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ORIGINAL RESEARCH

Iteration dependent interval based open-closed-loop iterative learning control for time varying systems with vector relative degree

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Abstract
For linear time varying (LTV) multiple input multiple output (MIMO) systems with vector relative degree, an open-closed-loop iterative learning control (ILC) strategy is developed in this article, where the time interval of operation is iteration dependent. To compensate the missing tracking signal caused by iteration dependent interval, the feedback control is introduced in ILC design. As the tracking signal of many continuous iterations is lost in a certain interval, the feedback control part can employ the tracking signal of current iteration for compensation. Under the assumption that the initial state vibrates around the desired initial state uniformly in mathematical expectation sense, the expectation of ILC tracking error can converge to zero as the number of iteration tends to infinity. Under the circumstance that the initial state varies around the desired initial state with a bound, as the number of iteration tends to infinity, the expectation of ILC tracking error can be driven to a bounded range, whose upper bound is proportional to the fluctuation. It is revealed that the convergence condition is dependent on the feed-forward control gains, while the feedback control can accelerate convergence speed by selecting appropriate feedback control gains. As a special case, the controlled system with integrated high relative degree is also addressed by proposing a simplified iteration dependent interval based open-closed-loop ILC method. Finally, the effectiveness of the developed iteration dependent interval based open-closed-loop ILC is illustrated by a simulation example with two cases on initial state.

KEYWORDS
intelligent control, iterative methods

1 | INTRODUCTION

Different from the model based control techniques [1–6], as an intelligent control scheme, iterative learning control (ILC) is usually utilised to impel the output of repetitive system to track a reference trajectory gradually along the iteration direction [7–10]. By employing the tracking information of preceding iterations, ILC adjusts the control input of subsequent iterations to make the tracking error reduce. One of the most conspicuous strengths of ILC is that less prior knowledge of the dynamical system is required in design process, which makes ILC popular in both theoretical and applicable fields with repetitive cases such as robot [11–13], high speed rack feeder [14], high speed train [15–17], pneumatic artificial muscle [18], and air conditioning system [19].

With the aim of achieving perfect tracking, most ILC methods require that the time interval at each iteration is fixed. However, in many actual circumstances, the controlled system may stop earlier or later due to external disturbances or system dynamics. This would lead to the time interval varies from iteration to iteration. Thus, some ILC designs for dynamical systems with iteration dependent interval have been investigated [14, 20–38]. And the authors in the work [39] have provided a review of the model and ILC method of the iteration dependent interval systems. In [20], a certain convergence is derived with the help of improved P-type ILC scheme for non-linear systems with iteration dependent interval initial state, and external disturbances. Other first order ILC schemes can be found in [21, 22], and [23]. Due to the iteration dependent interval, the tracking signal of last cycle would be lost as the actual time interval is less than the desired one. In this case, the mentioned first order ILC schemes, which only utilise the tracking signal of last cycle to adjust current control input, cannot obtain useful information from last iteration. To deal with the iteration dependent time interval, for linear discrete time systems, the iteration average operator is introduced in [24], where all former tracking signals are collected and employed to compensate the missing information. In [25], an iteratively moving average operator is introduced in ILC design, which uses the tracking information of latest iterations to compensate the signal that was missing at last iteration. For non-linear systems with initial state shifts and iteration dependent interval, a higher order ILC scheme using the information in some recent iterations is proposed in [26]. In [27], two ILC schemes are presented, which select valuable tracking information and filter out redundant information. Some other ILC schemes with the compensation mechanisms in iteration domain are presented for linear systems [28–30] and non-linear systems [31–37]. Combining flexible adaptive methods and two-dimensional methods, some novel ILC schemes have been addressed for a class of iteration dependent interval repetitive systems [35–38]. However, it is worth pointed out that these iteration based compensation methods [24–33] need to store lots of tracking information. As the time interval is shorter in many continuous iterations, the storage of ILC would be large, since the signal in many previous iterations are adopted in ILC law. Other compensation mechanisms should be considered for dynamical systems with iteration dependent interval.

On the other hand, most of the existing iteration dependent interval based ILC methods are developed for controlled systems with a relative degree of one [20–27, 30–33]. For dynamical systems represented in time domain, the relative degree reflects that the control input affects the system output connected with system matrices [40–43]. And for linear systems represented in frequency domain by a transfer function $U(s)/V(s)$, the relative degree is defined by $\overline{G} = \deg V - \deg U$, where $\deg(\cdot)$ denotes the order of polynomial with real coefficients. Nowadays, the ILC issue of dynamical systems with high relative degree ($\overline{G} > 1$) are studied [41–43]. The vector relative degree of multiple input multiple output (MIMO) systems reveals the relationship from control input to single system output, which can give more detailed information between them. Fortunately, in recent years, the iteratively moving average operator based ILC scheme with robustness analysis is proposed for linear continuous time systems with vector relative degree under iteration dependent input trail lengths and random initial state shifts [28]. For linear discrete time MIMO systems with vector relative degree and iteration dependent interval, an iteration average operator based PD-type ILC is presented, as well as P-type and D-type [29]. The authors in the work [34] used the modified error and iteratively moving average operator to design two iteratively variable pass lengths based sampled data ILC algorithms for non-linear continuous time systems with vector relative degree. However, the systems considered in [28, 29] are time invariant, and the missing information caused by iteration dependent interval is compensated by the signal of previous iterations [28, 29, 34]. More efforts should be made in iteration dependent interval based ILC scheme for time varying systems with vector relative degree.

In this paper, an iteration dependent interval based open-closed-loop ILC scheme including feedforward and feedback control structure is developed for linear time varying (LTV) MIMO systems with vector relative degree. As the tracking information of many continuous iterations is lost in a certain time interval, the signal of current iteration could be used for compensation with the help of feedback control item. Under the assumption that the initial state vibrates around the desired initial state uniformly in mathematical expectation sense, the expectation of ILC tracking error can converge to zero as the number of iteration tends to infinity. Under the circumstance that the initial state varies around the desired initial state with a bound, as the number of iteration tends to infinity, the expectation of ILC tracking error can be driven to a bounded range, whose upper bound is proportional to the fluctuation. It is demonstrated that the convergence condition is dependent on the feedforward gains only, while the appropriate feedback gains can improve the ILC convergence performance. As a special case, the controlled system with integrated high relative degree is also addressed by proposing a simplified iteration dependent interval based open-closed-loop ILC method. The major contributions of this paper are summarised as follows: (1) the existing iteration dependent interval based ILC studies for MIMO systems with vector relative degree requires that the system is time invariant [28, 29]. The LTV MIMO system with vector relative degree or integrated high relative degree is considered in this article. (2) In the ILC designs for dynamical MIMO systems with iteration dependent interval, the tracking information in previous iterations is adopted for compensation [28–37]. An iteration dependent interval based open-closed-loop ILC method containing feedforward and feedback control is proposed. With the help of feedback control, a compensation mechanism in time domain is presented, such that the missing tracking signal caused by iteration dependent interval can be compensated by the signal of current iteration. The proposed iteration dependent interval based open-closed-loop ILC method can accelerate the convergence speed with the help of appropriate feedback control gains. (3) To analyse the convergence of the proposed iteration dependent interval based ILC schemes, the λ-norm is widely used [22, 24, 25, 27, 30–34]. However, it is well known that the λ-norm might not be satisfactory measure of ILC tracking error [26, 29]. By employing mathematical induction in convergence analysis, λ-norm is avoided in this article.

The remaining sections of this article are organised as follows. In Section 2, the LTV MIMO system with vector degree and iteration dependent time interval is presented, as well as some preliminaries. Under the initial state vibrates around the desired initial state uniformly in mathematical expectation sense, the developed iteration dependent open-closed-loop ILC method and the convergence analysis are given in Section 3. When the initial state varies around the desired initial state with a bound, Section 4 provides the corresponding robustness analysis. Section 5 provides a simulation example to validate the effectiveness of the derived results. In the end, we conclude this article in Section 6.

2 | PROBLEM DESCRIPTION AND PRELIMINARIES

Consider the following LTV MIMO systems that operate repetitively,

$$\begin{cases} x_k(z+1) = A(z)x_k(z) + B(z)u_k(z), \\ y_k(z) = C(z)x_k(z), \end{cases} \tag{1}$$

where $k = 0, 1, \cdots$ is the iterative index, $z = 0, 1, \cdots, Z_k$ is the discrete time index, $x_k(z) \in R^p$ is the state, $u_k(z) \in R^r$ is the control input, and $y_k(z) \in R^m$ is the output. The matrices $A(z) \in R^{p\times p}$, $B(z) \in R^{p\times r}$, and $C(z) \in R^{m\times p}$. For system (1), $Z_k \left(\underline{Z} \le Z_k \le \overline{Z} \right)$ is the iteration dependent terminal time step at k-th iteration with upper bound $\overline{Z}$ and lower bound $\underline{Z}$. For iteration dependent interval based ILC problem, it is reasonable to assume that there exists a lower bound $\underline{Z} > 0$ and a upper bound $\overline{Z} > \underline{Z} > 0$, which are not required to be known in prior. In other words, the specific information of $\underline{Z}$ and $\overline{Z}$ is not included in the ILC scheme design.

Let $x_d(z)$ be the desired state, $y_d(z) = \left[y_d^{(1)}(z) \; y_d^{(2)}(z) \cdots y_d^{(m)}(z) \right]^T = C(z)x_d(z)$ is the corresponding output trajectory with $y_d^{(s)}(z)$, $z \in \{0, 1, \cdots, Z_d + G_s\}$, $s = 1, 2, \cdots, m$, where Z_d is the desired terminal time step and G_s is defined in Definition 1.

Definition 1. [26]: The vector relative degree of LTV MIMO system (1) is $G = (G_1, G_2, \cdots, G_m)$, if for all $1 \le s \le m$

$$\begin{cases} C_s(z+i) \prod\limits_{j=1}^{l-1} A(z+i-j)B(z+i-l) = 0, 1 \le l \le i, 1 \le l \le G_s - 1, \\ C_s(z+G_s) \prod\limits_{j=1}^{l-1} A(z+G_s-j)B(z+G_s-l) = 0, 1 \le l \le G_s - 1, \\ C_s(z+G_s) \prod\limits_{j=1}^{G_s-1} A(z+G_s-j)B(z) \ne 0\,. \end{cases} \tag{2}$$

where $G_s > 1$.

Assume that there is a unique control input $u_d(z) \in R^r$ for $0 \le z \le Z_d$, such that

$$\begin{cases} x_d(z+1) = A(z)x_d(z) + B(z)u_d(z) \\ y_d(z) = C(z)x_d(z) \end{cases} \tag{3}$$

The tracking error $e_k(z) = \left[e_k^{(1)}(z) \quad e_k^{(2)}(z) \quad \cdots \quad e_k^{(m)}(z) \right]^T$ at k-th iteration is

$$\begin{aligned} e_k(z) &= y_d(z) - y_k(z) \\ &= \left[y_d^{(1)}(z) - y_k^{(1)}(z) \quad y_d^{(2)}(z) - y_k^{(2)}(z) \quad \cdots \quad y_d^{(m)}(z) - y_k^{(m)}(z) \right]^T \end{aligned} \tag{4}$$

where $e_k^{(s)}(z) = y_d^{(s)}(z) - y_k^{(s)}(z)$, $s = 1, 2, \cdots, m$, $0 \le z \le \min\{Z_d + G_s, Z_k + G_s\}$.

Lemma 1. *If the vector relative degree of LTV MIMO system (1) is $G = (G_1, G_2, \cdots, G_m)$, then it is obtained that*

$$\begin{cases} y_k^{(s)}(z+i) = C_s(z+i) \prod\limits_{j=1}^{i} A(z+i-j)\, x_k(z), 0 \le i \le G_s - 1 \\ y_k^{(s)}(z+G_s) = C_s(z+G_s) \prod\limits_{j=0}^{G_s-1} A(z+j)\, x_k(z) + C_s(z+G_s) \prod\limits_{j=1}^{G_s-1} A(z+G_s-j)B(z)\, u_k(z) \end{cases} \tag{5}$$

where $y_k^{(s)}(z)$ is the s-th component of $y_k(z)$ at k-th iteration.

According to Definition 1, Lemma 1 can be directly proved by the solution of system (1). Then from (5) in Lemma 1, it is demonstrated that the output $y_k^{(s)}(z)$, $s \in \{1, 2, \cdots, m\}$ at the initial time interval $z \in \{0, 1, \cdots, G_s - 1\}$ is not influenced by the control input $u_k(z)$.

Lemma 2. *[10]: Assume a real sequence $\{\alpha_k\}$ for $k \geq 0$ is denoted as*

$$\alpha_{k+1} \leq \zeta \alpha_k + M \tag{6}$$

where M is a constant. If $\zeta < 1$, then

$$\limsup_{k \to +\infty} \alpha_k \leq \frac{M}{1 - \zeta} \tag{7}$$

In order to investigate the iteration dependent interval based ILC method for LTV MIMO systems with vector relative degree, let $E(\eta)$ be the mathematical expectation of the stochastic variable η. Then we give the following assumptions.

Assumption 1. [24]: The initial state $x_k(0)$ vibrates randomly with iteration while the mathematical expectation satisfies

$$E\{\|x_d(0) - x_k(0)\|\} = 0 \tag{8}$$

Assumption 2. The initial state $x_k(0)$ for any iteration fluctuates randomly while the mathematical expectation is bounded, that is

$$E\{\|x_d(0) - x_k(0)\|\} \leq \delta \tag{9}$$

Remark 1. In traditional ILC schemes, a common assumption is that the state $x_k(0)$ at each iteration is equal to the desired initial state $x_d(0)$ [25]. Comparing with it, Assumption 1 is relaxed, which allows $x_k(0)$ varying uniformly around $x_d(0)$ in mathematical expectation sense [24]. Assumption 2 is a general case of Assumption 1. It is worth noted that the strict requirement of identical initial condition in traditional ILC studies is relaxed under both Assumptions 1 and 2.

Moreover, the iteration dependent interval issue is also considered in this paper. In other words, the actual terminal time step Z_k is iterative varying and different from the desired one Z_d. For that case, a Bernoulli stochastic variable $\lambda_k(z)$ $(0 \leq z \leq \overline{Z})$ taking binary values 0 and one is introduced. On the one hand, $\lambda_k(z) = 1$ occurring with probability $q(z)$, $(0 \leq q(z) \leq 1)$ indicates that the ILC process of system (1) can continue to the time point z at k-th iteration. On the other hand, $\lambda_k(z) = 0$ occurring with probability $1 - q(z)$ indicates that the ILC process of system (1) cannot continue to the time point z at k-th iteration.

The modified tracking error $\tilde{e}_k(z) = \left[\tilde{e}_k^{(1)}(z) \quad \tilde{e}_k^{(2)}(z) \quad \cdots \quad \tilde{e}_k^{(m)}(z)\right]^T$ is defined as

$$\tilde{e}_k^{(s)}(z) = \lambda_k(z) \bullet e_k^{(s)}(z), 0 \leq z \leq Z_d + G_s (s = 1, 2, \cdots, m) \tag{10}$$

which implies

$$\tilde{e}_k^{(s)}(z) = \begin{cases} e_k^{(s)}(z), 0 \leq z \leq Z_k + G_s \\ 0, Z_k + G_s < z \leq Z_d + G_s \end{cases} \quad (s = 1, 2, \cdots, m) \tag{11}$$

when $Z_k < Z_d$ and

$$\tilde{e}_k^{(s)}(z) = e_k^{(s)}(z), 0 \leq z \leq Z_d + G_s, (s = 1, 2, \cdots, m) \tag{12}$$

when $Z_k \geq Z_d$.

3 | ITERATION DEPENDENT INTERVAL BASED OPEN-CLOSED-LOOP ILC DESIGN AND CONVERGENCE ANALYSIS

For LTV MIMO system (1) with vector relative degree $G = (G_1, G_2, \cdots, G_m)$, we design the iteration dependent interval based open-closed-loop ILC law as follows for $0 \leq z \leq Z_d$,

$$u_{k+1}(z) = u_{k+1}^f(z) + u_{k+1}^b(z) \tag{13}$$

$$u_{k+1}^f(z) = u_k(z) + \sum_{s=1}^{m} L_s \tilde{e}_k^{(s)}(z + G_s) \tag{14}$$

$$u_{k+1}^b(z) = \sum_{s=1}^{m} R_s \tilde{e}_{k+1}^{(s)}(z + G_s - 1) \tag{15}$$

Theorem 1. *For system (1) with vector relative degree $G = (G_1, G_2, \cdots, G_m)$ under Assumption 1, assume the desired output trajectory $y_d(z)$ is realized. Apply the iteration dependent interval based open-closed-loop ILC law (13)–(15). If there are learning gains $L_s (s = 1, 2, \cdots, m)$ to make*

$$\left\| I - \sum_{s=1}^{m} L_s C_s(z + G_s) \prod_{j=1}^{G_s - 1} A(z + G_s - j) B(z) \right\| \cdot q(z + G_s) + \|I\| \cdot [1 - q(z + G_s)] \leq \rho < 1 \tag{16}$$

then the mathematical expectation of tracking error $E\{\|e_k(z)\|\}$ will be driven to zero for $z \in \{G_s - 1, G_s, \cdots, G_s + Z_d\}$ as k goes infinity.

Proof. Let $\Delta u_k^f(z) = u_d(z) - u_k^f(z)$ and $\Delta u_k(z) = u_d(z) - u_k(z)$, subtracting both side of (14) with $u_d(z)$ and noting (10), it yields

$$\begin{aligned}\Delta u_{k+1}^f(z) &= \Delta u_k(z) - \sum_{s=1}^{m} L_s \tilde{e}_k^{(s)}(z + G_s) \\ &= \Delta u_k(z) - \sum_{s=1}^{m} L_s \lambda_k(z + G_s) e_k^{(s)}(z + G_s)\end{aligned} \tag{17}$$

Denote $\Delta x_k(z) = x_d(z) - x_k(z)$, according to (5), we get

$$\begin{aligned} e_k^{(s)}(z+G_s) &= y_d^{(s)}(z+G_s) - y_k^{(s)}(z+G_s) \\ &= C_s(z+G_s)\prod_{j=0}^{G_s-1} A(z+j)\Delta x_k(z) + C_s(z+G_s)\prod_{j=1}^{G_s-1} A(z+G_s-j)B(z)\Delta u_k(z) \end{aligned} \tag{18}$$

Substituting (18) into (17),

$$\begin{aligned} \Delta u_{k+1}^f(z) &= \Delta u_k(z) - \sum_{s=1}^{m} L_s\lambda_k(z+G_s)\left[C_s(z+G_s)\prod_{j=0}^{G_s-1} A(z+j)\,\Delta x_k(z) + C_s(z+G_s)\prod_{j=1}^{G_s-1} A(z+G_s-j)B(z)\Delta u_k(z)\right] \\ &= \left[I - \sum_{s=1}^{m} L_s\lambda_k(z+G_s)\,C_s(z+G_s)\prod_{j=1}^{G_s-1} A(z+G_s-j)B(z)\right]\Delta u_k(z) \\ &\quad - \sum_{s=1}^{m} L_s\lambda_k(z+G_s)\,C_s(z+G_s)\prod_{j=0}^{G_s-1} A(z+j)\,\Delta x_k(z) \end{aligned} \tag{19}$$

Subtracting both side of (13) with $u_d(z)$, and considering (15) and (10), it yields

$$\begin{aligned} \Delta u_k(z) &= \Delta u_k^f(z) - u_k^b(z) \\ &= \Delta u_k^f(z) - \sum_{s=1}^{m} R_s\lambda_k(z+G_s-1)e_k^{(s)}(z+G_s-1) \end{aligned} \tag{20}$$

Substituting (20) into (19),

$$\begin{aligned} \Delta u_{k+1}^f(z) &= \left[I - \sum_{s=1}^{m} L_s\lambda_k(z+G_s)\,C_s(z+G_s)\prod_{j=1}^{G_s-1} A(z+G_s-j)\,B(z)\right] \\ &\quad \times \left[\Delta u_k^f(z) - \sum_{s=1}^{m} R_s\lambda_k(z+G_s-1)\;e_k^{(s)}(z+G_s-1)\right] \\ &\quad - \sum_{s=1}^{m} L_s\lambda_k(z+G_s)\,C_s(z+G_s)\prod_{j=0}^{G_s-1} A(z+j)\Delta x_k(z) \\ &= \left[I - \sum_{s=1}^{m} L_s\lambda_k(z+G_s)\,C_s(z+G_s)\prod_{j=1}^{G_s-1} A(z+G_s-j)B(z)\right]\Delta u_k^f(z) \\ &\quad - \left[I - \sum_{s=1}^{m} L_s\lambda_k(z+G_s)C_s(z+G_s)\prod_{j=1}^{G_s-1} A(z+G_s-j)B(z)\right] \\ &\quad \times \sum_{s=1}^{m} R_s\lambda_k(z+G_s-1)\,e_k^{(s)}(z+G_s-1) \\ &\quad - \sum_{s=1}^{m} L_s\lambda_k(z+G_s)C_s(z+G_s)\prod_{j=0}^{G_s-1} A(z+j)\Delta x_k(z) \end{aligned} \tag{21}$$

Applying norm $\|\cdot\|$ and $E\{\cdot\}$ on both sides of (21), we have

$$
\begin{aligned}
E\left\{\left\|\Delta u_{k+1}^{f}(z)\right\|\right\} \leq\ & E\left\{\left\|I-\sum_{s=1}^{m} L_s\lambda_k(z+G_s)C_s(z+G_s)\prod_{j=1}^{G_s-1}A(z+G_s-j)B(z)\right\|\right\}E\left\{\left\|\Delta u_k^{f}(z)\right\|\right\} \\
& +E\left\{\left\|I-\sum_{s=1}^{m} L_s\lambda_k(z+G_s)\ C_s(z+G_s)\prod_{j=1}^{G_s-1}A(z+G_s-j)B(z)\right\|\right\}\sum_{s=1}^{m}\|R_s\| \\
& \times E\{\|\lambda_k(z+G_s-1)\|\}E\left\{\left\|e_k^{(s)}(z+G_s-1)\right\|\right\} \\
& +E\left\{\left\|\sum_{s=1}^{m} L_s\lambda_k(z+G_s)C_s(z+G_s)\prod_{j=0}^{G_s-1}A(z+j)\right\|\right\}E\{\|\Delta x_k(z)\|\}
\end{aligned}
\tag{22}
$$

According to the definition of Bernoulli stochastic variable $\lambda_k(z)$, there are

$$
\begin{aligned}
& E\left\{\left\|I-\sum_{s=1}^{m} L_s\lambda_k(z+G_s)C_s(z+G_s)\prod_{j=1}^{G_s-1}A(z+G_s-j)B(z)\right\|\right\} \\
& =\left\|I-\sum_{s=1}^{m} L_sC_s(z+G_s)\prod_{j=1}^{G_s-1}A(z+G_s-j)B(z)\right\|\cdot q(z+G_s)+\|I\|\cdot[1-q(z+G_s)]
\end{aligned}
\tag{23}
$$

and

$$
E\{\|\lambda_k(z+G_s)\|\}=q(z+G_s) \tag{24}
$$

On the other hand, from (23), (16) and (24), we can rewrite (22) as

$$
\begin{aligned}
E\left\{\left\|\Delta u_{k+1}^{f}(z)\right\|\right\} \leq\ & \rho E\left\{\left\|\Delta u_k^{f}(z)\right\|\right\}+\rho\sum_{s=1}^{m}\|R_s\|\cdot q(z+G_s-1)\cdot E\left\{\left\|e_k^{(s)}(z+G_s-1)\right\|\right\} \\
& +\sum_{s=1}^{m}\|L_s\|\cdot q(z+G_s)\cdot\|C_s(z+G_s)\|\prod_{j=0}^{G_s-1}\|A(z+j)\|\cdot E\{\|\Delta x_k(z)\|\} \\
\leq\ & \rho E\left\{\left\|\Delta u_k^{f}(z)\right\|\right\}+\rho m m_R\bar{q}E\left\{\left\|e_k^{(s)}(z+G_s-1)\right\|\right\}+m m_L\bar{q}m_C m_A^{G_s}E\{\|\Delta x_k(z)\|\} \\
\leq\ & \rho E\left\{\left\|\Delta u_k^{f}(z)\right\|\right\}+g_1\left[E\{\|\Delta x_k(z)\|\}+E\left\{\left\|e_k^{(s)}(z+G_s-1)\right\|\right\}\right]
\end{aligned}
\tag{25}
$$

where $\|L_s\|\leq m_L$, $\|R_s\|\leq m_R$, $(s\in\{0,1,\cdots,m\})$, $\|C(z)\|\leq m_C$, $\|A(z)\|\leq m_A$, $(z\in\{0,1,\cdots,Z_d+G_s\})$, and $g_1=\max\left\{m m_L\bar{q}m_C m_A^{G_s},\rho m m_R\bar{q}\right\}$ with $\bar{q}=\max_i q(i)$, $i\in\{0,1,\cdots,Z_d+G_s\}$.

Then, from (1), (3), (20) and (24), there is

$$
\begin{aligned}
E\{\|\Delta x_k(z)\|\} \leq\ & \|A(z-1)\|\cdot E\{\|\Delta x_k(z-1)\|\}+\|B(z-1)\|\cdot E\{\|\Delta u_k(z-1)\|\} \\
\leq\ & \|A(z-1)\|\cdot E\{\|\Delta x_k(z-1)\|\}+\|B(z-1)\|\cdot E\left\{\left\|\Delta u_k^{f}(z-1)\right\|\right\} \\
& +\|B(z-1)\|\cdot\sum_{s=1}^{m}\|R_s\|\cdot q(z+G_s-2)\cdot E\left\{\left\|e_k^{(s)}(z+G_s-2)\right\|\right\} \\
\leq\ & m_A E\{\|\Delta x_k(z-1)\|\}+m_B E\left\{\left\|\Delta u_k^{f}(z-1)\right\|\right\}+m m_B m_R q(z+G_s-2)E\left\{\left\|e_k^{(s)}(z+G_s-2)\right\|\right\}
\end{aligned}
\tag{26}
$$

According to (5) in Lemma 1 and (26), we have

$$\begin{aligned} E\left\{\left\|e_k^{(s)}(z+G_s-1)\right\|\right\} &= E\left\{\left\|y_d^{(s)}(z+G_s-1)-y_k^{(s)}(z+G_s-1)\right\|\right\} \\ &\le \|C_s(z+G_s-1)\| \prod_{j=1}^{G_s-1} \|A(z+G_s-1-j)\| E\{\|\Delta x_k(z)\|\} \\ &\le m_C m_A^{G_s} E\{\|\Delta x_k(z-1)\|\} + m_C m_A^{G_s-1} m_B E\left\{\left\|\Delta u_k^f(z-1)\right\|\right\} \\ &+ m_C m_A^{G_s-1} m m_B m_R q(z+G_s-2) E\left\{\left\|e_k^{(s)}(z+G_s-2)\right\|\right\} \end{aligned} \tag{27}$$

In consequence, from (26) and (27), we can get

$$\begin{aligned} & E\{\|\Delta x_k(z)\|\} + E\left\{\left\|e_k^{(s)}(z+G_s-1)\right\|\right\} \\ & \le \left(m_A + m_C m_A^{G_s}\right) E\{\|\Delta x_k(z-1)\|\} + \left(m_B + m_C m_A^{G_s-1} m_B\right) E\left\{\left\|\Delta u_k^f(z-1)\right\|\right\} \\ & + \left(1 + m_C m_A^{G_s-1}\right) m m_B m_R q(z+G_s-2) E\left\{\left\|e_k^{(s)}(z+G_s-2)\right\|\right\} \\ & \le g_2\left[E\{\|\Delta x_k(z-1)\|\} + E\left\{\left\|e_k^{(s)}(z+G_s-2)\right\|\right\}\right] + g_3 E\left\{\left\|\Delta u_k^f(z-1)\right\|\right\} \\ & \le \cdots\cdots \\ & \le g_2^z\left[E\{\|\Delta x_k(0)\|\} + E\left\{\left\|e_k^{(s)}(G_s-1)\right\|\right\}\right] + \sum_{l=0}^{z-1} g_2^{z-l-1} g_3 E\left\{\left\|\Delta u_k^f(l)\right\|\right\} \end{aligned} \tag{28}$$

where $g_2 = \max\left\{m_A + m_C m_A^{G_s}, \left(1 + m_C m_A^{G_s-1}\right) m m_B m_R \bar{q}\right\}$ and $g_3 = \left(1 + m_C m_A^{G_s-1}\right) m_B$ with $\|B(z)\| \le m_B$, $z \in \{0, 1, \cdots, Z_d + G_s\}$.

Since $E\{\|\Delta x_k(0)\|\} = 0$ holds from (8) of Assumption 1, it follows from (5) that

$$\begin{aligned} E\left\{\left\|e_k^{(s)}(G_s-1)\right\|\right\} &= E\left\{\left\|y_d^{(s)}(G_s-1) - y_k^{(s)}(G_s-1)\right\|\right\} \\ &\le \|C_s(G_s-1)\| \prod_{j=1}^{G_s-1} \|A(G_s-j-1) \\ &\|E\{\|x_k(0)\|\} \\ &= 0 \end{aligned} \tag{29}$$

Then, from (8) and (29), we can rewrite (28) as

$$\begin{aligned} & E\{\|\Delta x_k(z)\|\} + E\left\{\left\|e_k^{(s)}(z+G_s-1)\right\|\right\} \\ & \le \sum_{l=0}^{z-1} g_2^{z-l-1} g_3 E\left\{\left\|\Delta u_k^f(l)\right\|\right\} \end{aligned} \tag{30}$$

Consequently, substituting (30) into (25), it yields

$$\begin{aligned} E\left\{\left\|\Delta u_{k+1}^f(z)\right\|\right\} &\le \rho E\left\{\left\|\Delta u_k^f(z)\right\|\right\} \\ &+ g_1 \sum_{l=0}^{z-1} g_2^{z-l-1} g_3 E\left\{\left\|\Delta u_k^f(l)\right\|\right\} \end{aligned} \tag{31}$$

As $z = 0$ in (25), and considering (8) and (29), there is

$$E\left\{\left\|\Delta u_{k+1}^f(0)\right\|\right\} \le \rho E\left\{\left\|\Delta u_k^f(0)\right\|\right\} \tag{32}$$

Noting $\rho < 1$ shown in (16), we obtain

$$\lim_{k\to+\infty} E\left\{\left\|\Delta u_k^f(0)\right\|\right\} = 0 \tag{33}$$

As $z = 1$ in (31), we have

$$E\left\{\left\|\Delta u_{k+1}^f(1)\right\|\right\} \le \rho E\left\{\left\|\Delta u_k^f(1)\right\|\right\} + g_1 g_3 E\left\{\left\|\Delta u_k^f(0)\right\|\right\} \tag{34}$$

From (33) and (16), it follows that

$$\lim_{k\to+\infty} E\left\{\left\|\Delta u_k^f(1)\right\|\right\} = 0 \tag{35}$$

Assume that for $z = 1, \ldots, v-1$,

$$\lim_{k\to+\infty} E\left\{\left\|\Delta u_k^f(z)\right\|\right\} = 0 \tag{36}$$

As $z = v$, from (31), we have

$$E\left\{\left\|\Delta u_{k+1}^f(v)\right\|\right\} \le \rho E\left\{\left\|\Delta u_k^f(v)\right\|\right\} + g_1 \sum_{l=0}^{v-1} g_2^{v-l-1} g_3 E\left\{\left\|\Delta u_k^f(l)\right\|\right\} \tag{37}$$

Considering (33), (36), and (16), we obtain

$$\lim_{k\to+\infty} E\left\{\left\|\Delta u_k^f(v)\right\|\right\} = 0 \tag{38}$$

Thus, based on the mathematical induction, we can conclude that

$$\lim_{k\to+\infty} E\left\{\left\|\Delta u_k^f(z)\right\|\right\} = 0, z \in \{0, 1, \cdots, Z_d\} \tag{39}$$

According to (30), (39), and (8), for $z \in \{0, 1, \ldots, Z_d + 1\}$, it follows that

$$\lim_{k\to+\infty} [E\{\|e_k(z + G_s - 1)\|\} + E\{\|\Delta x_k(z)\|\}] = 0. \tag{40}$$

As a result,

$$\lim_{k\to+\infty} E\{\|e_k(z)\|\} = 0, z \in \{G_s - 1, G_s, \cdots, Z_d + G_s\}. \tag{41}$$

This completes the proof.

It is worth noted that in this article, MIMO systems with vector relative degree are studied, in which the relative degree of each output could be different from others. It is demonstrated from Theorem 1 that the initial time points are dependent on the relative degree. According to the convergence results, the system output $y_k^{(s)}(z)$, $s \in \{1, 2, \cdots, m\}$ in time interval $\{0, 1, \cdots, G_s - 1\}$ cannot be controlled, because the control input $u_k(z)$ does not affect the system output for $z \in \{0, 1, \cdots, G_s - 1\}$ as shown in Lemma 1. Therefore, the initial time points should be different for each system output. An example in practical ILC application is the system outputs consisting of acceleration, speed, and displacement. It is well known that the relationship among them is derivative, which is represented as difference in discrete time. Thus, as the control force is imposed, the acceleration is influenced firstly, then the speed is affected, finally the displacement, which means that each system output is affected by control input at different time points. Such multiple output system has vector relative degree.

If the LTV system (1) has an integrated high relative degree $\overline{G} > 1$, the Markov parameters satisfy

$$\begin{cases} C(z+i)\prod_{j=1}^{l-1} A(z+i-j)B(z+i-l) = 0, 1 \le l \le i, 1 \le l \le \overline{G} - 1, \\ C\left(z+\overline{G}\right)\prod_{j=1}^{l-1} A\left(z+\overline{G}-j\right)B\left(z+\overline{G}-l\right) = 0, 1 \le l \le \overline{G} - 1, \\ C\left(z+\overline{G}\right)\prod_{j=1}^{\overline{G}-1} A\left(z+\overline{G}-j\right)B(z) \ne 0\,. \end{cases} \tag{42}$$

Similar to Lemma 1, we obtain the following relationship according to (42) and (1),

$$\begin{cases} y_k(z+i) = C(z+i)\prod_{j=1}^{i} A(z+i-j)\, x_k(z), 0 \le i \le \overline{G} - 1 \\ y_k\left(z+\overline{G}\right) = C\left(z+\overline{G}\right)\prod_{j=0}^{\overline{G}-1} A(z+j)\, x_k(z) + C\left(z+\overline{G}\right)\prod_{j=1}^{\overline{G}-1} A\left(z+\overline{G}-j\right)B(z)\, u_k(z) \end{cases} \tag{43}$$

Therefore, the following Corollary 1 to Theorem 1 is derived.

Corollary 1. *For system (1) with integrated high relative degree $\overline{G} > 1$, under Assumption 1, assume the desired output trajectory $y_d(z)$ is realized. Apply the following simplified iteration dependent interval based open-closed-loop ILC law for $0 \le z \le Z_d$,*

$$u_{k+1}(z) = u_{k+1}^f(z) + u_{k+1}^b(z) \tag{44}$$

$$u^f_{k+1}(z) = u_k(z) + \overline{L}\tilde{e}_k(z+\overline{G}) \tag{45}$$

$$u^b_{k+1}(z) = \overline{R}\tilde{e}_{k+1}(z+\overline{G}-1) \tag{46}$$

If the learning gain $\overline{L}$ is chosen to make

$$\left\| I - \overline{L}C(z+\overline{G})\prod_{j=1}^{\overline{G}-1} A(z+\overline{G}-j)B(z)\right\| \cdot q(z+\overline{G}) + \|I\| \cdot [1-q(z+\overline{G})] \le \overline{\rho} < 1, \tag{47}$$

then the mathematical expectation of tracking error $E\{\|e_k(z)\|\}$ will be driven to zero for $z \in \{\overline{G}-1, \overline{G}, \cdots, \overline{G}+Z_d\}$ as k goes infinity.

4 | ITERATION DEPENDENT INTERVAL BASED OPEN-CLOSED-LOOP ILC DESIGN UNDER INITIAL STATE FLUCTUATION

In practice, the initial state in most ILC applications might not vibrate uniformly, which means that condition (8) of Assumption 1 does not satisfy. To address this case, a more relaxed initial condition should be considered. In this section, under the initial state fluctuation described in (9) of Assumption 2 rather than (8) of Assumption 1, the corresponding robustness analysis is provided.

Theorem 2. *For system (1) with vector relative degree $G = (G_1, G_2, \cdots, G_m)$ under Assumption 2, assume the desired output trajectory $y_d(z)$ is realized. Apply the iteration dependent interval based open-closed-loop ILC law (13)–(15). If there are learning gains $L_s(s = 1, 2, \cdots, m)$ to make (16) hold, then, as k goes infinity, the asymptotic bound of* $\limsup_{k\to+\infty} E\{\|e_k(z)\|\}$ *will be driven to a bounded range, whose upper bound is proportion to δ for $z \in \{G_s-1, G_s, \cdots, G_s+Z_d\}$.*

Proof. Since $E\{\|\Delta x_k(0)\|\} \le \delta$ holds from (9) of Assumption 2, it follows from (5) that

$$\begin{aligned} E\left\{\left\|e^{(s)}_k(G_s-1)\right\|\right\} &= E\left\{\left\|y^{(s)}_d(G_s-1) - y^{(s)}_k(G_s-1)\right\|\right\} \\ &\le \|C_s(G_s-1)\| \prod_{j=1}^{G_s-1} \|A(G_s-j-1) \\ &\quad \|E\{\|x_k(0)\|\} \\ &\le m_C m_A^{G_s-1}\delta. \end{aligned} \tag{48}$$

Then, from (9) and (48), we can rewrite (28) as

$$\begin{aligned} &E\{\|\Delta x_k(z)\|\} + E\left\{\left\|e^{(s)}_k(z+G_s-1)\right\|\right\} \\ &\le g_2^z\left(\delta + m_C m_A^{G_s-1}\delta\right) + \sum_{l=0}^{z-1} g_2^{z-l-1} g_3 E\left\{\left\|\Delta u^f_k(l)\right\|\right\} \\ &\le g_2^z\left(1 + m_C m_A^{G_s-1}\right)\ \delta + \sum_{l=0}^{z-1} g_2^{z-l-1} g_3 \\ &\quad E\left\{\left\|\Delta u^f_k(l)\right\|\right\}. \end{aligned} \tag{49}$$

Consequently, substituting (49) into (25), it yields

$$\begin{aligned} E\left\{\left\|\Delta u^f_{k+1}(z)\right\|\right\} &\le \rho E\left\{\left\|\Delta u^f_k(z)\right\|\right\} \\ &\quad + g_1 g_2^z\left(1 + m_C m_A^{G_s-1}\right)\ \delta \\ &\quad + g_1 \sum_{l=0}^{z-1} g_2^{z-l-1} g_3 E\left\{\left\|\Delta u^f_k(l)\right\|\right\} \end{aligned} \tag{50}$$

Denote $\theta = \frac{g_1\left(1+m_C m_A^{G_s-1}\right)\delta}{1-\rho}$ and $\vartheta = g_2 + \frac{g_1 g_3}{1-\rho}$. As $z = 0$ in (25), and considering (9) and (48), there is

$$E\left\{\left\|\Delta u^f_{k+1}(0)\right\|\right\} \le \rho E\left\{\left\|\Delta u^f_k(0)\right\|\right\} + g_1\left(1 + m_C m_A^{G_s-1}\right)\ \delta \tag{51}$$

Applying Lemma 2 to (51) and noting $\rho < 1$ shown in (16), we obtain

$$\limsup_{k\to+\infty} E\left\{\left\|\Delta u^f_k(0)\right\|\right\} \le \frac{g_1\left(1 + m_C m_A^{G_s-1}\right)\ \delta}{1-\rho} = \theta \tag{52}$$

As $z = 1$ in (50), we have

$$\begin{aligned} E\left\{\left\|\Delta u^f_{k+1}(1)\right\|\right\} &\le \rho E\left\{\left\|\Delta u^f_k(1)\right\|\right\} \\ &\quad + g_1 g_2\left(1 + m_C m_A^{G_s-1}\right)\ \delta \\ &\quad + g_1 g_3 E\left\{\left\|\Delta u^f_k(0)\right\|\right\} \end{aligned} \tag{53}$$

Applying Lemma 2 to (53) and considering (52) and (16), it follows that

$$\begin{aligned} \limsup_{k\to+\infty} E\left\{\left\|\Delta u^f_k(1)\right\|\right\} &\le \frac{g_1 g_2\left(1 + m_C m_A^{G_s-1}\right)\ \delta}{1-\rho} + \frac{g_1 g_3 \theta}{1-\rho} \\ &= \left(g_1 + \frac{g_1 g_3}{1-\rho}\right)\theta = \vartheta\theta \end{aligned} \tag{54}$$

Assume that for $z = 1, \ldots, v-1$,

$$\limsup_{k\to+\infty} E\left\{\left\|\Delta u^f_k(z)\right\|\right\} \le \vartheta^z \theta \tag{55}$$

As $z = v$, from (50), we have

$$E\left\{\left\|\Delta u_{k+1}^{f}(v)\right\|\right\} \le \rho E\left\{\left\|\Delta u_{k}^{f}(v)\right\|\right\} + g_1 g_2^{v}\left(1 + m_C m_A^{G_s-1}\right)\delta + g_1\sum_{l=0}^{v-1} g_2^{v-l-1} g_3 E\left\{\left\|\Delta u_{k}^{f}(l)\right\|\right\} \tag{56}$$

Applying Lemma 2 to (56) and considering (52), (55), and (16), we obtain

$$\begin{aligned}\limsup_{k\to+\infty} E\left\{\left\|\Delta u_{k+1}^{f}(v)\right\|\right\} &\le \frac{g_1 g_2^{v}\left(1 + m_C m_A^{G_s-1}\right)\delta}{1-\rho} + \frac{g_1\sum_{l=0}^{v-1} g_2^{v-l-1} g_3 E\left\{\left\|\Delta u_{k}^{f}(l)\right\|\right\}}{1-\rho}\\ &\le \frac{g_1 g_2^{v}\left(1 + m_C m_A^{G_s-1}\right)\delta}{1-\rho} + \frac{g_1 g_3\left(g_2^{v-1}\theta + g_2^{v-2}\vartheta\theta + \cdots + \vartheta^{v-1}\theta\right)}{1-\rho}\\ &= \left[g_2^{v} + \frac{g_1 g_3}{1-\rho}\left(g_2^{v-1} + g_2^{v-2}\vartheta + \cdots + \vartheta^{v-1}\right)\right]\theta\\ &= \left[\left(g_2 + \frac{g_1 g_3}{1-\rho}\right)\left(g_2^{v-1} + g_2^{v-2}\vartheta + \cdots + \vartheta^{v-1}\right) - g_2^{v-1}\vartheta - g_2^{v-2}\vartheta^2 - \cdots - g_2\vartheta^{v-1}\right]\theta\\ &= \left[\vartheta\left(g_2^{v-1} + g_2^{v-2}\vartheta + \cdots + \vartheta^{v-1}\right) - g_2^{v-1}\vartheta - g_2^{v-2}\vartheta^2 - \cdots - g_2\vartheta^{v-1}\right]\theta = \vartheta^{v}\theta\end{aligned} \tag{57}$$

Thus, based on the mathematical induction, we can conclude that

$$\limsup_{k\to+\infty} E\left\{\left\|\Delta u_{k}^{f}(z)\right\|\right\} \le \vartheta^{z}\theta, z \in \{0, 1, \cdots, Z_d\} \tag{58}$$

Then, considering (58), (48), and (9) in (28), it follows that for $z \in \{0, 1, \ldots, Z_d + 1\}$,

$$\begin{aligned}&\limsup_{k\to+\infty}\left(E\{\|\Delta x_k(z)\|\} + E\left\{\left\|e_k^{(s)}(z + G_s - 1)\right\|\right\}\right)\\ &\le g_2^{z}\left(1 + m_C m_A^{G_s-1}\right)\delta + \sum_{l=0}^{z-1} g_2^{z-l-1} g_3 \limsup_{k\to+\infty} E\left\{\left\|\Delta u_{k}^{f}(l)\right\|\right\}\\ &\le g_2^{z}\left(1 + m_C m_A^{G_s-1}\right)\delta + g_3\left(g_2^{z-1}\theta + g_2^{z-2}\vartheta\theta + \cdots + \vartheta^{z-1}\theta\right)\\ &= g_2^{z}\left(1 + m_C m_A^{G_s-1}\right)\delta + g_3\frac{g_1\left(1 + m_C m_A^{G_s-1}\right)\delta}{1-\rho}\left(g_2^{z-1} + g_2^{z-2}\vartheta + \cdots + \vartheta^{z-1}\right)\\ &= \left(1 + m_C m_A^{G_s-1}\right)\delta\left[g_2^{z} + g_3\frac{g_1}{1-\rho}\left(g_2^{z-1} + g_2^{z-2}\vartheta + \cdots + \vartheta^{z-1}\right)\right]\\ &= \left(1 + m_C m_A^{G_s-1}\right)\delta\left[\left(g_2 + \frac{g_1 g_3}{1-\rho}\right)\left(g_2^{z-1} + g_2^{z-2}\vartheta + \cdots + \vartheta^{z-1}\right) - g_2^{z-1}\vartheta - g_2^{z-2}\vartheta^2 - \cdots - g_2\vartheta^{z-1}\right]\\ &= \left(1 + m_C m_A^{G_s-1}\right)\delta\left[\vartheta\left(g_2^{z-1} + g_2^{z-2}\vartheta + \cdots + \vartheta^{z-1}\right) - g_2^{z-1}\vartheta - g_2^{z-2}\vartheta^2 - \cdots - g_2\vartheta^{z-1}\right] = \vartheta^{z}\left(1 + m_C m_A^{G_s-1}\right)\delta.\end{aligned} \tag{59}$$

Since $\limsup_{k\to+\infty} E\{\|\Delta x_k(z)\|\}$ is nonnegative, we can easily obtain that for $z \in \{0, 1, \ldots, Z_d + 1\}$

$$\limsup_{k\to+\infty} E\left\{\left\|e_k^{(s)}(z + G_s - 1)\right\|\right\} \le \vartheta^{z}\left(1 + m_C m_A^{G_s-1}\right)\delta \tag{60}$$

It implies that for $z \in \{G_s - 1, G_s, \ldots, G_s + Z_d\}$, the asymptotic bound $\limsup_{k\to+\infty} E\left\{\left\|e_k^{(s)}(z)\right\|\right\}$ is proportion to δ.

This completes the proof.

Remark 2. Theorem 2 gives the robustness analysis of iteration dependent interval based open-closed-loop ILC scheme under initial state fluctuation. From the theoretical results, one can see that the asymptotic bound of ILC tracking error is dependent on the bound of initial state fluctuation. In practice, we can drive the tracking error into an acceptable range by precisely setting the initial position.

Similar to Corollary 1, the following Corollary 2 to Theorem two is acquired.

Corollary 2. *For system (1) with integrated high relative degree $\overline{G} > 1$, under Assumption 2, assume the desired output trajectory $y_d(z)$ is realized. Apply the simplified iteration dependent interval based open-closed-loop ILC law (44)–(46). If the learning gain $\overline{L}$ is chosen to make (47) hold, then, as k goes infinity, the asymptotic bound of $\limsup_{k\to+\infty} E\{\|e_k(z)\|\}$ will be driven to a bounded range, whose upper bound is proportion to δ for $z \in \{\overline{G}-1, \overline{G}, \cdots, \overline{G}+Z_d\}$.*

Remark 3. In some ILC studies for linear MIMO system with integrated high relative degree, they require that the system is time invariant and operates in fixed time interval [42, 43]. Corollaries 1 and 2 gives the simplified iteration dependent interval based open-closed-loop ILC law for LTV MIMO systems with integrated high relative degree, which in fact extends the ILC designs for MIMO system with integrated high relative degree to a wider range.

5 | SIMULATION EXAMPLE

Example 1. To illustrate the effectiveness of the developed iteration dependent interval based open-closed-loop ILC method, consider the LTV MIMO system with vector relative degree $G = (1, 2)$ denoted as

$$\begin{cases} x_k(z+1) = \begin{bmatrix} 0.1 & -0.47 & -0.6e^{\frac{1-z}{150}} & 0.08 \\ \left(\frac{z}{80}\right)^2 - 0.9 & 0.08 & -0.02 & 0.05 \\ 0.5e^{\frac{1-z}{200}} & 0.04 & -0.05 & 0.01 \\ 0.01 & 0.08 & 3.01 & -0.02 \end{bmatrix} x_k(z) + \begin{bmatrix} \frac{z}{55} & -0.01 \\ 2.3e^{\frac{z}{100}} - 15 & 0.9 \\ \frac{z}{50}+1 & 2.6e^{\frac{1-z}{100}} + 5 \\ 0 & 0 \end{bmatrix} u_k(z) \\ y_k(z) = \begin{bmatrix} 0 & 0.7e^{\frac{-z}{400}} + 0.2 & 0 & 0 \\ 0 & 0 & 0 & 0.6e^{\frac{-z}{200}} \end{bmatrix} x_k(z) \end{cases} \tag{61}$$

where $z \in \{0, 1, \cdots, Z_k\}$. The desired output trajectories $y_d^{(1)}(z)$ and $y_d^{(2)}(z)$ of system (61) are given as follows:

$$\begin{cases} y_d^{(1)}(z) = 0.02z[1+\cos(4\pi z/100 - \pi)], z \in \{0, 1, \cdots, Z_d+1\}, \\ y_d^{(2)}(z) = 0.8[1+\sin(2\pi z/100 - \pi/2)], z \in \{0, 1, \cdots, Z_d+2\}, \end{cases} \tag{62}$$

where $Z_d = 100$. The iteration dependent terminal time step $Z_k \in \{95, 96, \cdots, 105\}$ is displayed in Figure 1. Afterwards, system (61) with ILC law (13)–(15) is simulated. The performance of ILC is evaluated by the following error index on the sum of absolute errors,

$$SE_k^{(s)} = \sum_{z=G_s-1}^{Z_d+G_s} E\left\{\left|y_d^{(s)}(z) - y_k^{(s)}(z)\right|\right\}, s = 1, 2. \tag{63}$$

The initial control input is taken as $u_0(z) = [0 \quad 0]^T$ for $z \in \{0, 1, \cdots, Z_d\}$. The feedforward control gains in (14) are set as $L_1 = \begin{bmatrix} -0.07 \\ 0.006 \end{bmatrix}$ and $L_2 = \begin{bmatrix} 0.0305 \\ 0.016 \end{bmatrix}$. The feedback control gains in (15) are selected as $R_1 = \begin{bmatrix} 0.015 \\ -0.1 \end{bmatrix}$ and $R_2 = \begin{bmatrix} 0.05 \\ 0.01 \end{bmatrix}$. To compare the proposed iteration dependent interval based open-closed-loop ILC law (13)–(15) with conventional ILC law without feedback control part, the P-type ILC law with control gains $L_1 = \begin{bmatrix} -0.07 \\ 0.006 \end{bmatrix}$ and $L_2 = \begin{bmatrix} 0.0305 \\ 0.016 \end{bmatrix}$ is also simulated.

In the simulation, two cases on initial state are provided under Assumption 1 and 2, respectively.

Case 1. The values of initial state $x_k(0) = \left[x_k^{(1)}(0) \; x_k^{(2)}(0) \; x_k^{(3)}(0) \; x_k^{(4)}(0)\right]^T \in R^4$ versus iteration k are shown in Figure 1, which satisfy $E\{\|x_d(0) - x_k(0)\|\} = 0$ with $x_d(0) = [0 \quad 0 \quad 0 \quad 0]^T$. The corresponding error indexes $SE_k^{(1)}$ and $SE_k^{(2)}$ along the iteration direction are provided in Figure 2. Figure 3 gives the resulting output trajectories at third and fifth iterations to desired trajectories by using law (13)–(15).

Case 2. The values of initial state $x_k(0) = \left[x_k^{(1)}(0) \; x_k^{(2)}(0) \; x_k^{(3)}(0) \; x_k^{(4)}(0)\right]^T \in R^4$ versus iteration k are shown in Figure 4, which satisfy $E\{\|x_d(0) - x_k(0)\|\} \le \delta$ with

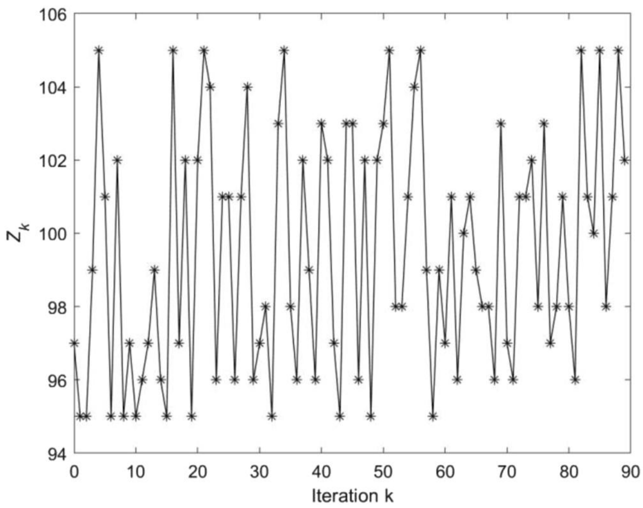

FIGURE 1 The variation of iteration dependent terminal time step Z_k versus iteration k.

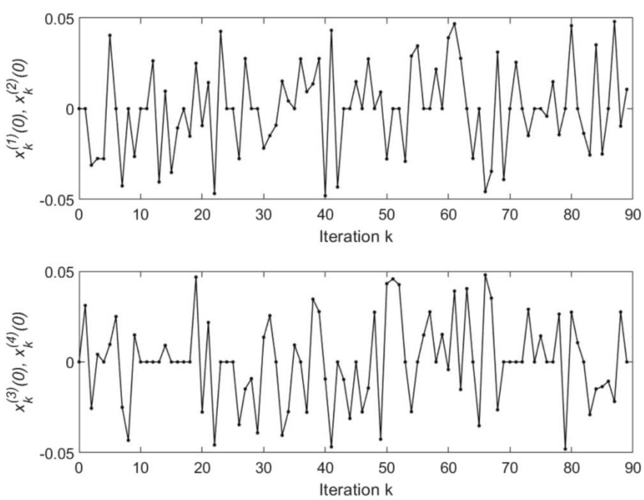

FIGURE 2 (case 1 of Example 1): The profiles of initial states $x_k^{(i)}(0)$, $(i=1,2)$ (above) and $x_k^{(i)}(0)$, $(i=3,4)$ versus iteration k as the values of initial state satisfy $E\{\|x_d(0)-x_k(0)\|\}=0$ with $x_d(0)=[0 \quad 0 \quad 0 \quad 0]^T$.

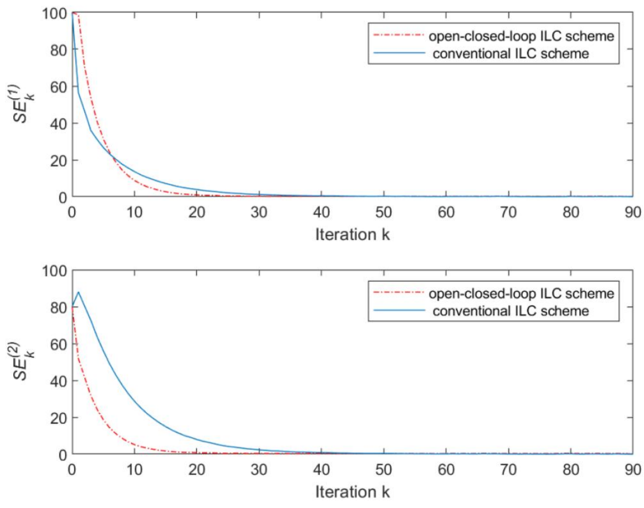

FIGURE 3 (case 1 of Example 1): The profiles of the error indexes $SE_k^{(1)}$ (above) and $SE_k^{(2)}$ (below) versus iteration k.

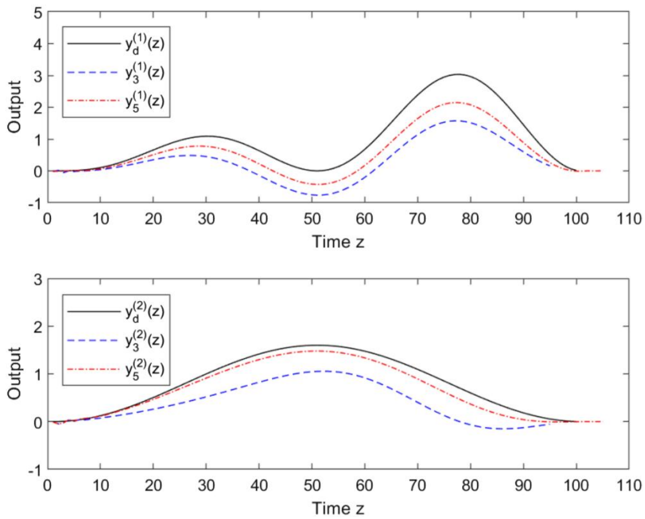

FIGURE 4 (case 1 of Example 1): The resulting output trajectories $y_k^{(1)}(z)$ to the desired output trajectory $y_d^{(1)}(z)$ (above) and $y_k^{(2)}(z)$ to the $y_d^{(2)}(z)$ (below) at iterations $k=3$ and $k=5$.

$x_d(0)=[0 \quad 0 \quad 0 \quad 0]^T$ and $\delta=1$. The corresponding error indexes $SE_k^{(1)}$ and $SE_k^{(2)}$ along the iteration direction are provided in Figure 5. Figure 6 gives the resulting output trajectories at third and fifth iterations to desired trajectories by using law (13)–(15).

Under the initial state $x_k(0)$ vibrates around the desired initial state $x_d(0)$ but satisfies $E\{\|x_d(0)-x_k(0)\|\}=0$ depicted in Figure 1, we can observe from Figure 2 that the error indexes $SE_k^{(1)}$ and $SE_k^{(2)}$ decrease fast at the first few iterations and then converge to zero asymptotically along the iteration axis. Theorem one is verified. As the initial state $x_k(0)$ fluctuates around the desired initial state $x_d(0)$ within a bound in mathematical expectation sense, that is, $E\{\|x_d(0)-x_k(0)\|\}\le\delta$ with bound $\delta=1$ displayed in Figure 1, Figure 5 shows that the error indexes $SE_k^{(1)}$ and $SE_k^{(2)}$ can be driven to a bounded range gradually as the number of iteration k tends to infinity. Theorem two is illustrated. From Figures 3 and 6, which show the system output trajectories $y_k^{(1)}(z)$ and $y_k^{(2)}(z)$ to the desired output trajectory $y_d^{(1)}(z)$ and $y_d^{(2)}(z)$ at different iterations, it can be found that though the time interval of system (61) is iteration dependent as shown in Figure 7, a progressively improved system outputs to track the desired output trajectories is achieved beyond the initial time points. Moreover, from Figures 2 and 5, it is shown that the proposed iteration dependent based open-closed-loop ILC law (13)–(15) has a faster speed than the traditional P-type ILC without feedback control part. This is because feedback

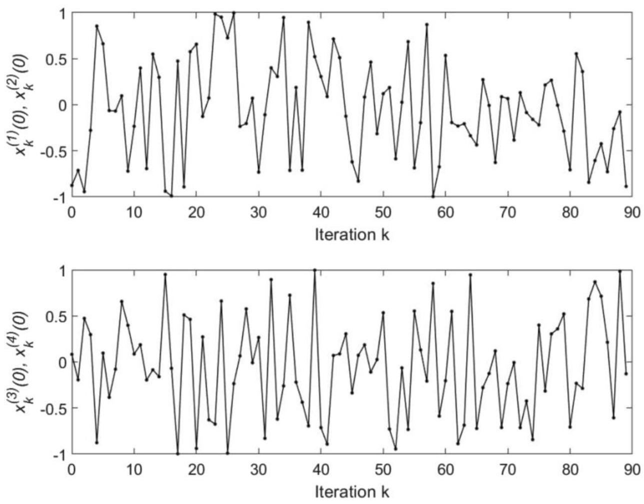

FIGURE 5 (case 2 of Example 1): The profiles of initial states $x_k^{(i)}(0)$, $(i = 1, 2)$ (above) and $x_k^{(i)}(0)$, $(i = 3, 4)$ versus iteration k as the values of initial state satisfies $E\{\|x_d(0) - x_k(0)\|\} \le \delta$ with $x_d(0) = [0 \quad 0 \quad 0 \quad 0]^T$ and $\delta = 1$.

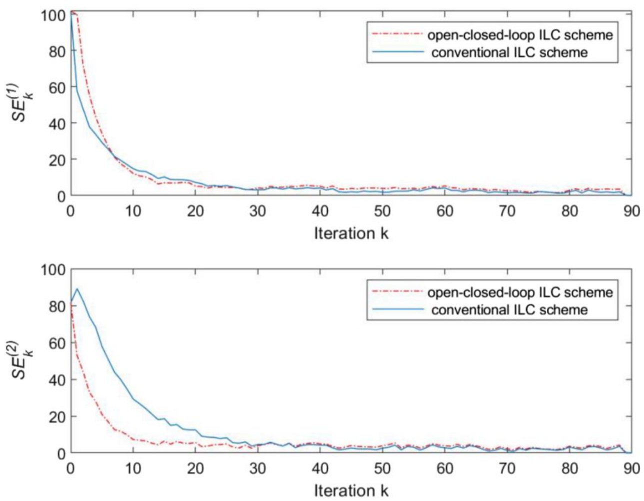

FIGURE 6 (case 2 of Example 1): The profiles of the error indexes $SE_k^{(1)}$ (above) and $SE_k^{(2)}$ (below) versus iteration k.

control item involved with ILC design can use the tracking error information of current iteration to compensate the missing signal due to the presence of iteration dependent time interval. Even if the open-closed-loop ILC law is a little more complicated than the traditional P-type ILC law for discrete time system, it can achieve a better transient response and convergence speed because there are more control gains to adjust.

Example 2. To illustrate the effectiveness of the simplified open-closed-loop ILC law (44)–(46), consider the following linear piezoelectric motor system with relative degree $\overline{G} = 2$ under iteration dependent interval, which is described as follows [44]

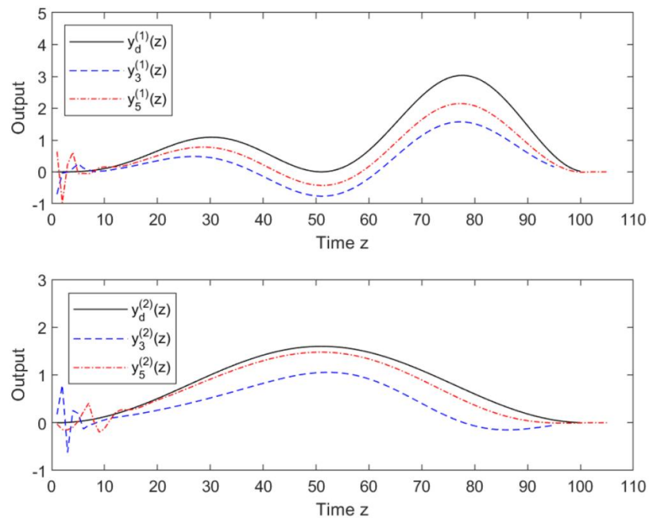

FIGURE 7 (case 2 of Example 1): The resulting output trajectories $y_k^{(1)}(z)$ to the desired output trajectory $y_d^{(1)}(z)$ (above) and $y_k^{(2)}(z)$ to the $y_d^{(2)}(z)$ (below) at iterations $k = 3$ and $k = 5$.

$$\begin{cases} \dot{\widehat{x}}_k^{(1)}(\widehat{z}) = \widehat{x}_k^{(2)}(\widehat{z}), \\ \dot{\widehat{x}}_k^{(2)}(\widehat{z}) = -\dfrac{\overline{V}}{\overline{M}}\widehat{x}_k^{(2)}(\widehat{z}) + \dfrac{\overline{F}}{\overline{M}}\widehat{u}_k(\widehat{z}), \\ \widehat{y}_k(\widehat{z}) = \widehat{x}_k^{(1)}(\widehat{z}), \end{cases} \tag{64}$$

where $\widehat{x}_k^{(1)}(\widehat{z})$ and $\widehat{x}_k^{(2)}(\widehat{z})$ are the motion position and the motion velocity, respectively, $\overline{M} = 1\ kg$ means the moving mass, $\overline{V} = 80\ N$ is the velocity damping factor, and $\overline{F} = 60\ N/V$ is the force constant. Let $u_k(z) = \widehat{u}_k(z \cdot T_s)$, $y_k(z) = \widehat{y}_k(z \cdot T_s)$, and $x_k(z) = \left[x_k^{(1)}(z) \quad x_k^{(2)}(z)\right]^T = \left[\widehat{x}_k^{(1)}(z \cdot T_s) \quad \widehat{x}_k^{(2)}(z \cdot T_s)\right]^T$. By using $\dot{\widehat{x}}_k^{(i)}(z \cdot T_s) \approx \frac{\left(\widehat{x}_k^{(i)}((z+1)\cdot T_s) - \widehat{x}_k^{(i)}(z \cdot T_s)\right)}{T_s}$ for $i = 1, 2$, the linear piezoelectric motor system (64) is discretised as

$$\begin{cases} x_k(z+1) = \begin{bmatrix} 1 & T_s \\ 0 & 1 - \dfrac{\overline{V}T_s}{\overline{M}} \end{bmatrix} x_k(z) + \begin{bmatrix} 0 \\ \dfrac{\overline{F}T_s}{\overline{M}} \end{bmatrix} u_k(z) \\ y_k(z) = [1 \quad 0]\, x_k(z) \end{cases} \tag{65}$$

where $T_s = 0.01\ s$ means the sampling time. The iteration dependent terminal time step Z_k in (65) varies between 95 and 105 as shown in Figure 8.The desired output trajectory $y_d(z)$ is described as

$$y_d(z) = 0.001\left(z - z^2\right) \tag{66}$$

where $z \in \{0, 1, \ldots, 100\}$ with $Z_d = 100$. The initial control input $u_0(z) = 0$ for $z \in \{0, 1, \ldots, 100\}$. In order to measure the ILC performance, the tracking error index is defined as

$$SE_k = \sum_{z=\overline{G}-1}^{Z_d+\overline{G}} E\{|y_d(z) - y_k(z)|\} \tag{67}$$

Apply the developed ILC law (13)–(15) with $\overline{G} = 2$, $L = 14.5$, and $R = 11$ to system (65). In the simulation, two cases on initial state are provided under Assumption 1 and 2, respectively.

Case 1. The values of the iterative initial state are set as $x_k(0) = x_d(0) = [0 \quad 0]^T$, which satisfy Assumption 1. The corresponding tracking error index SE_k along the iteration direction is depicted in Figure 9. Figure 10 provides the responding output trajectory $y_k(z)$ to the desired output trajectory $y_d(z)$ at iterations $k = 3$ and $k = 5$ by adopting the simplified open-closed-loop ILC law (44)–(46).

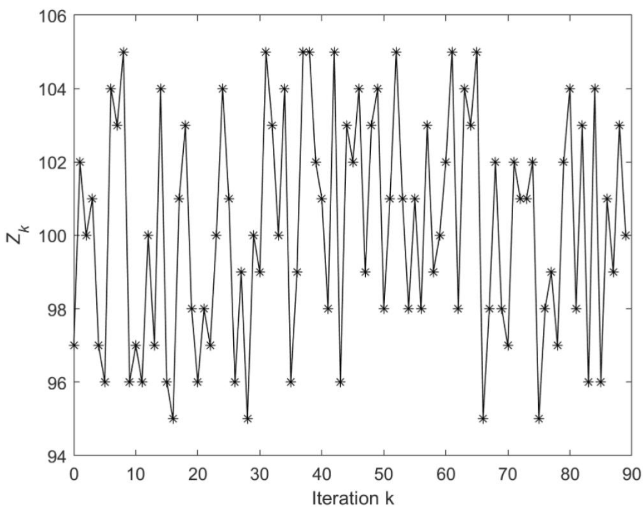

FIGURE 8 (Example 2): The variation of iteration dependent terminal time step Z_k versus iteration k.

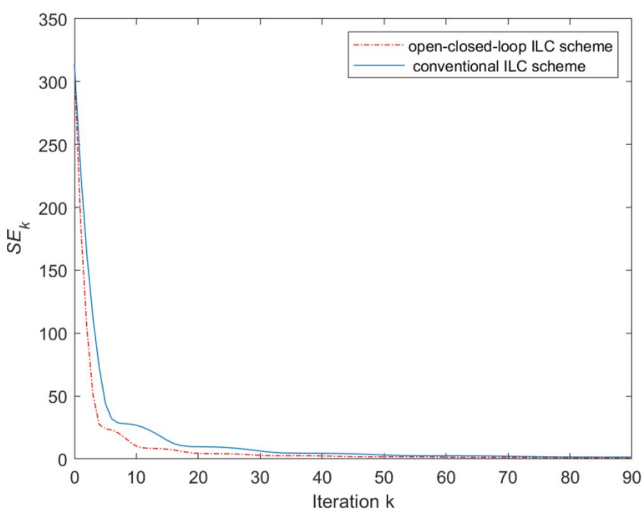

FIGURE 9 (case 1 of Example 2): The profiles of ILC tracking error index SE_k versus iteration k.

Case 2. The values of the iterative initial state $x_k(0) = \left[x_k^{(1)}(0) \quad x_k^{(2)}(0)\right]^T \in R^2$ are displayed in Figure 11, which satisfy $E\{\|x_d(0) - x_k(0)\|\} \le \delta$ with $x_d(0) = [0 \quad 0]^T$ and $\delta = 0.08$. The tracking error index SE_k along the iteration direction is given in Figure 12. Figure 13 shows the resulting output trajectory $y_k(z)$ to the desired output trajectory $y_d(z)$ at iterations $k = 3$ and $k = 5$ by adopting the simplified open-closed-loop ILC law (44)–(46).

For system (65) with relative degree $\overline{G} = 2$, as the iterative initial state $x_k(0) = x_d(0) = [0 \quad 0]^T$ satisfying Assumption 1, it is illustrated from Figure 9 that the tracking error index SE_k can be controlled to zero. From Figure 10, it can be seen that a

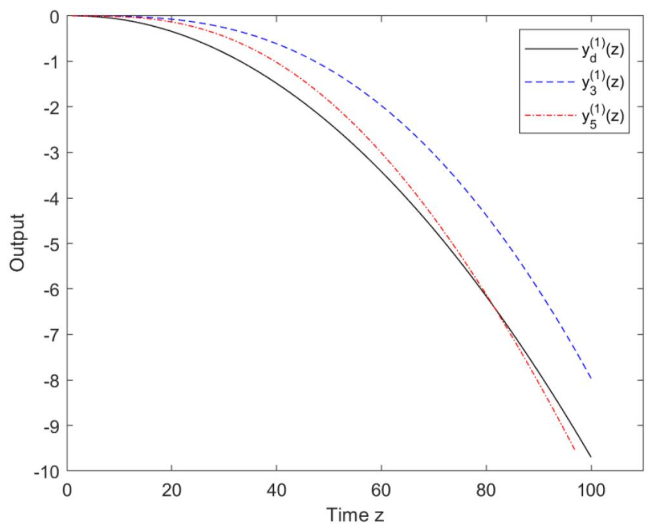

FIGURE 10 (case 1 of Example 2): The resulting output trajectory $y_k(z)$ to the desired output trajectory $y_d(z)$ at iterations $k = 3$ and $k = 5$.

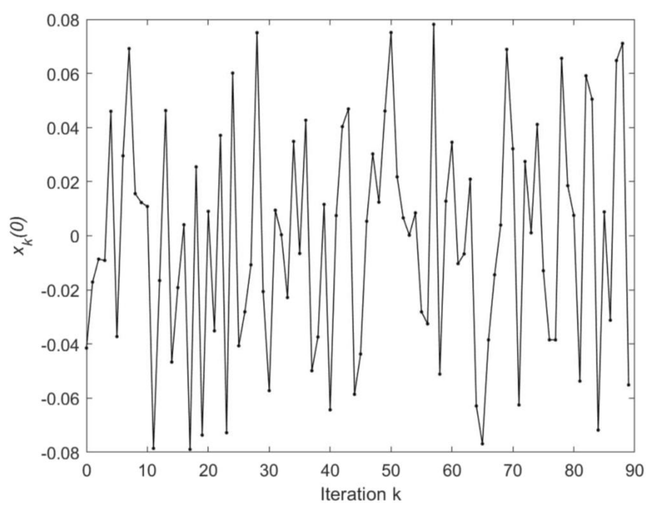

FIGURE 11 (Case 2 of Example 2): The profiles of iterative initial states $x_k^{(i)}(0)$, $(i = 1, 2)$ as the values of initial state satisfies $E\{\|x_d(0) - x_k(0)\|\} \le \delta$ with $x_d(0) = [0 \quad 0]^T$ and $\delta = 0.08$.

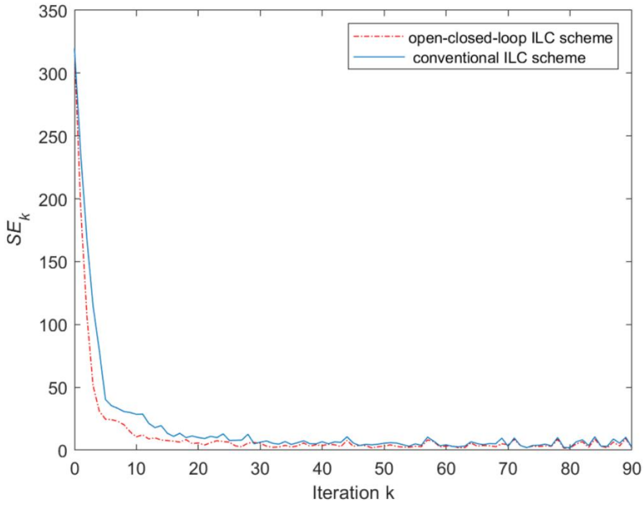

FIGURE 12 (Case 2 of Example 2): The profiles of ILC tracking error index SE_k versus iteration k.

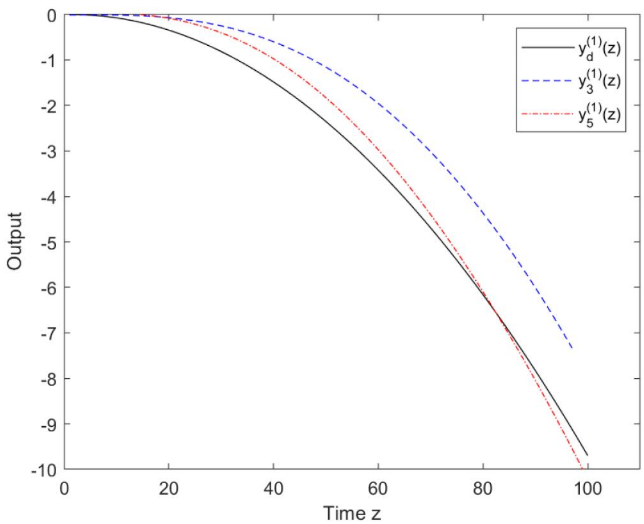

FIGURE 13 (Case 2 of Example 2): The resulting output trajectory $y_k(z)$ to the desired output trajectory $y_d(z)$ at iterations $k = 3$ and $k = 5$.

progressively improved tracking performance of system output $y_k(z)$ to desired output trajectory $y_d(z)$ has been accomplished for $z \in \{0, 1, \ldots, 100\}$. Corollary one is verified. For system (65) with relative degree $\overline{G} = 2$, as the iterative initial state varies around the desired initial state with a bound that is, $E\{\|x_d(0) - x_k(0)\|\} \leq \delta$ as shown in Figure 11 satisfying Assumption 2, Figure 12 illustrates that the control error index SE_k can be driven to a bounded range. From Figure 13, it is shown that the corresponding output trajectory can track the desired output trajectory progressively along the iteration direction. Corollary two is verified. In addition, Figures 9 and 12 demonstrate that the proposed simplified iteration dependent based open-closed-loop ILC law (44)–(46) has a faster speed than the conventional P-type ILC without feedback control part shown as (44) and (45).

6 | CONCLUSION

In this article, an iteration dependent interval based open-closed-loop ILC algorithm is developed for LTV MIMO systems with vector relative degree, in which the initial state can vibrate around the desired one with identical expectation. Theoretical analysis and simulation results have demonstrated that as the iteration number tends to infinity, the mathematical expectation of ILC tacking error can be controlled to zero beyond the initial time points, which depends on the vector relative degree of controlled system. Under initial state fluctuation, as the number of iteration tends to infinity, the proposed iteration dependent interval based open-closed-loop ILC law can drive the expectation of ILC tracking error to a bounded range, whose upper bound is proportional to the fluctuation. For the systems with integrated high relative degree, a simplified ILC law is given under these two initial conditions. It is shown that the convergence condition is determined by the feedforward gains, while the proper feedback control gains can expedite the convergence speed. Further works can focus on extending the developed open-closed-loop ILC approach to networked dynamical systems or non-repetitive systems.

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CONFLICT OF INTEREST STATEMENT

None.

DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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