay can be divided into first-order and higher-order effects. The first-order effect represents about 99.9 % of the total ionospheric error [26] and it is proportional to the TEC along the signal path and inversely proportional to the square of the signal frequency (f) [15]:

$$I = \frac{40.3 \cdot TEC}{f^2} \quad (26)$$

Higher-order effects, although much smaller (typically $<0.1\%$), become relevant in high-precision GNSS applications. These effects depend not only on the TEC but also on the orientation and strength of the Earth's magnetic field, as well as the plasma state of the ionosphere.

3.1.1 Mitigation of ionospheric delay

Given the potentially large and variable nature of ionospheric delays, several mitigation strategies have been developed along the years. These can be broadly categorized into dual-frequency combinations, empirical models, and network-based corrections, each with different levels of accuracy and complexity.

Dual-frequency observations

One of the most effective techniques for mitigating ionospheric effects is the use of dual-frequency GNSS receivers. Since the ionospheric delay is inversely proportional to the square of the frequency, Equation (26), measurements taken at two distinct frequencies can be combined to eliminate the first-order delay.

The known as ionosphere-free combination, also referred to as L3, is given by:

$$\Phi_3 = \frac{f_1^2}{f_1^2 - f_2^2} \Phi_1 - \frac{f_2^2}{f_1^2 - f_2^2} \Phi_2 \quad (27)$$

where:

- Φ_1 and Φ_2 are the carrier-phase measurements at frequencies f_1 and f_2 respectively
- Φ_3 is the resulting ionosphere-free observable

This linear combination effectively removes the first-order ionospheric term from the measurement equation, especially in double-differenced form. However, it comes at the cost of increased measurement noise and difficulties in estimating ambiguities compared to single-frequency solutions.

Empirical models

Alternatively, empirical models can be used to estimate and correct the ionospheric delay. The most commonly used is the Klobuchar model, which provides a simple approximation of the vertical ionospheric delay based on coefficients (α_n, β_n) broadcast in the GPS navigation message. Apart from these parameters, the model requires:

- User’s approximate geodetic coordinates
- Satellite’s azimuth and elevation angles

Using these inputs, the ionospheric delay is estimated using a cosine function modulated by the local time and the ionospheric pierce point. A full algorithmic description can be found in [27].

While the Klobuchar model corrects only about 50–60% of the absolute ionospheric error, it is computationally efficient and suitable for low-cost, single-frequency GNSS applications. For relative calculations, as the case of GBDM+, is also potentially applicable (see Section 5.2).

Network-based corrections

Another strategy for mitigating ionospheric effects is the use of network-based corrections, where data from reference stations are processed together to estimate and model atmospheric delays across a region. These corrections can then be transmitted to users in real time or near-real time, allowing improved positioning accuracy.

In this context, the ionospheric delay is treated as a spatially correlated error that can be interpolated from a reference network to the user’s location. This approach is particularly effective for eliminating the ionospheric component of the error in differential GNSS techniques, such as real-time kinematic (RTK).

Network-based methods are particularly beneficial during periods of ionospheric disturbance or in regions with high spatial variability in Total Electron Content (TEC). While these corrections are primarily effective for real-time applications, similar concepts can be used in post-processing scenarios.

3.2 Troposphere

The troposphere, or strictly neutral atmosphere, as introduced in Section 2.1.2, is the lowest part of the atmosphere, extending up to approximately 10–15 km above the Earth’s surface. In the context of microwave signal propagation (e.g., GNSS), it behaves as a non-dispersive medium, meaning that the delay caused by the atmosphere is independent of frequency. In this section, the tropospheric effect on microwaves signals is described, along with possible methods for its mitigation.

As introduced in Section 2.2, electromagnetic signals experience a path delay due to the refractive index of the atmosphere. This delay (ΔL) can be expressed as an integral of refractivity N along the signal path:

$$\Delta L = 10^{-6} \int N \cdot ds \quad (28)$$

This relationship, introduced in Equation (28), also applies to the troposphere. In case of microwaves signals, the refractivity is typically modelled using an expression that accounts for both dry gases and water vapour. This expression, derived from the general refractivity formulation, is:

$$N = k_1 \frac{P}{T} Z_d^{-1} + k_2 \frac{e}{T} Z_w^{-1} + k_3 \frac{e}{T^2} Z_w^{-1} \quad (29)$$

where:

- N is the refractivity [unitless]
- P is the pressure of dry air [hPa]
- e is the partial water vapour pressure [hPa]
- T is the temperature [K]
- Z_d is the compressibility factor for dry air [unitless]
- Z_w is the compressibility factor for water vapour [unitless]

Different values for the empirical constants k_1 , k_2 , and k_3 have been proposed by various authors. For example, assuming the usual CO_2 concentration of 375 ppm these values are $k_1 = 77.6890$, $k_2 = 71.2952$, and $k_3 = 375463$ [13]. However, it is important to consider that the CO_2 concentration is quickly increasing the last decades. These values are calibrated for microwave propagation and form the basis of most tropospheric models.

It is common to decompose the total refractivity into two components:

- Hydrostatic or dry refractivity, N_h , which accounts for the delay caused by dry gases and is relatively stable and predictable.
- Wet refractivity, N_w , which results from water vapour and exhibits strong spatial and temporal variability.

Accordingly, the total tropospheric delay can be expressed as the sum of the delays due to the dry (ΔL_h) and wet (ΔL_w) components:

$$\Delta L = \Delta L_h + \Delta L_w = 10^{-6} \int N_h \cdot ds + 10^{-6} \int N_w \cdot ds \quad (30)$$

The hydrostatic component, being more predictable, can be estimated accurately using surface pressure data and standard atmospheric assumptions. In contrast, the wet component is more challenging to model due to its strong dependence on local humidity, which varies significantly in both space and time.

Different strategies have been developed to approximate these integrals, leading to the formulation of various tropospheric delay models. These models typically aim to estimate

or correct ΔL , especially the wet component, which is difficult to model due to its high variability. Some of the most used tropospheric models are further detailed in Appendix A.1.

3.2.1 Mitigation of tropospheric delay

Tropospheric delay represents a significant error source in space geodetic techniques as Global Navigation Satellite Systems (GNSS). Its magnitude can reach approximately 2.3 meters in the zenith direction, while at low satellite elevation angles (e.g., 10°), the delay can exceed 20 meters [12]. Given the magnitude of this effect, effective correction or mitigation strategies are essential for achieving high-precision GNSS results.

Several approaches have been developed and have been widely adopted by the scientific community to account for tropospheric delay in GNSS processing. These methods vary in complexity, auxiliary data requirements, and achievable accuracy. Broadly speaking, they can be classified into three categories: empirical models, direct measurement techniques, and estimation strategies within the adjustment process itself.

The most commonly used approach involves the use of empirical or semi-empirical tropospheric delay models that approximate the tropospheric delay based on surface meteorological parameters or climatological data. These models generally decompose the total delay into two components: the hydrostatic (or dry) delay and the wet delay. The hydrostatic component, which accounts for roughly 85% of the total tropospheric delay [15], is relatively stable both in space and time. It is strongly correlated with atmospheric pressure and can therefore be estimated with high reliability using surface pressure measurements.

The wet component, in contrast, is significantly more difficult to model. It is caused by the presence of water vapour in the lower atmosphere, which exhibits high spatial and temporal variability. This component is sensitive to rapid changes in weather conditions and cannot be reliably inferred from surface pressure alone. Although several empirical models have been proposed to estimate the wet delay based on surface humidity and temperature (e.g., Saastamoinen, Askne and Nordius, Callahan, Berman TMOD), their accuracy remains limited in highly dynamic atmospheric scenarios. These limitations have led to the development of more advanced strategies for modelling or estimating the wet delay.

The modelling of tropospheric delay typically follows a two-step approach. First, both zenith hydrostatic delay (ZHD) and zenith wet delay (ZWD), are estimated using an appropriate model. Then, these zenith values are mapped to the actual signal direction using so-called mapping functions, which account for the elevation angle of the satellite signal. The total tropospheric path delay in the signal direction can be expressed as:

$$\Delta L = ZHD \cdot m_{f_h} + ZWD \cdot m_{f_w} \quad (31)$$

where mf_h is the hydrostatic or dry mapping function and mf_w the wet mapping function [28].

An alternative to model-based correction is the use of direct measurement techniques, which aim to retrieve the delay, particularly its wet component, through specific instruments. Among these water vapour radiometers (WVRs) are usually employed. These instruments estimate the integrated water vapour along the signal path by measuring sky brightness temperature at microwave frequencies. WVRs can provide real-time and high resolution estimates of the wet delay, which are particularly valuable in scientific and geodetic applications. However, their operational deployment is limited by their cost, calibration requirements, and sensitivity to local conditions, so their use is often limited to specific research or very specific applications [29].

A third strategy involves estimating the tropospheric delay directly as an unknown parameter within the GNSS data adjustment process. This approach is particularly common in high-precision GNSS techniques such as Precise Point Positioning (PPP) and Network Real-Time Kinematic (NRTK). In these methods, the tropospheric delay, often represented as a zenith total delay (ZTD), is estimated on a per-epoch, per-station basis. The advantage of this strategy is that it doesn't rely on external meteorological data and allows dynamic adaptation to actual atmospheric conditions. In the case of PPP, however, the estimation of ZTD contributes to the slow convergence time of the solution, as it is added to the number of parameters to be resolved and increases the dependency on precise satellite orbit and clock products.

In practice, the selection of a mitigation strategy depends on several factors: the required level of positioning accuracy, the availability of external data (e.g., meteorological observations), the operational context (real-time vs. post-processing), and the computational resources available. In general-purpose applications, empirical models offer simplicity and low data requirements, while estimation-based methods provide greater adaptability and accuracy, particularly in dynamic environments.

In the specific context of metrology, however, an additional consideration becomes crucial: the ability to assess and propagate the uncertainty associated with each correction applied, as it is the case of GBDM+, which is later analysed in this thesis. In this regard, methods that estimate the tropospheric delay within the GNSS processing, such as the PPP service CSRS-PPP [30], offer a clear advantage, as it delivers ZTD values along with their corresponding standard deviations, facilitating rigorous uncertainty evaluation and traceability. In contrast, empirical models rarely include information on the uncertainty of their predictions.

Tropospheric models

A wide range of models have been developed by the scientific community over the years to estimate the ZTD or its components (ZHD and ZWD). These models vary in complexity,

data requirements, and intended application. Broadly, they can be classified into two categories:

1. Empirical models using meteorological inputs: these models require surface meteorological data (e.g., temperature, pressure, humidity) and are typically based on empirical formulations derived from observational datasets. Usually provide zenith hydrostatic delay (ZHD) and zenith wet delay (ZWD) separately, which have to be projected into the slant direction using a mapping function. Examples of these models include:
 - Saastamoinen model: a widely used and relative simple model that estimates both the hydrostatic and wet delays [31].
 - Callahan and Askne-Nordius models: developed to provide improved estimation of the wet component based on refined empirical relationships [32, 33].
 - Berman models: models developed for accurate delay estimation under specific climatological conditions [34].
2. Blind models: these models do not require local meteorological observations. Instead, they rely on statistical climatology or numerical weather prediction (NWP) data to infer the necessary atmospheric parameters [35]. Blind models are particularly valuable in applications where meteorological data are unavailable or impractical to obtain. Examples of blind models are Global Pressure and Temperature (GPT) models.

Table 2 presents an overview of some of these models and their main characteristics, including what type of input data they require and their output.

Table 2: Tropospheric models resume

Model	Inputs	Outputs
Saastamoinen (complete)	T, P, e, β	ΔL
Saastamoinen (refined)	T, P, e, β	ΔL
Saastamoinen (dry)	P, φ, h	ZHD
Saastamoinen (wet)	T, e	ZWD
Chao	T, e, dT	ZWD
Callahan	T, e	ZWD
Berman70	T, e, dT	ZWD
Berman74	T, e	ZWD
BermanD/N	T, e	ZWD
BermanTMOD	T, e	ZWD
Hopfield (dry)	T, P	ZHD
Hopfield (wet)	T, e	ZWD
Black	T, P	ZHD
Askne and Nordius	T, e, dT, φ, doy	ZWD
Ifadis	T, P, e	ZWD
GPT	doy, φ, λ, h	P, T
GPT2	doy, φ, λ, h	$P, T, dT, e, \text{VMF1 coefficients}$

Input parameters in Table 2 are:

- T is temperature
- P is pressure
- e is water vapour pressure
- β is the zenith angle
- φ is geodetic latitude
- λ is geodetic longitude
- h is height
- dT is the temperature lapse rate (assumed constant decrease of temperate with heigh)
- doy is the day of year

In the context of this thesis some of these models have been selected to compute and analyse the tropospheric delay (see section 5.3). For the hydrostatic component (ZHD),

the Saastamoinen model is adopted because it is one of the most widely used in both scientific and engineering applications and several studies have reported its good agreement with observations [36, 37]. For the wet component (ZWD), multiple models are used to allow a comparative analysis: Saastamoinen (for consistency with the ZHD estimation), Callahan, Askne and Nordius, and Berman TMOD.

A detailed presentation of the mathematical expressions of each model used in this thesis is included in the Appendix A.1 where the notation and units have been unified for clarity, as the original sources often differ in how variables are defined and the units employed.

Tropospheric mapping functions

In the previous section, tropospheric models were introduced. Most of these models estimate zenith delays, which need to be projected to the slant direction using a mapping function. The tropospheric effect increases at lower satellite elevations (ε), since the signal traverses a longer atmospheric path.

Two types of mapping functions can be distinguished:

1. Mapping functions dependent on surface meteorological parameters: they require local measurements of temperature, pressure, and water vapour pressure. An example is the Herring mapping function.
2. Mapping functions independent of *in situ* measurements: they rely on pre-defined models or gridded data that account for seasonal and geographical variability. Notable examples include the Niell and Vienna Mapping Function 1 (VMF1) models.

Usually the dry mapping function (mf_h) and the wet mapping function (mf_w) are computed separately, using different formulation. Table 3 summarizes the characteristics of the most relevant mapping functions, highlighting their required inputs and the delays they model (hydrostatic and/or wet).

Table 3: Mapping functions comparison

Mapping function	Inputs	Outputs
Chao	ε	mf_h, mf_w
CfA-2.2	ε, T, P, e, dT	mf_h
Ifadis	ε, T, P, e	mf_h, mf_w
Herring	$\varepsilon, T, h, \varphi$	mf_h, mf_w
Niell	$\varepsilon, \varphi, doy, h$ ε, φ	mf_h mf_w
VMF1	$\varepsilon, \varphi, \lambda, h, doy$, grid files	mf_h, mf_w
VMF3	$\varepsilon, \varphi, \lambda, h, doy$, grid files	mf_h, mf_w
GMF	$\varepsilon, \varphi, \lambda, h, doy$	mf_h, mf_w

Notation of input parameters in Table 3 is the same as in the previous section. T is temperature, P is pressure, e is water vapour pressure, φ is latitude, λ is longitude, h is height, dT is the temperature lapse rate (assumed constant decrease of temperature with height), doy is the day of year, and ε is the elevation.

In the practical part of this thesis, VMF1, Niell, and Herring mapping functions have been implemented, owing to their widely accepted use and relevance in GNSS applications. VMF1, in particular, is used in CSRS-PPP, which is a tool employed for performance evaluation. Both VMF1 and Niell are independent of meteorological surface measurements, making them suitable for scenarios where real-time data is unavailable. Contrary, Herring mapping function requires local meteorological data.

The detailed formulations and coefficient definitions for each of these mapping functions are provided in Appendix A.2.

4. Atmospheric effects in terrestrial techniques

As introduced in Section 2.2, all geodetic techniques based on electromagnetic signals are influenced by atmospheric refraction, which alters the propagation speed and path of the signal. Terrestrial geodetic techniques, such as electronic distance measurement (EDM), and total stations (TS) use electromagnetic waves in the visible or near-infrared spectrum. As these waves travel through the atmosphere, their velocity and trajectory are affected by varying atmospheric conditions, introducing deviations in both distance and angle measurements.

To ensure accurate results, these atmospheric effects must be properly corrected or mitigated. This chapter explores the impact of atmospheric refraction on distance measurements and vertical angles, and presents several strategies for its reduction or compensation. Although atmospheric refraction also influences horizontal angles, this effect is typically negligible in open-air geodetic work. However, in specific scenarios such as tunnelling or confined environments, horizontal refraction can become significant and must be taken into account.

4.1 Effects over distances

Accurate terrestrial distance measurements using Electronic Distance Measurement (EDM) instruments or Total Stations (TS) are highly sensitive to atmospheric refraction. As electromagnetic signals travel through the atmosphere, variations in air temperature, pressure, and humidity affect the refractive index, which in turn alters the signal's speed and path. This section focuses specifically on the impact of atmospheric refraction on EDM observations. First the physical principles of EDMs are introduced and related with the refraction index. Then, the standard corrections are presented. Finally, practical approaches and advanced techniques for estimating or compensating for refractive effects over distances are discussed.

4.1.1 Principles of EDM and refractive index dependence

To better understand the effect of atmospheric refraction on distances measured by EDMs, it is essential to first introduce their basic working principles. EDM devices primarily operate by sending a beam, typically in the infrared or visible spectrum, toward a target reflector and then receiving the reflected signal back at the instrument [38]. Since the beam travels from the instrument to the target and back, the total distance travelled by the electromagnetic wave is twice the instrument-to-target distance.

The measured distance (D_0) can be expressed as a function of the travel time (Δt) [39]:

$$D_0 = \frac{1}{2} \cdot v \cdot \Delta t \quad (32)$$

where v is the velocity of the wave in the propagation medium.

This method of obtaining distance based on the measurement of the travel time is known as the pulse method. However, EDMs more commonly use continuous waves rather than pulses. In this case, the distance is determined by analysing the phase difference between the emitted and received signals.

In this approach, known as the phase difference method [39], the measured distance can be expressed as an integer number of full cycles ($m\lambda$) plus a fractional part ($g\lambda$):

$$D_0 = m \cdot \frac{\lambda}{2} + g \cdot \frac{\lambda}{2} \quad (33)$$

Several methods can be used to determine the integer m and fractional g components which are critical for calculating the distance based on Equation (33). These methods include:

- Phase resolution. The fractional part g is determined by measuring the phase angle ($\Delta\phi$) between the emitted and the received signals, giving:

$$D_0 = m \cdot \frac{\lambda}{2} + \frac{\Delta\phi}{2\pi} \cdot \frac{\lambda}{2} \quad (34)$$

The integer m is resolved by repeating the measurement at several frequencies.

- Path length modification. Some instruments eliminate the fractional part by adjusting the distance until the phase difference is zero, leading to [39]:

$$D_0 + \delta D = m \cdot \frac{\lambda}{2} \quad (35)$$

Again, m is found by repeating the measurement at different frequencies.

- Frequency modification. The frequency is adjusted until no phase difference is observed. When a zero phase difference is achieved, m is obtained by slightly modifying the frequency until another zero phase difference occurs, indicating $m+1$ half cycles:

$$D_0 = (m+1) \cdot \frac{\lambda_i}{2} \quad (36)$$

By using the two frequencies that yield zero phase differences, m is obtained.

Regardless of the specific method, the measured distance depends on the refractive index (n). From Equation (32), this dependence appears through the wave velocity v , which is influenced by the propagation medium. Recalling the definition of the refractive index (Equation (1)) introduced in Section 2.2, the velocity is:

$$v = \frac{c}{n} \quad (37)$$

where c is the speed of light in vacuum.

The relationship between the methods based on Equation (33) stems from the definition of wavelength, which depends on the propagation velocity v and the wave frequency f :

$$\lambda = \frac{v}{f} \quad (38)$$

Given that the measured distance depends directly on the wave velocity v , and thus on the refractive index n , the accurate determination of n becomes essential for precise distance measurements. Since n depends on atmospheric conditions, it can be expressed as a function of meteorological parameters such as pressure, temperature, and humidity.

Several empirical formulas have been developed by different authors to calculate the refractive index or refractivity from these meteorological parameters. Among these, the International Association of Geodesy (IAG) Resolution number 3 recommended formula provides a close-form expression suitable for visible and near-infrared wavelengths, achieving an accuracy of about 1 ppm in distance measurements [40]:

$$N = N_{gr} \cdot \frac{273.16}{1013.25} \cdot \frac{P}{T} - 11.27 \cdot \frac{e}{T} \quad (39)$$

where:

- N_{gr} is the group refractivity, a dimensionless, wavelength-dependent coefficient
- P is the atmospheric pressure [hPa]
- T is the temperature [K]
- e is the partial pressure of water vapour [hPa].

The wavelength dependence of N_{gr} (with λ in micrometers) is given by:

$$N_{gr} = 287.6155 + \frac{4.8866}{\lambda^2} + \frac{0.068}{\lambda^4} \quad (40)$$

For applications requiring even higher precision, according to the IAG Resolution number 3 it is recommended using the group refractive index computation method developed by Ciddor and Hill [41], which incorporates detailed spectroscopic data and atmospheric composition. However, during the 2020 EMPIR GeoMetre project, a sign error in the

annex of this method was identified, which causes a deviation of the group refractive index with the wrong sign. Pollinger published a correction to this error [42], IAG Resolution number 3 advises applying this correction to achieve the highest accuracy in refractivity calculations for EDM.

4.1.2 Classical corrections for distances

To correct for deviations from the standard atmospheric conditions under which EDM instruments are calibrated, several correction terms are usually applied to the measured distances.

First velocity correction

The distance measured by the EDM (D_0) depends on a reference refractive index (n_0) which does not necessarily coincide with the actual refractive index (n) at the time of the measurement, so it has to be corrected to obtain the true distance (D). Since the optical path remains constant, so:

$$D \cdot n = D_0 \cdot n_0 \quad (41)$$

The corrected distance can be written as:

$$D = D_0 \cdot \frac{n_0}{n} \quad (42)$$

The first velocity correction (c_1) is defined by approximating the current refractive index n with the mean refractive index between the path endpoints (i and j):

$$n = \frac{n_i + n_j}{2} \quad (43)$$

This correction (c_1) corresponds to the difference between the corrected distance and the distance measured by the instrument:

$$c_1 = D - D_0 \quad (44)$$

which, substituting Equation (42), yields:

$$c_1 = D_0 \cdot \left(\frac{n_0}{n} - 1 \right) \quad (45)$$

Second velocity correction

In the first velocity correction, the actual refractive index n is approximated by the mean of the refractive indices observed at the endpoints of the line. This procedure implicitly assumes that the variation of n along the path is linear. In reality, the vertical gradient of the refractive index is generally non-linear due to atmospheric stratification, so the averaging process always introduces a systematic discrepancy. This residual error is accounted for by the second velocity correction (c_2) given by [39]:

$$c_2 = \frac{k \cdot (1 - k) \cdot D_0^3}{12 \cdot R^2} \quad (46)$$

where k is the coefficient of refraction, which relates the vertical gradient of the refractive index to the curvature of the Earth's atmosphere (see Section 2.2), and R is the Earth's radius.

The second velocity correction is much smaller than the first velocity correction and is commonly neglected in practice for distances shorter than about 10 km. Assuming a value of $k = 0.13$ and an Earth radius of 6371 km, the correction for a distance of 10 km is approximately 0.23 mm. The correction reaches 1 mm at a distance of roughly 15–16 km.

Correction for the curvature of the path

Variations in the refractive index along the propagation path, primarily due to vertical temperature gradients, cause the light or EDM signal to follow a slightly curved trajectory rather than a straight line. Consequently, the distance measured along the curved wave path corresponds to an arc, which differs from the equivalent straight-line (chord) distance between the two endpoints. To obtain the rectilinear distance, an arc-to-chord reduction must be applied, which can be modelled by the following curvature correction:

$$c_3 = -\frac{k^2 \cdot D_0^3}{24 \cdot R^2} \quad (47)$$

where k is the refractive index coefficient, D_0 is the measured distance, and R is the Earth's radius.

This correction is generally negligible. For example, considering again a value of $k = 0.13$ and an Earth radius of 6371 km, the correction for a distance of 10 km is approximately 0.02 mm. Therefore, it is typically not applied in standard geodetic measurements.

4.1.3 Estimation and compensation techniques

The previously presented corrections are commonly applied to EDM measurements to compensate for atmospheric refraction effects. However, the challenge of accurately determining the actual refractive index along the measurement path remains unresolved. Therefore, a variety of estimation and compensation techniques have been proposed by various authors to address this critical issue and improve measurement accuracy.

Measurement of meteorological parameters

A widely used approach to estimate the refractive index is to measure meteorological parameters, such as temperature, pressure, and humidity, and compute the refractivity using

interpolation formulas like Equation (39). This method is straightforward to implement and is commonly used in practical applications.

The most common procedure involves collecting meteorological data at both ends of the measured line. When the wave path is approximately horizontal and atmospheric conditions vary smoothly, this approach provides a reasonably good estimate of the average refractive index along the path [39]. In general, that conditions involves distances below 1 km. In such cases, the integral of the refractive index on which the correction depends, as discussed in Section 2.2, can be well approximated by the average of the values measured at the two endpoints.

However, when the conditions along the wave path are not uniform, with significant height differences, abrupt changes in terrain properties or proximity to water bodies, atmospheric properties present local variations, so values measured at both ends may not accurately represent the entire path.

Moreover, in applications such as monitoring critical areas, target points are often inaccessible, so only meteorological data near the instrument can be measured. In those cases, the estimated refractive index becomes even less representative of the full signal path.

Ideally, meteorological parameters should be measured along the line of sight, at intervals of a few hundred meters [43], to more accurately reflect the true refractive index distribution. However, this is often not feasible in practice due to logistical and environmental limitations.

Refractometers

Refractometers are instruments that enable the direct measurement of the ambient refractive index without the need of measuring meteorological parameters. A common type is the dual-beam laser interferometer, which uses two laser beams: one travels through a vacuum chamber and the other through ambient air. By gradually evacuating the air from the chamber and observing the difference in the optical paths of the two beams, the instrument can determine the refractive index with high accuracy [39].

While refractometers are highly effective in controlled environments, such as laboratories or industrial settings, they are limited when used in outdoor applications. This is because they provide only local measurements, typically near the ground, where conditions are strongly influenced by the surface layer, and do not account for variations along the entire EDM path. As a result, while refractometers are valuable for calibration or reference purposes, they do not yield representative values for the full distance between instrument and target in field conditions.

Ratio method

The ratio method is a technique used to compensate for atmospheric refraction effects in electro-optical distance measurements without relying on meteorological data. It is based on the idea that if several distances are measured from the same instrument location, under similar environmental and topographic conditions, then the effect of atmospheric refraction will be approximately the same across those measurements. Therefore, any bias caused by refraction can be modelled as a common, unknown scale factor applied to all distances taken from that point [39].

One of the main advantages of the ratio method is that it allows for refraction correction without requiring direct atmospheric observations. However, its successful application depends on having a sufficient number of redundant measurements to preserve the network's degree of freedom. Additionally, it does not provide an absolute scale for the network and becomes less reliable in cases where the terrain or atmospheric conditions vary significantly between measured points.

A specific application of this method is the Local Scale Parameter Model (LSPM), which extends the ratio method by introducing a single unknown scale factor applied to all measured distances within a local network. This simplification reduces the number of parameters to estimate, improves the degree of freedom of the network adjustment, and facilitates field procedures [44].

Multiple wave-length method

An alternative strategy to mitigate the effects of atmospheric refraction in electro-optical distance measurements is to use instruments that operate with multiple wavelengths. As described in Section 2.2, the neutral atmosphere behaves as a dispersive medium in the visible and near infrared spectrum, meaning the refractive index of air varies with wavelength. This property allows the use of multi-wavelength techniques to infer and compensate for refractive effects: by simultaneously measuring the same path with two or more different wavelengths, it is possible to estimate the refractive index and correct the measured distance accordingly.

For a two-wavelength instrument, the corrected distance can be obtained using the following expression [39]:

$$D = D_1 - A \cdot (D_2 - D_1) \quad (48)$$

where D_1 and D_2 are the distances measured with the two wavelengths and A is a coefficient that depends on the specific pair of wavelengths used, as well as on atmospheric pressure and water vapour content.

The coefficient A is critical because it determines the resolution needed to measure the distance difference to achieve the desired accuracy. For instance, for blue and red light, A has a value of approximate 20, meaning that the distance difference must be measured

with 20 times higher resolution than the required final result, which makes the measurement challenging.

The separation between the selected wavelengths is also crucial. On the one hand, small separations results in minimal optical dispersion, requiring extremely high resolution that is difficult to achieve in field conditions [45]. On the other hand, large wavelength separations is also challenging due to the different limitations and considerations required for each wavelength region.

The theoretical basis of this method was proposed in the 1960s, with initial practical developments beginning in the 1970s. In the 1980s, the first prototype used in engineering projects two-wavelength EDM, the Terrameter, was developed, though only a limited number of units were produced, and its complex design and operational procedure led this instrument to be no longer available.

More recently, the *Conservatoire National des Arts et Métiers* (CNAM), in the context of the GeoMetre project, has made significant advances in this area. In [7] they introduced the *Arpent*, a two-wavelength instrument based on radio-frequency phase measurements of amplitude-modulated light. This device has demonstrated outdoor distance measurements up to 5 km with a relative uncertainty better than 10^{-7} . Additionally, [8] presented a high-accuracy multispectral EDM that use an optical super-continuum source, to enable dispersion-based inline refractivity compensation.

These developments highlight the renewed interest and potential of multi-wavelength techniques for achieving high-precision outdoor distance measurements. However, these instruments are complex to design and manufacture, and producing reliable, practical versions for real field use is still a challenge.

4.2 Effects over vertical angles

When electromagnetic signals travel through the atmosphere, not only the propagation speed is affected, but the signal path itself is bent due to vertical gradients in atmospheric refractivity. This curvature directly influences the measured vertical angle.

The refraction angle ($\Delta\varepsilon$) (Figure 49) is defined as the deviation between the actual signal direction and the straight line it would follow in the absence of atmospheric refraction, and it is directly related to the refractivity gradients perpendicular to the signal's trajectory.

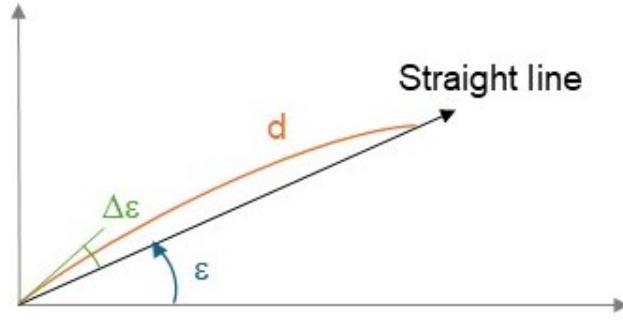


Figure 3: Refraction angle scheme (adapted from [3])

A general expression for $\Delta\varepsilon$ is given in [1]:

$$\Delta\varepsilon = \frac{10^{-6}}{l} \cdot \int_0^l \frac{dN}{du} \cdot (l - d) \cdot dx \quad (49)$$

where:

- l is the total signal path length.
- ε is the elevation angle (measured from the horizontal plane upwards).
- d is the distance from the source.
- u is the direction perpendicular to the trajectory.

The factor $(l - d)$ is included as scale factor, it accounts for the strongest influence of the closer points to the observation station on the refraction angle.

If the path is nearly horizontal, the perpendicular gradient to the line-of-sight (dN/du), can be approximated by the vertical refractivity gradient (dN/dh). Given that curvature is typically very small, (dN/du) can be approximated as:

$$\frac{dN}{du} = \cos \varepsilon \cdot \frac{dN}{dh} \quad (50)$$

Substituting this into Equation (49), a more complete formula is obtained [3]:

$$\Delta\varepsilon = \frac{10^{-6}}{l} \cdot \cos \beta \cdot \int_0^l \frac{dN}{dh} \cdot (l - d) \cdot dx \quad (51)$$

Assuming a constant gradient along the path, this simplifies to:

$$\Delta\varepsilon = \frac{1}{2} \cdot \cos \varepsilon \cdot \frac{dN}{dh} \cdot l \quad (52)$$

As previously mentioned, the refractivity, is related to meteorological parameters. Consequently, the refractivity gradient can be expressed in terms of these parameters. For electro-optical terrestrial techniques, humidity is often neglected because it has a much smaller influence compared to temperature T and pressure P . Thus, the gradient can be approximated as:

$$\frac{dN}{dh} = \frac{\partial N}{\partial T} \cdot \frac{dT}{dh} + \frac{\partial N}{\partial P} \cdot \frac{dP}{dh} \quad (53)$$

By differentiating the commonly used refractivity formula (Equation (39)) and making some simplifications, the refractivity vertical gradient can be expressed as:

$$\frac{dN}{dh} = \left((15.02e - 79.53P) \frac{1}{T^2} \right) \frac{\partial T}{\partial h} + \left(79.53 \frac{1}{T} \right) (-0.12) \quad (54)$$

The detailed derivation and assumptions are provided in Appendix B.

4.2.1 Corrections for vertical angles

In practice, instruments rarely measure the elevation angle (ε). Instead, they usually measure the zenith angle (β), defined as the angle between the line of sight and the vertical. Both are complementary:

$$\beta + \varepsilon = \frac{\pi}{2} \quad (55)$$

The correction to the measured zenith angle (δ_β) can be expressed as [46]:

$$\sin \delta_\beta = \frac{D}{2 \cdot R} \cdot k \quad (56)$$

where:

- D is the slope distance.
- R is the Earth's radius.
- k is the coefficient of refraction (introduced in Section 2.2).

It is essential to distinguish clearly between:

- Refraction angle ($\Delta\varepsilon$): it quantifies the actual angular deviation of the light path due to atmospheric curvature, typically positive when the ray bends downward.
- Correction to zenith angles (δ_β): the adjustment applied to the measured zenith angle. The correction has the opposite sign to the refraction angle:

$$\delta_\beta = -\Delta\varepsilon \quad (57)$$

To apply this correction, the coefficient of refraction k has to be estimated. As discussed in Section 2.2, this coefficient is influenced by vertical gradients in atmospheric refractivity, which in turn depend on meteorological conditions, specially on the vertical temperature gradient. Since k is not directly observable, various approaches have been developed to estimate it. These strategies can be broadly grouped in three categories:

1. Estimation of k from meteorological data. Some formulas estimate k directly from empirical relationships with parameters like pressure, temperature, and humidity. Others focus first on modelling or measuring the vertical temperature gradient, which is then used to compute k .
2. Alternative observational techniques to derive k directly from measurements, such as simultaneous reciprocal vertical angle observations.
3. Other correction strategies that do not require an explicit value of k , such as incorporating refraction effects into network adjustments or using multi-wavelength methods.

These methods are briefly discussed below.

(1) Estimation of k from meteorological data

Given the strong dependence on meteorological parameters, various formulas have been proposed to calculate the coefficient of refraction from meteorological data. One of the most widely used expression is [19]:

$$k = 503 \cdot \frac{P}{T^2} \cdot \left(0.0343 + \frac{dT}{dh} \right) \quad (58)$$

In this formula, P is the atmospheric pressure (in hPa), T is the absolute temperature [K] and $\frac{dT}{dh}$ is the vertical temperature gradient [K/m]. This gradient is the most critical component in Equation (58), as it is highly variable and is difficult to measure accurately in practice.

To estimate $\frac{dT}{dh}$ two main approaches can be used.

- Estimate temperature gradient by direct measurements. This involves placing temperature sensors at different heights above ground (h) and fitting the measured data to an analytical expression of the temperature profile. Several models have been proposed by different authors to obtain the temperature gradient, as shown in Table 4.

Table 4: Temperature gradient equations (based on [47])

Model	Temperature gradient equation
Kukkamaki	$b \cdot c \cdot h^{-1+c}$
Hugershoff	$2 \cdot b \cdot h$
Reissmann 1	$b + 2 \cdot c \cdot h$
Reissmann 2	$b + 2 \cdot c \cdot h + 3 \cdot f \cdot h^2$
Reissmann 3	$b + 2 \cdot c \cdot h + 3 \cdot f \cdot h^2 + 4 \cdot g \cdot h^3$
Heer	$b \cdot c \cdot e^{c \cdot h}$
Kharagani	$a + b \cdot c \cdot h^{-1+c}$
Linear	b

These models must be calibrated with field measurements by fitting the parameters (a, b, c, d, e, f). While this method provides a physically grounded estimate of the gradient, it presents two main difficulties: (1) selecting the best-fitting function, and (2) collecting accurate temperature data at varying heights, which requires complex instrumentation. Additionally, the gradients obtained from local measurements may not represent the entire wave path, making the method demanding and difficult to implement in practice.

- Modelling the temperature gradient. Another option is to estimate the temperature gradient using physical models of atmospheric behaviour, particularly within the boundary layer (Section 2.1.3).

An example is the Turbulence Transfer Model (TTM), which uses principles of energy flux between the Earth’s surface and the atmosphere to predict temperature profiles. This model is described in more detail in Section 6.1.1.

Alternatively, some authors propose empirical formulas that estimate k directly from meteorological variables (temperature, pressure, humidity) without requiring the temperature gradient. These models are often based on regression analysis using historical data. Examples include the formulas proposed in [48]:

- Linear model

$$k = -0.0077 \cdot P - 0.0066 \cdot T - 0.0024 \cdot RH + 6.2414 \quad (59)$$

- 2nd order polynomial model

$$k = 0.0004 \cdot (P + 0.86 \cdot T + 0.32 \cdot RH)^2 - 0.7182 \cdot (P + 0.86 \cdot T + 0.32 \cdot RH) + 287.46 \quad (60)$$

- Logarithmic model

$$k = -6.1343 \ln(P + 0.86 \cdot T + 0.32 \cdot RH) + 41.049 \quad (61)$$

- Exponential model

$$k = 2^{10} \cdot 21 \cdot e^{-0.0646 \cdot (P + 0.86 \cdot T + 0.32 \cdot RH)} \quad (62)$$

Another expression, proposed in [49], estimates the refraction coefficient based on easily accessible meteorological parameters:

$$k = 503 \cdot \frac{p}{T^2} \cdot \left(0.034 - 0.4343 \cdot B \cdot N \cdot \frac{1}{\beta} \cdot \sin a \right) \quad (63)$$

where:

- B is a coefficient experimentally determined (value = 1.70).
- N represents cloud cover (with $N = 0$ for a completely covered sky and $N = 1$ for a completely clear sky).
- a is the solar elevation angle.
- β is the zenith angle.

(2) Observational techniques: simultaneous and reciprocal observations

The method of simultaneous reciprocal vertical angle measurements is commonly applied to determine height differences, but it can also be used to estimate the coefficient of refraction. The principle consists of measuring zenith angles simultaneously from both ends of the line of sight, points A and B (Figure 4).

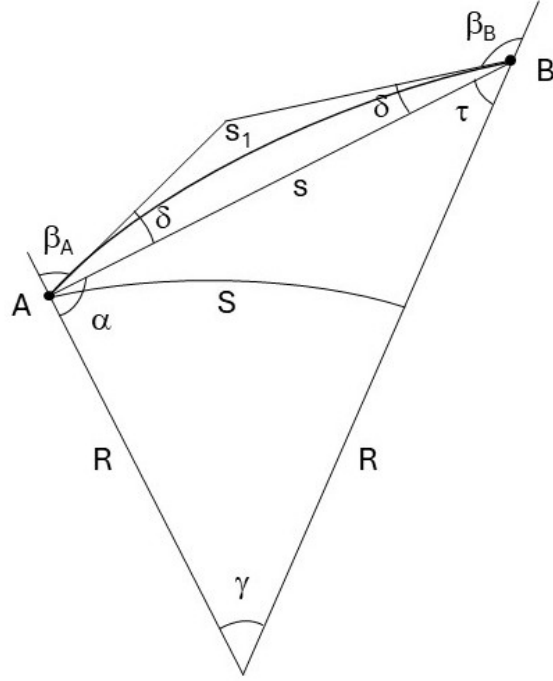


Figure 4: Basis of simultaneous and mutual observations (adapted from [50])

Following [50], the deviation of the true optical path from the straight line is denoted by δ . Assuming a spherically symmetric atmosphere, δ is identical at both ends, so the theoretical zenith angles β'_A and β'_B are obtained from the measured ones β_A and β_B as:

$$\beta'_A = \beta_A + \delta, \beta'_B = \beta_B + \delta \quad (64)$$

From Figure 4 it can be deduced:

$$\alpha + \tau + \gamma = \pi \quad (65)$$

Adding 2δ at both sides:

$$\begin{aligned} \alpha + \tau + \gamma + 2\delta &= \pi + 2\delta \\ (\alpha + \delta) + (\tau + \delta) + \gamma &= \pi + 2\delta \\ \pi - \beta_A + \pi - \beta_B + \gamma &= \pi + 2\delta \end{aligned} \quad (66)$$

So δ can be obtained as:

$$\delta = \frac{1}{2}(\pi + \gamma + \beta_B - \beta_A) \quad (67)$$

Angles α and τ can be computed as:

$$\alpha = \pi - (\beta_A + \delta), \tau = \pi - (\beta_B + \delta) \quad (68)$$

Finally, the expression to obtain δ from simultaneous and reciprocal observations is:

$$\delta = \frac{1}{2}(\pi + \gamma - \beta_A - \beta_B) \quad (69)$$

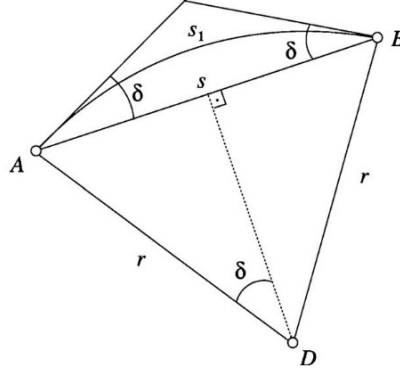


Figure 5: Spherical approximation of the actual optical ray path (Source: [50])

Under a spherical approximation of the actual optical ray path (Figure 5), δ can be geometrically related to the radius of curvature r of the bent ray:

$$\sin \delta = \frac{s/2}{r} \quad (70)$$

which allows expressing r as:

$$r = \frac{s}{2 \sin \delta} \quad (71)$$

Substituting this expression of r into the general definition of the coefficient of refraction (see Equation (7)) yields:

$$k = \frac{2R \sin \delta}{s} \quad (72)$$

Finally, replacing δ with its expression derived from the simultaneous reciprocal observations leads to a closed approximate formula for the estimation of k directly from the measured zenith angles:

$$k = 1 - \frac{R}{s}(\beta_A + \beta_B - \pi) \quad (73)$$

The complete derivation of this relationship is provided in [50].

This approach provides an average value of the coefficient of refraction along the entire line of sight, and therefore reflects the integrated effect of the atmospheric stratification

more realistically than point-based meteorological measurements.

(3) Other correction strategies

Unlike the previous approaches, the following correction strategies do not attempt to estimate the refraction coefficient. Instead, they rely on geometric or adjustment techniques to mitigate or compensate for atmospheric refraction effects.

These methods avoid the need for meteorological measurements, which are often difficult to obtain with sufficient accuracy along the entire optical path. However, they typically require specific observation configurations, a high volume of redundant measurements, or specific instrumentation and may be more complex to implement.

Network adjustment method

Another approach to accounting for refraction is to include its effects in the network adjustment process. By treating refraction as an unknown variable in the system of equations, the overall adjustment can incorporate atmospheric effects [46]. However, this method requires a large number of observations, and making the necessary assumptions for integrating these unknowns is challenging.

Multiple wave-length method

As with distances measurements, multiple wavelengths can be used to compensate for the effects of refraction on vertical angles. Since different wavelengths bend differently, the difference between the two apparent angles can be used to eliminate the refraction effect.

However, the challenge lies in accurately measuring the small angle differences caused by wavelength-dependent refraction. The resolution required to achieve accurate compensation is very high, making this method complex to implement in field conditions [51].

5. GBDM+: principles and tropospheric effect analysis

The determination of distances in outdoor environments over long ranges (several kilometres) with millimetric-level precision is required in various industrial and scientific fields. Traditional techniques such as electronic distance measurement (EDM) require intervisibility between endpoints and its precision is often limited by refraction effects. As an alternative, Global Navigation Satellite System (GNSS) technology offers significant advantages, including independence from line-of-sight visibility and suitability for large-scale deployments.

GNSS has demonstrated high precision in positioning and length determination. However, GNSS-based distance measurement also poses several challenges. First, traditional geodetic software is mainly oriented towards coordinate estimation, with limited flexibility for processing strategies. Second, various error sources influence the result and must be appropriately addressed. Finally, from a metrological perspective, its application is limited by the lack of well-defined uncertainty of the results (e.g. the estimated distance) that accounts for all significant error sources.

To overcome these limitations, the GNSS-Based Distance Meter (GBDM+) technique was developed at the Department of Cartographic Engineering, Geodesy and Photogrammetry of the *Universitat Politècnica de València* (DICGF-UPV). Its main objective is not only to provide precise distance estimations, but to rigorously propagate uncertainties from all relevant error sources into the final result, ensuring traceability to the SI metre. Unlike standard GNSS geodetic processing, which typically aims to estimate full 3D positions, GBDM+ adopts a dedicated methodology focused on baseline length determination.

The first studies related to high-precision GNSS-based length determination at the UPV focused on analysing distance reproducibility [52]. From this study, two main conclusions emerged. First, the use of identical antennas and mounting configurations in both baseline ends improves the achieved accuracy. Second, submillimetric consistency in the long term is feasible after some hour of observation over baselines up to a few hundred meters.

Subsequent investigations were focused on noise mitigation, particularly addressing the multipath error [53, 54, 55]. A mitigation methodology for L1/E1 carrier-phase double differences, based on sidereal filtering and a signal-to-noise ratio (SNR) discriminator, was tested on the UPV calibration baseline, yielding promising results. The results showed that multipath-induced residuals were typically reduced to below 1cm, with a bandwidth around 5mm [54].

A preliminary working scheme of a submillimetric GNSS-based distance meter for length metrology (GBDM) was presented in [5]. It focused on short horizontal baselines (up

to 1km) with negligible height differences, which are the general calibration baselines conditions. It was based on the following assumptions:

- Carrier-phase observations are provided with submillimetric noise level in modern receivers.
- Distances determined by GNSS are stable ($\approx 10^3$ km) at global scale at the level of $1ppb = 10^{-9}$.
- Distances can be determined with higher precision than their individual coordinate components.
- The high number of available GNSS satellites allows optimal satellite selection for length determination.

The GBDM processing approach was based on carrier-phase double differences, specifically tailored to length determination, with the following considerations:

- Only L1/E1 observables are used due to their lower noise compared to L2, L5, or E5 observables
- A free-ambiguity approach is used, eliminating the need for ambiguity resolution. This requires prior knowledge of coordinates of both ends with a few centimetres accuracy, which can easily be achieved through PPP processing.
- No tropospheric and ionospheric corrections are included. For distances under 1 kilometre in horizontal baselines, atmospheric effects are neglected as they are assumed to be nearly the same at both ends.
- Multipath mitigation is based on sidereal filtering combined with SNR-based discriminator [55].

This initial GBDM implementation was tested on the UPV calibration baseline, where its results were compared with EDM-based distance measurements carried out with a Mekometer ME5000. EDM measurements were done during the day, while GNSS measurements were performed during the night. This experiment demonstrated a consistency of the obtained distances with the SI-metre within a few tenths of a millimetre.

Despite of the promising results, the original GBDM approach was limited to relative horizontal baselines to up to 1 km. For longer distances, specially with strong height differences, the initial assumptions are no longer valid. In particular, atmospheric effects can no longer be neglected. To extend the method's applicability to longer baselines (up to 5 km) and scenarios with height variation, an improved version called GBDM+ was developed [6].

Developed within the framework of the European GeoMetre project [56, 4], GBDM+ aims to be a relatively simple strategy based on standard files and solutions. Unlike conventional GNSS processing, which does not provide a fully traceable uncertainty budget, GBDM+ rigorously propagates errors from various sources to the final distance measurement. This feature sets it apart from traditional geodetic methods and makes it suitable for metrological applications.

The GBDM+ approach provided the basis for the development of the Good Practice Guide on High-Accuracy GNSS-Based Distance Metrology, published as EURAMET Technical Guide No. 6 [57]. This guide summarizes the state of the art and establishes recommended procedures for high-accuracy GNSS-based length metrology.

This chapter is structured as follows: Section 5.1 introduces the fundamental working principles of GBDM+, including the methodology for uncertainty propagation. Section 5.2 outlines the strategies for error mitigation. Finally, Section 5.3 presents an analysis of tropospheric effects in the context of GBDM+, including the experimental results that form part of the research presented in this thesis.

5.1 GBDM+ processing approach

The method described in this section has been developed at the DICGF-UPV in the context of the GeoMetre project. It defines the theoretical basis and data processing strategy of GBDM+, which serves as the reference framework for the experimental analyses and results developed in this thesis and presented in later sections.

The basic carrier-phase observation equation for a receiver i and a satellite k , known as zero-differenced equation, is expressed as:

$$\lambda\varphi_i^k = \rho_i^k + \lambda N_i^k + cdt_i - cdt^k - I_i^k + T_i^k + MP_i^k + \delta_i - \delta^k + \epsilon_i^k \quad (74)$$

where λ is the carrier wavelength, φ_i^k is the carrier phase in cycles, ρ_i^k is the geometric distance between receiver and satellite, N_i^k is the integer ambiguity, c is the speed of light in vacuum, dt_i and dt^k are receiver and clock offsets, I_i^k is the ionospheric delay, T_i^k represents the tropospheric delay, MP_i^k is the multipath error, δ_i and δ^k are hardware biases, and ϵ_i^k represents the noise or random error ($\sim 1\text{mm}$).

GBDM+ is based on double-differenced equations, involving two receivers (i, j) and two satellites (k, l), so common errors are cancelled. This involves two sequential steps:

- **Single Differences (SD):** a single difference is formed by taking the difference between the observations from two receivers tracking the same satellite. This step eliminates satellite clock errors and reduces some receiver-specific biases.

- Double Differences (DD): a double difference is then obtained by taking the difference between the single differences from two different satellites observed by the same pair of receivers. This further removes receiver clock biases.

The resulting double-differenced carrier-phase equation can be written as:

$$\lambda\varphi_{ij}^{kl} = \rho_{ij}^{kl} + \lambda N_{ij}^{kl} - I_{ij}^{kl} + T_{ij}^{kl} + MP_{ij}^{kl} + \epsilon_{ij}^{kl} \quad (75)$$

By applying two simplifying assumptions, (1) fixing the coordinates of one receiver (e.g., i) and, (2) assuming equal partial derivatives of the satellite-to-receiver distances for both stations (assumption for baselines of a few kilometers), and grouping on the left-hand side of Eq.(75) all terms that can be either modelled or estimated (e.g., delays, multipath, ambiguities), the equation becomes (see [6] for details):

$$\lambda\varphi_{ij}^{kl} - \rho_{ij0}^{kl} - \lambda N_{ij}^{kl} + I_{ij}^{kl} - T_{ij}^{kl} - MP_{ij}^{kl} - \epsilon_{ij}^{kl} = \left(\frac{\partial \rho_{ij}^{kl}}{\partial X_j} \right)_0 dX_j + \left(\frac{\partial \rho_{ij}^{kl}}{\partial Y_j} \right)_0 dY_j + \left(\frac{\partial \rho_{ij}^{kl}}{\partial Z_j} \right)_0 dZ_j \quad (76)$$

Regarding the ambiguities, their resolution depend on the baseline characteristics. For short, horizontal baselines under low multipath conditions, a free-ambiguity approach can be adopted, where ambiguities are directly determined by rounding the phase-minus-range term to the nearest integer. Considering $I \approx T \approx MP \approx 0$ and neglecting the error ϵ becomes:

$$\lambda\varphi_{ij}^{kl} \approx \rho_{ij}^{kl} + \lambda N_{ij}^{kl} \quad (77)$$

so:

$$N_{ij}^{kl} \approx \varphi_{ij}^{kl} - \rho_{ij}^{kl} / \lambda \quad (78)$$

For longer baselines or in presence of non-negligible atmospheric effects, ambiguities are first estimated as float values and then fixed using simple resolution techniques, leveraging the accuracy of the initial PPP solution. In either case, ambiguities are treated as known or reliably estimated and included in the left-hand side of Equation (76). With equations of this type, a system of equations is mounted, with the form:

$$\vec{k} + \vec{r} = A \cdot \vec{x} \quad (79)$$

The vector $\vec{k}$ contains the value of all the left-hand parameters in Equation (75), $\vec{r}$ contains the residuals, A is the coefficient matrix build from the right-hand parameters and $\vec{x}$ represents the unknowns to be determined (the receiver j coordinates corrections in Earth-Centered Earth-Fixed (ECEF) system (dX_j, dY_j, dZ_j)).

Instead of solving this system directly, two matrices are introduced. The first one (R) is a rotation matrix relating the increments in the local geodetic system (x_{ij}, y_{ij}, z_{ij}) with the Earth-Centered Earth-Fixed coordinates (X_{ij}, Y_{ij}, Z_{ij}) as:

$$\begin{pmatrix} X_{ij} \\ Y_{ij} \\ Z_{ij} \end{pmatrix} = R \begin{pmatrix} x_{ij} \\ y_{ij} \\ z_{ij} \end{pmatrix} \quad (80)$$

with R given by:

$$R = \begin{pmatrix} -\sin \lambda_j & -\sin \varphi_j \cos \lambda_j & \cos \varphi_j \cos \lambda_j \\ \cos \lambda_j & -\sin \varphi_j \sin \lambda_j & \cos \varphi_j \sin \lambda_j \\ 0 & \cos \varphi_j & \sin \varphi_j \end{pmatrix} \quad (81)$$

The second one (J) relates the distance (D), azimuth (α) and height difference (Δh) with the coordinates increments in the local geodetic system (x, y, z) as:

$$\begin{pmatrix} dD_{ij} \\ d\alpha_{ij} \\ d\Delta h_{ij} \end{pmatrix} = J \begin{pmatrix} dx_{ij} \\ dy_{ij} \\ dz_{ij} \end{pmatrix} \quad (82)$$

with J built as:

$$J = \begin{pmatrix} \frac{\partial D_{ij}}{\partial x_{ij}} & \frac{\partial D_{ij}}{\partial y_{ij}} & \frac{\partial D_{ij}}{\partial z_{ij}} \\ \frac{\partial \alpha_{ij}}{\partial x_{ij}} & \frac{\partial \alpha_{ij}}{\partial y_{ij}} & \frac{\partial \alpha_{ij}}{\partial z_{ij}} \\ \frac{\partial \Delta h_{ij}}{\partial x_{ij}} & \frac{\partial \Delta h_{ij}}{\partial y_{ij}} & \frac{\partial \Delta h_{ij}}{\partial z_{ij}} \end{pmatrix} \quad (83)$$

Thus, the system in Equation (79), after renaming $B = ARJ^{-1}$ and defining a new unknown vector $\vec{x'}$ consisting on $dD_{ij}, d\alpha_{ij}, d\Delta h_{ij}$, is finally expressed as:

$$\vec{k} + \vec{r} = B \cdot \vec{x'} \quad (84)$$

The system can be solved by least-squares, using the weight matrix P , as:

$$\vec{x'} = (B^T P B)^{-1} B^T P \vec{k} \quad (85)$$

In the unknown $\vec{x'}$ the correction to the initial distance is obtained, as well as the corresponding precision in the first element of the covariance matrix.

5.1.1 Uncertainty propagation

To compute the uncertainty of the distance obtained, the error sources need to be characterized, and their uncertainties somehow estimated (section 5.2). These errors are estimated at zero-difference level and then propagated to the double differenced equations.

Some of the error sources are estimated in the zenith direction and then mapped to the receiver-satellite direction using a mapping function (such as the tropospheric or the

ionospheric delay). For these errors, the double-differenced uncertainty can be estimated as:

$$\sigma_{e_{ij}^{kl}}^2 = (m_j^l - m_j^k)^2 (\sigma_{e_j}^2 + \sigma_{e_i}^2) \quad (86)$$

where m_j^l and m_j^k are the mapping functions applied to zenith errors at receiver j for satellites l and k , respectively, and σ_{e_j} , σ_{e_i} are the standard deviations of the corresponding zenith error estimates at receivers i and j .

For those errors that don't need a mapping function to be applied, like multipath, the double differenced uncertainty is:

$$\sigma_{e_{ij}^{kl}}^2 = (\sigma_{e_j^l} + \sigma_{e_j^k})^2 + (\sigma_{e_i^l} + \sigma_{e_i^k})^2 \quad (87)$$

For a more detailed descriptions of the steps leading to Eq.(86) and Eq.(87), see [6].

The propagation of uncertainties from the input observations to the transformed parameters is carried out using standard linear error propagation techniques. Specifically, C_k denotes the covariance matrix of the observed data, while $C_{x'}$ represents the resulting covariance matrix of the transformed parameters:

$$C_{x'} = MC_k M^T \quad (88)$$

where M is $(B^T P B)^{-1} B^T P$ and C_k :

$$C_k = \begin{pmatrix} \sigma_{e_{ij}^{kl}}^2 & & \\ & \sigma_{e_{ij}^{km}}^2 & \\ & & \dots \end{pmatrix} \quad (89)$$

Finally, the estimated uncertainty for the distance, due to the propagated uncertainties of the error sources, is given by:

$$\sigma_{D_{ij}} = \sqrt{C_{x'}(1, 1)} \quad (90)$$

5.2 GBDM+ error mitigation

As discussed in previous sections, one of the basis of the GBDM+ methodology is the mitigation of the different errors and the estimation of their uncertainties to the rigorous

propagation to the final distance computation. This section provides an overview of major error sources, introduces the existing alternatives to its mitigation and discusses the specific strategies implemented in the GBDM+.

Ionospheric delay

In Section 3.1 the effects of the ionosphere on GNSS signal propagation were introduced and several mitigation strategies were presented.

Since the initial GBDM was initially designed for calibration baselines, the ionospheric delay was neglected due to the assumption that short baselines cancelled out its effects. However, as GBDM+ extends the baseline distance up to 5 km, with potentially significant height differences, this assumption is no longer valid.

During the GBDM+ development, two approaches were explored to mitigate ionospheric error: ionosphere-free combination and Klobuchar model (see Section 3.1). The Klobuchar model was finally selected as correction method as a compromise between ease of implementation and mitigation performance. This also avoids the noise that the iono-free combination introduces.

Generally it is assumed that the Klobuchar model corrects only the 50% of the ionospheric delay in absolute terms (i.e. zero differences), however, when double differences are used, the effect seems to cancel out and the results vary less than a millimetre if compared with the use of L3 [58].

However, no explicit uncertainty model is included in the current version of the GBDM+, and this remains a potential source of unquantified error.

Tropospheric delay

Similar to the ionosphere effects, tropospheric effects were not considered in the original GBDM due to the limited length and approximately horizontal of baselines, which are the usual characteristics of calibration baselines. In GBDM+, however, the troposphere becomes a relevant error source, particularly due to the height differences among baseline ends.

As discussed in Section 3.2.1, several options exist to deal with this error, such as applying standard atmospheric models, estimating tropospheric parameters during processing, or using external corrections. The selected approach in GBDM+ is to use tropospheric corrections provided by the CSRS-PPP service. This solution was selected for its practical advantages:

- **Easy implementation.** This service provides precomputed tropospheric delay estimates, ready for integration into the processing workflow.

- Uncertainty estimation. CSRS-PPP includes an associated uncertainty, which can be propagated through to the final baseline distance.

One of the main contributions of this thesis is the comparison and analysis of the tropospheric delays provided by the CSRS-PPP service with those estimated using different tropospheric models. The main limitation of tropospheric models is that they do not provide a direct estimation of the associated uncertainty. However, they allow the integration of local meteorological measurements (such as temperature, pressure, and humidity), which makes the delay estimation more representative of the actual atmospheric conditions and can potentially enhance overall performance. This analysis is presented in Section 5.3.

Multipath

The multipath effect occurs when GNSS signals reflect off nearby surfaces before reaching the antenna, causing them to arrive at the antenna via multiple paths. It is a highly localized and geometry-dependent effect that persists even after applying double differencing and it is often considered one of the main limiting factor of the accuracy in high-precision applications, as it is highly dependent on the local environment, varies with satellite geometry, and introduces systematic, site-specific biases that are difficult to model or predict.

To mitigate multipath, several techniques have been proposed in the literature, ranging from hardware improvements to signal processing strategies. The most commonly used methods include:

- Improved antenna design (e.g., choke ring antennas). These antennas are specifically engineered to minimize signals arriving from low elevation angles or reflected off nearby surfaces, reducing the reception of multipath components.
- Observation weighting using signal-to-noise ratio (SNR). Multipath typically degrades signal quality, which is reflected in lower SNR values. These values can be used to down-weight potentially corrupted observations during adjustment. This method is computationally efficient and applicable in real time. However, SNR is influenced by other factors (e.g., satellite elevation, receiver type), so the correlation with multipath is not always straightforward.
- Sidereal filtering. Based on the periodic nature of satellite orbits, this method assumes that multipath errors repeat with a sidereal day cycle. By analysing residuals from consecutive days, multipath patterns can be identified and subtracted from the observations. This technique can provide site and satellite specific multipath corrections along with an associated uncertainty estimate. Its main disadvantage is the requirement of multiple days datasets, and it is primarily applicable to GPS due to its stable sidereal repeat cycle.

In the context of GBDM, several studies related with multipath mitigation have been carried out [5, 55, 54]. Sidereal filtering, in combination with SNR estimator, was proposed

as potential solution to mitigate multipath.

The sidereal filtering in GBDM+ uses observation residuals obtained after Precise Point Positioning (PPP) processing, specifically using the CSRS-PPP service. Residuals from at least three consecutive days are matched to construct a multipath model for each specific site and satellite.

This model provides:

- A multipath correction, based on the average value of the residuals.
- An uncertainty estimate, derived from the experimental dispersion of the values from different days.

This uncertainty is then propagated to the final baseline distance estimation, as part of the overall GBDM+ error budget.

Antenna calibration

The antennas used to acquire GNSS data introduce errors that need to be corrected. Signals are received at a point of the antenna that cannot be mechanically identified, as it varies with frequency, azimuth, and elevation of the received signal. This variation is characterized by two parameters.

Firstly, the Antenna Phase Center Offset (PCO) is defined as the difference between the Antenna Reference Point (ARP), the intersection of the vertical axis of the antenna and the bottom), and the mean electrical phase center [29]. This value is fixed for an specific antenna model and is typically provided as a set of 3D coordinates.

Since the actual reception point depends on the azimuth and the elevation of the satellite, Antenna Phase Center Variations (PCV) are also required. PCV is defined as the difference between the electrical phase centre of individual measurements and the mean electrical phase center [29].

Both PCO (as 3D coordinates) and PCV (as azimuth-elevation tables) are provided in antenna files (ANTEX format). These files can be specific to an individual antenna if it has been calibrated, or generic files provided by the International GNSS Service (IGS) can be used, containing values for different antenna models.

With the information in the antenna files, the error can be corrected or mitigated. However, in the GBDM+ methodology, we are interested not only in the correction of the error but also in the assessment of its uncertainty. To do this, GBDM+ proposes comparing two different calibrations, with the difference taken as the uncertainty and propagated

to double differences to the final distance [6]. These two calibrations could be individual antenna calibrations obtained through different methodologies or, alternatively, an individual calibration respect the generic ones provided by the IGS.

5.3 Analysis of the tropospheric effect in the GBDM+ context

This section presents the experimental field campaigns in which the GBDM+ methodology, developed at the UPV, has been tested and validated. Although the development of GBDM+ itself is not part of this thesis, the analysis of the tropospheric effects observed during these experiments constitutes an original contribution of this work.

After introducing the main characteristics of the campaigns and summarising their results in terms of baseline accuracy and measurement uncertainty, this thesis focuses on the evaluation of tropospheric delays using the data collected. The analysis includes both zero-difference and double-difference perspectives, providing a detailed assessment of the troposphere's impact on GBDM+ performance and contributing to a more rigorous understanding of atmospheric influences in high-precision GNSS applications.

5.3.1 Description of the field campaigns

This section introduces the 3 field campaigns where the GBDM+ methodology was tested: the Cortes de Pallás geodetic network, the EURO5000 calibration baseline, and the CERN geodetic network. These campaigns serve as the experimental foundation for the tropospheric analysis developed in this thesis. While some results from these campaigns, such as baseline distances and measurement uncertainties, are briefly presented, the focus here is to describe the experimental setups, instrumentation, and data characteristics. The detailed analysis of tropospheric effects using this data is carried out in the subsequent sections.

Cortes de Pallás geodetic network

The improved GBDM+ methodology, as presented in [6], was initially tested in the Cortes de Pallás geodetic network. The working area and the geodetic network are further described in Section 6.3.1.

The application case presented in [6] corresponds to the observation campaign conducted on July 21-23, during which the network was measured with a Kern Mekometer ME5000 during daytime and with GNSS during nighttime. GNSS observations were performed using two Leica GS10 receivers and two individually calibrated Leica AR25 choke-ring antennas.

Results for the baseline 8001-8007, with a distance of approximately 2 km and a height difference of 350 meters, revealed uncertainties up to 60 mm in the double-differenced tropospheric delay when using the uncertainty provided by CSRS-PPP propagated to the DD (Equation (87)). The uncertainty in the final distance, propagated from this delay, was highly dependent on the observation time span. For one-hour data spans, the distance uncertainty exceeded 1 mm; for four hours, it dropped below 0.6 mm and stabilized at around 0.55 mm from six hours onward.

The total uncertainty budget for the 8001–8007 baseline, propagated from different error sources and various processing time spans, is summarized in Table 5.

Table 5: Uncertainties in mm in 8001-8007 baseline for different time spans

Time span (h)	$u_{tropo.delay}$	$u_{multipath}$	$u_{antenna.calibration}$	u_{TOTAL}
1	1.08	0.49	0.81	1.44
2	0.75	0.33	0.53	0.98
3.5	0.56	0.26	0.40	0.74
5	0.46	0.22	0.33	0.61
10	0.42	0.20	0.30	0.55

These results underscore that the tropospheric delay was the dominant source of distance uncertainty for this baseline.

For the purposes of this thesis, data from a subsequent campaign in Cortes de Pallás, conducted in July 2019, is used to further analyse the tropospheric delay. GNSS observations were again conducted at night. Two baselines were measured, as summarized in Table 6.

Table 6: Summary of baselines measured during the July 2019 campaign

Baseline	Distance (km)	Height difference (m)	Observation Duration
8001–8007	2.0	356	6.5 hours
8001–8010	1.2	33	7.5 hours

EURO5000 Calibration Baseline

The EURO5000 calibration baseline, located in the Pieniny Klippen Belt (Poland), is a novel primary reference baseline in Europe, established in the context of the GeoMetre project, intended to serve as a European reference standard for distances up to 5 km.

The site comprises concrete pillars with 30 cm diameter, enabling baselines of approximately 1, 2, 3, 4, and 5 km. A schematic of the pillar layout is shown in Figure 6.

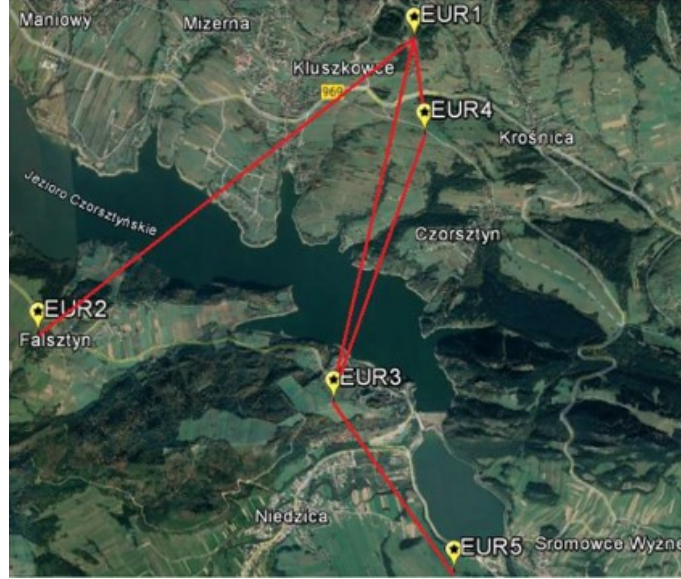


Figure 6: EURO5000 baseline (Source: [58])

A preliminary campaign was conducted by the Warsaw University of Technology (WUT) from September 5 to 9, 2021. The results of this campaign, published in [58], were focused on the longest baseline (EUR1–EUR2) and compared three different measurement strategies:

- EDM length determination
- GNSS geodetic processing using TBC software
- GBDM+ methodology

The main findings from this study included:

- Neglecting the double-difference (DD) tropospheric delay introduces a significant bias of around 8 mm in the baseline distance.
- Distances obtained using GBDM+ with processing blocks of 4, 6, and 8 hours were consistent with results from the other methodologies, demonstrating the robustness of the GBDM+ approach.

Regarding the uncertainty estimated from each source of error and the total final uncertainty of the distance, the obtained results are shown in Table 7, where it can be noticed that the tropospheric error has the biggest impact onto the final distance uncertainty.

Table 7: Uncertainties in mm (k=2) EUR1-EUR2 baseline

$u_{tropo.delay}$	$u_{multipath}$	$u_{antenna.calibration}$	$u_{antenna.height}$	u_{TOTAL}
0.56	0.20	0.29	<0.01	0.66

A second campaign was carried out from May 9 to 12, 2020, as a joint effort between the *Universitat Politècnica de València* (UPV), responsible for GBDM+ measurements, and the *Conservatoire National des Arts et Métiers* (CNAM) in France, deploying a two-color refraction-compensated Arpent distance meter prototype. The GNSS data from this campaign is used in the present thesis for tropospheric analysis (Section 5.3.3).

The baselines measured during the May 2022 campaign are summarized in Table 8.

Table 8: Summary of baselines measured during the May 2022 campaign

Baseline	Distance (km)	Height difference (m)	Observation duration (h)
EUR1–EUR4	1.0	149	6 hours
EUR3–EUR5	2.3	103	55 hours
EUR3–EUR4	3.0	76	6 hours
EUR1–EUR3	4.0	149	17 hours
EUR1–EUR2	5.0	69	18 hours

During the GNSS campaign, meteorological sensors were installed on each pillar to collect atmospheric data and support tropospheric model testing. GNSS choke ring antennas were used, all oriented toward the north, ensuring consistent and high-quality data collection for subsequent analysis (Figure 7).



Figure 7: EURO5000 pillar set-up

CERN geodetic network

The CERN geodetic network, part of the European Organization for Nuclear Research, is being extended for the eventual construction of the Future Circular Collider (FCC), requiring extremely high-accuracy geodetic measurements. In the context of the GeoMetre project, a subnetwork of five pillars was selected for evaluation (Figure 8).

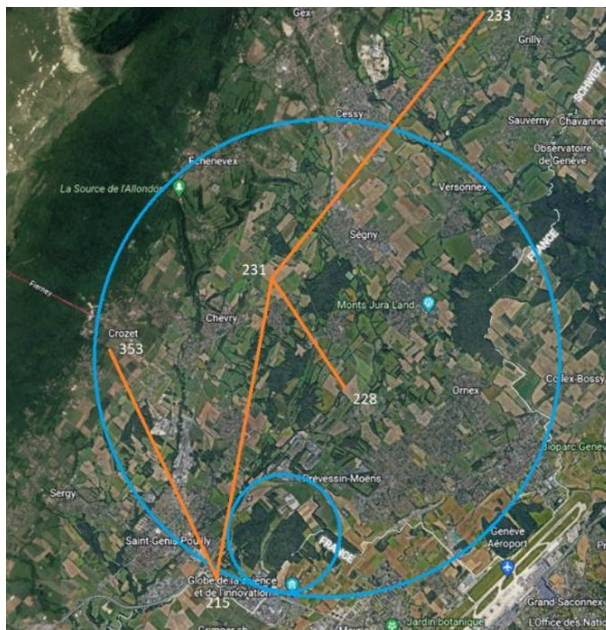


Figure 8: CERN geodetic network (Source: [59])

A joint field campaign was conducted in July 2022, involving several institutions and measurement techniques:

- Arpent, a two-color refractivity-compensated distance meter (CNAM, France)
- GBDM+ (UPV, Spain)
- Kern Mekometer ME5000, a high-precision electronic distanciometer (Figure 9, right)

Table 9: Summary of baselines measured during the July 2022 campaign

Baseline	Distance (km)	Height difference (m)	Observation duration (h)
231-228	2.2	54	73 hours
215-353	4.7	47	73 hours
231-215	6.0	49	68 hours
233-231	6.5	211	72 hours

Five Leica GR25 GNSS receivers with five individually calibrated Leica AR25 choke ring antennas were used. Meteorological sensors were also installed on each pillar (Figure 9, left).



Figure 9: CERN pillar set-up (left), Mekometer measurements (right)

Initial results for this campaign were presented in [59], showing the uncertainty of the 231-228 baseline for different time blocks (Table 10). The results demonstrated that the tropospheric effect had the highest contribution to the total uncertainty, and a time span of at least six hours was needed to achieve uncertainties below 1 mm.

Table 10: Uncertainties in mm for 231-228 baseline for different time spans

Time span (h)	$u_{tropo.delay}$	$u_{multipath}$	$u_{antenna.calibration}$	u_{TOTAL}
1	1.79	0.86	0.90	2.24
2	1.21	0.58	0.61	1.50
3	0.95	0.46	0.48	1.18
4	0.84	0.41	0.42	1.04
6	0.67	0.34	0.34	0.83
8	0.58	0.30	0.29	0.72

The final distances obtained with GBDM+ and their corresponding uncertainties are provided in Table 11.

Table 11: GBMD+ results: summary of baseline distances and corresponding uncertainties, values in m (k=2)

Baseline	Distance	u_{TOTAL}
231-228	2235.6911	0.0007
215-353	4753.1237	0.0006
231-215	6019.4459	0.0008
233-231	6542.0130	0.0008

Results obtained with the Mekometer, calibrated at the UPV calibration lab, are shown in Table 12. For the Mekometer measurements, the index of refraction was determined at baselines endpoints using Testo 176P1 meteorological sensors. These measurements were limited by using only end meteorological measurements and the lack of calibration for the reflector.

Table 12: ME5000 results: summary of baseline distances and corresponding uncertainties, values in m (k=2)

Baseline	Distance	u_{TOTAL}
231-228	2235.6934	0.0013
215-353	4753.1223	0.0017

The comparison between GBDM+ and Arpent measurements, presented in [60], revealed a difference of less than 0.7 mm for three of the four measured distances, underscoring the consistency and high accuracy of the GBDM+ methodology. A summary of the obtained

distances and their corresponding uncertainties obtained by both the GBDM+ and the Arpent is shown in Table 13.

Table 13: GBDM+ and Arpent results: summary of baseline distances and corresponding uncertainties, values in mm (k=1)

Baseline	GBDM+ distance	u_{GBDM+}	Arpent distance	u_{Arpent}
231-228	2235691.07	0.32	2235690.69	0.225
215-353	4753123.72	0.32	4753121.35	0.403
231-215	6019445.93	0.42	6016446.31	0.376
231-233	6542012.96	0.38	6542012.28	0.723

5.3.2 Zero-differences tropospheric analysis

This section presents one of the original contributions of this thesis: the analysis of tropospheric delays using zero-difference (ZD) GNSS observations. The focus is on comparing the zenith tropospheric delay (both ZTD and ZWD) obtained from the CSRS-PPP service with values computed using several empirical models. In addition, the impact of different mapping functions on retrieving the slant tropospheric delay is evaluated.

Comparison of zenith tropospheric delay models

To assess the influence of different approaches on tropospheric delay estimation, CSRS-PPP results are compared with several empirical models. For the hydrostatic component (ZHD), the Saastamoinen model is applied. The wet component (ZWD) is computed using the Saastamoinen, Callahan, Askne and Nordius, and Berman TMOD models. The formulation of these models is presented in Appendix A.1.

In order not to overload the analysis with the treatment of all campaigns, the analysis focuses on stations 231 and 233 from the CERN campaign. These stations were selected as representative examples of the range of altitudes encountered across campaigns.

Figure 10 shows the evolution of ZHD at stations 231 and 233 over two full days, comparing PPP-derived values with those obtained from the Saastamoinen hydrostatic model.

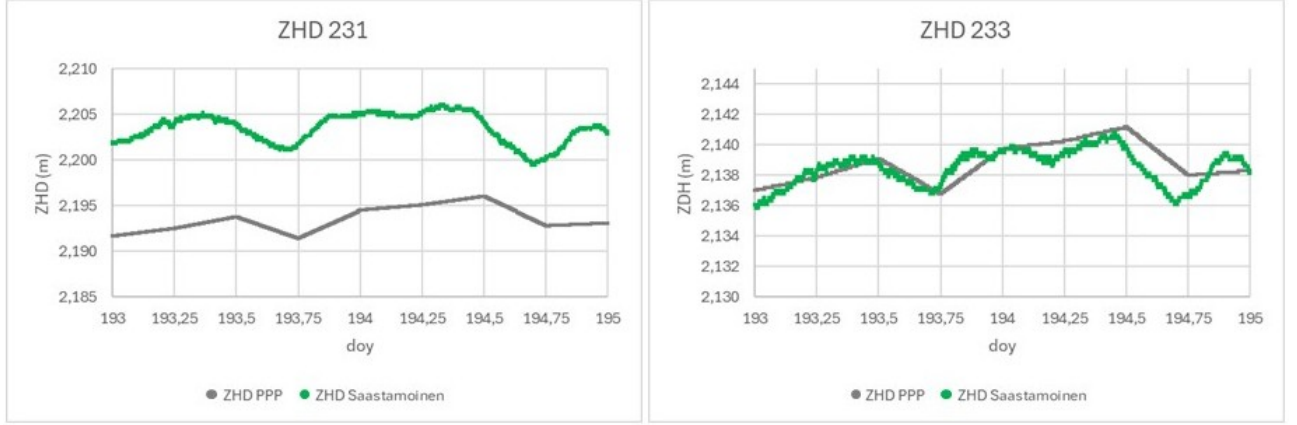


Figure 10: ZHD from PPP and Saastamoinen model in stations 231 and 233

The ZHD component shows a good agreement between PPP and model-based estimates. At station 231, small absolute offsets can be observed, while at station 233 the PPP and Saastamoinen values are nearly identical. This correspondence is expected, as the CSRS-PPP service also relies on the Saastamoinen formulation for ZHD computation, but using atmospheric data from global models. By contrast, the empirical estimates rely on locally measured meteorological data. As a result, PPP-derived ZHD evolves more smoothly, whereas the Saastamoinen estimates reflect small short-term variations.

For the wet component, Figure 11 presents the evolution of ZWD from PPP and the different empirical models.

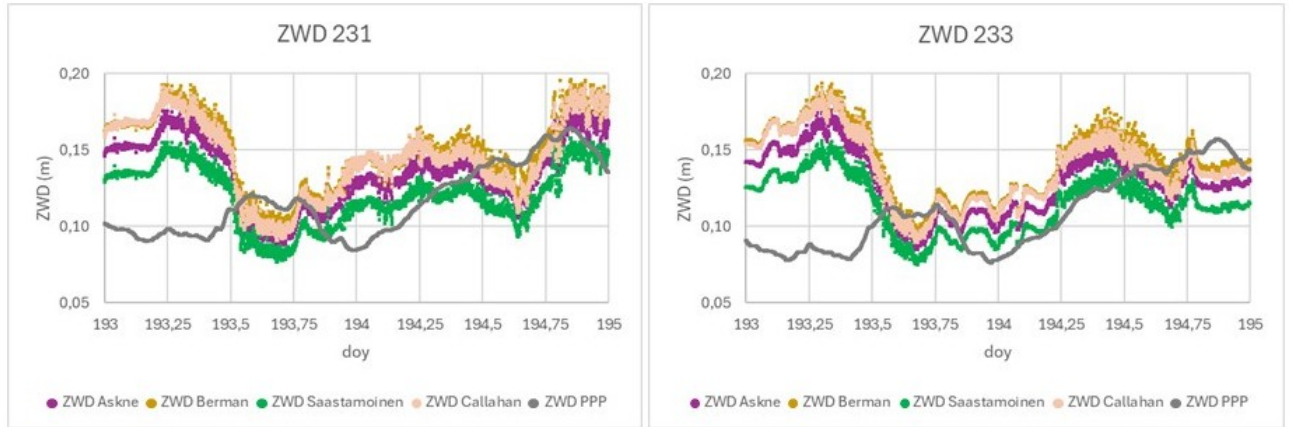


Figure 11: ZWD from PPP and tropospheric models in stations 231 and 233

In the wet component, discrepancies are more evident. While the different empirical models yield similar temporal trends among themselves, they differ from the PPP-derived

ZWD.

To do deeper into these discrepancies, Figure 12 shows the differences between PPP and Saastamoinen for both ZHD and ZWD.

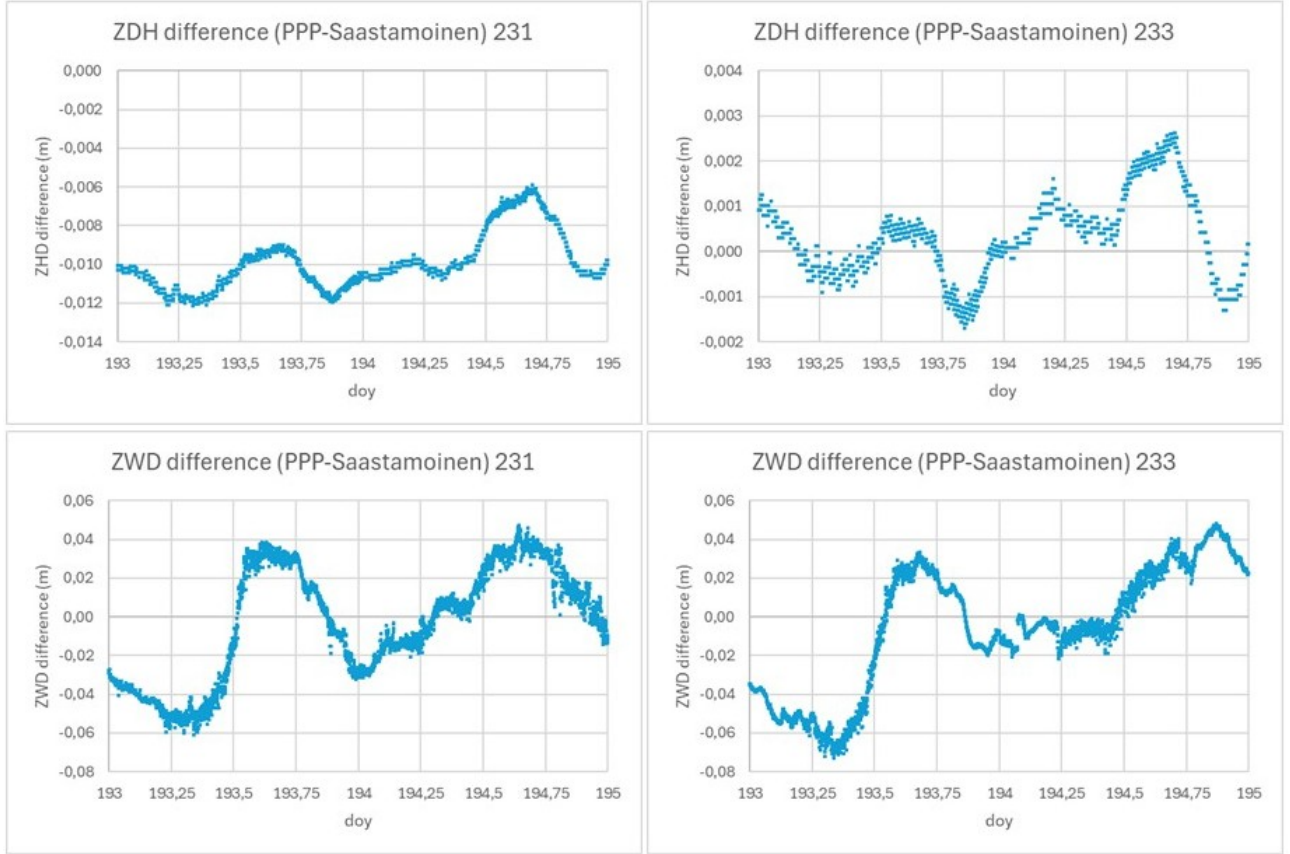


Figure 12: ZHD differences between PPP and Saastamoinen in stations 231 and 233

Differences in ZHD between PPP and Saastamoinen range from 6 to 12 mm at station 231, and from 1 to 3 mm at station 233. For the ZWD, discrepancies are larger, reaching 4–6 cm. These results confirm that the wet delay is the most critical and uncertain component, as expected.

As shown in Figure 11, the different wet-delay models present very similar temporal evolution, but differ in absolute values. Figure 13 highlights these differences by comparing ZWD from the Saastamoinen model with those from the other models.

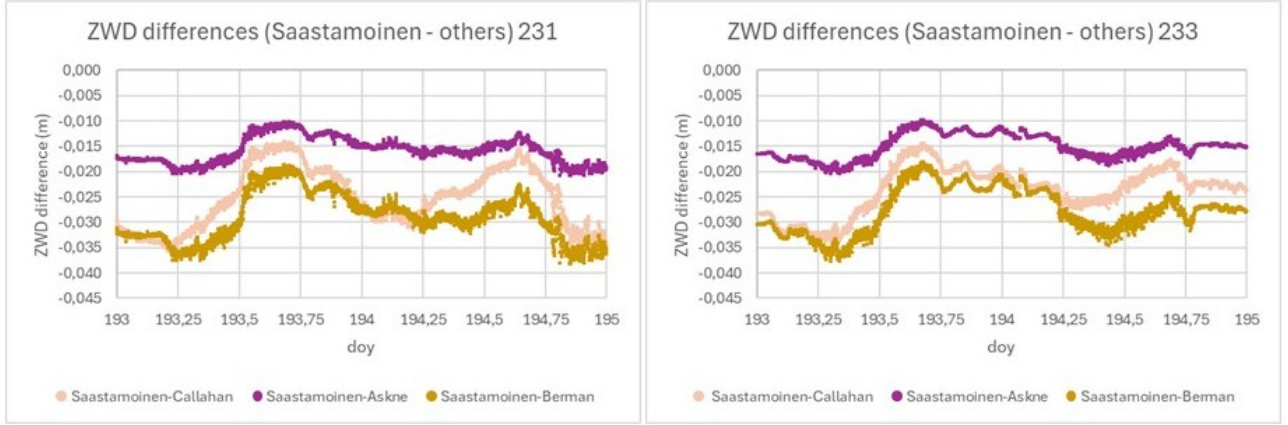


Figure 13: ZWD differences between Saastamoinen and other models in stations 231 and 233

Differences between Saastamoinen and the other wet-delay models range from 1 to 4 cm. These findings show that the choice of model has a non-negligible impact when working with zero-difference observations. The consequences of this choice on double-difference processing are further analysed in Section 5.3.3.

Impact of mapping functions on slant delay

To convert ZTD into slant tropospheric delay, several mapping functions can be used (see Appendix A.2). In this thesis, three widely used functions have been selected: VMF1, Niell, and Herring. The choice is motivated by their broad adoption in geodetic applications and by their complementary characteristics. The Herring function relies on locally measured meteorological data. VMF1 is the mapping function used by the CSRS-PPP service, ensuring consistency with the PPP-derived values. Finally, the Niell mapping function is an empirical model that does not require external meteorological data and is commonly used in GNSS processing.

The tropospheric delays obtained with these mapping functions are very similar. For this reason, only one representative example is shown, corresponding to station 231 and satellite G01.

Figure 14 shows the slant tropospheric delay obtained PPP-derived ZHD and ZWD, combined with the different mapping functions. The left panel presents its temporal evolution, while the right panel shows the relationship with the satellite's elevation angle.

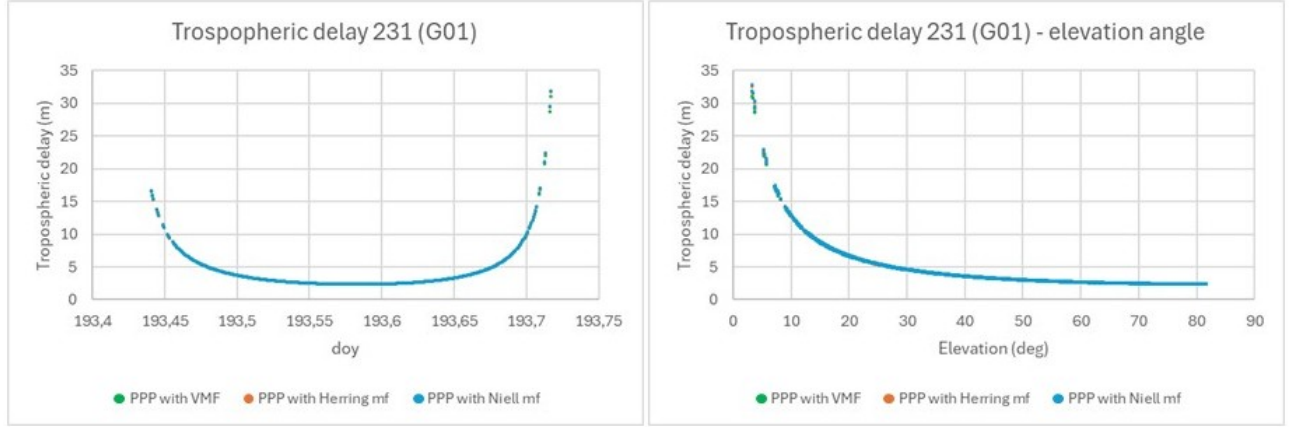


Figure 14: Tropospheric delay with different mapping functions, station 231, satellite G01

At first sight, differences between the mapping functions are not noticeable. To better illustrate them, the differences with respect to VMF1 (used as the reference) have been computed. Figure 15 shows the temporal evolution of these differences.

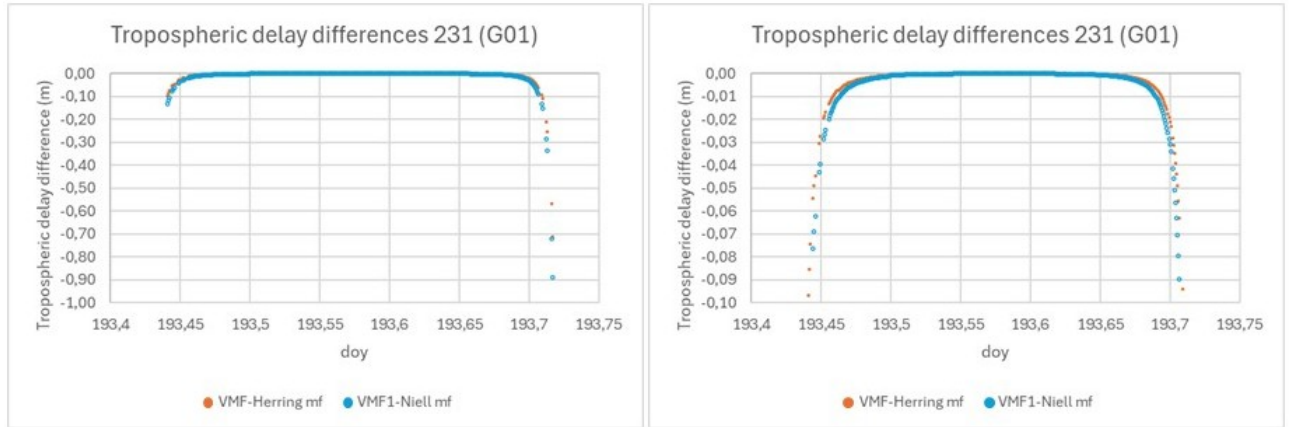


Figure 15: Tropospheric delay differences between mapping functions, station 231, satellite G01

As it can be seen, few differences reach 1 m, while the bulk of the period presents differences under 1 cm. The right panel, shown with a reduced vertical scale, highlights that the majority of differences are in fact below 2 cm. To examine the dependence on elevation, Figure 16 plots the differences as a function of the satellite's elevation angle.

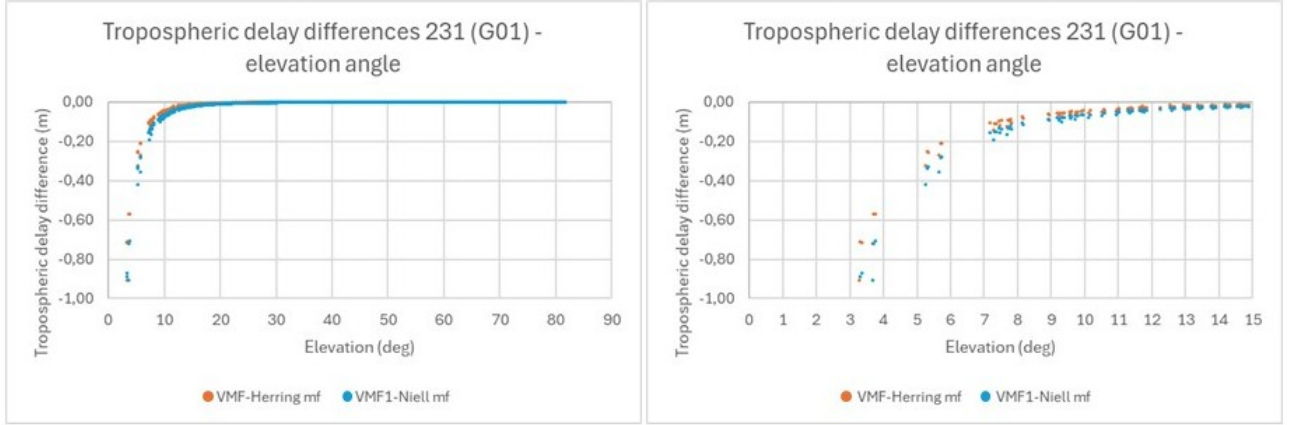


Figure 16: Tropospheric delay differences between mapping functions respect elevation, station 231, satellite G01

The largest discrepancies appear at lower elevation angles, as expected. However, with the commonly applied elevation mask of 10° , the differences between mapping functions remain small. For elevations above 15° , the discrepancies fall consistently below 2 cm.

5.3.3 Double-differences tropospheric analysis

This section presents the analysis of the tropospheric delay in double-differenced (DD) observations. The basics of DD were described in Section 5.1. Here, the focus is on analysing the results of DD tropospheric delay on the different experimental campaigns.

In this thesis, the double differences were generated sequentially, meaning that each satellite was compared with the next one according to their order of appearance in the dataset. This approach differs from the more common strategy of selecting a single reference satellite, typically the one with the highest elevation. Using the highest-elevation satellite as reference has the advantage of reducing the tropospheric effect on the reference signal, as such satellites experience smaller atmospheric delays and generally provide more stable observations.

The sequential strategy adopted here has its own benefits. By not fixing all double differences to a single satellite the correlation of residuals associated with the reference is reduced. This can be especially useful in multipath-affected or highly variable atmospheric conditions, where relying on a single satellite might propagate systematic effects across all DD combinations.

The results are presented in three parts. First, a preliminary analysis examines how the use of different tropospheric models and mapping functions affects the DD residuals. Next, the results of each campaign are presented in terms of double-differenced tropospheric delay. Finally, a joint analysis combining all campaigns is carried out, with the

objective of assessing the relationship between baseline length, height difference, and the magnitude of the tropospheric double differences.

Implications for double-differences processing

The previous section highlighted significant discrepancies between tropospheric delay estimates obtained from different models, particularly in the wet component. In this section, we analyse the effect of forming double differences (DD), which can largely mitigate these discrepancies.

As an example, Figure 17 shows the differences in DD for satellites G01-G03 in baseline 231-233. Saastamoinen model has been used as a reference to compute the difference with the other models.

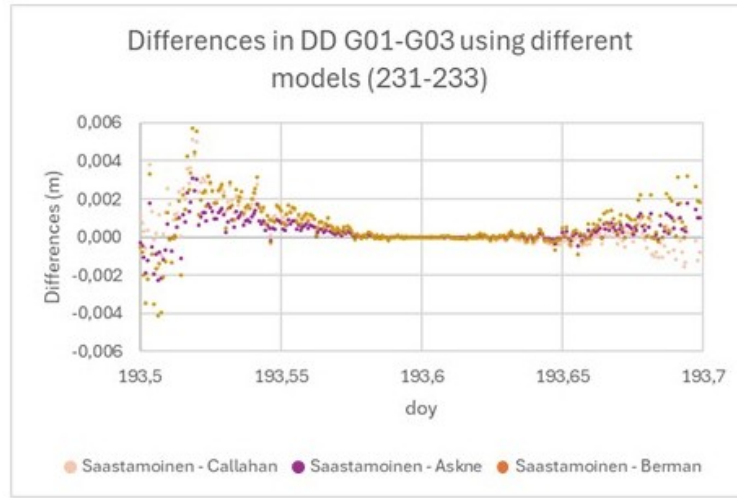


Figure 17: Differences in DD tropospheric delay, satellites G01-G03, using different models in baseline 231-233

Differences between models are below 6 mm. The higher values are the extreme ones, probably at low elevations. The rest of the differences computed are almost null. Thus, while zero-difference analysis shows significant variability among models, double-differencing largely cancels out these discrepancies.

Regarding the mapping functions the same analysis can be done. To do this, the slant delay using PPP-based ZHD and ZWD and VMF1 is used as a reference to compute differences with the obtained applying other mapping functions. The results for satellites G01 and G03 for 231-233 baseline is shown in Figure 18.

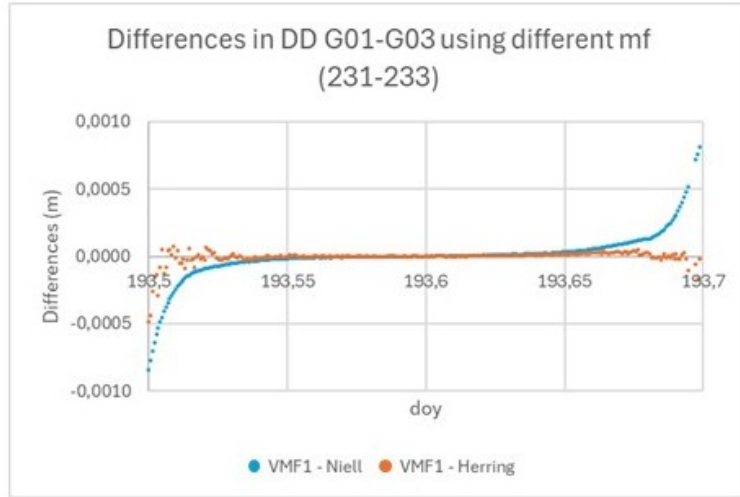


Figure 18: Differences in DD tropospheric delay, satellites G01-G03, using different mapping functions in baseline 231-233

Using VMF1 of other mapping functions yields differences in the double differenced tropospheric delay below 1 mm. Generally, these differences can be neglected.

With this analysis it can be concluded that, even some discrepancies between models and mapping functions can be seen at ZD level, after computing DD they become much less important. This justifies the choice of using only the CSRS-PPP and Saastamoinen models with the VMF1 mapping function in the double-differences analysis presented in this section.

Cortes de Pallás geodetic network

The double-differenced tropospheric delay for the campaign in Cortes de Pallás July 2019 is displayed in Figure 19.

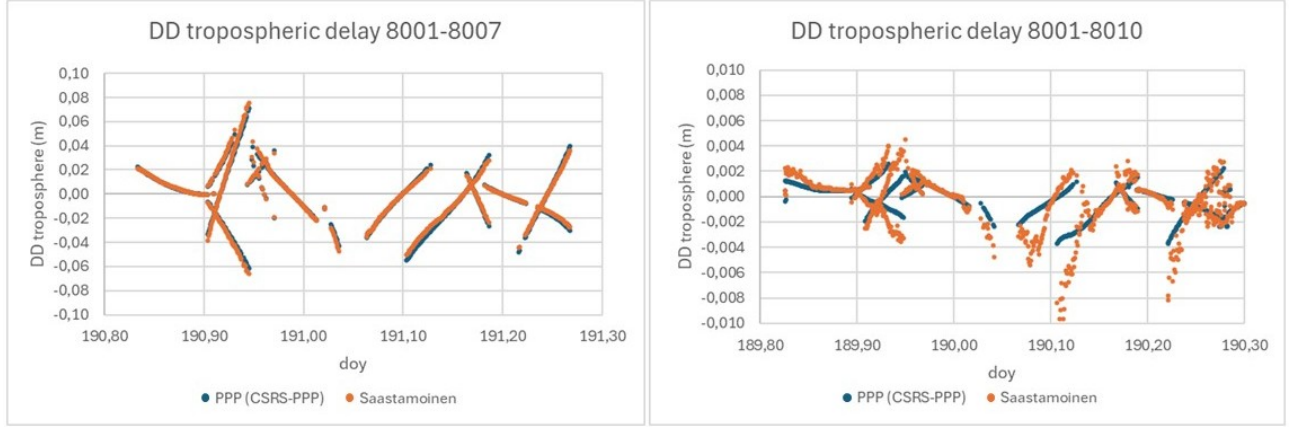


Figure 19: Double differenced tropospheric delay, baselines 8001-8007 and 8001-8010

The results show that double differences of tropospheric delay reach values up to 0.08 m in the 8001-8007 baseline, while in the baseline 8001-8010 values remain below 0.004 m. This difference is significant, with the 8001-8007 baseline showing an order of magnitude higher values. This can be attributed to the longer distance (2 km compared to 1.2 km for the other baseline) and, most notably, the considerable height difference between the baseline endpoints: 356 m for 8001-8007 compared to just 33 m for 8001-8010.

Another notable observation from Figure 19 is the difference in results when using tropospheric delay estimated from Precise Point Positioning (PPP) and the Saastamoinen model. Although the obtained values are generally similar, the Saastamoinen results for the 8001-8010 baseline exhibit greater dispersion, particularly in extreme values. This can be attributed to outliers or inconsistent meteorological measurements, as the Saastamoinen model relies on meteorological data that may contain errors or variability.

EURO5000 Calibration Baseline

The results for the EURO5000 campaign are shown in Figure 20 (baseline EUR1-EUR2) and in Figure 21 (baselines EUR1-EUR4, EUR3-EUR5, EUR3-EUR4, and EUR1-EUR3). The observation period shown is the common observation time for all baselines (EUR3-EUR5 slightly adjusted to ensure the same time span).

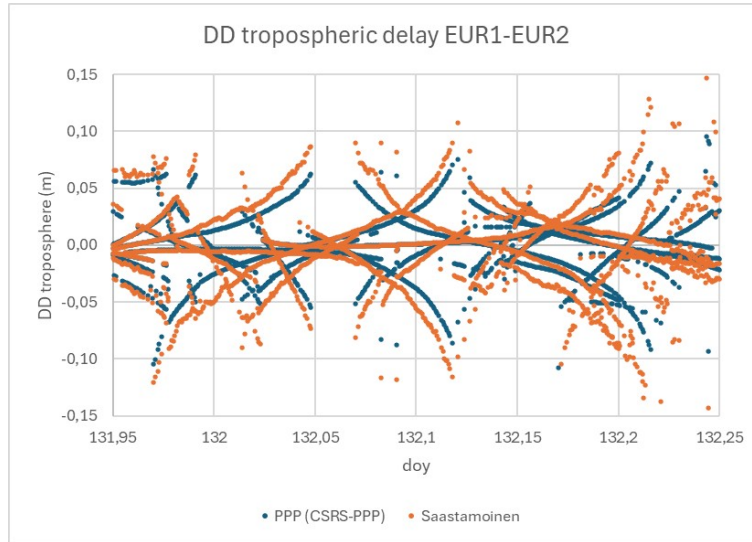


Figure 20: Double differenced tropospheric delay, baseline EUR1-EUR2

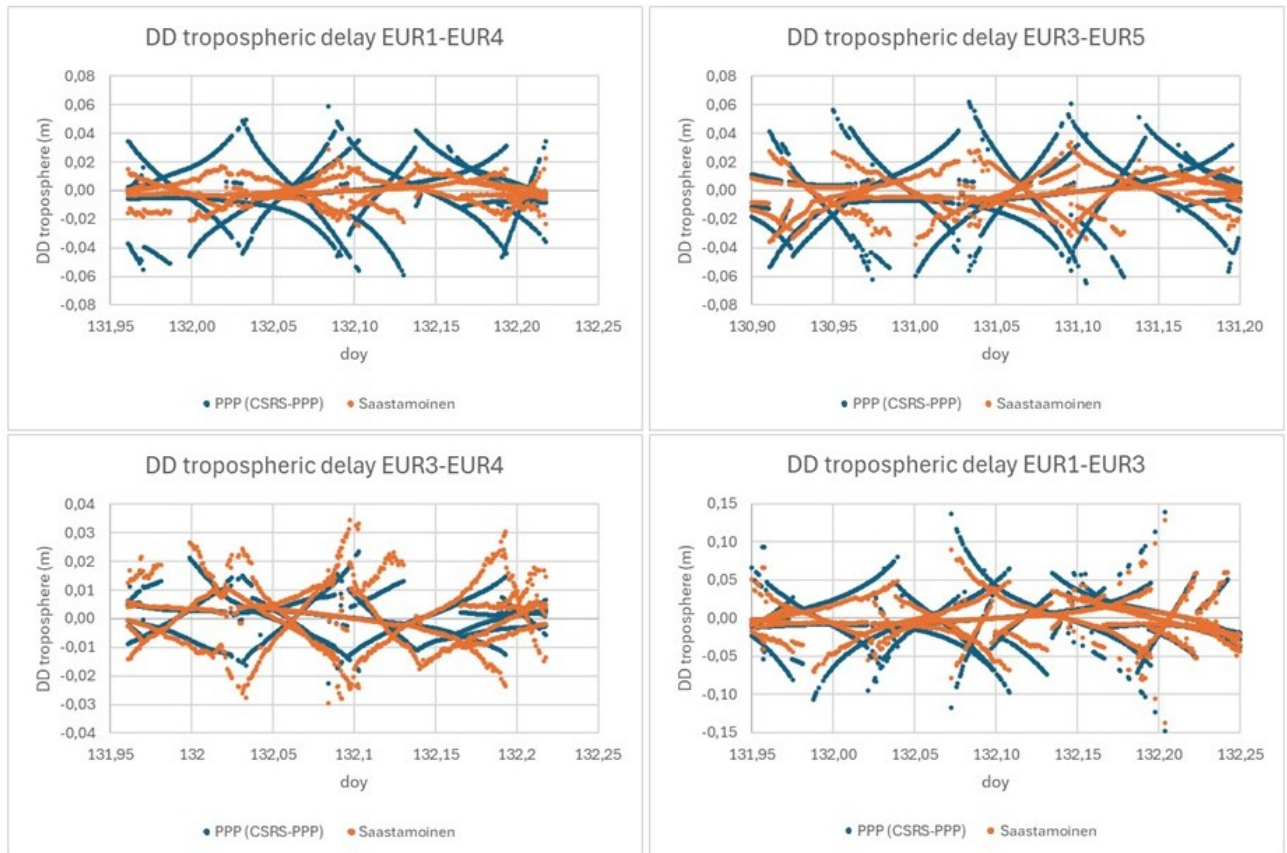


Figure 21: Double differenced tropospheric delay, baselines EUR1-EUR4, EUR3-EUR5, EUR3-EUR4, and EUR1-EUR3

The results show variations in the magnitude of double-differenced tropospheric delays depending on baseline characteristics. For instance, the EUR1-EUR2 and EUR1-EUR3 baselines exhibit the highest variations, reaching up to ± 0.1 m. This is expected given their longer distances and height differences. On the other hand, shorter baselines like EUR3-EUR4 present much lower double differences, remaining within ± 0.03 m. The baselines EUR1-EUR4 and EUR3-EUR5 show intermediate values, up to ± 0.05 m and ± 0.06 m, respectively. This confirms the trend observed in the previous campaign: longer baselines and significant height differences contribute to larger residual tropospheric delays even after forming double differences.

Regarding differences between results using CSRS-PPP and the Saastamoinen model, it can be noticed that in baseline EUR1-EUR4, the Saastamoinen model results in lower double-differenced delays. The same occurs in EUR3-EUR5, but with less difference. For the other baselines, the results of CSRS-PPP and Saastamoinen are similar. However, Saastamoinen-based double differences have higher dispersion or outliers, particularly in extreme values, which may be due to inaccuracies or variability in the meteorological data used for the model.

CERN geodetic network

The results for the CERN campaign are shown in Figure 22, which covers one full day of observations to ensure common time spans across all baselines.

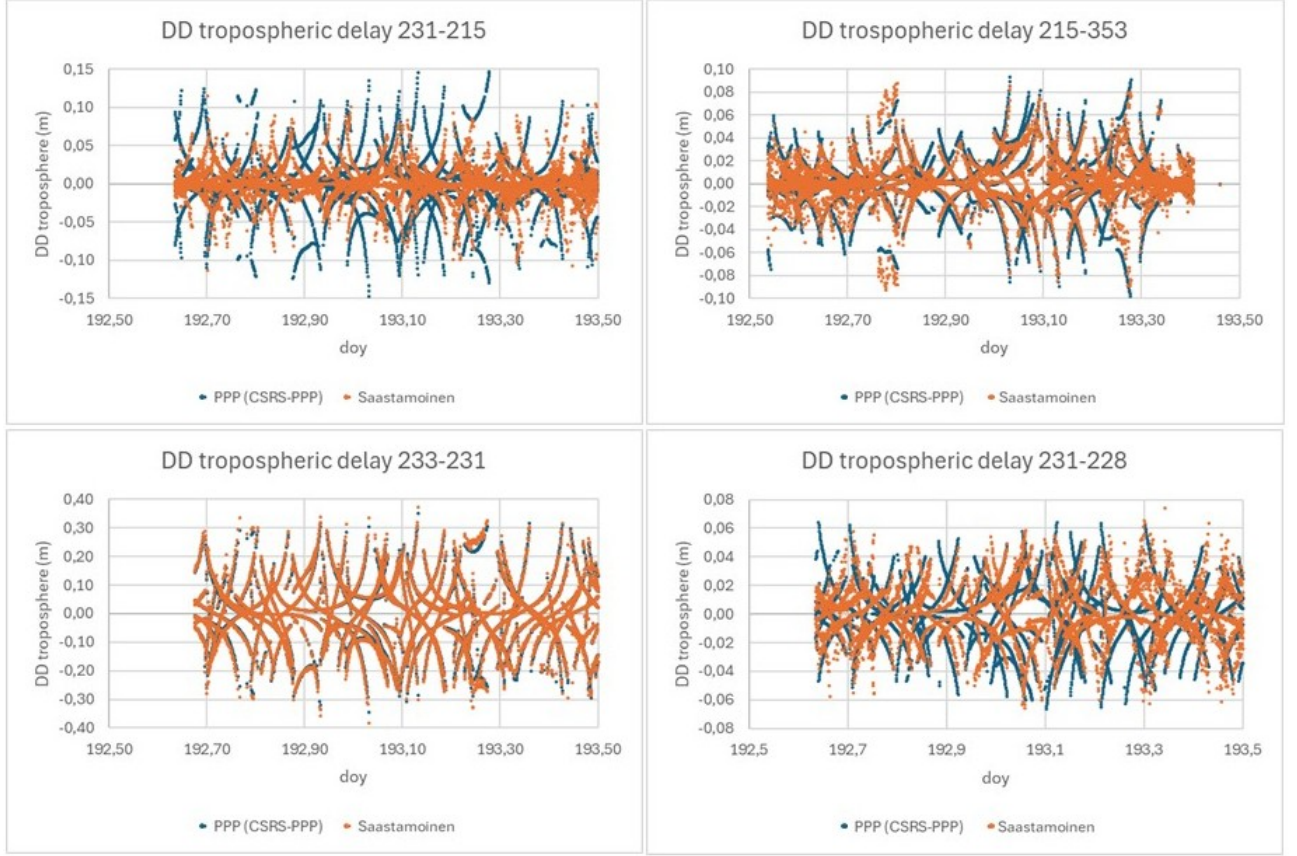


Figure 22: Double differenced tropospheric delay, baselines 215-353, 231-215, 233-231, and 231-228

The analysis reveals a clear dependence of the double-differenced tropospheric delay on baseline length and height difference. For the shortest baseline (231-228), the values remain within ± 0.06 m. Baseline 215-353 shows slightly larger variations, up to ± 0.10 m, while baseline 231-215 reaches about ± 0.15 m. The largest variations are observed for baseline 233-231, where values reach ± 0.30 m; this is the longest baseline in the set. These results are consistent with those obtained in previous campaigns and confirm that both longer baselines and greater height differences lead to larger residual tropospheric delays, even after applying double differencing.

Regarding the comparison between tropospheric modelling approaches, no significant differences are found between the CSRS-PPP estimates and those obtained with the Saastamoinen model. For the longest baseline (233-231), the two methods yield almost identical results. For the shorter baselines, small differences are observed, with the Saastamoinen-based delays being slightly lower than those derived from CSRS-PPP.

Joint analysis across campaigns

The results across all campaigns confirm the combined influence of baseline length and height difference on the double-differenced (DD) tropospheric delay. A summary of the results is shown in ascending order of baseline length in Table 14.

Table 14: DD tropospheric delay for different baseline lengths and height differences

Baseline	Distance [km]	Height difference [m]	DD tropospheric delay [m]
EUR1-EUR4	1	149	± 0.005
8001-8010	1.2	33	± 0.004
EUR3-EUR5	1.5	56	± 0.06
8001-8007	2	356	± 0.08
231-228	2.2	55	± 0.06
EUR3-EUR5	2.3	103	± 0.08
EUR3-EUR4	3	76	± 0.03
EUR1-EUR3	4	149	± 0.1
215-353	4.7	47	± 0.09
EUR1-EUR2	5	69	± 0.1
231-215	6	49	± 0.1
233-231	6.5	211	± 0.3

In general, longer baselines tend to show larger DD tropospheric delays, as expected. Nevertheless, the role of height difference is also clearly observable when comparing baselines of similar length. For instance:

- 231–228 vs. 8001–8007: although the first (2.2 km, 55 m) is slightly longer than the second (2.0 km, 356 m), it shows smaller DD values (± 0.06 m vs. ± 0.08 m), reflecting the stronger impact of the larger height difference.
- 215–353 vs. EUR1–EUR3: despite being shorter (4.7 km, 47 m vs. 4 km, 149 m), 215–353 yields slightly smaller DD values (± 0.09 m vs. ± 0.1 m), again pointing to the amplifying effect of a greater height difference.
- EUR1–EUR4 vs. 8001–8010: the shorter baseline (1 km, 149 m) presents slightly higher DD values than the longer one (1.2 km, 33 m), also explained by its greater height difference.

The most pronounced case is baseline 233–231 (6.5 km, 211 m), where both a long baseline and a large height difference combine, leading to the maximum observed DD values of ± 0.3 m.

Regarding the differences between zenith tropospheric delay from CSRS-PPP and the Saastamoinen model, discrepancies tend to be more noticeable in shorter baselines. Shorter baselines may be more sensitive to local atmospheric conditions. Small errors or variability in these conditions could potentially cause a greater impact on the results. For longer baselines, the tropospheric delay is averaged over a greater distance, which could reduce the impact of local atmospheric variations, leading to more similar results between the two methods. In these cases, both the CSRS-PPP service and the Saastamoinen model provide more consistent results as the influence of local atmospheric errors is minimized. Furthermore, in shorter baselines, Saastamoinen-based double differences tend to show more variability or outliers. This might be due to the use of unfiltered meteorological data, leading to occasional discrepancies.

5.3.4 Conclusions

The previous section has examined the influence of tropospheric delays on GNSS-based distance estimation within the GBDM+ framework, focusing on both zero-difference (ZD) and double-difference (DD) levels.

The results show that, at the ZD level, there are notable differences between values estimated with PPP processing (via the CSRS-PPP service) and those computed with empirical models. The zenith hydrostatic delay (ZHD) shows a similar evolution in both PPP estimations and tropospheric model-based values, and it strongly depends on atmospheric pressure. However, the zenith wet delay (ZWD) varies significantly between PPP-derived values and empirical models. These differences are mainly driven by humidity and temperature variations, and can reach several centimetres in the wet component alone. While differences in absolute values between models are observed, their temporal evolution is generally consistent.

The effect of mapping functions used to convert zenith delays to slant delays has been also assessed. At the ZD level, differences between mapping functions (such as VMF1, Niell, and Herring) were mostly limited to low satellite elevation angles and generally remained within a few centimetres. However, once applied in DD processing, these differences became practically negligible, demonstrating the strong cancellation effect of DD formulations on mapping function dependencies.

Although DD processing efficiently mitigates most of the tropospheric delay variability, the analysis confirms that residual tropospheric effects still persist under certain conditions. In particular, when the baseline exceeds 3 to 4 kilometres, or when there is a significant height difference between receivers (e.g., greater than 100 meters), residual delays in the DD domain can reach up to ± 0.3 meters. This was observed consistently across all test environments, including Cortes de Pallás, the EURO5000 dataset, and the campaigns conducted at CERN.

PPP-derived delays and empirical models using measured meteorological parameters show

generally similar behaviour for long baselines. However, for short baselines, greater differences were observed between the two approaches, likely due to local atmospheric variability and higher sensitivity to input data.

In conclusion, while double-differencing remains a robust method for reducing atmospheric errors in GNSS processing, it does not eliminate tropospheric effects entirely. For applications such as GBDM+, which aim to achieve millimetre-level precision and traceability, these residuals are not negligible. The findings underscore the need for refined tropospheric modelling and explicit uncertainty propagation in the processing workflow, especially for scenarios involving long baselines or complex terrain.

6. 3D refractivity model for distance and vertical angle correction

As discussed in previous chapters of this thesis, atmospheric refraction remains a limiting factor for high-precision geodetic measurements, especially in complex terrains. Traditional correction methods, which rely on meteorological observations at both ends of the baseline, are often insufficient because such point measurements do not adequately represent the atmospheric conditions along the entire propagation path.

To overcome this limitation, and as one of the main objectives of this research, a 3D Refractivity Model (3D-RM) has been developed as a cost-effective alternative. The proposed model combines data from a network of meteorological sensors that enable automatic data recording, with a revisitation of classical atmospheric physics concepts, such as the Turbulence Transfer Model (TTM), and incorporates terrain information derived from a Digital Terrain Model (DTM).

An initial version of the 3D-RM and its application was presented in [61]. In this thesis, the model is further developed and described in greater detail, including its theoretical background, implementation aspects, an analysis of the influence of different variables. This chapter provides a comprehensive description of the 3D-RM, beginning with its physical foundations, followed by the steps required for its implementation, and concluding with its application to field experiments for performance assessment.

6.1 Model basis

In this section, the main concepts underlying the developed model are introduced, focusing on the Turbulence Transfer Model (TTM) and the sensible heat flux parameter (H), together with an overview of the available data products.

These concepts are directly applicable to the atmospheric boundary layer which is the lowest part of the atmosphere where terrestrial geodetic measurements are typically conducted and where turbulent and convective processes strongly influence refractivity (see Section 2.1.3 for further details). Since the TTM is designed to model heat and momentum transfer within this layer, and the sensible heat flux quantifies one of its dominant energy exchanges, both provide the physical framework for representing refractive effects.

6.1.1 Turbulence Transfer Model (TTM)

The Turbulence Transfer Model (TTM), originally formulated by Priestly and later expanded by Dodson and Zaher [62], describes how temperature varies in the lower atmosphere by considering the dynamics of turbulent heat exchange. The model provides a

framework to estimate vertical potential temperature gradients based on ground-level atmospheric parameters.

An important concept in the TTM is atmospheric stability, which can be related with the sensible heat flux (H), a measure of heat exchange between the ground and the air. When H is zero, the atmosphere is neutral; a positive value indicates unstable conditions (typically during daytime), whereas a negative value suggests a stable atmosphere (commonly at night).

In neutral conditions, the potential temperature (θ) remains constant with height, so the potential temperature gradient ($\partial\theta/\partial h$) is 0. However, under stable or unstable conditions, the model defines specific expressions to calculate the temperature gradient across different layers of the atmosphere:

- Stable atmosphere

$$\frac{d\theta}{dh} = -\frac{2 \cdot 10^{-3} \cdot H}{U_* \cdot h} \left(1 + \frac{5h}{L}\right) \quad (91)$$

- Unstable atmosphere

$$\frac{d\theta}{dh} = -\frac{H}{C_p \cdot \rho \cdot U_* \cdot vk} \cdot h^{-1}, \quad for \quad 0 < h < 0.03 \cdot L \quad (92)$$

$$\frac{d\theta}{dh} = -0.027 \cdot H^{2/3} \cdot h^{-4/3}, \quad for \quad 0.03 \cdot L < h < L \quad (93)$$

$$\frac{d\theta}{dh} = 0, \quad for \quad h > L \quad (94)$$

where:

- H is the sensible heat flux [W/m^2]
- h is the height above the surface [m]
- vk is the Von Karman constant [dimensionless, typically 0.4]
- C_p and ρ are the specific heat and density of air respectively
- U_* is the friction velocity [dimensionless]
- L is the Obukhov length [m]

The friction velocity (U_*) acts as scaling parameter for turbulent momentum exchange near the surface [17]. It is computed from the wind speed (U), the height at which the wind is measured (h_0), and the surface roughness parameter (z_0), using the following relationship:

$$U_* = \frac{vk \cdot U}{\ln(h_0/z_0)} \quad (95)$$

The Obukhov length L represents the height above which heat flux no longer influence the atmospheric temperature profile. It is derived as:

$$|L| = -87 \cdot 10^3 \cdot \frac{U_*^3}{H} \quad (96)$$

The actual temperature gradient (dT/dh) can be obtained from the potential temperature gradient ($d\theta/dh$) using meteorological parameters at the surface:

$$\frac{dT}{dh} = \left(\frac{P}{1000} \right)^{0.286} \cdot \left(\frac{d\theta}{dh} + 2.06 \cdot \frac{T}{P^{1.286}} \cdot \frac{dP}{dh} \right) \quad (97)$$

where P represents the atmospheric pressure [hPa], T corresponds to the temperature [C], and dP/dh is the pressure vertical gradient [hPa/m].

Finally, as discussed in the theoretical chapters, since refractivity depends on both temperature and pressure, the vertical gradient of atmospheric refractivity can be derived by combining the temperature and pressure gradients (see Equation (53) in Section 4.2). The temperature gradient is computed using Equation (97) while the pressure gradient can be approximate to a constant rate.

6.1.2 Sensible heat flux (H)

As outlined in Section (6.1.1), the sensible heat flux (H) plays a crucial role in determining the thermal structure of the atmosphere and in deriving vertical gradients of temperature and refractivity. This section introduces the concept of sensible heat flux, discusses its physical basis and importance, and presents an overview of different techniques available for its estimation. Then, several available remote sensing products from which values of sensible heat flux can be retrieved are presented.

Sensible heat flux represents the direct transfer of thermal energy between the Earth's surface and the atmosphere, primarily driven by turbulence air motion. It accounts for the portion of energy that results in temperature changes without involving phase transitions (as opposed to latent heat). These processes are fundamental to surface energy budgets and are critical for understanding land-atmosphere interactions, weather dynamics, and

microclimatic phenomena.

In the context of atmospheric modelling, the sensible heat flux contributes to the surface energy balance equation [1]:

$$R_{NET} = H + G + E \quad (98)$$

where:

- R_{NET} is the net radiation at the surface (incoming minus outgoing radiation at the surface).
- G is the ground heat flux (energy conducted into the soil).
- E is the latent heat flux associated with evapotranspiration.

Among these components, H is particularly influential in modulating near-surface temperature gradients, especially in dry conditions where latent heat exchange is minimal.

A range of methodologies has been developed for quantifying sensible heat flux, each with its own assumptions, instrumentation needs, and levels of accuracy. The most relevant techniques for environmental and geodetic applications include:

- **Eddy-Covariance Method.** This technique is widely accepted as one of the most accurate methods for obtaining sensible heat flux. It is based on the high-frequency measurements of vertical wind velocity and air temperature, using the covariance between these two parameters [63].

Although it is highly precise, this method required sophisticated equipment, such as fast-response anemometers and temperature sensors, and extensive post-processing to account for instrumental and atmospheric corrections. These factor limit its applicability in large-scale geodetic surveys.

- **Optical Scintillometry.** Scintillometers are instruments that measure the structure parameters of the refractive index structure parameter (C_n^2) by analysing the fluctuations of two laser beams [64]. This parameter is related with the sensible heat flux (H) by the Monin-Obukhov similarity theory (MOST).

Scintillometers enables spatially integrated flux measurements over large areas. However, their dependency on precise calibration and sensitivity to environmental conditions make it impractical in complex terrains.

Given the logistical and technical constraints of direct measurement methods, reanalysis datasets and remote sensing products have become increasingly popular alternatives for

estimating sensible heat flux across wider spatial and temporal scales. These datasets combine satellite observations, surface measurements, and numerical weather prediction models to produce coherent time series of atmospheric parameters. Several widely used datasets include:

- **ERA5.** ERA5, developed by the European Centre for Medium-Range Weather Forecasts (ECMWF) as part of the Copernicus Climate Change Service (C3S), provides a detailed record of global atmospheric, land surface and ocean parameters since 1950. It has a spatial resolution of 31 km with hourly temporal resolution.

Comparisons between ERA5 and its predecessor ERA-Interim, indicate that ERA5 provide more accurate estimates compared to direct measurements [65].

- **ERA5 land.** ERA5 land is a derivative product of ERA5, focused on land variables with enhanced spatial resolution of 9 km, with hourly temporal resolution.

Although ERA5 land offers better spatial resolution compared to ERA5, some studies suggest that its sensible heat flux estimations may be not necessary better [66]. An experimental assessment of ERA5 and ERA5 land is shown in section 6.3.5.

- **JRA-55.** The Japanese 55-Year Reanalysis dataset by the Japan Meteorological Agency, with global coverage, provides data from 1958, with a spatial resolution of approximately 140 km and 6-hourly temporal resolution.
- **NCEP.** The National Centers for Environmental Prediction (NCEP) provide two widely used reanalysis datasets: NCEP1, with a spatial resolution around 209 km and 6-hourly temporal resolution and NCEP2, with improved model physics and data assimilation.
- **MERRA.** The Modern-Era Retrospective Analysis for Research Applications produced by the Global Modeling and Assimilation Office (GMAO) from NASA has hourly temporal resolution and its spatial resolution is approximately 50 x 67 km.

Each of these datasets presents specific advantages and constraints, with their suitability depending on factors such as spatial and temporal resolution, historical coverage, and consistency with ground-based observations. An experimental comparative between ERA5 and ERA5 land and in-situ measurements is presented in Section 6.3.4.

6.2 Proposed 3D refractivity model (3D-RM)

This section introduces the 3D Refractivity Model (3D-RM), designed to estimate atmospheric refraction effects by combining vertical atmospheric characterization with spatial refractivity modelling. As discussed in Section 2.1.3, in the atmospheric boundary layer

turbulent and convective processes produce strong spatial and vertical gradients in temperature, the primary driver of refractivity variability. The 3D-RM addresses these effects by integrating ground-based meteorological observations with physically based vertical modelling.

The model is based on two fundamental ideas (Figure 23):

1. Spatial interpolation of refractivity. Meteorological variables, especially temperature and humidity, often vary significantly over relatively short distances. To capture this variability, a network of meteorological sensors with automatic registration is deployed within the working area and then the computed refractivity can be interpolated to estimate refractivity at arbitrary locations within the study area, at each predefined height level.
2. Layered atmospheric modelling: near-surface meteorological measurements are limited in representing atmospheric conditions at varying heights, due to the significant influence of the surface. To extend refractivity estimates vertically, the atmosphere is conceptualized as stratified into layers parallel to the ground. The Turbulence Transfer Model (TTM) is applied to estimate refractivity gradients, enabling the propagation of refractivity values from ground-based measurements to upper layers.

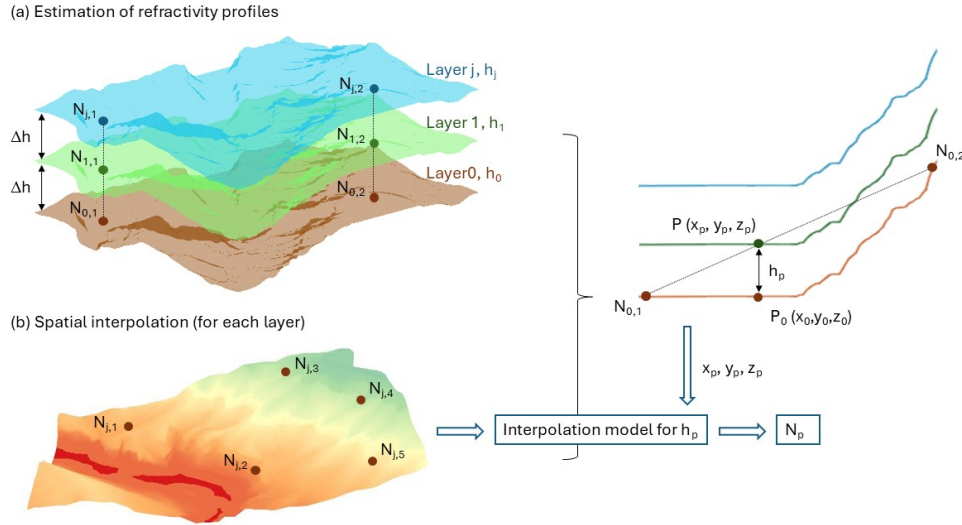


Figure 23: Model basis: (a) layered atmospheric modelling, (b) spatial interpolation [61]

Meteorological sensors are installed throughout the area, at specific points at varying heights above ground level, denoted as $h_{0,i}$ for each sensor i . For most applications, these heights are considered approximately uniform, denoted as h_0 . However, if height variations are significant relative to the vertical discretization interval (Δh), an additional step

is needed to adjust the estimated refractivity to a common reference height h_0 , as done in step 1 using the TTM.

Each station provides the meteorological data that is used to calculate the refractivity at the reference height h_0 ($N_{0,i}$) using Equation (39). Using TTM (or alternatively, pre-defined gradient profiles), refractivity values are then propagated upward to construct vertical profiles specific to each station.

Subsequently, refractivity at each altitude layer is interpolated across the area using Multiple Linear Regression (MLR) models, creating a layered 3D refractivity field. The computed model for each layer can then be used to estimate refractivity at any location within the layer.

The workflow to implement the 3D-RM includes the following 4 steps:

1. Generate vertical refractivity profiles at each station using the Turbulence Transfer Model (TTM).
2. Perform spatial interpolation of refractivity for each vertical layer using Multiple Linear Regression models (MLR).
3. Apply the model to observations to estimate refractivity at intermediate points of the Line Of Sight (LoS).
4. Compute corrections based on modelled refractivity and refractivity gradients.

The overall workflow is illustrated in Figure 24, with the symbol legend provided in Figure 25.

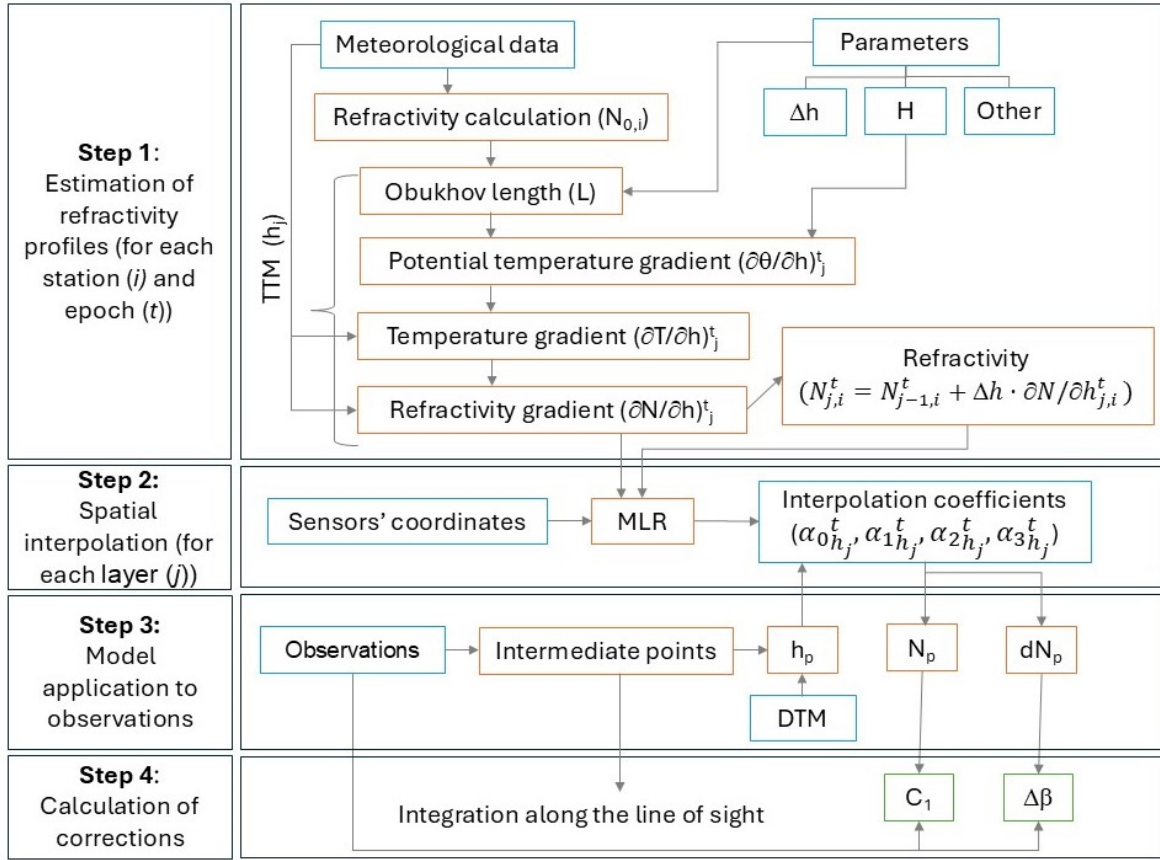


Figure 24: Overall working flow of the model (Source [61])

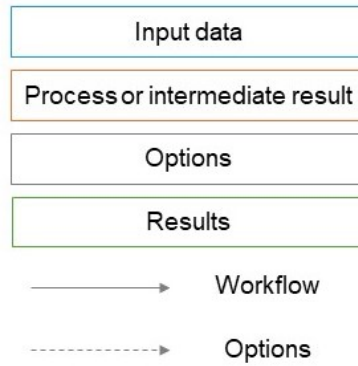


Figure 25: Model's schemes legend

A detailed description of each step is provided in the upcoming sections.

6.2.1 Step 1 - Preparation of vertical information

The first step in the implementation of 3D-RM consists of generating vertical refractivity distributions at each meteorological station. These vertical profiles are essential for characterizing the atmosphere in a layered manner and are constructed from near-surface meteorological data using either pre-defined temperature gradients or the Turbulence Transfer Model (TTM).

Several approaches are available in the model for estimating the refractivity gradients required to build these profiles:

1. Pre-defined fixed gradients. This method relies on standard temperature gradients sourced from the literature or prior empirical studies. While simple, it may not reflect local atmospheric conditions.
2. TTM with fixed sensible heat flux values. A more refined approach involves applying the TTM with constant sensible heat flux values (H).
3. TTM using time-varying H data. The most advanced method incorporates heat flux data from external sources (e.g., ERA-5), enabling a time-dependent computation of refractivity gradients using the TTM.

At each station i , and for every epoch t , the refractivity value at the sensor height (h_0), denoted $N_{0,i}^t$, is first calculated using measured meteorological parameters (Equation (39)). Subsequently, refractivity values at higher elevations are estimated using the computed refractivity gradients (dN/dh). Refractivity values are computed ranging from the station's height h_0 to the maximum specified height (h_{max}) for an established vertical interval (Δh).

If the TTM is used, the following sub-steps are applied (see section 6.1.1 for details):

- Auxiliary variables as friction velocity (U_*) and Obukhov length (L) are estimated from measured wind speed, sensor height, estimated roughness parameter (z_0), and sensible heat flux, using Equations (95) and (96).
- Based on atmospheric stability, the potential temperature gradient ($\partial\theta/\partial h$) is calculated using the appropriate formulation in Equations (91) to (94).
- The temperature gradient (dT/dh) is then computed using Equation (97), assuming a constant vertical pressure gradient of -0.12 hPa/m.
- Finally, the refractivity gradient is determined using Equation (54). As mentioned in Section 4.2, the derivation of this expression can be seen in Appendix B.

Once refractivity gradients are computed, vertical profiles are iteratively constructed for each epoch (i.e each minute). Starting from h_0 and its associated refractivity N_0 , the value at the next layer is obtained by incrementing with the product of the gradient and

vertical step Δh . This process is repeated until reaching h_{max} , producing a complete vertical profile per station and epoch. These results are stored to be used in subsequent stages of the model.

When using external heat flux data (e.g., hourly ERA-5 values), temporal alignment with the model's epoch resolution is required. The following temporal interpolation strategies are considered:

1. No interpolation: uses the nearest available heat flux value.
2. Linear interpolation: applies linear interpolation between discrete values.
3. Polynomial interpolation: Employs pre-fitted fifth-degree polynomials for greater continuity, with separate fits for day and night periods (i.e., two polynomials per day).

The decision on which method to use may depend on the availability and resolution of the flux data and the required model precision.

Figure 26 illustrates the logical workflow of this initial step of the model.

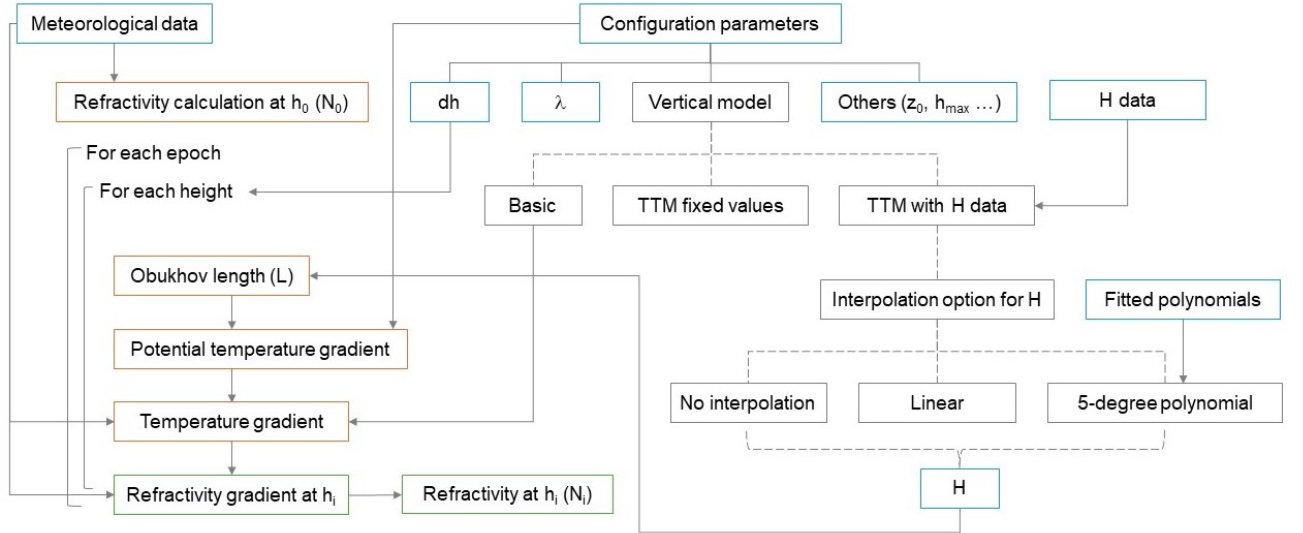


Figure 26: Workflow of step 1 of the model - preparation of vertical information

Note that the scheme in Figure 26 represents the general working flow for step 1, while in the overall scheme (Figure 24) is a simplified version which only includes the option of using TTM with heat flux information.

Table 15 summarizes the required parameters to compute step 1 of 3D-RM, offering an overview of options and data dependencies.

Table 15: Parameters needed in step 1

Parameter	Description	Options	Additional information
Vertical model	How vertical layers are defined	Fixed gradients	Gradients to apply
		TTM with fixed values	H values to apply
		TTM with H data	File with H values per hour
H interpolation	Temporal interpolation of H	no interpolation	-
		linear interpolation	-
		5-degree polynomial interpolation	File with pre-computed coefficients
λ	instrument wavelength	value	-
dH	height interval to define layers	value	-
h_{max}	higher layer	value	-
u	wind velocity	value	-
z_0	roughness parameter	value	-

6.2.2 Step 2 - Refractivity interpolation

The subsequent step of the 3D-RM involves the spatial interpolation of both the refractivity (N) and its vertical gradient (dN/dh) at specific heights above ground (h_j), ranging from the sensor level h_0 up to the maximum height h_{max} with a regular vertical increment of Δh .

To achieve this, Multiple Linear Regression (MLR) models are employed. This method was selected after comparing various interpolation approaches in the study area of Cortes de Pallás, where MLR provided the best performance in terms of root mean square error (RMSE) and coefficient of determination (R^2) [67].

Each MLR model relates the refractivity (or refractivity gradient) at a given height and time to the spatial position of the meteorological stations. Specifically, the dependent variable is modeled as a function of 3D spatial coordinates (x, y, z) in a local Cartesian coordinate system, where:

- x, y are the planimetric coordinates.
- z is the altitude above a reference plane.

The general form of the regression model is:

$$N_{h_j}^t = \alpha_{0h_j}^t + \alpha_{1h_j}^t \cdot x + \alpha_{2h_j}^t \cdot y + \alpha_{3h_j}^t \cdot z \quad (99)$$

A unique set of coefficients $(\alpha_{0h_j}^t, \alpha_{1h_j}^t, \alpha_{2h_j}^t, \alpha_{3h_j}^t)$ is computed for each height level h_j and epoch t , using the refractivity values obtained in Step 1 for each station. The same process is repeated independently for the refractivity gradient values (dN/dh) .

The use of 3D coordinates is crucial to adequately capture the spatial variability of refractivity, especially in regions with varying terrain elevation. This approach ensures that both spatial variability (including elevation) and the influence of the ground (through specific models for different heights above ground) are considered.

The spatial domain for interpolation is typically bounded by the extent of the sensor network. Points within this domain are interpolated, while points outside may result in extrapolation. However, extrapolation is generally avoided to minimize errors, as it involves estimating values beyond the range of observed data.

Each model's performance is evaluated using two metrics:

- The root mean squared error ($RMSE$), indicating the average interpolation error.
- Coefficient of determination (R^2), measuring the proportion of variance explained by the model.

Both metrics are computed and stored for every layer and epoch to track model accuracy and serve future validation or sensitivity analyses.

Figure 27 shows a scheme of the information stored for subsequent steps of the model.

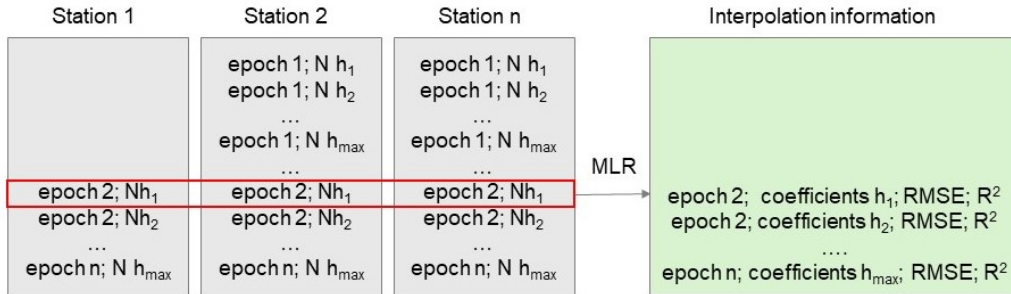


Figure 27: MLR interpolation scheme

Only those epochs for which the selected meteorological stations provide valid data are considered during interpolation.

A summary of the input parameters required in this step is provided in Table 16.

Table 16: Parameters needed in step 2

Parameter	Description
Coordinates	Coordinates for each station
N	Refractivity computed for each epoch, height, and station
$\frac{dN}{dh}$	Refractivity gradient computed for each epoch, height, and station

6.2.3 Step 3 - Model application to observations

Once the interpolation coefficients have been obtained for each height level, the 3D-RM can estimate both the refractivity (N) and its vertical gradient ($\partial N/\partial h$) at any point within the study area, provided that the point lies within the interpolation domain with a height below h_{max} . This functionality is designed to support the correction of distance and vertical angle measurements typically collected using terrestrial geodetic instruments, such as total stations (TS) or terrestrial laser scanners (TLS).

To apply the model, the coordinates of the station and the target point must be known or estimated. In standard geodetic workflows, measurements are taken from a known station to an unknown point. In such cases, the coordinates of the target are derived from observed distance, vertical angle, and horizontal angle data, assuming the orientation of the station is known.

The process to compute the refractivity at a specific epoch (t) for a point with known coordinates and at a specific height above the ground is as follows:

1. The corresponding epoch t is accessed in the interpolation file.
2. The height above ground (h_p) at the point is computed.
3. The model retrieves the coefficients associated with the nearest lower height layer (h_j) relative to h_p .
4. These coefficients are then applied to the 3D coordinates of the point to estimate refractivity and its vertical gradient.

This procedure is applied similarly when estimating the refractivity gradient, using the gradient-specific interpolation coefficients.

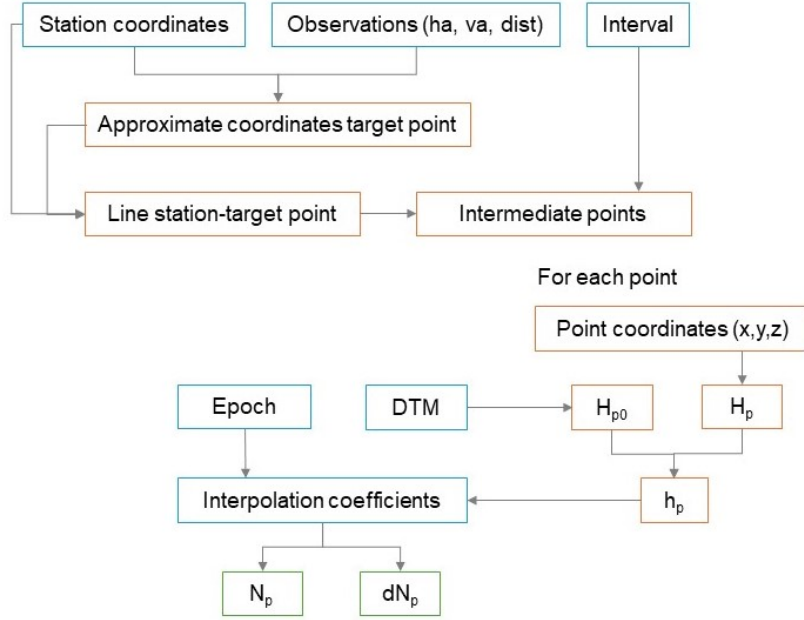


Figure 28: Working flow of step 3 of the model - application to observations

To compute accurate corrections, refractivity is needed at several points along the line of sight (LoS) between the station and the target point. To account for spatial variations, the straight LoS is discretised into intermediate points at a defined resolution (e.g., every 100 m). The workflow for each of these intermediate points is illustrated in Figure 28 and involves the following steps:

1. 3D coordinates calculation: the 3D coordinates (x_p, y_p, z_p) of the intermediate point are calculated in the local system (or the system in which station and target point are expressed).
2. Terrain elevation extraction: the corresponding altitude at the (x_p, y_p) location, referred to as H_{DTM} , is determined using a Digital Terrain Model (DTM). The DTM provides altitude in a specific system, usually orthometric height. This step is crucial because the 3D-RM layering is based on height above ground, thus the DTM is needed for its estimation.
3. Height above ground computation: the height above ground (h_p) is calculated as the difference between the point's z-coordinate (z_p) and the DTM-provided ground elevation (H_{DTM}) , as illustrated in Figure 29). Consistency between the coordinate system used for the points and the DTM is crucial. If different systems are used, such as local coordinates for the points and orthometric height in the DTM, a transformation must be applied to relate the two systems. In the experiments presented in this thesis, the local coordinates x, y, z are converted into UTM30 X_{UTM}, Y_{UTM}, H to calculate height above ground.

4. Retrieval of interpolation coefficients: based on the computed height above ground (h_p) and the observation epoch (t), interpolation coefficients for the nearest layer (h_j) are retrieved from interpolation files
5. Estimation of refractivity and its gradient: the retrieved coefficients are applied to the point's coordinates to compute both N and $\partial N/\partial h$ at that location and time

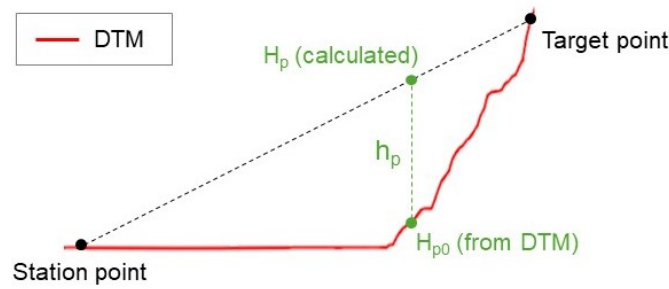


Figure 29: Calculation of height above ground with DTM information

A complete list of the input parameters required for Step 3 is provided in Table 17.

Table 17: Parameters needed in step 3

Parameter	Description	Options	Additional information
Observations	field observations	TS or TLS	Observation file
Interval	Interval to compute the intermediate points	value	-
Station	Station's data	-	Files from step 2 (N and dN)
Ellipsoid*	Reference ellipsoid	-	Ellipsoid parameters
Geoid*	Reference geoid	-	Grid file
DTM	Digital terrain model	-	DTM file
$\alpha_0, \alpha_1, \alpha_2, \alpha_3$	Interpolation coefficients	-	Interpolation files

* Ellipsoid and Geoid information are needed to perform coordinate system transformations, depending on the coordinate system used, this may be not necessary

6.2.4 Step 4 - Correction computation

The final step of the model involves computing the distance and vertical angle corrections for each observation, using the refractivity and refractivity gradient values previously estimated along the line of sight. The model is focused on distance and vertical angle corrections.

Distance correction

As discussed in Section 4.1.2, the conventional method for correcting distances is the first velocity correction (see Equation (45)), where the mean refractive index n_m is traditionally approximated by averaging the value at both ends of the observed baseline. However, this simplification does not reflect the actual atmospheric conditions along the path. A more accurate estimation involves computing the mean refractive index n_m along the entire line of sight.

Thanks to the results from Step 3, where refractivity values were computed at intermediate points along each observation line, this integral can now be numerically approximated. The model applies the trapezoidal method for this integration, using m equally spaced intermediate points (illustrated in Figure 30).

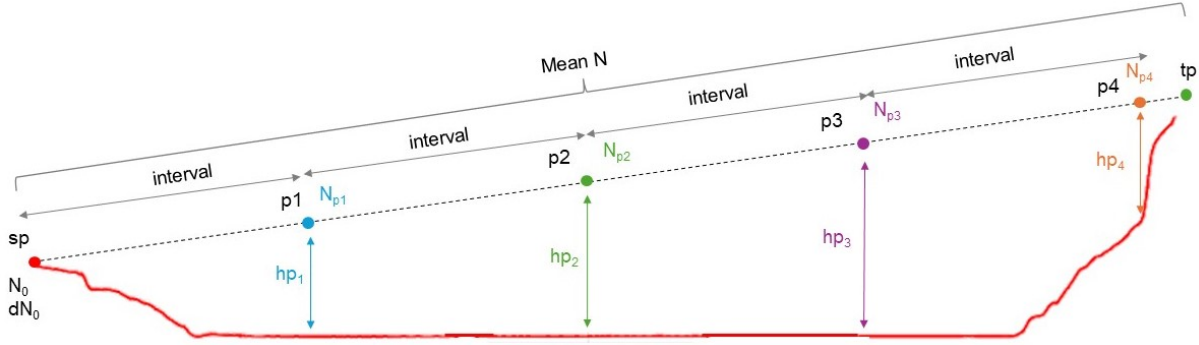


Figure 30: Integration example along one line

The general expression of the trapezoidal method is:

$$\int_a^b f(x)dx = \frac{\Delta x}{2} \cdot [f(x_0) + 2f(x_1) + \dots + 2f(x_{m-1}) + f(x_m)] \quad (100)$$

Applied to this context, the integral becomes:

$$\int_0^l n(x)dx = \frac{\Delta s}{2} \cdot [n_0 + (2n_{p1} + \dots + 2n_{pm-1}) + n_{pm}] \quad (101)$$

Here, Δs refers to the fixed spacing between intermediate points. The correction applied has the form of the first velocity correction (Equation (45)). In this case, as the term n_m is obtained by integrating values along the line of sight, no further refinement (e.g., second velocity correction) is required.

Vertical angle correction

To correct the vertical angle, the refraction angle is estimated as outlined in Section 4.2.1 (see Equation (49)).

The refraction angle is computed by integrating the refractivity gradient along the line of sight, again using the trapezoidal method, using the refractivity gradients interpolated at each intermediate point of the line of sight. The calculated refraction angle is then added to the measured zenith angle to correct it.

Table 18 summarizes the parameters and information needed to compute the step 4 of the model.

Table 18: Parameters needed in step 4

Parameter	Description
Measurements	Field measurements (distance and vertical angle)
n_0	Reference refraction index
Interval	Interval of the intermediate points
N_i	Refractivity value estimated for each intermediate point
dN_i	Refractivity gradient estimated for each intermediate point

This step finalizes the 3D-RM model by applying refractivity-based corrections to distance and vertical angle measurements. Through numerical integration along the line of sight, the model aims to improve accuracy of field observations under varying atmospheric conditions.

6.3 Cases of study: application of 3D-RM to TS measurements

In this section, experimental studies conducted to evaluate the 3D-RM are presented. The section is organized as follows:

1. The working area where the field campaigns were conducted is described.
2. A summary of the different campaigns carried out during the development of this thesis is provided.
3. The main results related to the 3D-RM are presented.
4. Some intermediate results and parameter assessment are included.

6.3.1 Working area: Cortes de Pallás

Cortes de Pallás is a village in Valencia (Spain). The specific area of interest is a cliff known as La Muela, where a hydroelectric power complex operates. This area, which is a Cretaceous limestone region, presents recurrent geotechnical issues. In 2015, part of the cliff collapsed, resulting in damage to the hydroelectric power plant and nearby roads, isolating the village. After the refurbishment and consolidation works the *Universitat Politècnica de València (UPV)* in collaboration with the Department of Roads and Infrastructures of the *Diputació de València*, started a three-year project to monitor the cliff to prevent future rockfalls. This project included the establishment of a geodetic network to serve as a reference frame, along with the installation of permanent prisms on the hillside for continuous monitoring.



Figure 31: Overview of the working area (Source: [68])

Figure 31 shows an overview of the working area, the geodetic network's boundaries are marked in yellow, while the critical area to be monitored is highlighted in red.

Geodetic network

The geodetic network in Cortes de Pallás comprises 10 geodetic pillars (labelled from 8001 to 8010) and was designed as a pure 3D trilateration network [69]. After the installation

of the pillars, three measurement campaigns were conducted over three consecutive years (2018, 2019, 2020) using a high-precision EDM Kern Mekometer ME5000 (Figure 32). To ensure high accuracy, the measurements were supported by meteorological data collected with 10 calibrated data loggers.



Figure 32: Mekometer ME5000 measurements

The computation of pillar coordinates involved applying various corrections, including refraction, EDM frequency drift, and geometric reductions. Detailed descriptions of these corrections are available in [69]. Subsequently, a rigorous least-squared adjustment was performed, yielding a set of coordinates with standard deviations ranging from 0.1 to 1.2 mm for each pillar, with 85.7% of residuals below 1 mm [69]. Figure 33 shows the free-network solution for year 2019, including the standard error ellipses in red and the vertical precision in blue.

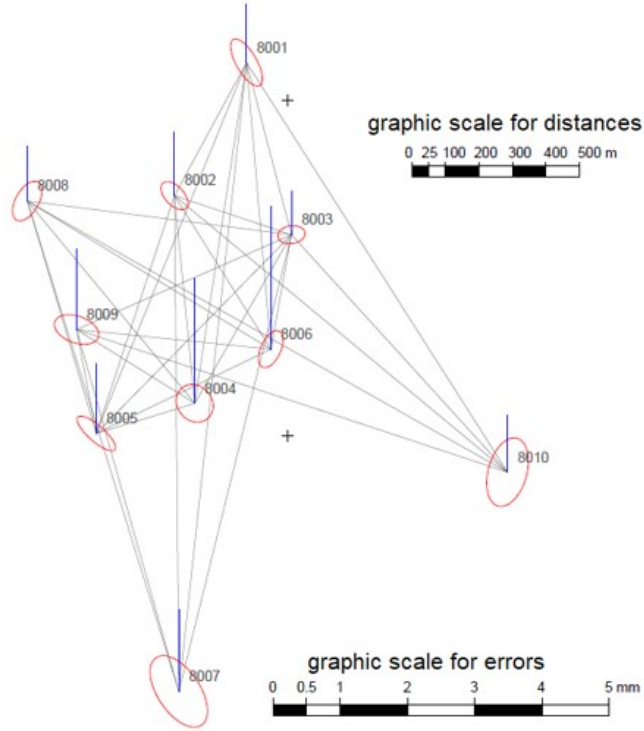


Figure 33: Free-network solution 2019 (Source: [68])

To ensure the reliability of the reference frame for deformation monitoring, a thorough analysis of its stability was performed using data from the three-year measurement campaigns. The results, which are detailed in [68], revealed horizontal displacements at all points between 2018 and 2020, particularly near the water reservoir, where displacements reached 2-3 mm. Vertical displacements in these areas showed downward shifts ranging from -13.22 to -16.25 mm.

The reference coordinates in the local system CP2017 (detailed further) are presented in Table 19.

Table 19: Local coordinates of the geodetic network pillars

Pillar	x(m)	y(m)	z(m)
8001	159.4083	94.2080	144.9715
8002	536.2871	341.2314	46.7024
8003	285.0310	608.8918	106.5783
8004	776.2580	914.5217	14.9182
8005	1077.0223	854.3889	74.0872
8006	500.4831	879.7765	69.8557
8007	1224.7750	1647.0658	499.7153
8008	929.5876	147.6371	155.1183
8009	981.7671	554.0390	10.4536
8010	53.8649	1536.3211	467.0790

Local coordinate system

A conventional Cartesian coordinate system, named CP2017, was defined in Cortes de Pallás to facilitate further deformation analysis and also the processing and integration of other techniques, such as image-based ones [69].

The xz-plane of the local system was established to be approximately coincident with the cliff of interest (Figure 34).

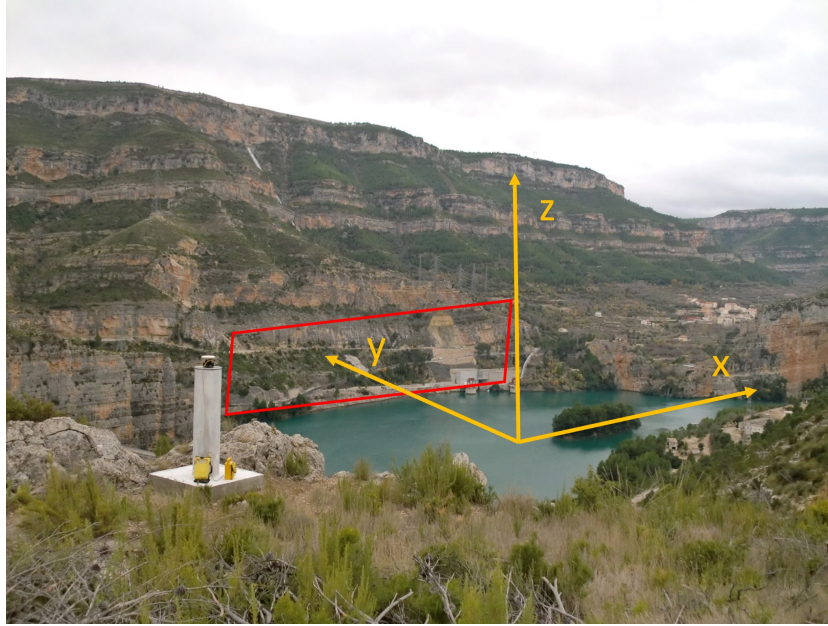


Figure 34: Local coordinate system. The critical area is marked in red

The definition of this system is shown in Table 20. Geodetic coordinates are given in

ETRS89.

Table 20: Definition of the CP2017 local system

Parameter	Value
x_0	800 m
y_0	800 m
x_0	150 m
ϕ_0	39 °14' 47.98063"
λ_0	-0 °55' 55.55670"
h_0	520.57344 m
α_0	230 gon

With this conventional definition, coordinates and precisions obtained with different measurements and adjustments can be converted into different coordinate systems with no loss of accuracy. Coordinates of the different campaigns conducted in the working area are usually obtained in the local system CP2017 (x, y, z) , in geodetic coordinates ETRS89 (ϕ, λ) , and UTM30 (E, N) . Both ellipsoidal (h) and orthometric (H) height can be obtained.

Target points

In addition to the 10-pillars geodetic network, 15 target points were also permanently installed at critical locations on the cliff (named 1001-1015). These target points consist on a Leica 360 reflector together with a 145 mm diameter target sphere, intended for use with image-based techniques (Figure 35).



Figure 35: Target point consisting on reflector and sphere

The coordinates of the reflectors were also determined using the Mekometer ME5000 measurements (Table 21).

Table 21: Local coordinates of the target points

Point	x(m)	y(m)	z(m)
1001	912.8108	1005.2084	144.7208
1002	908.1700	1007.6356	139.8012
1003	901.4871	1010.6389	145.7455
1004	922.0150	999.5838	118.6362
1005	917.9428	999.3877	114.3216
1006	926.8292	996.9338	111.7634
1007	894.6261	988.8788	86.0142
1008	874.9508	983.5045	71.6587
1009	873.2529	991.7038	85.6273
1010	619.6238	950.3059	85.5166
1011	495.6261	952.8132	138.6876
1012	480.6799	954.6206	158.9148
1013	479.1463	946.6025	141.6632
1014	488.0169	952.9289	156.1855
1015	900.0689	983.5999	80.8598

Figure 36 shows how the target points are distributed in the cliff, specifically placed in critical areas as it can be seen in the red-squared zoom.



Figure 36: Distribution of target points

Auxiliary points

Apart from the primary network and target points, additional points were included as needed. A point designated as 9000 was defined on the ground with a permanent mark, providing a reference for tripod installations. An additional pillar, 8011, was added to enhance the network's geometry for meteorological sensor placement and also to install target spheres for TLS registration; however, this pillar is considered second-order due to its lower precision. Finally, an auxiliary point, 8012, was temporarily added for a specific field campaign to install a meteorological sensor on it.

The overall distribution of these points is shown in Figure 37.

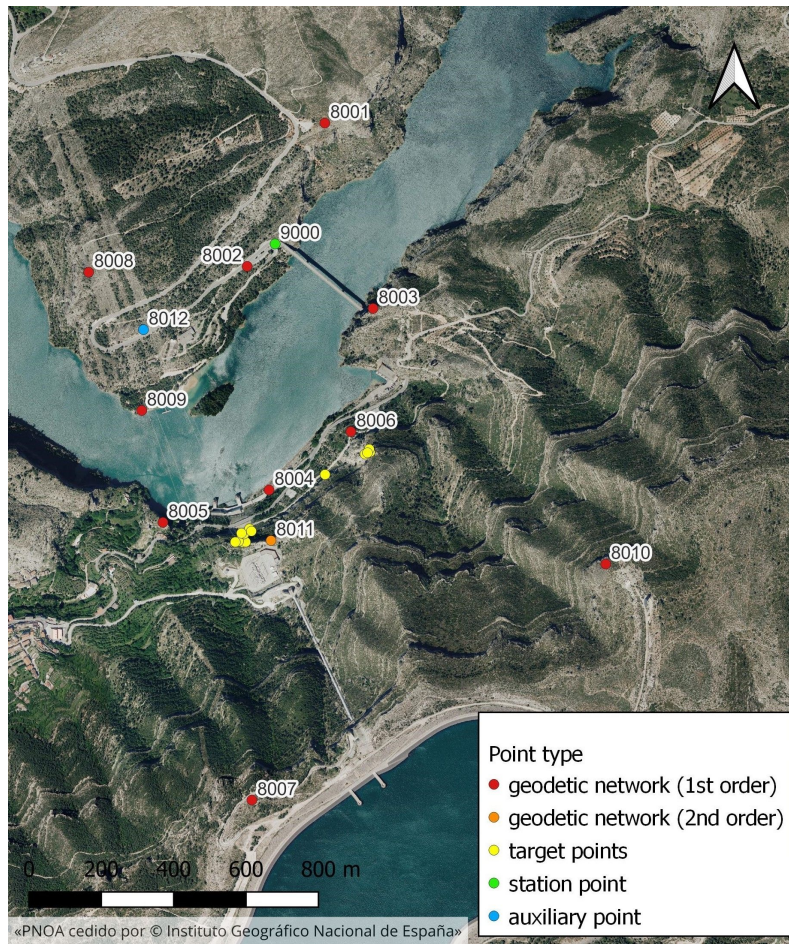


Figure 37: Point distribution in Cortes de Pallás

Figure 38 shows a detailed layout of the target points.



Figure 38: Target points in the cliff

Local coordinates for these points are shown in Table 22.

Table 22: Local coordinates of additional points

Point	x(m)	y(m)	z(m)
9000	438.4706	323.5775	38.2975
8011	837.8354	1040.1494	157.1011
8012	871.9447	359.0876	80.5792

6.3.2 Description of field campaigns

During the development of this thesis, several campaigns were conducted to get different types of data for the development and testing of the 3D-RM. Table 23 provides a summary of the main characteristics of each campaign, including the instruments used, the locations of the meteorological sensors, and the points observed.

Table 23: Field campaigns summary

Campaign	Instruments	Meteorological sensors	Target points
February 2023	TS (9000)	8001, 8002, 8005, 8006, 8009, 8011	1001-1015
July 2023	TS (9000)	8001, 8002, 8003, 8004, 8005, 8006, 8007, 8009, 8011, 9000	1001-1015
	TLS (8002, 8009)		working area
June 2024	TS (8009)	8002, 8004, 8005, 8006, 8009, 8011, 8012	1001-1009, 1015, 8004, 8005, 8006, 8011

February 2023

This campaign took place on February 28th 2023. It was conducted as a preparatory step to collect the initial data for developing the model. The objectives were:

- Provide coordinates for point 8011.
- Assess the influence of meteorological sensor ventilation.
- Collect the first data set using the Leica TM30 total station.

In this campaign, only six ventilated shelters were available and they were installed at the points shown in Table 23. Additionally, two non-ventilated shelters were installed at points 8004 and 9000.

To provide coordinates for point 8011, measurements were taken using the TM30 from points 8003, 8002, and 8009. The target points on the cliff were measured from point 9000, with the TM30 mounted on a tripod. A total of four series of five measurements to each prism were measured between 13:00 and 18:00.

July 2023

This second campaign was on July 23rd 2023. The main objectives of the campaign were:

- Acquire point cloud data using TLS.
- Collect measurement data using the Leica TM30 total station.

In this campaign, ten meteorological sensors were installed using ventilated shelters at points listed in Table 23.

For the TLS measurements, five half-meter diameter target spheres were installed at points 8003, 8004, 8005, 8006, and 8011, covering all the critical area. These target sphere were used for registration, to integrate TLS point clouds into the CP1017 local reference frame. Different point cloud acquisitions were performed at various times from pillars 8002 and 8009 to have data acquired under different atmospheric conditions.

Similarly to the February campaign, the TS was set up at point 9000 using a tripod. In this case, three series of five measurements were taken to the target points from 08:00 to 11:00. No more data were collected because the atmospheric conditions were almost constant.

June 2024

The final campaign took place on June 25, 2024, and was designed to acquire total station measurements over an extended period. Its main objectives were:

- Collecting continuous data during the daytime to enable a more detailed analysis of refractivity and validation of the model.
- Expanding the range of vertical angles observed.

For this purpose, an auxiliary point (8012) was equipped with a meteorological sensor. This point, set on a tripod with RTK-determined GNSS coordinates, allowed for a flexible configuration and contributed to an improved geometry of the meteorological sensor network.

In total, seven meteorological sensors were deployed using ventilated shelters. The total station was installed at point 8009, which was closer to the target points, providing coverage over a wide range of vertical angles. Measurements were carried out systematically: a series of five observations was taken every hour from 7:00 to 22:00, with a battery replacement at 15:00.

Instruments and setting up

The following instruments were used during the campaigns:

- Leica TM30 robotic total station (EDM 0.6 mm + 1 ppm, angular appreciation 0.5").
- Leica 360 prisms: apart from the target points permanently installed on the cliff, additional prisms were set up at various points as needed.
- Kern RMO5035 prism: used for highly accurate measurements, such as those taken to provide coordinates for point 8011.

- Testo 176P1 atmospheric data-loggers: calibrated at the UPV calibration laboratory, with an accuracy of ± 0.2 K for temperature and ± 1.8 hPa for air pressure.
- Self-ventilated shelters: used to protect meteorological sensors, connected to batteries to ensure ventilation for accurate measurements.
- Simple shelters: used during the first campaign due to a shortage of ventilated shelters, allowing for a comparison of results.
- Leica ScanStation P50 long-range terrestrial laser scanner (3 mm + 10 ppm in 570 m / > 1 km mode, 8").
- Half-meter diameter spheres: installed at designated pillars when TLS was used to serve as reference points to point cloud georeferencing.

Figure 39 shows the main instrumentation used during the field campaigns: (a) half-meter sphere installed with meteorological sensor used as target point for TLS; (b) 360 prism installed on a pillar with a meteorological sensor used as reference; (c) Leica TM30 TS mounted on tripod; (d) Leica P50 TLS installed on pillar with meteorological sensor.

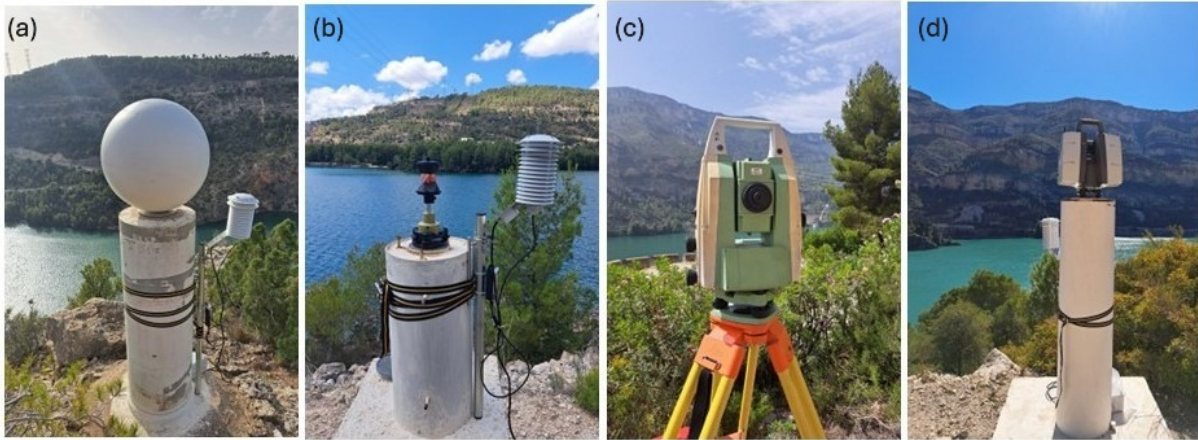


Figure 39: Instruments used in field campaigns

6.3.3 Selected parameters for the application of 3D-RM

This section describes the configuration of the 3D-RM model adopted for the analysis of the field campaigns. The selected parameters from those discussed in Section 6.2 and based in the evaluation of their performance, are further analyzed in Section 6.3.5. The aim is to define a consistent and realistic setup that ensures comparability across all experiments.

- Interval: intermediate points along the line of sight were generated every 100 m. Ideally, meteorological parameters should be measured at this resolution [43], so this interval was considered appropriated.
- Δh : the vertical interval between vertical layers was set to 1 m. This choice is particularly relevant near the surface, where atmospheric stratification is strongest. For altitudes above 20–30 m, a coarser resolution could be used.
- h_{max} : the maximum height above ground was fixed at 200 m, in line with the terrain profiles of the study area.
- z_0 : the surface roughness length was set uniformly to 0.02 m, representative of open terrain [70].
- Wind speed: since wind conditions were relatively stable, a single representative value (2–3 m/s) was used for each campaign.
- Vertical model: hourly sensible heat flux values obtained from remote-sensing products were used to represent the vertical structure of the atmosphere, offering a more realistic characterization than fixed-gradient models.
- Sensible heat flux data: ERA5 was chosen as the optimal source for heat flux information.
- Temporal H interpolation: linear interpolation was selected for temporal interpolation of H, as the use of 5-degree polynomials did not significantly improve results while increasing computational complexity.
- Spatial interpolation: multiple linear regression (MLR) was used for spatial interpolation of refractivity in each vertical layer.
- The DTM used is from the National Geographic Institute (IGN) with a spatial resolution of 2 meters.

6.3.4 Results of the application of 3D-RM

This section presents the results obtained from applying the 3D-RM model to the data collected during the field campaigns, using the parameter configuration described in Section 6.3.5. Two main aspects are analysed: a) Distance corrections, evaluated both with measurements and simulated data, and b) Vertical corrections results.

a) Distance corrections results

This section presents the results obtained using the 3D-RM to compute distance corrections. The analysis is organized into two main parts: first, tests performed on simulated data; and second, corrections computed using actual field measurements from various

campaigns. These results are used to assess both the model's precision, through the standard deviation of corrected measurements, and its accuracy, by comparing them to reference distances derived from high-precision coordinates.

- Results over simulated points

Target points on the cliff are concentrated in specific locations, leading to limited coverage of the entire area and relatively similar conditions among them. To explore how the 3D-RM behaves under different topographic and geometric conditions, four auxiliary points were simulated. These points, while not measured in the field, were strategically placed approximately 820 meters from station 8002 (Figure 40).



Figure 40: Simulated points in the working area (in green)

Despite their similar distance from the station point, the line of sight to each point varies significantly. Figure 41 shows the topographic profiles for auxiliary points 2 and 4. As shown, the LoS between station 8002 and point 4 is nearly horizontal, while station 8002 and point 2 have a considerable height difference. This allowed us to evaluate the model's response to refractive variability driven by terrain and geometry, rather than only distance.

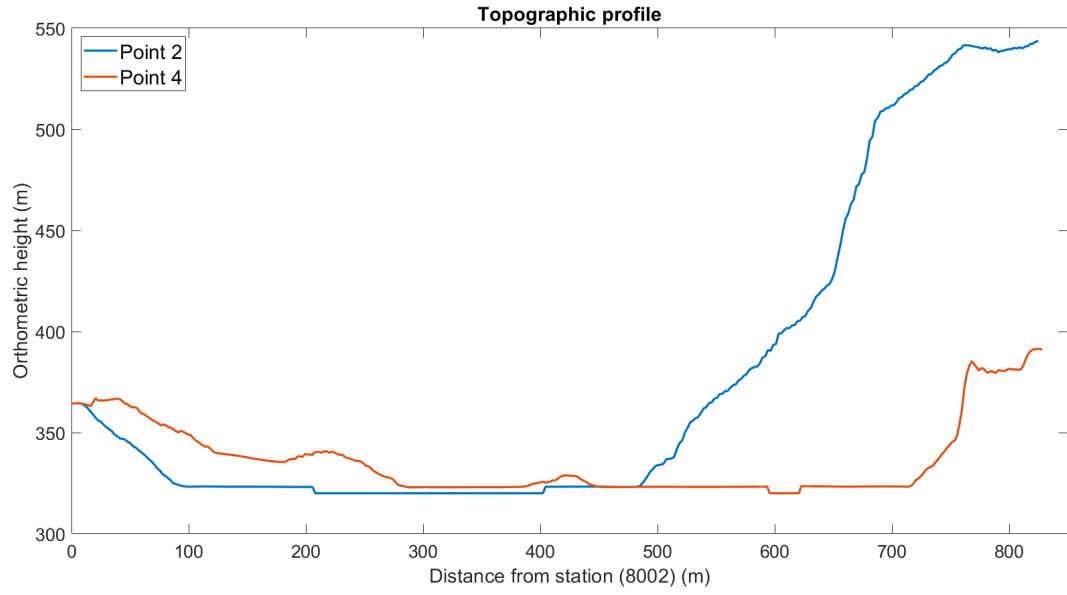


Figure 41: Topographic profiles from simulated points 2 and 4

To estimate the distance corrections, known coordinates of station 8002 and the auxiliary points were used to simulate the observations. Using meteorological data from the July 2023 campaign, two correction strategies were compared: one using only data recorded at the station point (traditional approach), and another applying the 3D-RM with all the meteorological information. The results are presented in Figure 42.

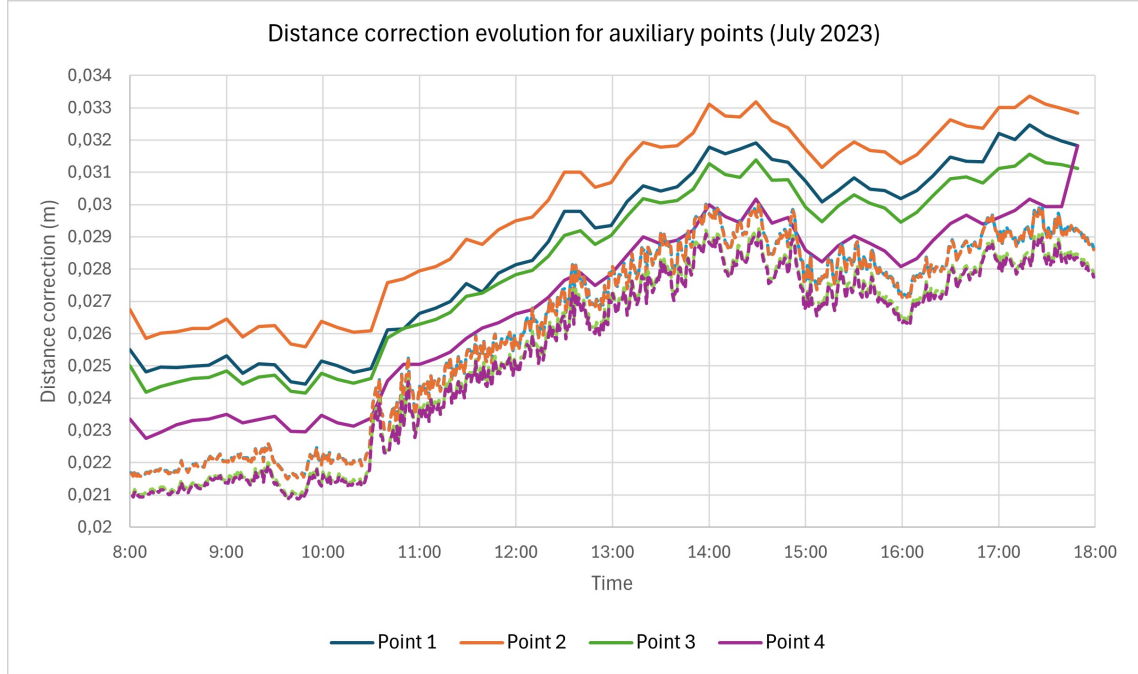


Figure 42: Distance corrections obtained for simulated points. Solid lines: 3D-RM results, dashed lines: only meteorological data at station point used

The results confirm that corrections based on station-only data are nearly identical across all simulated points, with minor variations due to small differences in distance. In contrast, the 3D-RM yields noticeable different corrections depending on LoS profile. Distance corrections differ by up to 3.5 mm between points, particularly between points 2 and 4. The maximum discrepancy between both approaches reaches 4 mm, which can be considered critical for high-precision geodetic instruments.

This demonstrates the 3D-RM's ability to produce corrections that are sensitive to actual atmospheric and geometric variability, unlike the standard method which only reflects conditions at the station.

- Results obtained for measured distances

The 3D-RM was then applied to real data from three field campaigns: February 2023, July 2023, and June 2024. Results were compared to those obtained via the traditional correction method, which uses only meteorological data from the station point (denoted as N_0). Since the target points were inaccessible, no sensors could be installed at their locations.

Additionally, the 3D-RM was tested under a simulated scenario in which no meteorological sensor was installed at the station. In campaigns where the station was located at 9000, both stations 9000 and 8002 were excluded from the model due to their proximity. This

alternative approach, referred to as 3D-RM 2, is evaluated for two main reasons:

1. To simulate real-world situations where placing a sensor near the station is not feasible or optimal.
2. To improve flexibility when multiple stations are used. By relying on meteorological data from a network of representative points rather than the station itself, 3D-RM 2 eliminates the need to relocate sensors when instruments are repositioned within the area, making the method more practical and efficient.

The mean distance corrections for all campaigns and methods are presented in Table 24.

Table 24: Mean distance correction for different campaigns (in mm)

Point	February 2023			July 2023			June 2024		
	N_0	3D-RM	3D-RM 2	N_0	3D-RM	3D-RM 2	N_0	3D-RM	3D-RM 2
1001	12.1	13.3	13.3	22.2	24.3	24.3	11.9	13.6	13.8
1002	12.0	13.1	13.1	22.2	24.2	24.2	11.9	13.6	13.8
1003	12.0	13.3	13.2	22.2	24.3	24.3	12.0	13.8	14.0
1004	12.0	12.7	12.6	22.2	23.8	23.8	11.5	12.9	13.1
1005	11.9	12.5	12.5	22.1	23.6	23.6	11.5	12.8	13.1
1006	12.0	12.5	12.5	22.2	23.7	23.7	11.4	12.7	12.9
1007	11.6	11.6	11.5	21.5	22.5	22.5	11.2	12.2	12.5
1008	11.4	11.1	11.0	21.0	21.8	21.8	11.1	12.0	12.2
1009	11.5	11.5	11.4	21.2	22.2	22.2	11.4	12.4	12.6
1010	9.4	9.5	9.4	17.4	18.2	18.1	-	-	-
1011	9.1	10.2	10.2	17.0	18.4	18.4	-	-	-
1012	9.2	10.6	10.6	17.1	18.8	18.8	-	-	-
1013	9.1	10.2	10.2	16.8	18.3	18.3	-	-	-
1014	9.2	10.5	10.6	17.1	18.8	18.7	-	-	-
1015	11.5	11.5	11.5	21.4	22.4	22.3	11.1	12.0	12.2

As described in Section 6.3.2, the station point was located at point 9000 during the February 2023 and July 2023 campaigns, while in June 2024, the station point was at pillar 8009. The distances between station 9000 and the target points are greater than those between station 8009 and the same targets, resulting in an expected lower refraction effect in the June 2024 campaign.

From Table 24, it can be observed that, in general, 3D-RM corrections are slightly larger than those using only the meteorological data at the station (N_0). The corrections in July 2023 are significantly higher than those in February 2023, likely due to higher temperatures in July than in February and longer baseline lengths (~ 800 m) compared to the June 2024 campaign (~ 450 m). The corrections obtained when applying the 3D-RM including all meteorological stations are very close to those obtained when excluding

the station points (3D-RM 2), particularly in the February 2023 and July 2023 campaigns.

Temporal evolution of corrections and measured distances.

To further evaluate the performance of the 3D-RM we need to analyse not only the mean correction obtained but its evolution over time. For the sake of conciseness, only the results for the target points 1001 and 1007 are analysed as representative examples.

Figures 43 and 44 show the temporal evolution of both the measured distances and the corresponding corrections for the February 2023 and July 2023 campaigns. These two campaigns cover similar time intervals, allowing a direct comparison of the model's behaviour under different atmospheric conditions.

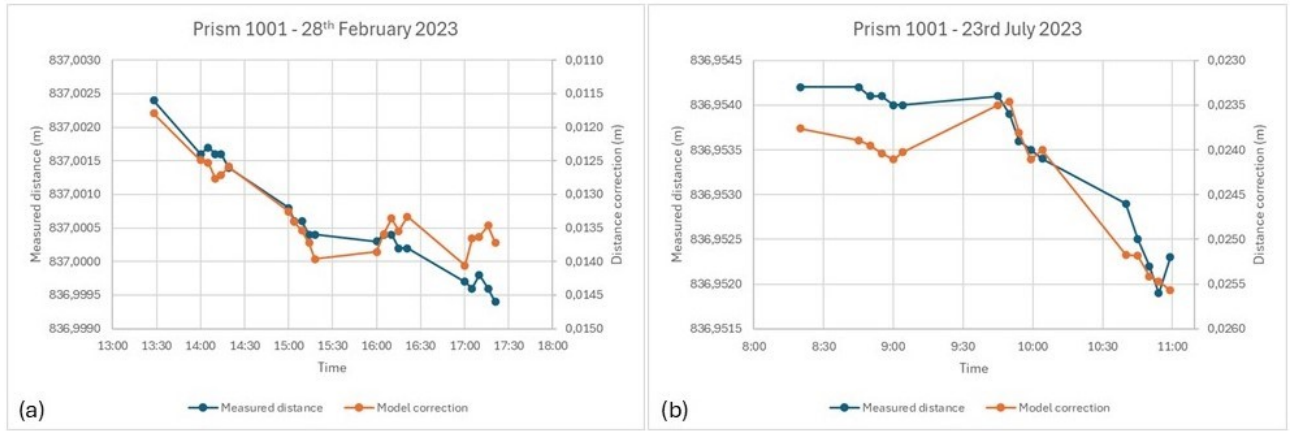


Figure 43: Distance and correction evolution for prism 1001. (a) February 2023, (b) July 2023

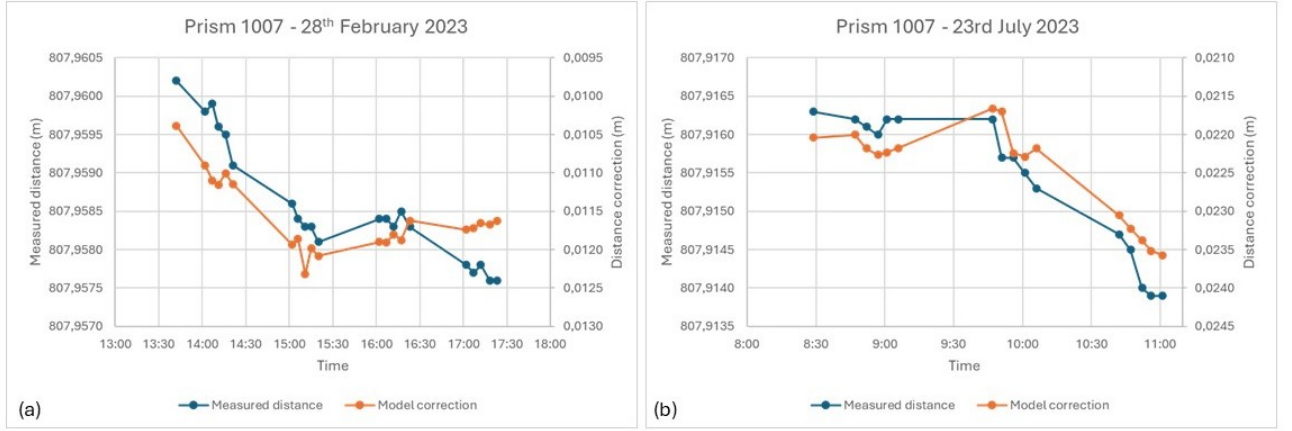


Figure 44: Distance and correction evolution for prism 1007. (a) February 2023, (b) July 2023

In all cases, the 3D-RM successfully captures the trends and fluctuations observed in the raw measurements, with a close match between the variation in distance and the corresponding correction. Differences between measured and corrected distances remain within ± 0.5 mm throughout the entire observation periods. Even subtle variations, such as the slight increase observed around 9 AM in the July 2023 campaign, are reflected in the model's corrections, demonstrating its sensitivity and adaptability.

Figures 45 and 46 present the same analysis for the June 2024 campaign (prism 1001 and 1007, respectively). In contrast to the earlier campaigns, this campaign covers an extended time period (from sunrise to sunset) allowing for a more comprehensive analysis of the diurnal evolution of atmospheric refraction.

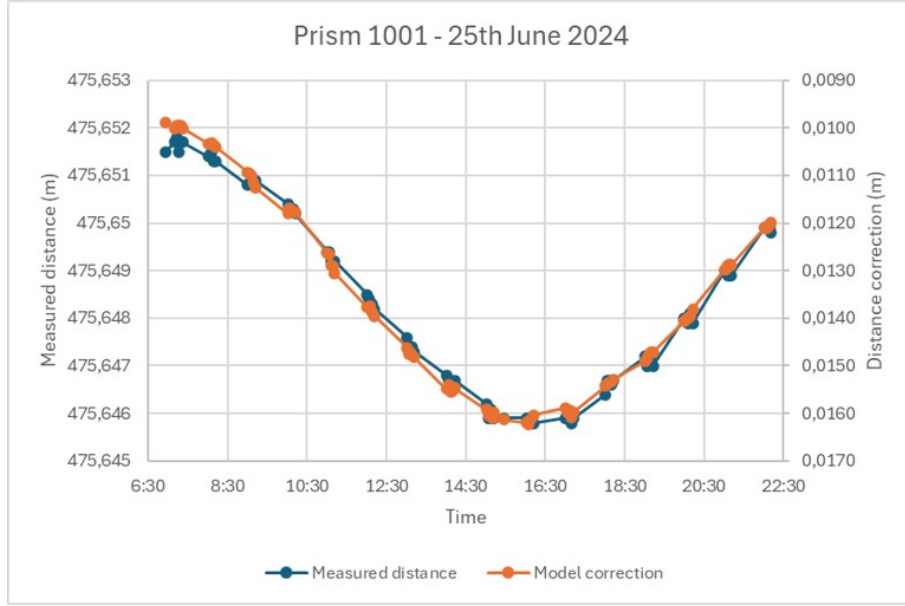


Figure 45: Distance and correction evolution for prism 1001 (June 2024)

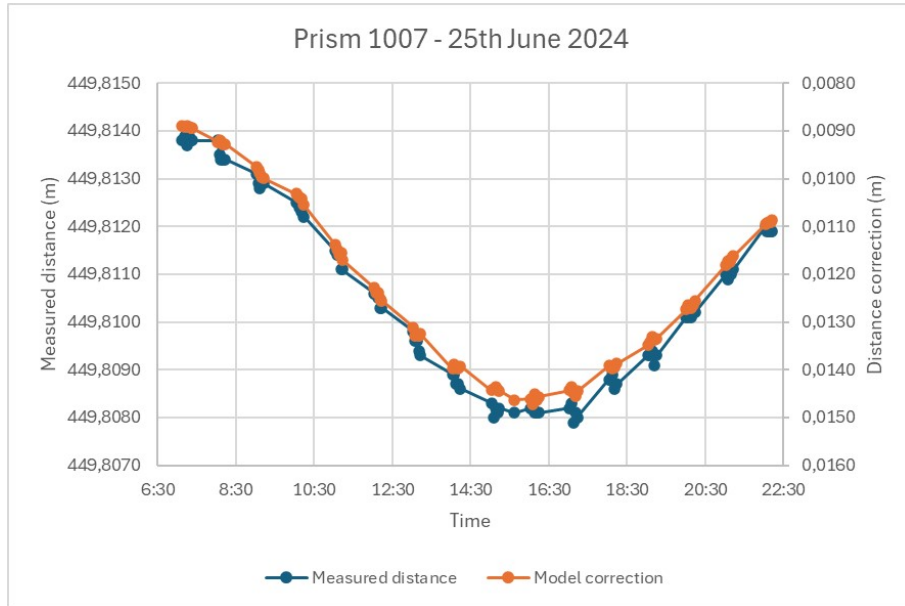


Figure 46: Distance and correction evolution for prism 1007 (June 2024)

In this case, the expected temporal behaviour of refraction is clearly observed. Lower corrections at sunrise, a steady increase as temperatures rise throughout the morning, a peak around noon (local time is UTC+2), and a gradual decrease in the afternoon. The amplitude of the variation in measured distances reaches approximately 6 mm, confirming the strong influence of diurnal atmospheric refraction. Despite this variability, the model remains consistent, with differences between raw measurements and corrected distances

consistently under 0.5 mm.

Due to the extended coverage of the June 2024 campaign, a more detailed view is provided in Figure 47, which breaks the results for prism 1001 into four-hour intervals. This visualization allows for closer inspection of the correction trends and further confirms the model's ability to capture the atmospheric dynamics throughout the day.

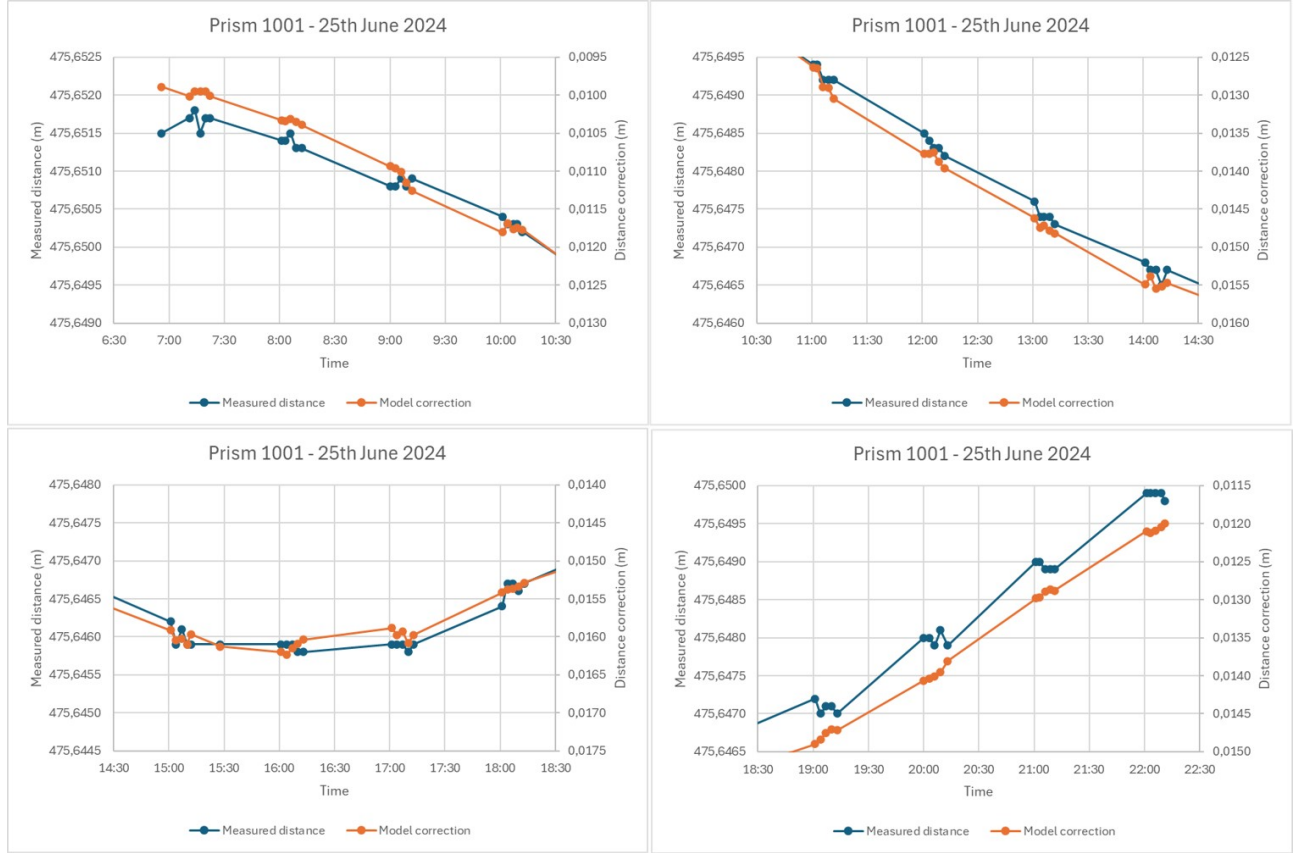


Figure 47: Distance and correction evolution for prism 1001 4-hour periods (June 2024)

All results presented in Figures 43, 44, 45, and 46 demonstrate a strong agreement between the evolution of measured distances and the model's estimated corrections. The temporal correlation confirms that the 3D-RM accurately models the atmospheric refraction and provides corrections that align closely with observed field variations.

Results in terms of standard deviation.

Since atmospheric refraction changes over time, it causes measurable variations in distances throughout the day. As the actual positions of the points are fixed during the observation window, any distance fluctuation should be primarily attributed to refraction effects. Therefore, a model that effectively mitigates atmospheric influence should reduce

the variability in the distance series. This improvement is quantitatively assessed using the standard deviation.

Table 25 shows the standard deviations for the raw distances (D_0) and those corrected using the three tested methods: the traditional station-based approach (N_0), the 3D-RM model, and the reduced version of the model (3D-RM 2). For clarity and readability, the nomenclature for the different correction approaches has been abbreviated compared to previous tables.

Table 25: Standard deviations of distances (mm)

Point	February 2023				July 2023				June 2024			
	D_0	N_0	RM	RM 2	D_0	N_0	RM	RM 2	D_0	N_0	RM	RM 2
1001	0.8	1.2	0.4	0.4	0.8	0.5	0.3	0.3	2.5	0.2	0.2	0.3
1002	0.8	1.2	0.4	0.3	0.8	0.5	0.3	0.3	1.6	0.1	0.2	0.3
1003	0.8	1.2	0.4	0.3	0.9	0.5	0.4	0.4	1.3	0.3	0.1	0.3
1004	0.8	1.2	0.5	0.4	0.9	0.5	0.4	0.4	2.2	0.3	0.2	0.3
1005	0.8	1.1	0.4	0.4	0.9	0.5	0.4	0.4	2.0	0.2	0.2	0.3
1006	0.8	1.1	0.5	0.4	0.9	0.5	0.4	0.4	1.9	0.2	0.2	0.3
1007	0.8	1.2	0.5	0.4	0.9	0.4	0.4	0.4	2.0	0.2	0.2	0.2
1008	0.8	1.0	0.6	0.5	0.8	0.5	0.4	0.4	1.9	0.2	0.2	0.2
1009	0.7	1.0	0.6	0.5	0.8	0.5	0.4	0.4	1.9	0.2	0.2	0.3
1010	0.6	0.8	0.4	0.4	0.7	0.4	0.4	0.4	-	-	-	-
1011	0.6	0.8	0.4	0.4	0.7	0.4	0.3	0.3	-	-	-	-
1012	0.6	0.9	0.4	0.3	0.7	0.4	0.3	0.3	-	-	-	-
1013	0.6	0.8	0.3	0.4	0.7	0.4	0.3	0.3	-	-	-	-
1014	0.6	0.9	0.3	0.4	0.7	0.4	0.3	0.3	-	-	-	-
1015	0.7	1.1	0.5	0.5	1.1	0.3	0.5	0.5	1.9	0.2	0.2	0.2

It is important to note that the standard deviation is a reliable measure when there is a sufficiently large dataset. In cases where the number of data is small, such as in of February 2023 and July 2023 campaigns, the standard deviation may be less reliable, and might not fully capture the true variability in the measurements. However, with a larger dataset, as in the June 2024 campaign, the standard deviation becomes a more meaningful indicator of measurement variability.

As displayed in Table 25, the standard deviation of the raw measured distances rises up to 2.5 mm, with an average of around 2 mm in June 2024. These values are reduced when applying the traditional approach of using meteorological data at the station, yielding values around 0.2 mm. Similar results are obtained when the 3D-RM is applied, with no significant differences. The reduced model also shows no substantial difference in terms of standard deviation.

In the February 2023 campaign, however, the corrected distances exhibit higher variability than the raw ones, deviating from the general pattern observed in the other campaigns. This can be explained by the fact that the meteorological data at the station were discarded, as they were collected with a non-ventilated shelter, which can introduce systematic biases in meteorological readings (see Section 6.3.5). As an alternative, data from station 8002 were used. Although this station is relatively close, its meteorological records may not have been fully representative of the conditions at the station or along the line of sight, which could explain the increased dispersion in the corrected distances.

Overall, these results indicate that, with the exception of February 2023, all three correction approaches are effective in mitigating the variability in distance measurements caused by refraction. However, it is important to interpret these results with caution, especially for campaigns with fewer data points, where the standard deviation may not fully represent the model's performance.

Results in terms of closeness to reference values.

While the previous analysis focused on the standard deviation to assess the precision of the measurements, evaluating the accuracy (how close the corrected distances are to known reference values) of the distance measurements is equally important. This involves comparing the raw and corrected distances obtained through different approaches with a reference value to determine their closeness.

Reference values were derived from high-precision coordinates of the points, which were obtained with high precision using a Mekometer ME5000 and rigorous adjustment procedures as detailed in Section 6.3.1. Since some of the cliff prisms are installed in critical and potentially unstable areas, certain points may have experienced movement since the last monitoring campaign, potentially affecting their current positions relative to the reference coordinates.

Table 26 presents the distance errors computed as the differences between the measured or corrected values and the reference distance for the different campaigns.

Table 26: Distance error respect theoretical value (mm)

Point	February 2023				July 2023				June 2024			
	D_0	N_0	RM	RM 2	D_0	N_0	RM	RM 2	D_0	N_0	RM	RM 2
1001	-16.7	-4.6	-3.4	-3.4	-26.5	-4.2	-2.1	-2.1	-17.5	-5.5	-3.8	-3.5
1002	-13.8	-1.8	-0.6	-0.6	-25.4	-3.2	-1.2	-1.2	-14.0	-2.0	-0.4	-0.1
1003	-17.9	-5.9	-4.6	-4.7	-27.4	-5.2	-3.1	-3.1	-12.5	-0.3	1.4	1.7
1004	-9.8	2.2	2.8	2.7	-24.7	-2.6	-1.0	-0.9	-15.5	-3.9	-2.5	-2.2
1005	-9.8	2.1	2.7	2.6	-25.5	-3.4	-1.9	-1.9	-15.2	-3.5	-2.2	-1.9
1006	-9.8	2.2	2.7	2.7	-25.1	-2.9	-1.5	-1.4	-15.5	-4.0	-2.7	-2.4
1007	-10.7	0.9	0.9	0.8	-24.3	-2.8	-1.9	-1.8	-11.5	-0.2	0.8	1.1
1008	-10.2	1.1	0.9	0.8	-24.5	-3.5	-2.8	-2.8	-12.9	-1.6	-0.8	-0.5
1009	-8.4	3.1	3.1	3.1	-21.7	-0.5	0.5	0.4	-10.7	0.8	1.9	2.1
1010	-21.6	-12.3	-12.1	-12.2	-32.6	-15.2	-14.5	-14.5	-	-	-	-
1011	-24.0	-14.9	-13.8	-13.8	-37.0	-20.0	-18.6	-18.6	-	-	-	-
1012	-27.5	-18.3	-16.9	-17.0	-38.2	-21.0	-19.4	-19.4	-	-	-	-
1013	-26.6	-17.5	-16.4	-16.4	-38.3	-21.5	-20.0	-20.0	-	-	-	-
1014	-27.6	-18.3	-17.0	-17.0	-38.3	-21.7	-20.0	-20.0	-	-	-	-
1015	-7.9	3.7	3.6	3.6	1.1	-21.7	-0.3	0.6	-9.8	1.4	2.3	2.5

The results indicate that applying correction methods generally reduces the distance error compared to the raw measurements, enhancing the accuracy of the distance.

In the February 2023 campaign, the differences between using only station-based meteorological data (N_0) and applying the 3D-RM model are not significant for most points.

In contrast, the July 2023 campaign shows differences between the N_0 approach and the 3D-RM around 2 mm. Given the precision of instruments like the Leica TM30, such differences are significant and highlight the superior performance of the 3D-RM under these conditions. The improved accuracy can be attributed to better quality and distribution of meteorological sensors during this campaign compared to February 2023, where only six ventilated sensors were available, potentially limiting the effectiveness of the corrections.

An important observation arises from the results associated with points 1010 to 1014, which display significantly higher differences, reaching several centimetres even after the refraction correction. As these prisms are installed in a critical areas and show elevated errors both in the February 2023 and July 2023 campaigns, it is plausible to conclude that these points have experienced movement over time. This potential displacement need further investigations and deformation analysis. For this reason, these points were excluded from the June 2024 campaign.

In the June 2024 campaign, improvements over the traditional approach are more variable. For some points, such as 1001 and 1002, the model correction brings the measurements

closer to the reference. For others, results are slightly less accurate. This can be attributed to the shorter distances measured in this campaign—approximately 450 m—where atmospheric effects are less severe, and the traditional method already provides a reasonably good correction.

Finally, the results of the reduced 3D-RM model (3D-RM 2), which excludes the meteorological station at the total station point, show no substantial differences compared to the full 3D-RM model. This finding suggests that a well-distributed network of meteorological sensors across the study area can adequately support the model’s performance without the necessity of installing sensors at specific locations like the station point. Consequently, this approach can make measurement campaigns more efficient and cost-effective by reducing set-up complexity.

b) Vertical angle results

This section presents the vertical angle results. Unlike distance measurements, vertical angle measurements cannot be directly and fully evaluated in this study due to limitations related to the instrument’s internal processing. Specifically, modern total stations equipped with Automatic Target Recognition (ATR) systems apply internal filtering and adjustment algorithms that are not publicly documented. These routines aim to stabilize angle readings and improve target tracking but can suppress small variations caused by atmospheric refraction. As a result, measured vertical angles may not accurately reflect the true impact of refractive bending.

Due to this limitation, the results presented in this section are not intended as a comprehensive validation of the 3D-RM for angle correction. Instead, they provide an illustrative example of the model’s output, highlighting the expected diurnal evolution of the computed refraction angles. Although direct comparisons with the measured angles are not meaningful in this context, the results help demonstrate that the 3D-RM produces physically consistent corrections based on atmospheric dynamics.

The June 2024 campaign was selected for this purpose, as it offers a complete diurnal cycle and stable measurement conditions. The evolution of both the measured vertical angle and the computed refraction angle for prism 1001 is shown in Figure 48.

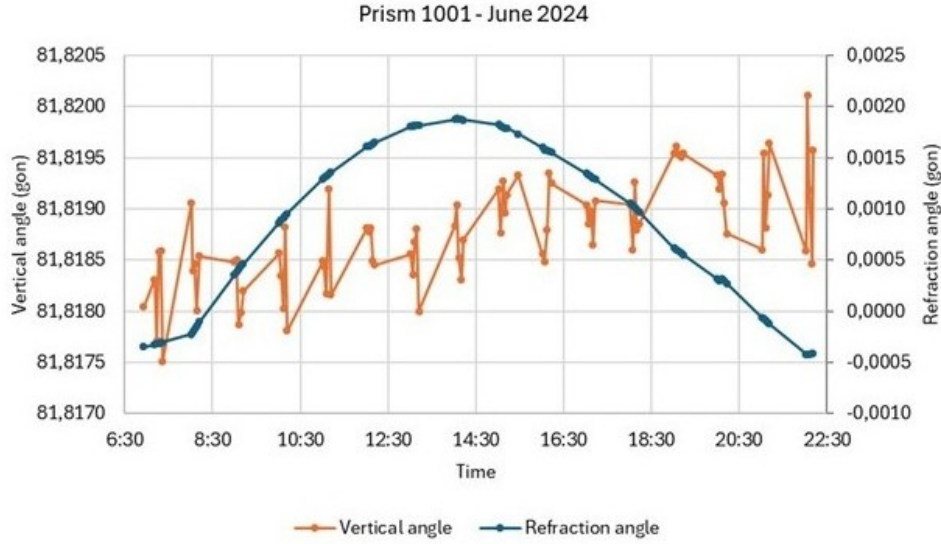


Figure 48: Vertical angle and correction evolution for prism 1001 (June 2024)

The refraction angle values range from approximately -0.5 mgon to 2 mgon. These values are consistent with the typically expected for this kind of environment and measurement configuration, where long-distance lines of sight and daytime heating induce noticeable but moderate vertical bending.

The evolution of the refraction angle follows a plausible diurnal pattern. Low values early in the morning, increasing with daytime heating, peaking around noon, and gradually decreasing again towards sunset.

In contrast, the measured vertical angle series shows limited variability throughout the day and does not exhibit the same temporal trend as the refraction angle. The series remains nearly constant, with short-term fluctuations that do not correspond to the expected atmospheric evolution. This behaviour is likely attributable to the influence of the ATR system, which stabilizes the vertical angle output using internal estimation routines.

As noted in previous studies [19], the ATR system in modern total stations is designed to enhance precision by combining real-time tracking, image-based recognition, and internal adjustment models. While this results in high repeatability under stable conditions, it also limits the sensitivity to gradual atmospheric effects, making it difficult to directly observe the influence of refraction on angle measurements. Therefore, in field applications, vertical angle measurements may not be as suitable as distances for assessing refraction effects unless raw, unprocessed data were accessible.

Nonetheless, the correction values obtained by the 3D-RM are consistent with physical expectations and follow the atmospheric dynamics throughout the day. This supports the model's capability to estimate vertical bending based on the distribution of refractivity

gradients. However, further validation of the model is still needed.

6.3.5 Assessment and influence of selected parameters in 3D-RM

As previously stated, the 3D-RM involves several parameters and inputs that can be modified. Section 6.3.4 presented the main results obtained using a specific set of parameters. However, a deeper examination of the intermediate processing steps is necessary to understand how specific choices impact the accuracy and stability of the corrections. This section analyses the effect of several parameters and modelling decisions, offering empirical justification for the configurations ultimately adopted in the study.

Interpolation of refractivity

A crucial component of the 3D-RM is the spatial interpolation of refractivity at different vertical layers. As discussed in Section 6.2.2, this interpolation is performed at each designated vertical layer using MLR. This section briefly introduces the motivation for choosing MLR as the interpolation method and presents the results of this interpolation across different campaigns. All interpolation methods tested have been implemented in our home-made software.

A initial study, presented in [67], compared various interpolation methods. This study used meteorological data from nine data loggers installed in Cortes de Pallás (CP) on November 26th, 2019, as an initial attempt to investigate refraction in the area. Refractivity was calculated based on the meteorological data from each station, to interpolate these refractivity values to specific coordinates. After evaluating and discarding certain interpolation methods such as Kriging or co-Kriging, two interpolation methods were selected for further analysis: inverse distance weighting (IDW) using both 2D and 3D distances, and multiple linear regression (MLR). The effectiveness of these methods was assessed using the root-mean-square error (RMSE) through cross-validation.

The study concluded that MLR was the optimal method for refractivity interpolation in the area, as it consistently yielded lower RMSE values compared to IDW. During the observation period, the RMSE for MLR was below 1 at most epochs, and the R^2 values were above 0.85 for most epochs.

Subsequent campaigns in the same area further validated the use of MLR for refractivity interpolation. Initially, the interpolation was performed using the computed refractivity at each station, assuming all sensors were installed at approximately the same height above ground, as it was the case in the study referenced in [67].

The RMSE results for the different campaigns are summarized as follows:

- CP February 2023: RMSE values were below 1.2 for all observation epochs, with approximately 80% of epochs presenting RMSE values under 0.8.

- CP July 2023: RMSE values were below 1 for the entire observation period.
- CP June 2024: RMSE values were below 1.6 for all observation epochs, with approximately 80% of epochs presenting RMSE values under 1.

In terms of the coefficient of determination R^2 :

- CP February 2023: R^2 values were above 0.9 for all observation epochs.
- CP July 2023: R^2 values were higher than 0.85, exceeding 0.95 in approximately 75% of the epochs.
- CP June 2024: R^2 values were higher than 0.75 and higher than 0.9 in approximately 85% of the epochs.

The spatial interpolation is conducted at each designated height above the ground or vertical layer. However, for simplicity, only the results at the reference height (h_0) are presented here. The results at other height levels are similar and consistent with those presented, further supporting the use of MLR as the optimal interpolation method for the refractivity required in the 3D refractivity model.

Use of non-ventilated shelters

During the February 2023 campaign, six meteorological stations were equipped with ventilated shelters, while two stations (8004 and 9000) used non-ventilated ones. The evolution of refractivity across all meteorological stations is shown in Figure 49. Stations 8004, in red, and 9000, in black, require further analysis as they were the points with non-ventilated sensors.

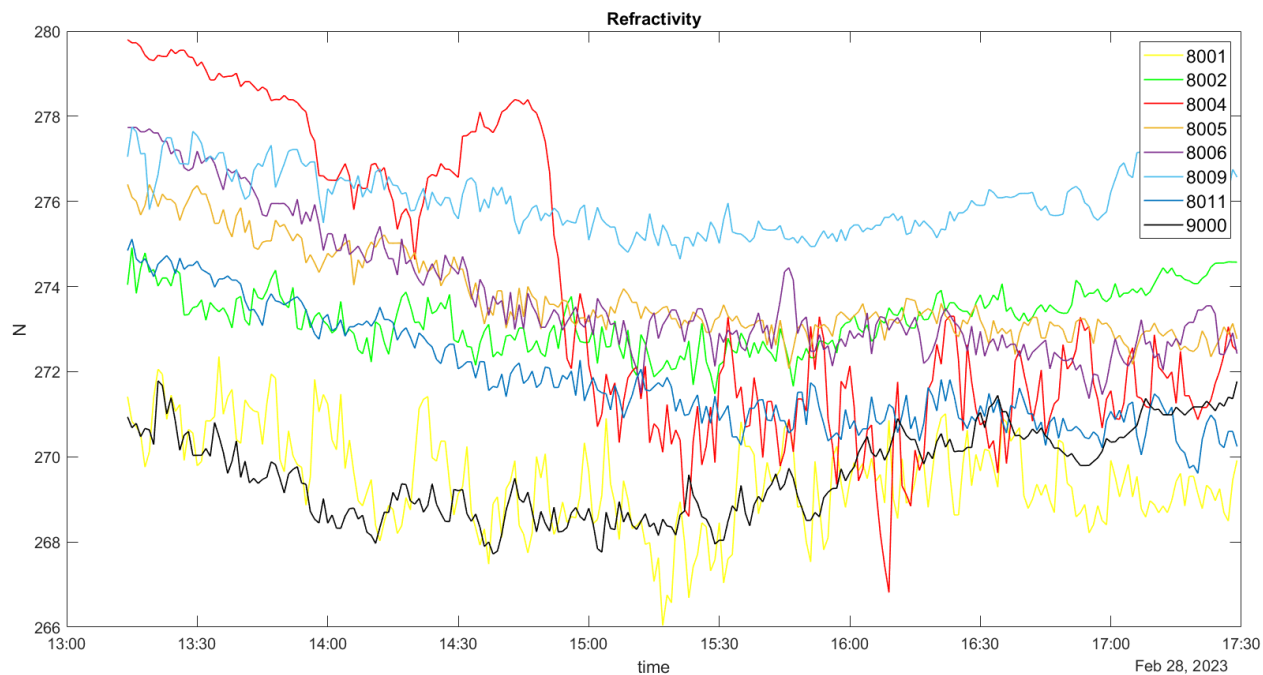


Figure 49: Refractivity evolution for all stations (February 2023)

Figure 50 compares these problematic stations with their nearest ventilated stations: station 8004 is compared to station 8008 and 9000 is plotted together with station 8002.

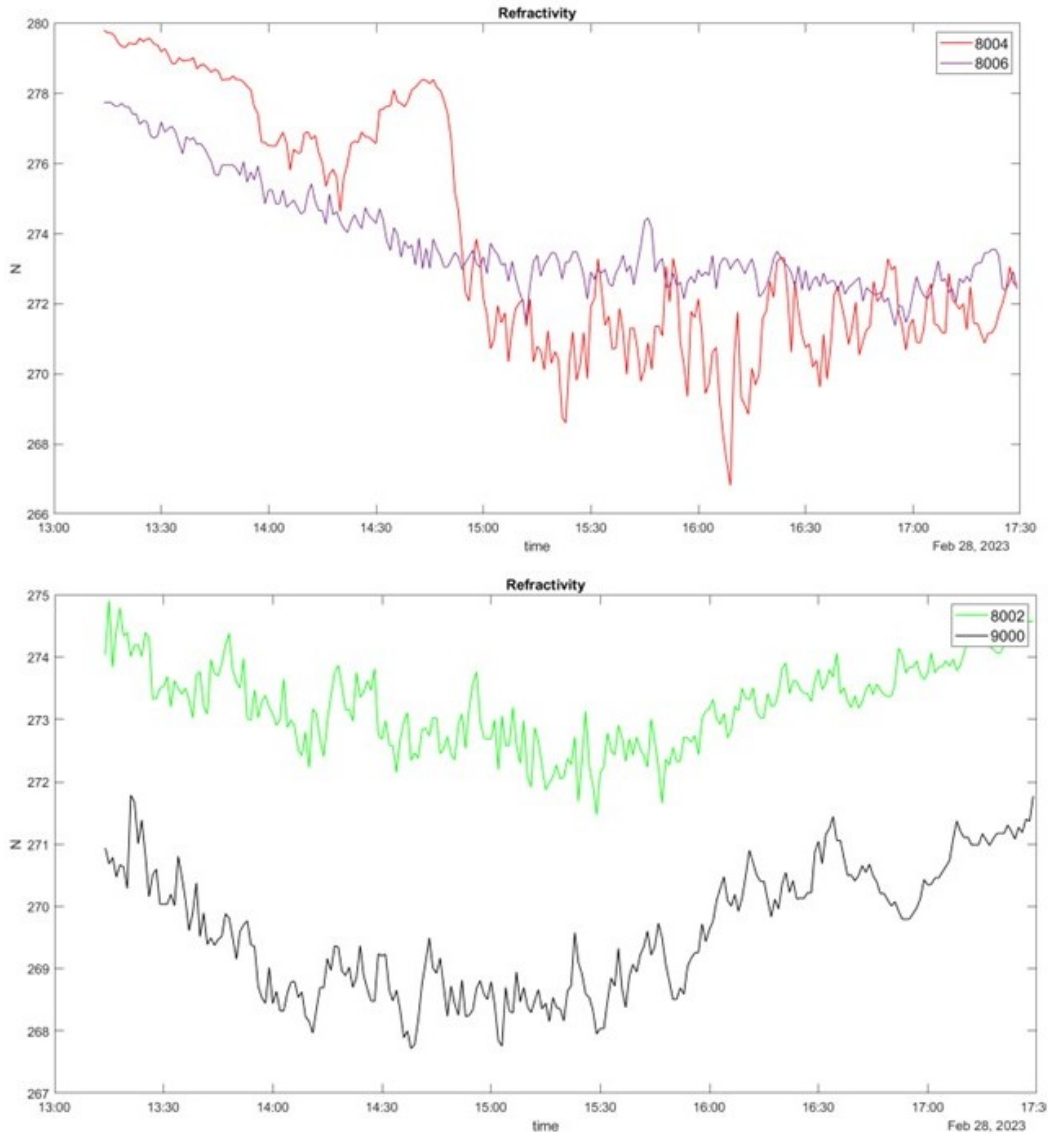


Figure 50: Refractivity evolution for stations 8008 and 8004 (up) and 8002 and 9000 (down) (February 2023)

At station 8004 (in red), a sharp change around 14:30 followed by strong oscillations occurred when the parasol shielding the sensor was displaced by wind. This event likely accounts for the abrupt change and the subsequent high variability in the data, as the sensor was directly exposed to the sun.

At station 9000, the issue is less apparent, as no obvious anomalies are present. However, when compared with station 8002, which is located approximately 100 meters away under similar conditions, a systematic offset becomes evident. The temperature at station 9000, being the parameter with the most significant impact on computed refractivity, is consistently 4-5 C higher than at station 8002, likely due to insufficient ventilation.

Including data from these two stations in the refractivity interpolation results in a mean RMSE of approximately 2 and an R^2 value of 0.62. These values are notably worse than the results obtained when excluding these two stations (1.2 and 0.8, respectively). Consequently, these points were excluded from the final analysis for this campaign, and only data from the six ventilated sensors were considered.

This analysis emphasizes the importance of sensor standardization and proper ventilation of meteorological sensors in field deployments.

Assesment of ERA5 and ERA5 land products

The 3D-RM incorporates sensible heat flux (H) as an input to apply the Turbulence Transfer Model (TTM). As introduced in Section 6.1.2, various remote sensing datasets offer estimations of sensible heat flux. Among them, ERA5 and ERA5 land, both developed by the European Centre for Medium-Range Weather Forecasts (ECMWF), stand out due to their relatively high spatial and temporal resolutions and public accessibility. ERA5 offers a horizontal spatial resolution of approximately 31 km, which is coarser than some specialized applications might require, but sufficient for estimating large-scale atmospheric gradients, as targeted in this model.

According to existing literature, ERA5 tends to provide more accurate estimations of surface fluxes, including sensible heat flux, when compared to ERA5 land. However, to complement this literature-based assessment and to evaluate the practical performance of both datasets under specific conditions, an experimental comparison was conducted.

A validation campaign was conducted in Interlaken, Switzerland, on September 23–24, 2023. A BLS900 scintillometer from Scintec (Tübingen, Germany) was deployed to measure the sensible heat flux over a 600-meter stretch of asphalt. The total duration of measurements spanned approximately 28 hours, capturing both diurnal and nocturnal flux dynamics.

Figure 51 presents the raw time series of sensible heat flux as recorded by the scintillometer, alongside values extracted from ERA5 and ERA5 land.

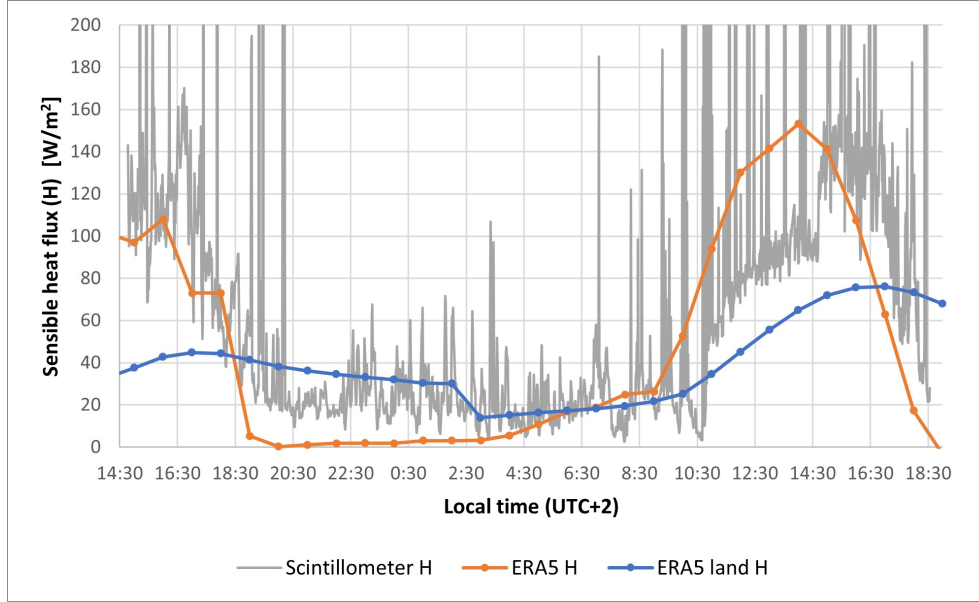


Figure 51: Heat flux evolution 23-24 September 2023, Interlaken (Switzerland)

Due to a high level of noise in the measured data, a preprocessing step was applied to improve visualization and interpretability. First, a threshold of 300 was applied, followed by a moving average filter. The resulting smoothed dataset is shown in Figure 52.

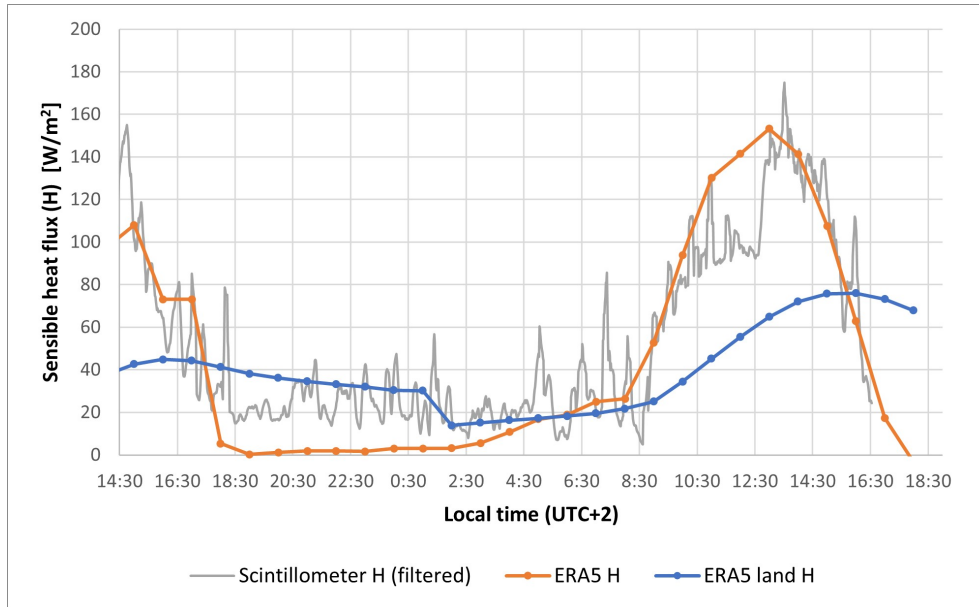


Figure 52: Heat flux evolution 23-24 September 2023, Interlaken (Switzerland) - filtered

As seen in the filtered results (Figure 52) ERA5 generally tracks the measured values more accurately during daylight hours, while during the night, it tends to slightly underestimate the flux. In contrast, ERA5-Land produces higher night-time fluxes than both ERA5 and

the measurements. These findings are consistent with previous literature suggesting that ERA5 provides a more balanced representation of surface energy dynamics under diverse atmospheric conditions.

Overall, the ERA5 sensible heat flux demonstrates good agreement with the measured values, confirming its potential as a reliable approximation for the sensible heat flux required to apply the Turbulence Transfer Model (TTM).

It is important to note that this validation experiment was conducted over an asphalt surface, whereas the main field campaigns in this thesis involve measurements over open water. Given the distinct thermal properties and heat storage capacities of these surfaces, sensible heat flux behaviour may differ considerably. Nonetheless, the experiment serves as a valuable indicator of the overall consistency of ERA5 flux estimates.

While the results are specific to the test conditions and cannot be broadly generalized, they provide empirical support for the use of ERA5 in this thesis. The combination of strong agreement with field measurements, established accuracy in the literature, operational ease of access, and adequate spatial resolution for this application justifies the selection of ERA5 as the primary source of sensible heat flux values for the implementation of the Turbulence Transfer Model (TTM) in this work.

Temporal interpolation of sensible heat flux

Since the sensible heat flux data from ERA5 and ERA5 land products have a temporal resolution of 1 hour, interpolation is necessary when using this data in the 3D refractivity model. Two different interpolation strategies have been implemented. The first strategy is a simple linear interpolation of H , where the value of H at the desired time is calculated from the values at the preceding and following whole hours. The second strategy involves fitting a 5-degree polynomial to each 12-hour period (0-12 and 12-24 hours).

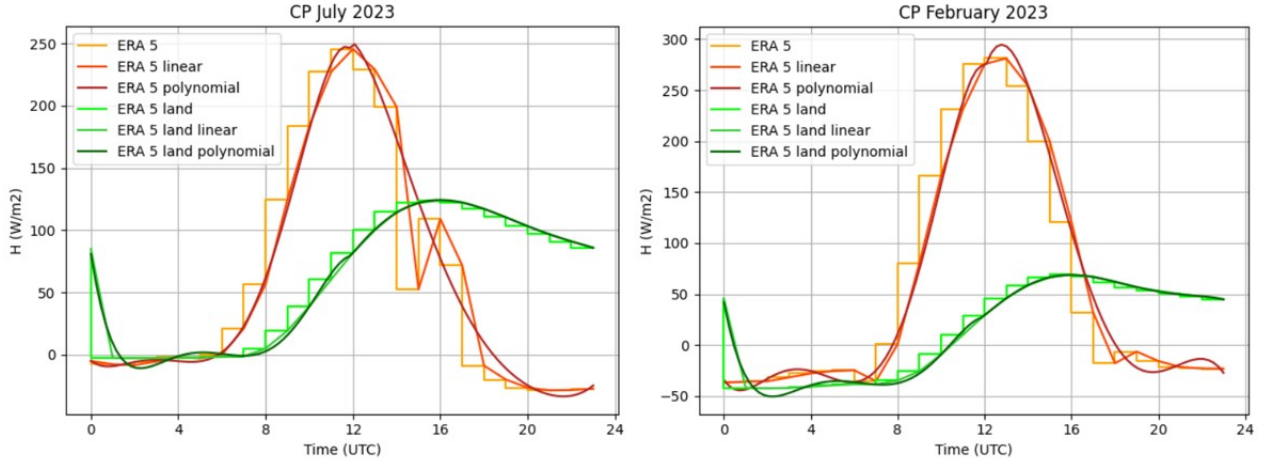


Figure 53: Heat flux with different interpolations for CP July 2023 (left) and CP February 2023 (right)

Figure 53 shows the evolution of sensible heat flux from both ERA5 and ERA5 land, including the original data and the interpolated values using both linear and 5-degree polynomial interpolation for 24 hours for two different days corresponding to two campaigns in Cortes de Pallás. It can be appreciated that both linear and polynomial interpolations are similar, with small variations at specific times.

The results in terms of the coefficient of determination (R^2) for different campaigns are shown in Table 27. These campaigns include Cortes de Pallás February and July 2023 (see Section 6.3.2) and the experiment conducted in Interlaken (Switzerland) (previously introduced in this section).

Table 27: R^2 5-degree interpolation of H

Campaign	Date	R^2 ERA5	R^2 ERA5 land
CP July 2023	20/07/2023 0:00 - 12:00	0.998	0.997
	20/07/2023 12:00 - 24:00	0.934	0.999
CP February 2023	28/02/2023 0:00 - 12:00	0.995	0.974
	28/02/2023 12:00 - 24:00	0.991	0.998
Interlaken 2023	23/09/2023 0:00 - 12:00	0.996	0.999
	23/09/2023 12:00 - 24:00	0.953	0.994
	24/09/2023 0:00 - 12:00	0.992	0.996
	24/09/2023 12:00 - 24:00	1	0.999

6.3.6 Conclusions

The analysis and experiments presented in this chapter demonstrate that the 3D-RM is a reliable and effective tool for correcting atmospheric refraction in terrestrial geodetic measurements. Across multiple field campaigns, the model consistently improved the quality of distance measurements, reducing the standard deviation of raw distances and improving their agreement with reference values derived from high-precision geodetic instruments.

Although a full validation of vertical angle corrections was not possible due to limitations associated with the ATR system used in the available total station, the modeled refraction angles followed an expected diurnal trend, consistent with atmospheric behaviour. These results suggest that the 3D-RM can also be used to estimate angle corrections, although further studies are required to fully confirm this application.

A significant advantage of the proposed model lies in the comparison between the full version 3D-RM and the reduced configuration 3D-RM 2, in which no meteorological sensor is installed at the total station location. The results obtained with 3D-RM 2 were almost identical to those from the complete version. This confirms that a well-distributed network of meteorological sensors can sufficiently characterize the atmospheric conditions of the entire area without the need for placing sensors at every measurement station. This flexibility enables more efficient setups for long-term monitoring or field campaigns where total stations are frequently moved within the area of interest.

Compared to the traditional approach, which uses only the meteorological parameters at the station point, the 3D-RM provides improved results, particularly for longer distances. For baselines exceeding several hundred meters, the differences between the traditional and 3D-RM corrections reached a few millimetres. In high-precision applications, such discrepancies can be critical and directly affect the accuracy and traceability of the results. In contrast, for short distances (e.g., 400–500 meters), the traditional method yields results closer to those of the 3D-RM, as atmospheric conditions along the path are more homogeneous.

Several aspects of the model have been individually evaluated and confirmed through dedicated analyses. The use of Multiple Linear Regression (MLR) for interpolating refractivity was validated with low RMSE values and strong correlation coefficients across all campaigns. The importance of using ventilated meteorological shelters was also demonstrated, as non-ventilated sensors produced biased temperature readings that significantly degraded the quality of the interpolation and overall model performance.

Furthermore, the selection of ERA5 as the input dataset for sensible heat flux estimation proved adequate. Despite its relatively coarse spatial resolution (31 km), ERA5 provided values that aligned well with those measured by a scintillometer during a dedicated experiment in Interlaken, Switzerland. The implementation of polynomial or linear temporal

interpolation of the heat flux was also found to be essential to ensure continuity and smoothness in the calculated corrections.

In summary, the 3D-RM is shown to be a cost-effective, modular, and adaptable model that can significantly enhance the precision of terrestrial distance measurements. Its ability to account for spatial and vertical variations in refractivity represents a clear improvement over traditional correction methods and offers a valuable contribution to high-precision geodesy, particularly in environments where conventional approaches are insufficient.

6.4 Proof of concept: application of 3D-RM to TLS

6.4.1 Motivation and context

Atmospheric refraction, as explained throughout this thesis, affects all electromagnetic sources. In this chapter, the refractivity correction model 3D-RM has been presented, developed for distance and vertical angle electro-optical measurements. In Section 6.3, the 3D-RM was tested over TS measurements, demonstrating its potential to effectively correct for refraction. However, TS measurements are not the only ones affected by this phenomenon, other electro-optical systems, such as Terrestrial Laser Scanners (TLS), are also influenced by atmospheric refraction.

In TLS applications, the impact of atmospheric refraction becomes particularly critical in long-range observations or deformation monitoring scenarios. Under such conditions, uncorrected refractive effects can introduce systematic errors in both distance and angular measurements, leading to apparent displacements that may reach several centimetres or even decimetres [51]. These distortions are primarily caused by bending of the laser beam, which results in the scanner registering surface points that are not located along the actual line of sight. This effect intensifies as the distance increases and as the beam crosses regions with strong thermal gradients, typically near the ground on sunny days.

Unlike TS measurements, which involve a defined and fixed target (e.g., a prism), TLS acquires dense point clouds where the exact surface point impacted by the laser pulse is unknown and continuously varies. This makes the application of classical correction models difficult, as they generally assume known, discrete target points. Furthermore, high-precision TLS measurements are influenced by additional factors that introduce errors into the results, such as the targets used for registration or the stability of the setup.

Despite these challenges, the inclusion of refractive effects in TLS error modelling is increasingly acknowledged as essential, particularly for high-precision or long-range applications. Kerekes [71] developed an error model for TLS in which atmospheric influences are represented stochastically or spatially, taking into account local topography and stratified atmospheric layers. Friedli [51], after demonstrating the influence of refraction on TLS

point clouds, recommended acquiring TLS data near sunset, when refractive effects are reduced.

Given these considerations, this work explores the feasibility of extending the 3D Refractivity Model (3D-RM), originally developed for total station measurements, to TLS data. The model’s formulation is easily adaptable to TLS, provided that the point cloud can be expressed as a set of distance and angle measurements. It must be emphasised, however, that a complete validation of its application to TLS requires further research, as the treatment of other error sources and TLS-specific processing considerations lies beyond the scope of this thesis.

To investigate this possibility, a tailored methodology for applying the 3D-RM to TLS point clouds is proposed in Section 6.4.2, followed by a preliminary assessment of its performance in Section 6.4.3. These results are not intended as a definitive solution to the complex problem of atmospheric correction in TLS, but to serve as a proof of concept to establish the basis for future developments in this direction.

6.4.2 Processing methodology

In order to apply the 3D-RM refractivity correction model to terrestrial laser scanner (TLS) data, a dedicated processing workflow was developed and implemented in custom software as a proof of concept. The main requirement of this workflow is to maintain strict geometrical control over all applied transformations, ensuring that the instrument’s position remains fixed at its known coordinates. This constraint is essential, as many commercial TLS processing packages adjust the instrument’s position during georeferencing, either to minimise residuals or to improve the overall registration, which can lead to discrepancies between the nominal and actual instrument location. The proposed methodology includes the following steps:

1. **Time stamp to each point.** Assuming the laser scanner stores the measured points sequentially during the scanning process, the acquisition time of each point can be estimated based on the known scan start and end times. A time stamp is interpolated and assigned to each point of the original point cloud. This temporal information is essential for applying the specific model parameters corresponding to the acquisition time.
2. **Point cloud cleaning.** A cleaned version of the point cloud is produced using any point cloud processing software to remove noise and isolate the area of interest. However, time stamps must be preserved from the original unfiltered cloud. Therefore, the cleaned cloud is matched back to the original version via point-by-point comparison (e.g., using coordinates), and only the corresponding points with valid timestamps are retained. This two-step process is required because applying timestamps directly to the cleaned cloud would lead to incorrect time associations due to reordering or data loss during filtering.

3. **Georeferencing via 6-parameter transformation.** The cleaned point cloud, expressed in the local scanner coordinate system, is georeferenced to the working coordinate system (in this case the so-called CP2017). To do this, the centres of the reference spheres using during scanning are extracted (again using standard commercial software), and a six-parameter Helmet transformation (three rotations and three translations, with no scale) is computed and applied using in-house software.

Many commercial georeferencing tools are available to do this. However, these tools usually estimate scanner position as a free parameter. In our case, the scanner's position is assumed to be fixed and known. This preserves the geometric consistency of the transformation, which is essential to study distortions introduced by atmospheric refraction.

After the transformation, the local Cartesian coordinates of each are converted to polar coordinates (zenith angle, azimuth, and distance), which are required for the application of the 3D-RM.

4. **Application of the 3D-RM.** Once the point cloud is expressed in polar coordinates and each point has an associated time stamp, the 3D-RM is applied in the same way as for total station data. The model estimates the refractivity corrections in distance and vertical angle for each point individually, taking into account its acquisition time and line-of-sight geometry, and using the pre-computed model parameter per epoch and height.
5. **Back-transformation to Cartesian coordinates.** The corrected polar coordinates, simulating measured angles and distances, are transformed back into Cartesian coordinates (both in the working frame and in the local scanner system). This enables the visualization of the corrected point cloud in any standard 3D point cloud software and allows the comparison with the original data.
6. **Iterative georeferencing refinement.** Since atmospheric refraction also affects the observed position of the reference spheres, the georeferencing transformation parameters may absorb part of the refractivity. To mitigate this, an iterative refinement is proposed: after the first correction, the centres of the reference spheres are re-extracted from the corrected cloud, and a new transformation is computed and applied. This process helps ensure that the final georeferencing does not compensate for atmospheric effects, thereby preserving the integrity of the correction applied via the 3D-RM model.

Figure 54 summarizes the workflow developed to apply the 3D-RM to laser scanner data.

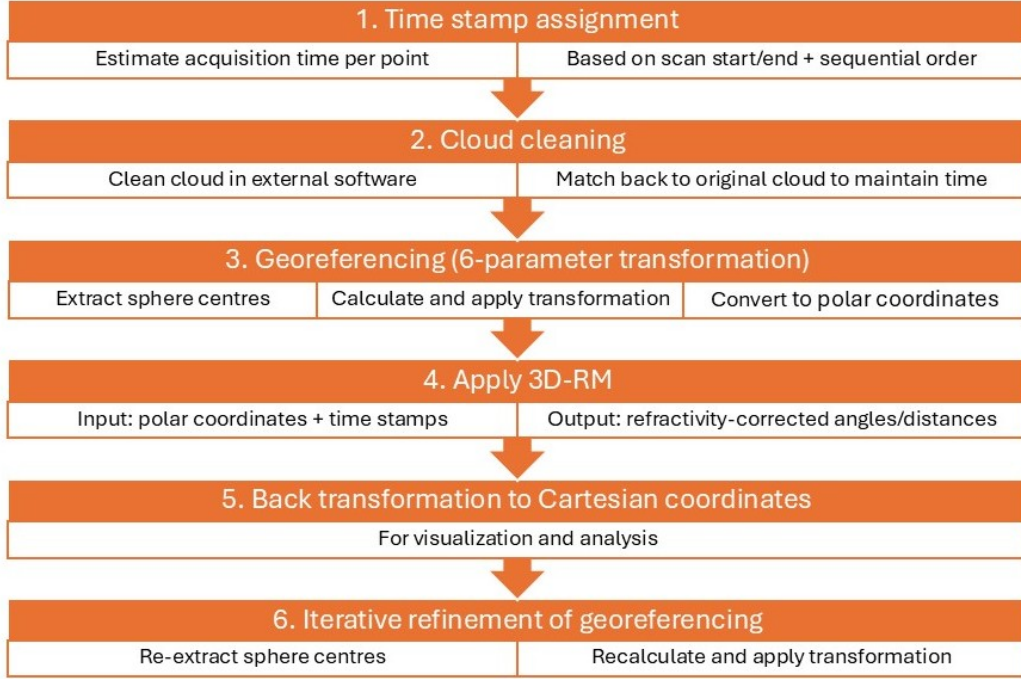


Figure 54: Workflow to apply 3D-RM to TLS data

6.4.3 Preliminary results

This section shows the preliminary results of applying the 3D-RM refractivity correction model to TLS point clouds. The working area was again Cortes de Pallás, and the measurement campaign took place on 23 July 2023, simultaneously with the TS observations.

A Leica ScanStation P50 long-range TLS ($3\text{mm} + 10\text{ppm}$ in 570 m / > 1 km mode, 8") was installed on pillar 8002. For georeferencing, 5 half-metre spheres were placed on pillars 8003, 8004, 8006, 8005, and 8011. In addition, meteorological sensors were deployed throughout the working area to provide input data for the 3D-RM.

To minimise registration-related errors, the entire scene was captured in a single scan. Two different point clouds were acquired: the first at 7:00 am and the second at 13:00 pm, ensuring that measurements were taken under different atmospheric conditions.

Both point clouds were processed following the methodology described in Section 6.4.2, resulting in two new datasets corresponding to the refraction-corrected versions of the originals.

The computed refraction effect was evaluated by comparing the original and corrected point clouds using a cloud-to-cloud distance analysis in *CloudCompare (v.2.13 alpha)*

(<https://www.cloudcompare.org/>). Figure 55 shows the results for the 07:00 am dataset, and Figure 56 for the 13:00 pm dataset.

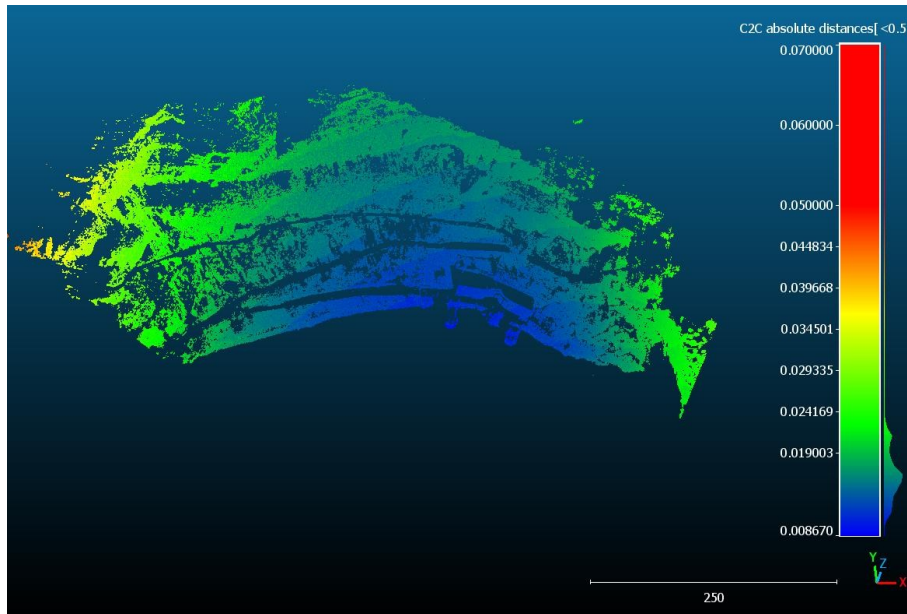


Figure 55: Comparison between original and refraction corrected point clouds at 7:00 am

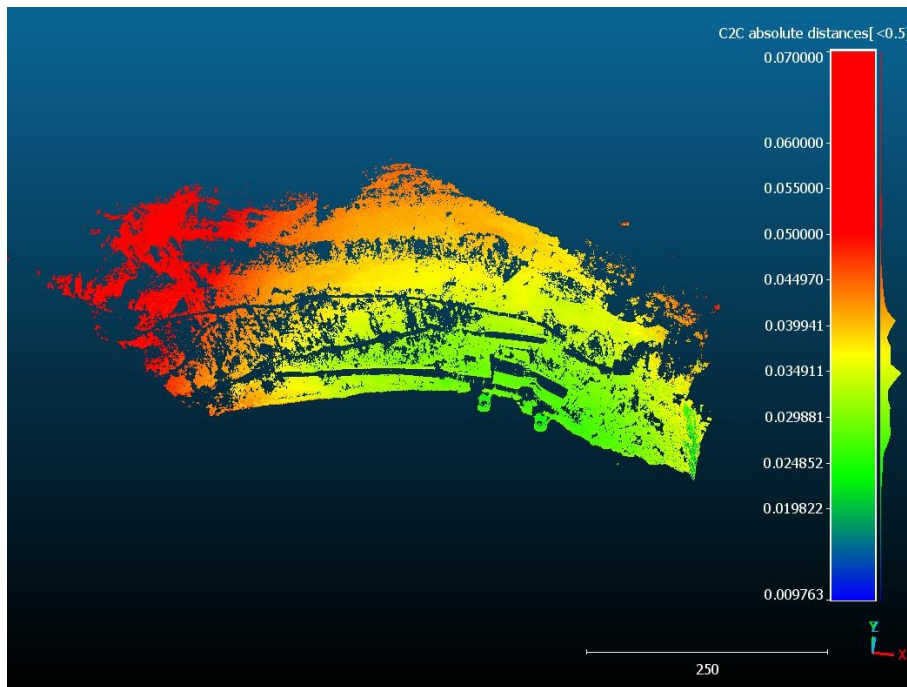


Figure 56: Comparison between original and refraction corrected point clouds at 13:00 pm

As shown in Figure 55, the differences between the original and refraction-corrected point clouds in the early morning scan were below 2.4 cm. In contrast, for the midday scan (Figure 56), differences ranged from 2 cm to 5 cm. These magnitudes are significant for high-precision applications, especially in deformation monitoring, where comparing point clouds acquired under different atmospheric conditions could lead to apparent displacements and potentially incorrect conclusions.

Although these results are preliminary, they clearly demonstrate that atmospheric refraction can have a measurable and non-negligible impact on TLS point clouds. This supports the underlying motivation of this proof of concept: to explore the feasibility of adapting the 3D-RM to TLS workflows as a step toward systematic, physically based refraction correction in laser scanning applications.

6.4.4 Conclusions

This proof of concept has introduced a dedicated processing methodology for applying the 3D-RM refractivity correction model to TLS point clouds. The approach ensures geometrical control of transformations, maintaining the instrument position fixed in its known coordinates, and allows TLS data to be treated as a set of distance and angle measurements suitable for physical refraction modelling.

The preliminary results presented in Section 6.4.3 confirm that atmospheric refraction can induce systematic shifts of several centimetres in TLS point clouds. While these magnitudes are significant for high-precision long-range applications, further work is needed before a complete validation can be achieved.

During the campaign, several limitations were identified that may have affected the accuracy of the results. In particular, the precision of the automatic detection of the reference spheres was not optimal, as some spheres were not perfectly sharp in the scans. Additionally, their spatial distribution was not ideal, potentially reducing the robustness of the georeferencing. For these reasons, no direct comparison between the two uncorrected point clouds (07:00 am vs. 13:00 pm) was performed, as they contain other error sources in addition to atmospheric refraction.

Future work should address these issues by improving reference target placement, ensuring optimal visibility in all scans, and refining detection accuracy. Once these aspects are resolved, a more rigorous validation of the 3D-RM for TLS can be undertaken.

7. Conclusions and outlook

This thesis explores how atmospheric refraction affects the accuracy of high-precision geodetic measurements. Despite current improvements in geodetic instruments and data processing techniques, atmospheric refraction is still a key source of error, particularly in applications that require millimetre-level accuracy over relative long distances in outdoor critical applications such as length metrology or deformation monitoring.

The study focused on two complementary lines of research. The first part examined tropospheric delays in GNSS-based distance estimation, specifically analysing the behaviour of residual errors at the double-difference (DD) level. The goal was to understand under which conditions, such as long baselines or large height differences, these residual effects become non-negligible, potentially limiting the precision of techniques like the Ground-Based Distance Meter (GBDM+) developed at the DICGF-UPV.

The second part of the thesis introduced a new 3D refractivity model (3D-RM) aimed at improving upon the traditional correction method used in terrestrial geodetic measurements. By combining a network of meteorological sensors with vertical atmospheric profiling and remote sensing data, the 3D-RM was designed to more accurately represent atmospheric conditions along the entire line of sight. The model was developed as a practical, flexible, and cost-effective alternative that could deliver better correction performance, particularly in complex terrain where standard approaches may be insufficient.

Together, these efforts contribute to a better understanding and treatment of atmospheric refraction in both satellite-based and terrestrial measurement systems, bringing actual measurement performance closer to the high precision that instruments are designed to deliver.

7.1 Conclusions

This thesis has examined atmospheric refraction in high-precision geodesy from two complementary perspectives: GNSS-based distance estimation and terrestrial optical techniques. The work combines theoretical analyses, methodological developments, and experimental validation campaigns, with the common goal of improving the traceability and accuracy of geodetic measurements.

At the GNSS level, the study analysed tropospheric effects at both zero-difference (ZD) and double-difference (DD) stages. At the zero-difference (ZD) level, significant discrepancies were found between tropospheric delays estimated through Precise Point Positioning (PPP) and those obtained from empirical models, particularly in the zenith wet delay (ZWD), which is highly sensitive to local humidity and temperature conditions. These differences can reach several centimetres. Once DD processing was applied, discrepancies

were strongly reduced.

Additionally, the study compared several mapping functions (VMF1, Niell, and Herring) and found that their impact on DD-level results was minimal. Differences among them remained below 1 mm, even at low satellite elevation angles, suggesting that the choice of mapping function becomes less critical once DD is formed.

However, the analysis demonstrated that DD residuals persist under certain geometries: for baselines exceeding 3–4 km or with vertical separations greater than 100 m, residual delays up to ± 0.3 m were consistently observed. These findings reinforce the importance of accounting for the troposphere, even when using DD processing. For applications like GBDM+, where distance estimates must meet strict traceability requirements, tropospheric effects must be explicitly modelled and corrected. Both empirical models and PPP-based estimates provide viable strategies. However, PPP is particularly well suited in the GBDM+ context, as it not only estimates tropospheric delays but also quantifies their uncertainty, allowing these values to be directly integrated into rigorous uncertainty budgets, as required in metrology.

The thesis has introduced and validated a novel three-dimensional refractivity model (3D-RM) that integrates distributed meteorological measurements, ERA5 remote sensing data, vertical profiling with the Turbulence Transfer Model (TTM), and terrain information.

Across three independent field campaigns, 3D-RM consistently improved distance measurement quality: standard deviations were reduced from the millimetre level to a few tenths of a millimetre, and agreement with reference distances improved by up to 2 mm on long sight lines. A key finding was the excellent performance of the simplified configuration (3D-RM2), which excludes the need for a sensor at the total station location. This demonstrates that a well-distributed meteorological network can capture the main atmospheric variability, making the model scalable and practical for diverse real-world scenarios.

Although vertical angle correction could not be fully evaluated due to limitations with the ATR system of the total station, the modelled refraction angles followed a consistent and expected diurnal pattern. This indicates that the model holds potential for angle correction, pending further validation with raw, unfiltered angular measurements.

The thesis has also presented a proof of concept for applying the 3D-RM to Terrestrial Laser Scanner (TLS) point clouds. Preliminary results confirmed that atmospheric refraction can induce systematic shifts of several centimetres in long-range TLS data, which is critical for high-precision applications. While limitations in target detection and georeferencing prevented a full validation, the methodology developed provides a clear path for integrating physical refraction modelling into TLS workflows.

In summary, this thesis provides two main original contributions to the study and mit-

igation of atmospheric refraction in high-precision geodesy. First, it offers a systematic evaluation of residual tropospheric delays at the double-difference level, with direct implications for the traceability and uncertainty modelling in GNSS-based distance estimation, particularly within the GBDM+ framework. Second, it the development and validation of the 3D-RM, which demonstrating its practical benefits over traditional correction methods and confirming its performance in real-world conditions.

More broadly, the results confirm that atmospheric refraction remains a critical source of error in high-precision geodesy. However, through careful modelling, data integration, and experimental validation, its impact can be substantially mitigated, bringing practical field performance closer to the nominal specifications of modern geodetic instruments.

7.2 Outlook

While this thesis has addressed key challenges in atmospheric modelling for geodetic measurements, several aspects remain open for further research and development.

In the GNSS domain, future work should include a systematic evaluation of how different tropospheric models influence final distance estimations, not only at the DD level but also in the context of rigorous uncertainty propagation. Additional experiments under diverse meteorological conditions would help quantify residual DD delays more accurately. Special attention should be given to baselines with strong height differences and with a distance of more than 3 or 4 km. Additional analysis of long-term datasets could also shed light on seasonal variability and its implications.

For the 3D-RM, several aspects remain to be explored. A priority is the full validation of angle corrections, either by operating total stations without ATR filtering or by employing datasets that include raw angular measurements. Testing the model over longer baselines, more complex mountainous terrains, and heterogeneous land covers would provide further insights into its generalizability.

Incorporating surface-dependent parametrizations, that is distinguishing between water, asphalt, vegetation, or urban structures, into the Turbulence Transfer Model could improve the representation of refractivity gradients in mixed environments. Similarly, while ERA5 proved suitable for sensible heat flux estimation, targeted experiments over water bodies and mixed surfaces would help confirm whether its spatial resolution is sufficient in all contexts. A systematic comparison with other remote sensing products or high-resolution in-situ data would also clarify the sensitivity of the model to input source characteristics.

The proof of concept developed in this thesis demonstrated the potential of 3D-RM to mitigate centimetric refraction-induced biases in long-range TLS point clouds. However, further validation is required before operational use can be recommended. Future

campaigns should focus on improving the distribution and quality of reference targets, ensuring optimal visibility and accurate detection. Once these limitations are addressed, systematic comparisons between corrected and uncorrected point clouds will allow a more rigorous quantification of the model's benefits.

Finally, this thesis highlights the benefits of integrating data and methodologies across GNSS and terrestrial domains. Future work could explore hybrid strategies that combine PPP-based tropospheric estimates with 3D-RM spatial modelling, potentially enabling cross-validation and improved robustness in multi-instrument monitoring networks.

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A Mathematical formulation of tropospheric models and mapping functions

A.1 Tropospheric models

This appendix presents the formulation of the selected tropospheric models. For detailed derivations, please refer to the original sources (as cited in the text). The equations have been harmonised to use consistent symbols and units:

- Zenith delay (ZTD) and slant delay (ΔL) are in meters [m]
- Temperature (T) is in Kelvin [K]
- Water vapour pressure (e) and pressure (P) are in hectopascals [hPa]
- Height (h) is in meters [m]

The subscript $_0$ denotes surface values; $_h$ and $_w$ refer to hydrostatic and wet components, respectively.

Saastamoinen models

Saastamoinen developed a total tropospheric delay model based on gas laws and the following assumptions [31]:

- The atmosphere is considered a mixture of two ideal gases: dry air and water vapour, in hydrostatic equilibrium
- Temperature decreases linearly with height; a constant vertical temperature gradient is assumed
- Refractivity constants are taken from Essen and Froome

Local gravity can be determined using the expression:

$$g_m = 9.784 \cdot (1 - 0.0026 \cdot \cos(2\varphi) - 2.8 \cdot 10^{-7} \cdot h) \quad (102)$$

where φ is the user's latitude.

For average conditions in the middle altitudes, the vapour pressure is determined from surface value as:

$$e = e_0 \cdot \left(\frac{T}{T_0} \right)^{\frac{v \cdot g}{R_d \cdot dT}} \quad (103)$$

where:

- v is a numerical coefficient derived from local observation.
- R_d is the molar constant for dry air.
- dT is the temperature lapse rate (rate in which temperature decreases with height).

Finally, the expression for the Saastamoinen's total delay model is:

$$\Delta L = 0.002277 \cdot \sec \beta \left[P + \left(\frac{1255}{T} + 0.05 \right) \cdot e - 1.16 \cdot \tan^2 \beta \right] \quad (104)$$

where:

- β is the zenith angle
- The 1.16 factor is height-dependent. Saastamoinen [31] provides different values depending on height, the value above corresponds to sea level.

This model was subsequently refined by taking into account two correction terms: one height-dependent (B) and other that depends on both the height and the zenith angle (δR). This refinement was introduced by Bauersima and leads to:

$$\Delta L = 0.002277 \cdot \sec \beta \left[P + \left(\frac{1255}{T} + 0.05 \right) \cdot e - B \cdot \tan^2 \beta \right] + \delta R \quad (105)$$

Values B and δR can be interpolated from Tables 28 and 29

Table 28: Correction term B in refined Saastamoinen model [29]

Height [km]	B [mb]
0.0	1.156
0.5	1.079
1.0	1.006
1.5	0.938
2.0	0.874
2.5	0.813
3.0	0.757
4.0	0.654
5.0	0.563

Table 29: Correction term δR in refined Saastamoinen model [29]

Zenith angle	Station height [km]							
	0	0.5	1.0	1.5	2.0	3.0	4.0	5.0
60°00'	0.003	0.003	0.002	0.002	0.002	0.002	0.001	0.001
66°00'	0.006	0.006	0.005	0.005	0.004	0.003	0.003	0.002
70°00'	0.012	0.011	0.010	0.009	0.008	0.006	0.005	0.004
73°00'	0.020	0.018	0.017	0.015	0.013	0.011	0.009	0.007
75°00'	0.031	0.028	0.025	0.023	0.021	0.017	0.014	0.011
76°00'	0.039	0.035	0.032	0.029	0.026	0.021	0.017	0.014
77°00'	0.050	0.045	0.041	0.037	0.033	0.027	0.022	0.018
78°00'	0.065	0.059	0.054	0.049	0.044	0.036	0.030	0.024
78°30'	0.075	0.068	0.062	0.056	0.051	0.042	0.034	0.028
79°00'	0.087	0.079	0.072	0.065	0.059	0.049	0.040	0.033
79°30'	0.102	0.093	0.085	0.077	0.070	0.058	0.047	0.039
79°45'	0.111	0.101	0.092	0.083	0.076	0.063	0.052	0.053
80°00'	0.121	0.110	0.100	0.091	0.083	0.068	0.056	0.047

Zenith hydrostatic delay

Assuming hydrostatic equilibrium and ideal gas behaviour, the zenith hydrostatic delay can be obtained by integrating the refractivity. Using the empirically derived gravity calculation of Saastamoinen (see Equation (102), the expression obtained is:

$$ZHD = 0.0022768 \cdot \frac{P_0}{9.784 \cdot (1 - 0.0026 \cdot \cos(2\varphi) - 2.8 \cdot 10^{-7} \cdot h)} \quad (106)$$

This formulation follows [72], slightly different versions can be found in literature.

Saastamoinen wet zenith delay model

A wet zenith delay model can also be derived from the Saastamoinen formulation. By applying the variation of water vapour pressure described in Equation (103) a simplified expression for the wet zenith delay is obtained through integration [37]:

$$ZWD = 0.002277 \cdot \left(\frac{1255}{T_0} + 0.05 \right) \cdot e_0 \quad (107)$$

Callahan model

This empirical model estimates the wet zenith delay using surface meteorological data, assuming water vapour pressure varies with height as [32]:

$$e = e_0 \cdot \exp(-a \cdot h - b \cdot h^2) \quad (108)$$

The Equation (108) was empirically found to describe the water vapour pressure distribution in Western Europe, with $a = 0.288$ and $b = 0.0480$. The resulting expression to obtain the water vapour pressure leads to:

$$e = e_0 \cdot \exp(-0.288 \cdot h - 0.0480 \cdot h^2) \quad (109)$$

The range error due to water vapour, that is the wet component, is based on Chao's consideration of adiabatic atmosphere. Assuming a lineal temperature lapse rate and the exponential behaviour of the water partial pressure Equation (109), the resulting expression can be integrated [32].

A complete derivation of the Callahan model is very complicated. Using nominal values, the expression for calculating the wet zenith delay in a simplified form is [37]:

$$ZWD = \frac{1035 \cdot e_0}{T_0^2} \quad (110)$$

Berman models

Berman derived several models for the zenith wet tropospheric delay. The original **Berman 70** model assumes a linear temperature lapse rate, constant relative humidity and zero wet refractivity at the tropopause [37]. The expression for this model is:

$$ZWD = \frac{373}{dT(B - A \cdot C)} \cdot \left(1 - \frac{C}{T_0}\right)^2 \cdot e_0 \quad (111)$$

with constants $A = 17.1485$, $B = 4684.1$, $C = 38.45$ and dT representing the temperature lapse rate).

Several refined versions were developed based on correlations between wet/hydrostatic delays and refractivities [11], named Berman 74, Berman D/N and Berman TMOD [34]. Berman TMOD model attempts to model the systematic diurnal variations.

All these models can be expressed as:

$$ZWD = 10.946 \cdot K \cdot \frac{e_0}{T_0} \quad (112)$$

The value of the constant K for each model is shown in Table 30.

Table 30: Berman models constants

Model	Constant value (K)
Berman 74	0.3224
Berman TMOD	0.3281
Berman D/N (day case)	0.2896
Berman D/N (night case)	0.3773

Askne and Nordius model

This model assumes water vapour pressure decreases with height similarly to pressure, described as ([33]):

$$e = e_0 \cdot \left(\frac{P}{P_0} \right)^{\omega+1} \quad (113)$$

where ω is a parameter related to seasonal and latitudinal variations. This value should be determined from local atmospheric profiles, but usually mean values are used. For example, in Table 31 values for Northern Hemisphere suggested by Smith are shown.

Table 31: Seasonal and latitudinal ω variation [73]

Latitude	winter	spring	summer	annual average
0-10	3.37	2.85	2.80	2.91
10-20	2.99	3.02	2.70	2.91
20-30	3.60	3.00	2.98	3.12
30-40	3.04	3.11	2.92	3.00
40-50	2.70	2.95	2.77	2.78
50-60	2.52	3.07	2.67	2.79
60-70	1.76	2.69	2.61	2.41
70-80	1.60	1.67	2.24	2.03
80-90	1.11	1.44	1.94	1.62
Northern hemisphere average	2.52	2.64	2.62	2.61

It is assumed that temperature decreases with height at a fairly constant rate (temperature lapse rate dT):

$$T = T_0 - dT \cdot dH \quad (114)$$

The simplified expression for calculating the wet zenith tropospheric delay is [37]:

$$ZWD = 10^{-6} \left(k'_2 + \frac{k_3}{T_m} \right) \cdot \frac{R_d}{(\omega + 1) \cdot g_m} \cdot e_0 \quad (115)$$

where:

- k'_2 and k_3 are refractivity constants
- R_d is the molar constant for dry air.

and T_m is obtained as:

$$T_m = T_0 \left(1 - \frac{dT \cdot R_d}{(\omega + 1) \cdot g_m} \right) \quad (116)$$

Saastamoinen's gravity formula is typically used for g_m in this model.

A.2 Tropospheric mapping functions

This appendix presents the formulation of the selected mapping functions. For detailed derivations, please refer to the original sources (as cited in the text). The equations have been harmonised to use consistent symbols and units:

- Temperature (T) is in degrees Celsius [C]
- Water vapour pressure (e) and pressure (P) are in hectopascals [hPa]
- Height (h) is in kilometres [km]
- Elevation angle (ε)
- Latitude (φ)

Subscript $_0$ refers to the surface value of the corresponding parameter. Subscripts $_h$ and $_w$ refer to the hydrostatic and wet components, respectively.

Herring mapping function

The Herring mapping functions (wet and hydrostatic mapping function) are location and temperature dependent.

They follow the continued fraction formulation originally proposed by Marini [74], who introduced it to model the mapping of tropospheric delay in satellite laser ranging. Herring adapted this formulation, truncated to three terms and normalized to unity at the zenith, for use in very long baseline interferometry (VLBI) [29]. Both hydrostatic and wet mapping functions are expressed as:

$$m(\varepsilon) = \frac{\frac{1}{1 + \frac{a}{1 + \frac{b}{1 + c}}}}{\sin(\varepsilon) + \frac{a}{\sin(\varepsilon) + \frac{b}{\sin(\varepsilon) + c}}} \quad (117)$$

The coefficients (a , b , and c) were obtained by least squares fitting of ray-traced delays through atmospheric profiles (temperature and water vapour) from radiosondes launched near ten VLBI stations in the United States [75].

The hydrostatic coefficients are computed as:

$$\begin{aligned} a_h &= [1.2320 + 0.0139 \cos \varphi - 0.0209 \cdot h_0 + 0.00215 \cdot (T_0 - 10)] \cdot 10^{-3} \\ b_h &= [3.1612 - 0.1600 \cos \varphi - 0.0331 \cdot h_0 + 0.00206 \cdot (T_0 - 10)] \cdot 10^{-3} \\ c_h &= [71.244 - 4.293 \cos \varphi - 0.149 \cdot h_0 + 0.0021 \cdot (T_0 - 10)] \cdot 10^{-3} \end{aligned} \quad (118)$$

And the wet coefficients as:

$$\begin{aligned} a_w &= [0.583 - 0.011 \cos \varphi - 0.052 \cdot h_0 + 0.0014 \cdot (T_0 - 10)] \cdot 10^{-3} \\ b_w &= [1.402 - 0.102 \cos \varphi - 0.101 \cdot h_0 + 0.0020 \cdot (T_0 - 10)] \cdot 10^{-3} \\ c_w &= [45.85 - 1.91 \cos \varphi - 1.29 \cdot h_0 + 0.015 \cdot (T_0 - 10)] \cdot 10^{-3} \end{aligned} \quad (119)$$

Niell mapping function

The Niell mapping functions (hydrostatic and wet) use the same continued fraction as in Equation 117, normalised to unity at the zenith [76]. The main improvement compared to the Herring function is the inclusion of a height correction term in the hydrostatic model [77].

Unlike the Herring model, Niell's mapping functions do not rely on local meteorological data. Instead, coefficients a , b , and c are interpolated from look-up tables (Tables 32 and 34) derived from the U.S. Standard Atmosphere profiles [78].

Niell mapping functions were developed under the following assumptions [76]:

- Northern and Southern Hemispheres are temporally antisymmetric.
- The equatorial region is represented by the 15° profile.
- The polar region is represented by the 75° profile.

Hydrostatic mapping function

The coefficients (a , b and c) are computed as:

$$a = a_{avg} + a_{amp} \cos \left(2\pi \frac{doy - 28}{365.25} \right) \quad (120)$$

where a_{avg} and a_{amp} are interpolated from Table 32 according to the station's latitude, doy is day of year, and the phase of the mapping function was adopted as day of year 28.

Table 32: Niell hydrostatic mapping function coefficients

	Latitude				
	15°	30°	45°	60°	75°
a_{avg}	$1.2769934 \cdot 10^{-3}$	$1.2683230 \cdot 10^{-3}$	$1.2465397 \cdot 10^{-3}$	$1.2196049 \cdot 10^{-3}$	$1.2045996 \cdot 10^{-3}$
b_{avg}	$2.9153695 \cdot 10^{-3}$	$2.9152299 \cdot 10^{-3}$	$2.9288445 \cdot 10^{-3}$	$2.9022565 \cdot 10^{-3}$	$2.9024912 \cdot 10^{-3}$
c_{avg}	$62.610505 \cdot 10^{-3}$	$62.837393 \cdot 10^{-3}$	$63.721774 \cdot 10^{-3}$	$63.824265 \cdot 10^{-3}$	$64.258455 \cdot 10^{-3}$
a_{amp}	0.0	$1.2709626 \cdot 10^{-5}$	$2.6523662 \cdot 10^{-5}$	$3.4000452 \cdot 10^{-5}$	$4.1202191 \cdot 10^{-5}$
b_{amp}	0.0	$2.1414979 \cdot 10^{-5}$	$3.0160779 \cdot 10^{-5}$	$7.2562722 \cdot 10^{-5}$	$11.723375 \cdot 10^{-5}$
c_{amb}	0.0	$9.0128400 \cdot 10^{-5}$	$4.3497037 \cdot 10^{-5}$	$84.795348 \cdot 10^{-5}$	$170.37206 \cdot 10^{-5}$

In the Southern Hemisphere, the seasonal phase is shifted by half a year.

A height correction is also applied to account for the variation in atmospheric thickness:

$$\Delta m f(\varepsilon) = h \cdot \frac{1}{\sin(\varepsilon)} - f(\epsilon, a_{ht}, b_{ht}, c_{ht}) \quad (121)$$

where $f(\epsilon, a_{ht}, b_{ht}, c_{ht})$ again refers to Equation (117), but with the height correction coefficients (Table 33).

Table 33: Niell height correction coefficients

coefficient	value
a_{ht}	$2.53 \cdot 10^{-5}$
b_{ht}	$5.49 \cdot 10^{-3}$
c_{ht}	$1.14 \cdot 10^{-3}$

Wet mapping function

Coefficients are directly interpolated from Table 34. No height correction is applied, as water vapour is not in hydrostatic equilibrium and its vertical distribution is highly variable [76].

Table 34: Niell wet mapping function coefficients

	Latitude				
	15°	30°	45°	60°	75°
a	$5.8021897 \cdot 10^{-4}$	$5.6794847 \cdot 10^{-4}$	$5.8778019 \cdot 10^{-4}$	$5.9727542 \cdot 10^{-4}$	$6.1641693 \cdot 10^{-4}$
b	$1.4275268 \cdot 10^{-3}$	$1.5138625 \cdot 10^{-3}$	$1.4572752 \cdot 10^{-3}$	$1.5007428 \cdot 10^{-3}$	$1.7599082 \cdot 10^{-3}$
c	$4.3472961 \cdot 10^{-2}$	$4.6729510 \cdot 10^{-2}$	$4.3908931 \cdot 10^{-2}$	$4.4626982 \cdot 10^{-2}$	$5.4736038 \cdot 10^{-2}$

VMF1

The Vienna Mapping Function 1 (VMF1) is an improved version of the earlier Vienna Mapping Function (VMF), which was based on ray-tracing through Numerical Weather Models (NWMs) and used fixed or climatological coefficients derived for specific latitudes [79]. In VMF1, the mapping function was refined to improve accuracy and temporal resolution.

VMF1 continues to use the continued fraction formulation of Equation (117). However, in this version, the coefficients are derived using ERA-40 reanalysis data from the European Centre for Medium-Range Weather Forecasts (ECMWF), which provides global, high-resolution atmospheric data¹ [80]. This allows the coefficients to vary seasonally and asymmetrically between hemispheres.

Wet mapping function

¹Some values in original source are wrong, corrected parameters are provided in: [https://vmf.geo.tuwien.ac.at/documentation/Boehm%20et%20al.,%202006a%20%20%20\(VMF1\).pdf](https://vmf.geo.tuwien.ac.at/documentation/Boehm%20et%20al.,%202006a%20%20%20(VMF1).pdf)

For the wet mapping function, the coefficients b and c are fixed and equal to the values from the Niell Mapping Function for latitude 45° , because wet zenith delays are relatively small compared to hydrostatic delays, and the impact of variability in these coefficients is considered negligible. The adopted values are $b = 0.00146$, $c = 0.04391$.

Only coefficient a is determined dynamically from numerical weather models. These a values are computed from rigorously ray-traced delays at 3° elevation using global atmospheric data from the European Centre for Medium-Range Weather Forecasts (ECMWF). They are provided every six hours in global grid files with a resolution of 2.5° longitude $\times$ 2.0° latitude [81].

Hydrostatic mapping function

For the hydrostatic component, b coefficient is set to 0.0029.

The coefficient c is computed as a function of day of year (doy) and latitude (φ), using the following expression:

$$c = c_0 + \left[\left(\cos \left(\frac{doy - 28}{365} \cdot 2\pi + \psi \right) + 1 \right) \cdot \frac{c_{11}}{2} + c_{10} \right] \cdot (1 - \cos \varphi) \quad (122)$$

where ψ is a parameter used to specify the hemisphere. This equation introduces both seasonal and hemispheric asymmetry based on the ERA-40 climatology. The values of the parameters c_0 , c_{10} , and c_{11} depend on the hemisphere and are summarized in Table 35.

Table 35: VMF1 hydrostatic parameters [80]

Hemisphere	c_0	c_{10}	c_{11}	ψ
northern	0.062	0.001	0.005	0
southern	0.062	0.002	0.007	π

As in the wet component, coefficient a is determined from ray-traced delays using ECMWF NWMs and is provided in the same global grid files, along with the zenith hydrostatic delay (ZHD) and zenith wet delay (ZWD).

To apply the VMF1 at a given location and time, the user must first interpolate temporally between the two nearest grid files, and then apply a bilinear spatial interpolation using the surrounding four grid points in latitude and longitude.

B Refractivity gradient derivation

In Section 4.2, we introduced the effect of atmospheric refraction on vertical angle measurements, which depends on the vertical gradient of atmospheric refractivity. For practical applications using electro-optical instruments, this gradient can be estimated using meteorological data, through the following expression:

$$\frac{dN}{dh} = \frac{\partial N}{\partial T} \cdot \frac{dT}{dh} + \frac{\partial N}{\partial P} \cdot \frac{dP}{dh} + \frac{\partial N}{\partial e} \cdot \frac{de}{dh} \quad (123)$$

Although atmospheric refractivity also depends on humidity, its influence in the visible and near infrared range is relatively small (typically on the order of 0.04 ppm, according to [71]). Therefore, in most practical applications with electro-optical instruments, the effect of humidity is neglected. However, this simplification may no longer be valid when working at very high accuracies, below 0.1 ppm.

Neglecting the humidity term leads to a simplified version of the expression:

$$\frac{dN}{dh} = \frac{\partial N}{\partial T} \cdot \frac{dT}{dh} + \frac{\partial N}{\partial P} \cdot \frac{dP}{dh} \quad (124)$$

To compute this gradient, we start from a commonly used formula for atmospheric refractivity in the visible and near-infrared domain, as recommended by the International Association of Geodesy (IAG) [40]:

$$N = N_{gr} \cdot \frac{273.16}{1013.25} \cdot \frac{P}{T} - 11.27 \cdot \frac{e}{T} \quad (125)$$

For simplification, N_{gr} is taken as 295, corresponding to standard atmospheric conditions. Defining constants A and B to group the coefficients:

$$N = A \cdot \frac{P}{T} - B \cdot \frac{e}{T} \quad (126)$$

with $A \approx 79.53$ and $B \approx 15.02$.

We now compute the partial derivatives:

- Partial derivative of N with respect T :

$$\frac{\partial N}{\partial T} = \frac{-A \cdot P}{T^2} - \left(\frac{-B \cdot e}{T^2} \right) = \frac{B \cdot e - A \cdot P}{T^2} \quad (127)$$

- Partial derivative of N with respect P :

$$\frac{\partial N}{\partial P} = \frac{A}{T} \quad (128)$$

Substituting these into the expression for the vertical gradient of refractivity:

$$\frac{\partial N}{\partial h} = \left(\frac{B \cdot e - A \cdot P}{T^2} \right) \cdot \frac{dT}{dh} + \frac{A}{T} \cdot \frac{dP}{dh} \quad (129)$$

The vertical pressure gradient ($\frac{dP}{dh}$) can be approximate to -0.12 hPa/m. This approximation is valid for altitudes up to approximately 2–3 km above sea level under standard atmospheric conditions. However, it should not be used in high altitudes, extreme weather conditions, or when high precision is required.

By substituting this approximation and the constant values of A and B , the final expression becomes:

$$\frac{\partial N}{\partial h} = \left((15.02e - 79.53P) \frac{1}{T^2} \right) \frac{dT}{dh} + \left(79.53 \frac{1}{T} \right) (-0.12) \quad (130)$$

This expression allows estimating the vertical refractivity gradient using local meteorological measurements of temperature and pressure. The main difficulty lies in accurately determining the vertical temperature gradient ($\frac{dT}{dh}$). The use of a mean lapse rate is not sufficiently accurate, particularly near the ground, where electro-optical instruments typically operate. In this layer, the temperature profile can vary significantly due to surface effects, radiation balance, and local atmospheric conditions, making average values unreliable. For this reason, the use of measured or modelled values of the vertical temperature gradient becomes essential to achieve accurate results.