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FEM Edge Effect and Capacitance Evaluation on Cylindrical Capacitors

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Abstract: The purpose of this paper is to show the influence of the edge-effect on the electric field distribution, and hence on the inner capacitance and outer capacitance of a cylindrical capacitor surrounded by an insulating medium. To generalize the results, a two-dimensional axisymmetric finite element model of a cylindrical capacitor has been generated and the problem has been resolved taking into account the distance between the conductors for a complete set of dimensions. The available obtained results have been compared with previous published works. Finally, using statistical tools, the mathematical expression for computing the relationship between capacitance and insulation gap and cylindrical plates dimensions has been obtained.

Key words: Capacitance, electric field effects, simulation, space charge.

1. Introduction

In the field of electrical engineering, cylindrical capacitors are used frequently. The manufacture of these capacitors, from a technological point of view, does not show any unusual difficulty, but from the designer point of view, it turns out a complex matter the accurate calculation of the capacitance [1-3]. The calculation of the capacitance of a cylindrical capacitor, when it is a cylindrical capacitor of finite height h, but great with respect to the R_1 radius and to the thickness of the dielectric between cylindrical plates d, does not involve any difficulties, whenever the disturbance of the electric field near the bases of the cylinders, also called edge effect [4-7], is neglected. Nevertheless, if the length of the cylinders is similar to the radius of the cylinders that compose the capacitor, the edge effect is not despicable and therefore the problem involves a high level of complexity [8-10].

The aim of this paper is to calculate the effect of the edges on the capacitance of a cylindrical capacitor in vacuum for several geometrical configurations.

With this objective, a base model has been generated using the FEM (finite element method), that allows to calculate the capacitance of a cylindrical capacitor varying the thickness, the length and the relative position of the cylinders.

2. Ideal Cylindrical Capacitors

2.1 Theoretical Capacitance

The capacitance of a cylindrical capacitor [11, 12] of height h finite, but great with respect to the R_1 radius and to the thickness of the dielectric between plates d, as shown in Fig. 1, is given by Eq. (1). This equation is valid whenever the edge effect is disregarded [13].

Eq. (1) shows that the capacitance of this type of capacitor is inversely proportional to the natural logarithm of the distance between the plates d that constitute the capacitor and is directly proportional to the capacitor height h. Graphically, the aspect of this ideal behavior of the capacitance is shown in Fig. 2. In order to generate this figure, a number of values for R_1, d and h have been taken. These values are shown in Table 1.

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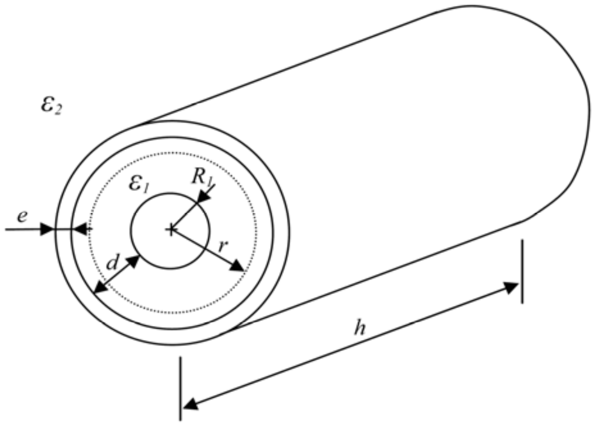

Fig. 1 Cylindrical capacitor geometric model. Where: *e* is the external conductor thickness, *d* is the distance between cylinders, *h* is the conductor's length, R_1 is the radius of inner conductor, ε_1 is the relative dielectric permittivity of the dielectric medium between conductors, and ε_2 is the relative dielectric permittivity of the dielectric medium around the external conductor.

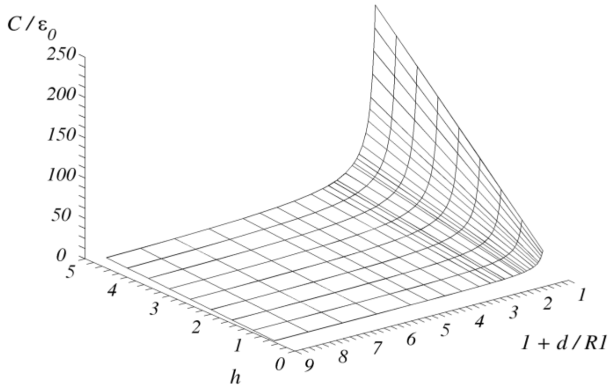

Fig. 2 Capacitance of an ideal cylindrical capacitor.

Table 1 Values of geometric variables.

R_1 (m)		*d* (m)		*h* (m)	
min	max	min	max	min	max
0.5	4.5	0.5	4.5	1.12	9.0

$$C_{ideal} = \frac{2\times\pi\times\varepsilon_0\times h}{\ln\left(1+\frac{d}{R_1}\right)} \quad (1)$$

In Fig. 2, the effect of the logarithm function is remarkable for values of 1 + d/R_1 near the unit. This fact causes that the interval of values between 1 and 2 is the zone where more varies the capacitance vs. small increases of the geometric variables ratio. In accordance, the results obtained in this research are focused in this zone (Fig. 3).

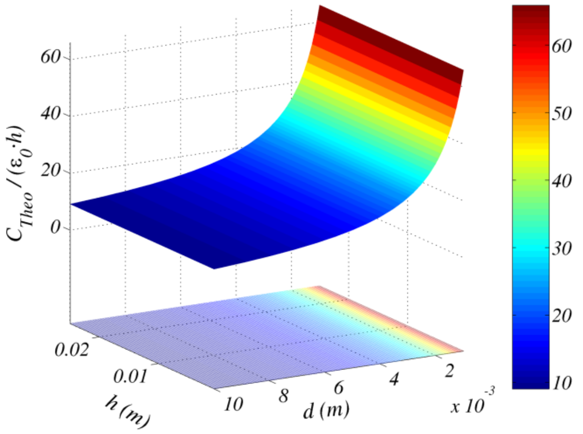

Fig. 3 Capacitance of an ideal cylindrical capacitor in the investigated geometric region.

2.2 Model Capacitance

In this section, the internal capacitance of an ideal cylindrical capacitor by means of a model based on FEM is calculated. By means of this calculation, the FEM model is validated provided that the error compared to the ideal case is very limited.

For the case of a cylindrical capacitor, an axisymmetric 2D model is used because this type of capacitors is cylindrically symmetrical. In addition, the resulting geometry is symmetrical, so the model is reduced to one half. The resulting model is shown in Fig. 4.

The use of a reduced model diminishes the number of nodes that take part in the FEM calculation and, therefore, the time of calculation. However, it must be taken into account that, if the mesh is not well adapted, the error in the solution may be very high.

The relative error has been evaluated by means of Eq. (2).

$$Error(\%) = \left(\frac{C_{simulation}}{C_{theoretical}} - 1\right)\times 100 \quad (2)$$

After the simulation, the obtained error is between 0.0028% and 0.0015%. This small error provides a primary validation of the FEM calculation model.

2.3 Electric Field

A secondary result of the used model is the calculation of the electric field between the plates of the cylindrical capacitor.

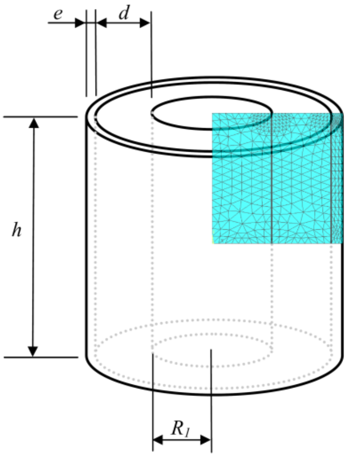

Fig. 4 2D axisymmetric ideal capacitor model.

In Fig. 5, the distribution of the electric field in the dielectric material located between the conductors of the capacitor is shown.

The obtained values of the electric field in the simulation agree with those obtained using Eq. (3), that is, with the theoretical expression for the calculation of the electric field between the conductors of a cylindrical capacitor (Fig. 6).

$$EF\left(r\right)=\frac{C\times\Delta V}{2\times\pi\times\varepsilon_0\times h\times r} \tag{3}$$

where:

EF(r) is the electric field between the plates of the cylindrical capacitor;

C is the capacitance calculated with Eq. (1);

ΔV is the difference of potential between plates (100 V);

h is the height of the cylinders;

r is the distance to the center of the capacitor (Fig. 1).

3. Real Cylindrical Capacitors

3.1 Model Capacitance

The capacitance of a real capacitor [14, 15] is the sum of its inner capacitance and its outer capacitance [16, 17], as it is stated in Eq. (4).

$$C = C_{in} + C_{out} \tag{4}$$

The inner capacitance is the capacitance between the plates that conform the capacitor, and agree generally with the calculated ideal capacitance by means of Eq. (1). On the other hand, the outer capacitance is the capacitance that appears in the outside of the plates due to the deformation of the electric field lines. This deformation is a function of the dielectric permittivity and by the effect of the edges of the cylinders.

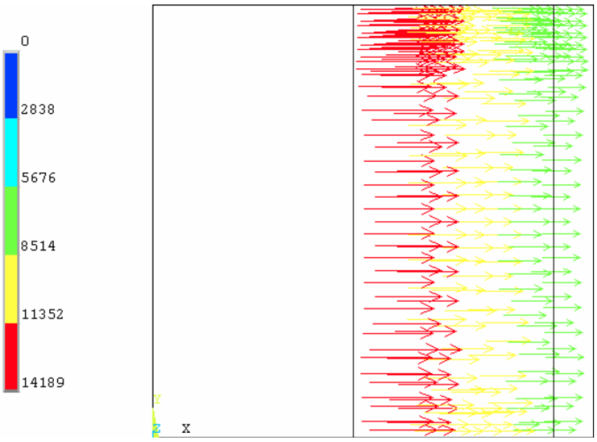

Fig. 5 Electric field distribution in the dielectric medium between conductors. Where: R_1 = 0.01 m, *d* = 0.01 m, *e* = 2 E-3 m and *h* = 0.044 m.

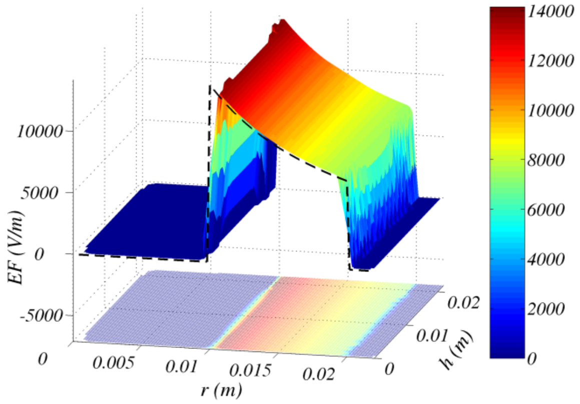

Fig. 6 Electric field distribution in simulated model. Where: R_1 = 0.01 m, *d* = 0.01 m, *e* = 2 E-3 m and *h* = 0.044 m. The dashed line is EF calculated with Eq. (3).

In order to calculate accurately the values of the capacitance of a cylindrical capacitor, to the validated model that has been used previously, an area that surrounds externally to the conductors as shown in Fig. 7, has been added. This area allows modeling the outer dielectric material.

The results obtained in the simulation (Fig. 8) for different models generated from the values from the variables of Table 2 have been statistically analyzed to obtain a mathematical expression that allows to calculate the capacitance of a real cylindrical capacitor [18].

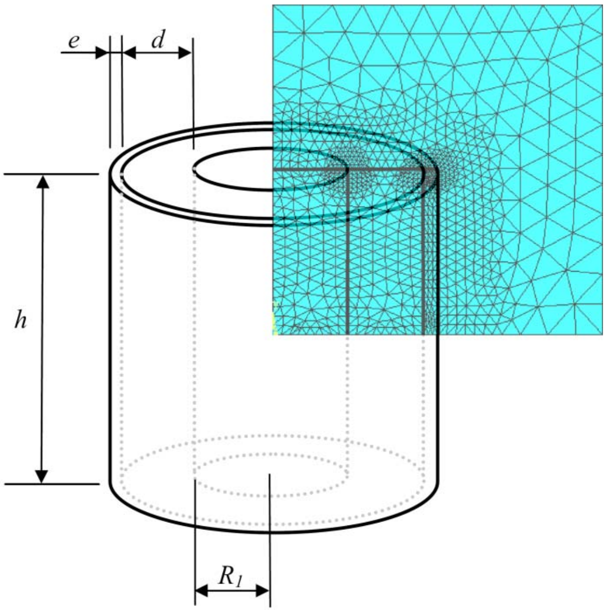

Fig. 7 2D axisymmetric real capacitor model.

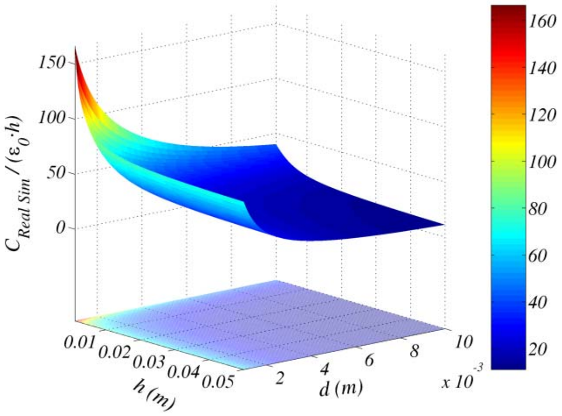

Fig. 8 Simulated real capacitance/length ε_0 related to distance and length.

In Fig. 8, it can be observed that the simulated real capacitance grows very quickly with respect to the theoretical capacitance when the variable length/thickness is reduced and to a lesser extent, when the variable separation/thickness increases.

The outer capacitance has been calculated with the Eq. (4), and the ratios of simulated real capacitance/theoretical capacitance and outer capacitance/inner capacitance are shown in Figs. 9 and 10, respectively.

A detailed statistical analysis of the 9,261 calculated values shows that the best fit is obtained relating the quotients of capacities with separation d and length h.

Table 2 Values of the geometric variables.

e (m)	d (m)	h (m)
0.100000 E-2	0.100000 E-2	0.100000 E-2
0.112202 E-2	0.112202 E-2	0.121604 E-2
0.125893 E-2	0.125893 E-2	0.147876 E-2
0.141254 E-2	0.141254 E-2	0.179823 E-2
0.158489 E-2	0.158489 E-2	0.218672 E-2
0.177828 E-2	0.177828 E-2	0.265915 E-2
0.199526 E-2	0.199526 E-2	0.323364 E-2
0.223872 E-2	0.223872 E-2	0.393224 E-2
0.251189 E-2	0.251189 E-2	0.478176 E-2
0.281838 E-2	0.281838 E-2	0.581482 E-2
0.316228 E-2	0.316228 E-2	0.707107 E-2
0.354813 E-2	0.354813 E-2	0.859871 E-2
0.398107 E-2	0.398107 E-2	0.104564 E-1
0.446684 E-2	0.446684 E-2	0.127154 E-1
0.501187 E-2	0.501187 E-2	0.154625 E-1
0.562341 E-2	0.562341 E-2	0.188030 E-1
0.630957 E-2	0.630957 E-2	0.228653 E-1
0.707946 E-2	0.707946 E-2	0.278051 E-1
0.794328 E-2	0.794328 E-2	0.338122 E-1
0.891251 E-2	0.891251 E-2	0.411170 E-1
0.100000 E-1	0.100000 E-1	0.500000 E-1

The radius of the inner conductor R_1 is 0.01 m, the relative dielectric permittivity of the dielectric medium between conductors ε_1 and the relative dielectric permittivity of the dielectric medium around the external conductor ε_2 is equal to 1. The assumed dielectric permittivity of the medium is 8.8541878 E-12 F/m.

The relation of the quotient of capacities to the separation is slightly parabolic almost linear. The relation of the quotient of capacities with the variable length is hyperbolic.

The simplest Eq. (5) that adjusts simulated to ideal values is:

$$\frac{C_{fitting}}{\varepsilon_0} = \sqrt[2]{k_1 \cdot h^2 + k_2 \cdot h} \cdot d^{k_3} \qquad (5)$$

where $k_1 = 0.622088$, $k_2 = 0.00546281$, $k_3 = -0.615925$ and ε_0 is the dielectric permittivity of the medium, air in this case. With this adjustment r-Squared is 98.97%.

3.2 Electric Field

A secondary result of the used model is the calculation of the distribution of the electric field between plates of the cylindrical capacitor and in the space that surrounds it. This distribution is shown in Fig. 11.

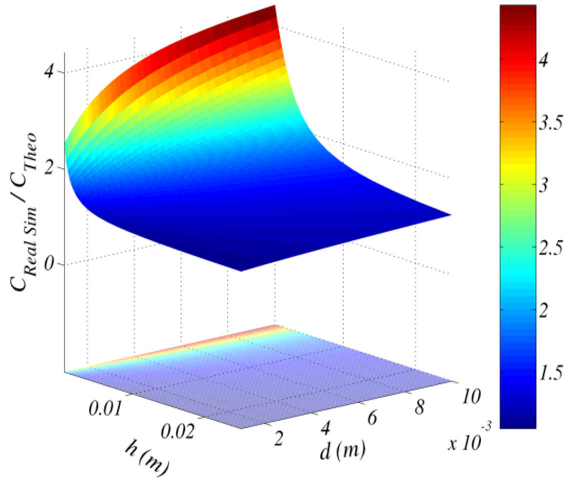

Fig. 9 Ratio of the simulated real capacitance and the theoretical capacitance related to distance and length.

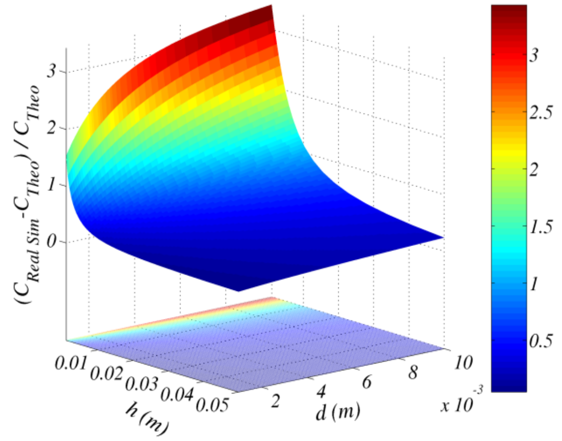

Fig. 10 Simulated real capacitance minus theoretical capacitance related to the theoretical capacitance as a function of the distance and length.

The obtained values of the electric field between plates agree very well with those obtained using Eq. (3) that is the theoretical expression for the calculation of the electric field between the conductors of a cylindrical capacitor (Fig. 12).

It is necessary to notice that the electric field is very high in the edges of the capacitor. The reason of this increase is the edge effect: the surface charge density increases in the edges and this causes an increase on the electric field [19-21]. This is an interesting result because it clearly shows that the lines of electric field outside of the inner dielectric are non-negligible [4, 5].

These two combined effects confirm the validity of Eq. (4).

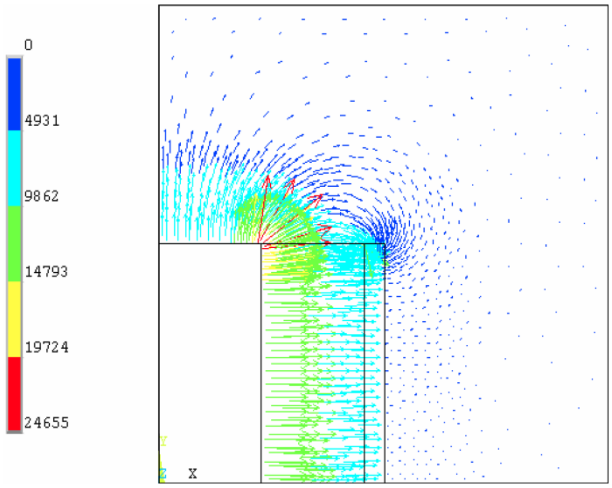

Fig. 11 Electric field distribution in the dielectric medium. Where: R_1 = 0.01 m, d = 0.01 m, e = 2 E-3 m and h = 0.044 m.

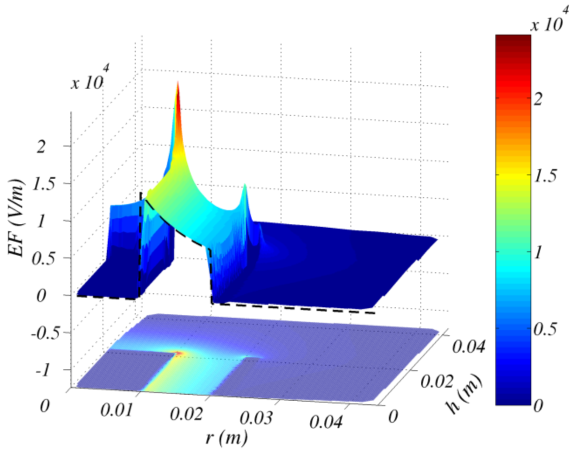

Fig. 12 Electric field distribution in simulated model. Where: R_1 = 0.01 m, d = 0.01 m, e = 2 E-3 m and h = 0.044 m. The dashed line is EF calculated with Eq. (3).

4. Conclusions

In this paper, the validity of the method of the finite elements has been demonstrated to calculate the capacitance of a cylindrical capacitor. Firstly, the capacitance between two conductors has been calculated and an expression has been obtained that allows calculating the real capacitance of a cylindrical capacitor.

It has been observed that in plates of small dimensions, the thickness of the conductors dramatically modifies the value of the capacitance compared to an ideal capacitor.

It has been shown that it is possible to calculate the capacitance of any system of conductors for the industrial elaboration of multiwire complex systems.

Nowadays, different iterative algorithms are used to calculate the capacitance of any model [8, 22-26], thanks to the performance that offer the computers. These algorithms are designed to obtain the surface charge distributions, the charge density and the capacitance. The disadvantage of these algorithms is that they are designed for a few specific cases, so that if the model changes, the algorithm must be reconstructed almost entirely.

In contrast to the disadvantage of the custom iterative algorithms, there are commercial software based on the FEM (finite elements method), which integrate generic algorithms applicable to the most popular problems of electrostatics, independently of the geometry. This increases the flexibility while cost and time of development are minimized. In this work, the authors have used the commercial software ANSYS [27] since this software has a specific module to solve problems of electrostatics.

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