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Analysis of Oscillations in a Cableway: Wind Load Effects

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Abstract

El propósito de este trabajo de carácter didáctico es el de desarrollar e investigar un modelo no lineal para el análisis de la reacción de una cabina de auto-desmontable teleférico monocable expuesto a una desaceleración repentina y a las fuerzas del viento. La ley y las ecuaciones diferenciales de primer y segundo lugar de Newton son las herramientas básicas para la construcción del modelo. Además algunas consideraciones básicas se han hecho sobre el aire. Todos los datos numéricos utilizados para la simulación fueron tomados desde un teleférico en la estación de esquí de Ravascletto-Zoncolan en el noreste de Italia.

The purpose of this paper is to develop and investigate a non-linear model for analysing the reaction of a self-detachable cabin monocable ropeway exposed to a sudden deceleration and wind forces . The First and Second Newton's Law and Differential Equations are the basic tools for building the model. Furthermore a few basic considerations have been made about the air "dragging and lifting" forces that induce oscillations and vibrations in mechanical systems alike. All the numerical data used for the simulation was taken from a ropeway in the skiing site of Ravascletto-Zoncolan in the North- East of Italy.

Keywords: Teleférico, Leyes de Newton, ecuaciones diferenciales.
Cableway, Newton laws, differential equations

1 Introduction

Ropeways are widely spread in mountain resorts, not only because they are convenient and fast transportation for the maintenance of ski runs, but because they are economic and ecological transportation to hard-to-reach places. Cabin ropeways are preferred in resorts that need high transport capacity. The question of the wind influence and oscillations on a cableway in different conditions is of great importance. It strongly affects the comfort of the passengers as well as the safety engineering criteria during the initial calculation and later on during the periodical checks of the transport system.

Figure 1: Self detachable aerial cableways. Suspended on a hauling and sustaining rope, the cabins need to face big stresses from inertial forces and additional wind loads.

It is necessary to predict when the cabins need to be detached and stored in the storehouse, since huge wind loads can harm the complex cable structural system. This paper will study the basics of the phenomena. Furthermore it's interesting to study both the air induced oscillations and the consequences they can have on the cableway.

It is hard to predict this type of motion theoretically and so there are few theoretic publications discussing this dynamic problem. Most of the publications have been made in consideration of a suspended cable only, without any forces suspended on it.

The tensile forces and stresses in the ropes caused by static loads are known from experiments made in situ on real cableways. The dynamic background of the problem is not included. An analysis of the static behaviour of ropeways is developed, assuming a full non-linear structural model.

Self-detachable cableways

The main difference between continuous movement monocable aerial ropeways and bicable aerial ropeways is simply the number of rope systems. While with a monocable ropeway one or two ropes assume the carrying and hauling function (carry-hauling cable), with a bicable aerial ropeway the cars are carried by one or more ropes (track ropes) and moved by a further rope system (haul rope). Both the monocable aerial ropeways and the bicable aerial ropeways are equipped with operationally releasable clamping devices. In the station, the cars are separated from the haul rope, decelerated and guided onto an overhead monorail which leads the cars through the embarkation and disembarkation area at low speed.

In mountainous regions, continuous movement monocable aerial ropeways are very commonly used. As the high carrying capacity is independent of track length and intermediate stations along the route are easy to implement, this ropeway system is admirably suitable for covering long distances not only in touristic developed areas but also in inner urban areas.

2 Model

The problem of theoretical modelling of the motion of a ropeway excited by the wind force is solved by using a Nonlinear Method. In this first part of the paper, we will simply study the motion of a cable oscillating if exposed to external inertial forces that induce the oscillation. In the following part we will study the wind effects on them.

The author creates a mathematical model of a one column span of a cabin ropeway. In this model the neighbouring column spans are not considered to be influencing the motion and the dynamic reaction of the span under analysis. One of the aims of this didactic paper is to show that the spring model we're about to study describes well a rope oscillation with distributed masses suspended under it (cabins).

In order to find the oscillation of the rope- ideal mass-spring-damper system, we will consider the following forces acting. However, all the values in dynamical motions and the behaviour itself are often difficult to predict, because several variables are involved in the dynamic.

By treating the mass as a free body and by considering t the time variable, x the displacement of the spring's end from its equilibrium position, m the sum of the cabin masses, the Newton's Second Law is simply given by:

$$F_i = ma = m\frac{d^2x}{dt^2}. \tag{1}$$

which is considered to be the inertial force of the system. The displacement is relative to a fixed point of reference.

According to the Second Newton's Law [1], by considering k the spring constant the spring force is given by:

$$F_s = kx. \tag{2}$$

This force is an oscillatory restoring force exerted by the spring on that end, which acts in the opposite direction of the gravity.

In order to have a system that takes count of the air friction, by considering c the damping coefficient of the air viscosity, there's a damping force given by:

$$F_c = cv = c\frac{dx}{dt}. \tag{3}$$

The damping reduces the oscillations' amplitudes in an oscillatory system. It is directly proportional to the velocity. By summing all of the forces as a result of their action on the system and by considering that the vibration is free, the applied force must be considered zero (e.g. when you let go the masses of it).

$$F_i + F_c + F_s F_{tot} = 0. \tag{4}$$

This leads to a Linear Homogeneous Differential Equation with constant coefficients.

$$0 = m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx. \tag{5}$$

This is a case of an under-damped oscillation, with a frequency smaller than in the non-damped case; this means that it has an amplitude decreasing with time.

By having considered these mathematical equations and the following numerical data, let's make a simulation.

Simulation of the oscillation

In order to study the behaviour of the system, we have considered some data that have been obtained from experimental researches.

As for the masses m of the cabins, we have considered the typical values obtained from "Sigma" cabins, which have a range of 500 kg up to 1000 kg per cabin. We have chosen a 700 kg cabin. On the span there are 5 cabins alike.

By considering Hooke's Law, as for the elastic coefficient k of the rope, the author has considered the values in the direction of the rope.

We need to consider the speed of the cabin in order to make an estimation of the boundary conditions that induce the inertial forces in (1). Cableways can have a speed up to 6 m/s which is the speed that we will consider in our simulations. This speed might lead to an acceleration of the cabin due to the decelerations of 3 m/s^2.

The damping coefficient c is the most difficult to estimate, since the damping is made by the air friction only, no other mechanical dampers are involved in the mechanism. By considering the fluid resistance we will simply consider experimental data we got from the researches made in the skiing resort of Zoncolan in Italy. In order to get a coefficient of c we used the experimental data in the following expression:

$$\frac{F_c}{dx/dt} = c. \tag{6}$$

We estimated the force, time and the displacement and we got a $c = 290$ N s/m value.

Table I	**Symbol**	**Value**	**Unit**
Cabins masses on the rope	mm	7000	kg
Cabin - rope speed	v	6	m/s
Elastic coefficient of the rope	k	7000	N/m
Damping coefficient - air friction	c	290	N s/m

Table 1: Summary of the coefficients.

It has been considered a short deceleration time that induces the motion, which can occur in so-called emergency stops.

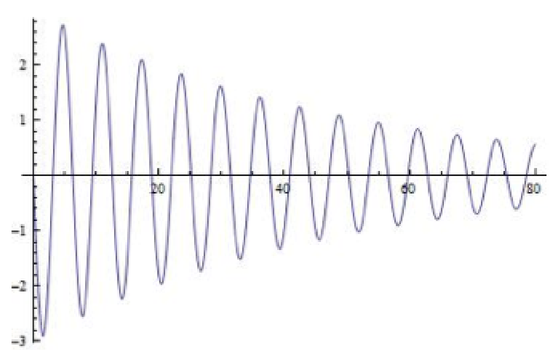

Figure 2: Displacement of the cabin due to the oscillation of the cabin as a consequence of a sudden deceleration of the motion.

In addition to that, the following boundary conditions have been considered:

$$x(0) = 0, \quad x'(0) = -3,$$

which can be indeed interpreted as the displacement at $t = 0$ is 0 (initial position) and the acceleration in $t = 0$ is -3 m/s^2. The results of the simulation are the following.
Let's notice that in the y axis there's the displacement [m] from the equilibrium position. In the x axis there's the time [s].

The main characteristic of the under-damped oscillator is that it approaches the zero value and keeps oscillating in infinitesimal quantities about that zero ([6]).

The behaviour shown by the graph appears to be quite realistic. It shows the wide oscillations that take place in the moment when there's a fast deceleration. By considering those values, the magnitude of the movement appears to be quite wide, which can influence the rope's integrity in time. The peak to peak amplitude is about 3 metres, which again appears to be quite realistic.

These are the ideal oscillating conditions. These kind of situations occur when there's the necessity to have a fast deceleration in the stations, e.g. when there are some troubles in passengers' embarkations or disembarkations, especially the ones involving children.

3 Wind load on the cabin

There are many factors that can influence a cableway's behaviour: the engine's pumping and, which is quite common, aerodynamical wind induced resistances and forces. In this part of the paper, we will analyse the forces acting on the systems and the consequences they have on the cableway.

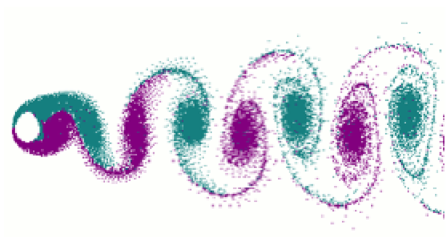

Figure 3: The Von Kármán street is a repeating pattern of swirling vortices caused by the unsteady separation of flow of a fluid over bluff bodies [3].

A body being invested by an airstream experiences a force that depends on the relative velocity. If the motion of the body is time dependent, i.e. the situation is that of an unsteady flow, the relation between the motion and the forces generated may become quite complex. However some simplifications arise from the fact that, in practice, the most frequent type of body motion is a quite simple periodic motion, often a sinusoidal one, and in one direction only. In such a case the motion becomes an oscillation and is governed by the classical second order "mass-stiffness-damping" differential equation.

The important part for the aerodynamicists is then to correctly evaluate and express the flow induced forces. The ideal example, since it is simple and happens quite frequent in real conditions, is that of a 2-dimensional cylindrical body oscillating in a transverse uniform steady flow.

Wind forces induce a "drag and lift forces", which are the consequence of the so called fluid induced turbulences. This phenomena is basically responsible for the singing and oscillation of all kind of suspended cables.

4 Model

The model we're about to study is a simplified model, since it has a good working in the case there's a distribution of the weight along the cable. In our case we have several cabins distributed on.

Figure 4: The researched case considers 5 cabins simultaneously on the same span. The distribution of the weight along the span is . therefore considered to be constant all along the span.

We consider the taut string of length L placed transversally to a spacially varying current of velocity V.
With reference to the model used above and to the figure below the equation for the body motion can be written as [2]:

$$F_{wind}(x,t) = m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx. \tag{7}$$

where F_{wind} is the horizontal aerodynamical force, to be determined.

Referring again to the figure, using the conventional approach for the expression of the forces as function of the velocity, we might write them down as:

$$F_{wind}(x,t) = \frac{1}{2}\rho AV^2 c_l \cos(wt). \tag{8}$$

In this case c_l is the lift coefficient, which is responsible for the lifting of the element invested by the wind flow which has a speed of V.

It is however interesting to notice that all the forces depend totally on the area invested by the stream (in this case the cable span with an additional surface given by the cabins' surfaces), the air stream speed and the density of the fluid investing the mass (which is air in our study case).

In order to making the simulation, it's necessary to determinate the parameters. We adjust the fixed simulation parameters to both the experimental conditions made in situ and the informations obtained from the literature. In correspondence with the experiment, we assume a clear sky with an air density of $\rho = 1.3163$ kg/m^{-3} at -5C$^\circ$ [5]. The lifting (dragging) coefficient c_l will have varying values, so it's possible to analyse different kind of cabin shapes.
A is the area of the body that is invested by the wind which can have a range of 2 up to 4 sq m per cabin [4]. In this case, we have to consider the area (as mentioned above) of all the cabins involved. We we will consider a 4 m^2 area per cabin. There are 5 cabins on the span. The total area of the surface involved is then 20 m^2.

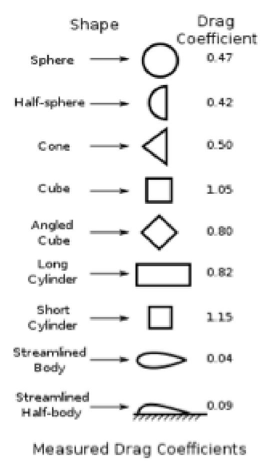

Figure 5: In the figure above there are some values of the drag coefficients for different object shapes. Let's notice that a streamlined body can have a great change on the overall value of the coefficient. In real cases, sphere and cubed cabins are the most used.

Figure 6: A Sigma company produced cabin. The airstream is investing this kind of cabin, with a total surface of 4 sq m.

Figure 7: A quasi oval cabin, used in the skiing resort of Alpe di Siusi in Italy. These cabins have better aerodynamic performance. However, they don't let skiers to put their skies outside the cabin, as happens in the Sigma cabins.

The angle of incidence of the wind is the angle by which the air stream invests the body and, basically, the spatial varying excitation frequency. This will be a sinusoidal dragging and lifting force. We chose a value of 1.8 rad.

Table II	**Symbol**	**Value**	**Unit**
Lift coefficient	c_l	0.5-1.15 [7]	[1]
Spatially varying excitation frequency	ω	1.8	rad
Density of the fluid	ρ	1.3163	Kg/ m^3
Total area of the cabins	A_{tot}	20	m^2
Relative speed of the wind	V	$10 - 30$	m/s

Table 2: Summary of the fixed parameters.

In order to study the phenomena, the author preferred to study the aerodynamical-wind forces displacement separately from the displacement caused by the free oscillations of the mass-spring-damper system (6).

Therefore the equation that gives us the displacement caused by the wind is the following

$$F_{wind}(x,t) = \frac{1}{2}\rho A V^2 c_l \cos(wt) = m\frac{d^2x}{dt^2}. \tag{9}$$

This equation gives us the displacement x. In this case m is the total mass. There will be the following boundary conditions considered:

$$x(0) = 0, \quad x'(0) = 0.$$

Simulation of the oscillations

By using the numerical values of the first section of the paper (6) , we will now analyse the behaviour of the wind forced oscillations, by using the values in Table 2.

Let's notice that in the y axis there's the displacement [m] from the equilibrium position. In the x axis there's the time [s].

1) Wind speed 10 m/s

In the case of a weak 10 m/s wind and an oval cabin (c_l 0.5) the response of the cabin to the wind is basically zero. The circular shape has a very good aerodynamic response to the wind. The wind induces some small perturbations when the self oscillations of mass- spring- damper cable, get lower peak to peak oscillations. However, what can be noticed is that even a weak wind can make peak to peak amplitudes wider.

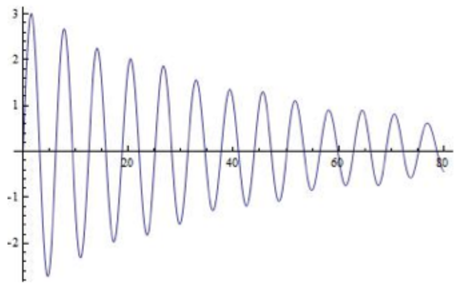

Figure 8: Dragging coefficient 0.5.

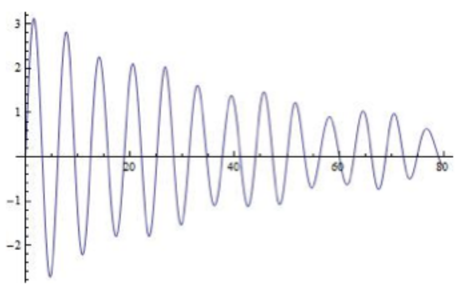

Figure 9: Dragging coefficient 1.15.

A dragging coefficient 1.15 represents a cabin that case a cubic shape. The results show clearly that this kind of surface doesn't influence the peak to peak amplitudes, but it can have a slightly bigger influence on the oscillating behaviour, especially when the self oscillation of the mass- spring- damper system gets smaller.

2) Wind speed 20 m/s

In a 20 m/s wind speed effects have a slightly bigger influences on the behaviour of the oscillations. Peak to peak amplitudes are still quite small.

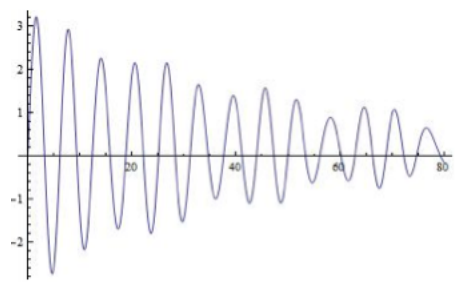

Figure 10: Dragging coefficient 0.5.

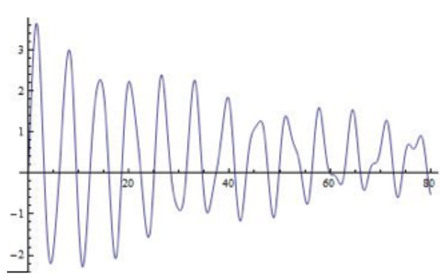

Figure 11: Dragging coefficient 1.15.

A 20 m/s wind with cube-shaped cabins have quite a considerable influence on the behaviour of the system. Peak to peak amplitudes get wider. And oscillations are less constant.

3) Wind speed 30 m/s

The results show that this wind speed has, obviously, a very high influence on the behaviour of the oscillating. Cabins are completely driven by the wind which can cause severe damages to the system. The continuous oscillations that the cabins are exposed with constant wind are very dangerous for the stability of the vibrating string.

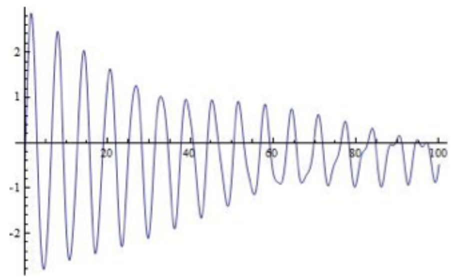

Figure 12: Dragging coefficient 0.5.

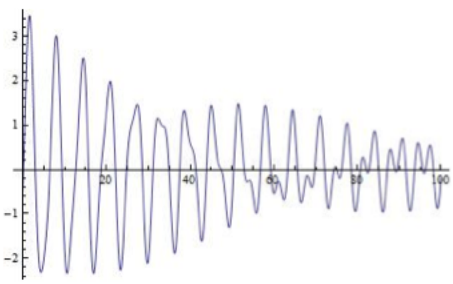

Figure 13: Dragging coefficient 1.15.

A 1.15 dragging coefficient has a very high influence. Peak to peak amplitudes are wider. The wind force has a high percentage of driving.

5 Conclusions and outlook

Recently, many models of the oscillations of the cables and the wind effects acting on them have been developed. The fast growing market is calling for a necessity to have a better knowledge of the behaviour of these transport systems, that move every year millions of people. Most of these systems are located in windy and "unfriendly" areas, so a further knowledge of the topic is needed. There's still a lack of researches on the behaviour of cableways systems, since the research can be made in restricted areas (i.e. Alpine area).

We modelled the oscillating cableway as a mass on a spring, which is damped by the air friction. This very simple model describes very well the oscillating conditions, even though some conditions must be fulfilled (the model works for a unique cabin on the span). However, let's consider that this is a very basic model that has a very general description of the phenomena. In any case, it can be used as a good base for further researches. The author suggests the following studies of the topic:

1) A more specific study of the parameters of the mass-spring-damper model. It could be made by making some experiments in order to find a better correspondence by a spring model and a suspended long cable.

2) Shall we get a more realistic oscillation model, further mathematical models should be made to describe a complex oscillating system, where there's a more realistic description of the punctual force on the rope due to the cabin suspended.

3) A further research on wind shear forces.

4) The author suggests considering a further vibrating string model by comparing the results.

However, this model shows that with huge wind loads cabins are supposed to be detached and put into the storehouse.

This is what actually is done in real conditions. Normally cabins are detached and stored in the storehouse for wind speeds above 90 km/h. So, these results show the good correspondence with the real cases.

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