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4 **ON THE RELATION BETWEEN EXTERNAL STRUCTURE AND INTERNAL**
5 **CHARACTERISTICS IN THE NEAR-NOZZLE FIELD OF DIESEL SPRAYS.**

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16
17 **ABSTRACT**

18 In this paper, a high-resolution visualization technique has been used in combination
19 with an extensively validated OD model in order to relate the external structure of a
20 diesel spray to the internal properties in the vicinity of the nozzle. For this purpose,
21 three single-hole convergent nozzles with different diameters have been tested for
22 several pressure conditions. The analysis of the obtained images shows that the spray
23 width significantly changes along the very first millimeters of the spray. From the high
24 resolution images captured, two parameters have been evaluated. The first one is the
25 external non-perturbed length, where droplet detachment has not been observed. The
26 second one is a transitional length, defined as the axial position where the spray width

27 increases linearly after a transient behavior, making it possible to establish a spray cone
28 angle definition. Furthermore, the internal liquid core length has been estimated for
29 these nozzles using an extensively validated zero-dimensional model. The intact liquid
30 core length has proved to be correlated with both the transitional length and the non-
31 perturbed length with a very high degree of reliability. In the case of the transitional
32 length, a quadratic correlation has been observed, whereas a linear relationship has been
33 confirmed between the intact core length and the non-perturbed length. The results
34 presented here may help to shed light on better understanding of such a complex
35 process as atomization.

36 **KEYWORDS:** Diesel spray, atomization, near-nozzle, high pressure injection, breakup
37 length, intact liquid core.

38

39 1. INTRODUCTION

40 The knowledge of the atomization process in diesel sprays is valuable due to the fact
41 that combustion efficiency and emissions are directly related to spray atomization and
42 fuel-air mixing processes. During the last decade, several tools were developed aiming
43 at the analysis of diesel spray behavior [1-8]. Nevertheless, due to the complexity of the
44 problem, there are still a lot of uncertainties on spray formation and break-up. Research
45 activities have been made in the last years to characterize sprays under high injection
46 pressure conditions by using visualization techniques focused on the nozzle vicinity
47 zone. In this sense, the authors analyzed the transient structures in the first millimeters
48 of diesel sprays using different optical techniques [9]. They observed that, during the
49 initial stage of the injection, the spray consists of a non-perturbed liquid column and an
50 umbrella-shaped structure in the nozzle tip. Linne et al. [10] studied the first 3
51 millimeters of the spray identifying and evaluating periodic structures on the spray
52 contour. Kastengren et al. [11, 12] used X-Ray techniques to measure the projected
53 mass distribution up to the first 5 millimeters of the spray.

54 Spray atomization has also been assessed by using numerical simulations, either using a
55 Reynolds-averaged Navier-Stokes (RANS) equations approach for turbulence modeling
56 [13, 14] or even using direct numerical simulations (DNS) [15, 16, 17] despite the high
57 computational cost of this kind of simulations. Som et al. [13] showed the differences in
58 the combustion process between two breakup models: the KH model and the KH-ACT
59 model, which consists of an improvement to the KH model by also considering
60 cavitation and turbulence phenomena. The inclusion of these improvements enhanced
61 the primary breakup process, causing smaller droplet sizes and a decrease in liquid
62 penetration. With regard to the flame lift-off length, the KH-ACT model predicted a lift-

63 off length closer to the experimental values. Shinjo et al. [15, 16] and Ménard et al. [17]
64 studied the diesel spray at low injection velocity. In [15, 16], the formation of ligaments
65 and droplets was studied at 30, 50 and 100 m/s. Lebas et al. [14] used the DNS
66 calculations of Menard et al. [17] to set the parameters and constants of an ELSA
67 (Eulerian-Lagrangian Spray Atomization) model, which was successfully tested with
68 experimental data in terms of liquid and vapor penetration.

69 It is well known that spray characteristics are highly influenced by the flow features at
70 the nozzle outlet [18, 19, 20, 21]. However, their study is very complicated due to the
71 small orifice diameters, the high velocities and the cavitation phenomenon that can take
72 place inside the nozzle, especially in non-convergent nozzles [22, 23, 24, 25, 26].
73 Additionally, many researchers have observed an important increase in the atomization
74 level and the spray angle connected to cavitation phenomenon [5, 19, 24, 27, 28].

75 In the present paper, the atomization process of Diesel sprays has been assessed by
76 visualizing the spray in the first millimeters. To this end, three single-hole convergent
77 nozzles with different diameters have been tested for a wide range of pressure
78 conditions. The tests have been carried out with a diffused backlighting technique,
79 performing the acquisition at two different image resolutions in order to focus in
80 different regions of spray. With all, the study region ranges from the nozzle tip to 5.5
81 mm away in the axial direction.

82 Two different parameters have been evaluated. The external non-perturbed length has
83 been obtained from the best resolution images, whereas a transitional length indicating
84 the axial position after an initial transient zone from which the spray width spreads
85 linearly with the axial position has been determined from the lowest resolution images.
86 From this transitional length onwards, a spray cone angle definition can be established

87 if the droplets in the border of the spray with axial position higher than the transitional
88 length are used. In parallel, the potential of a 0-dimensional model previously validated
89 for a wide range of conditions has made it possible to characterize the internal liquid
90 core for the different injection conditions tested on these nozzles. This parameter, which
91 is not experimentally accessible with the optical technique used in this investigation, has
92 been compared and correlated with the non-perturbed length and the transitional length.

93 The paper is divided into 5 sections. In Section 2, the visualization facility, the optical
94 setup and the technique used to process the images have been described. Section 3
95 includes the results and analysis of the images taken from visualization. Afterwards, a
96 theoretical model for the liquid core length is obtained in Section 4, where the results
97 from this model are also compared with previous experimental results in order to link
98 both the internal and external parameters in a spray. Finally, the most important
99 conclusions of the study have been pointed out in Section 5.

100

101 **2. EXPERIMENTAL TOOLS**

102 A Bosch common-rail fuel injection system with a solenoid-valve operated injector has
103 been used. A standard commercial diesel fuel has been chosen for the study. The main
104 physical and chemical characteristics of this fuel are reported in Table 1.

105 **2.1 Determination of the internal geometry of the nozzles**

106 A methodology based on silicone molding [29] has been employed to get information
107 on the internal geometry of the nozzles used. The results of the values obtained applying
108 this technique are displayed in Table 2, in which the values of diameter at the inlet and

109 at the outlet of the nozzle are shown. As it can be noted, the three nozzles are strongly
110 conical and therefore not prone to cavitate [19]. The degree of conicity of each nozzle is
111 evaluated by means of the *k-factor*, defined as:

$$k - factor = \frac{D_i[\mu m] - D_o[\mu m]}{10} \quad (1)$$

112 **2.2 Visualization setup**

113 The tests have been carried out with a diffused backlighting technique combined with
114 an optical setup that includes a biconvex lens, making it possible to achieve high
115 amplification ratios. A scheme of this optical setup is shown in Fig. 1. As it can be seen,
116 the laser source used for illumination and the CCD camera are placed in opposite sides
117 of the visualization test rig. The laser, the camera and the lens are aligned. A specific
118 drawing of the visualization test rig is shown in Fig. 2. It mainly consists of a stainless
119 steel cylinder with two optical windows. The upper cover contains the injector holder,
120 whereas the cover in the bottom contains the backpressure regulation system, which
121 makes use of N₂. The maximum pressure in the chamber is limited to 6 MPa due to
122 mechanical tolerances. A Nd-YAG laser operating in pulsed mode has been used as an
123 illumination source since it offers the possibility of using small shot duration (around 7
124 ns), which is needed to freeze the image and capture the structures of the spray. The
125 purpose of the optical diffuser placed after the laser (Fig. 1) is to produce uniform
126 illumination and to filter the high intensity, avoiding damages in the camera sensor. The
127 facility makes it possible to set the distances between the camera, lens and test rig in
128 order to get the pictures with the desired magnification ratio. These distances depend on
129 the size of the required visualization window, the characteristics of the lens, the size of
130 the CCD sensor and the refractive index of the fluid that fills the visualization chamber.

131 The characteristics of the lens are displayed in Table 3. These magnitudes are related to
132 the following equations:

$$\frac{1}{d_1} + \frac{1}{d_2} = \frac{1}{FL} \quad (2)$$

$$M = \frac{h_s}{h_w} = \frac{d_2}{d_1} \quad (3)$$

133 where FL is the focal length, d_1 is the distance from the spray axis to the lens, d_2 is the
134 distance from the CCD sensor to the lens, M is the magnification ratio, h_w is the size of
135 the visualization window and h_s is the camera sensor size (=7 mm.). The distances used
136 in the current investigation for both configurations are displayed in Table 4.

137 **2.3 Experimental methodology and acquired image processing**

138 As it has been mentioned, an optical facility has been used for visualizing Diesel sprays
139 at steady conditions, injecting in a pressurized chamber. With this aim, a set of 20
140 pictures has been acquired at the time instant that corresponds to full needle-lift
141 conditions, so that the flow characteristics are stabilized. Two picture resolutions have
142 been used: 250 pixels/mm (visualization window of 4.2x5.5 mm) and 1000 pixels/mm
143 (visualization window of 1.2x1.5 mm). The first resolution level has made it possible to
144 characterize the external spray morphology up to about 5.5 millimeters, and it has been
145 used to analyze the evolution of spray width. The second one has been useful to obtain
146 more specific information of the spray structure in the first 3 mm of the spray.

147 An injection pressure of 50 MPa has been tested for different values of chamber density,
148 which has been modified by controlling the chamber pressure. The values of chamber
149 pressure and their corresponding densities are shown in Table 5.

150 Pictures obtained from the visualization tests have been processed using an on-purpose
151 software developed and implemented in Matlab. This software uses an algorithm based
152 on Otsu's method [30] to detect the intensity threshold that defines the spray. This
153 method has proved to be useful for pictures that clearly show two regions (liquid and
154 gas) with different intensity levels [1, 6, 18, 19, 20, 21]

155

156 **3. EXPERIMENTAL RESULTS AND ANALYSIS**

157 Fig. 3 shows two samples of the pictures acquired for the highest resolution
158 configuration. They belong to nozzles A and C at an injection pressure of 50 MPa and a
159 chamber pressure of 1 MPa. This resolution allows the visualization of the first 1.5 mm
160 of the spray. These images are analyzed afterwards, but at first glance it can be seen that
161 there is a region near the nozzle where the spray width is practically constant and equal
162 to the outlet diameter. Additionally, it can be seen that this region is longer for the
163 Nozzle C, which has a larger nozzle diameter.

164 Fig. 4 displays two samples of pictures using the lowest resolution. This kind of image
165 makes it possible to visualize the spray up to a distance of around 5 mm from the nozzle
166 in the axial direction. These images belong to the nozzle A, at the injection pressure of
167 50 MPa and two different backpressures of 1 MPa (left) and 2.5 MPa (right). In this
168 case, it can be clearly noted that an increase in the chamber pressure leads to a higher
169 spray width due to the influence of chamber density on the air-fuel mixing process.

170 The spray width has been determined by the images processing algorithm. Its axial
171 evolution has been analyzed for all the nozzles and experimental conditions tested in
172 order to study the near-nozzle field structure. As a sample, the contour obtained for

173 Nozzle A for the backpressure of 1 MPa is displayed in the bottom part of Fig. 5.
174 Additionally, a linear fit applied to the spray contour points located far from the nozzle
175 is depicted as a solid line. According to the contour shape, it is possible to distinguish
176 three different zones in the spray:

- 177 - Zone 1 (until ~ 0.4 mm): the spray width is constant and equal to the nozzle
178 outlet diameter. It defines the non-perturbed length (L_{np}).
- 179 - Zone 2 (from ~ 0.4 mm until ~ 2.2 mm): atomization takes place and the evolution
180 of spray width with the axial position is not linear. The distance from the nozzle
181 to the end of this zone is called transitional length (L_t).
- 182 - Zone 3 (from ~ 2.2 mm onwards): the contour profile follows a linear fit with
183 high accuracy.

184 As shown in the upper part of Fig. 5, in addition to the transitional length and the non-
185 perturbed length, there is a third parameter related to the internal liquid core length, L_c .
186 This parameter is not possibly determined by the visualization technique carried out in
187 this investigation. Other techniques might be used in order to assess this internal
188 characteristic length, such as X-ray measurements [11, 12]. In the current study, a 0-
189 dimensional model able to predict the liquid core length and the axial velocity drop
190 along the spray axis has been used in order to compare the intact liquid core length with
191 the non-perturbed length and the transitional length experimentally determined. This
192 model has been previously validated using Particle Doppler Anemometry [31] and X-
193 ray measurements of mass distribution in the primary break-up zone of the spray [7, 8].

194 In order to precisely characterize the non-perturbed length, the pictures with the best
195 resolution have been used. The transitional length and the spray cone angle have been
196 determined using the lowest resolution pictures.

197 **3.1. Spray cone angle analysis**

198 The spray cone angle is normally used to assess the efficiency of the mixing process. Its
199 value is mainly dependent on nozzle geometry [18], the presence or absence of
200 cavitation phenomenon [5, 19] and chamber density [21, 31, 32, 33]. This parameter is
201 usually determined taking the assumption that the spray is similar to a cone, performing
202 a linear fit to both the upper and the lower parts of the spray contour and determining
203 the angle formed by both lines. However, this fit would only be accurate from the
204 transition length onwards. To solve this problem, the contour is treated as shown in Fig.
205 6. It is first divided in segments of 50 pixels, starting from the end of the image. From
206 these segments, a series of b_i vectors is defined, including the coordinates of the contour
207 points corresponding to each of the segments in a cumulative way (i.e. the b_1 vector
208 includes the points of the first segment of the contour, the b_2 vector includes the ones
209 corresponding to the first and the second segment, and so on). With the information of
210 each vector it is possible to obtain a linear fit over both the upper and the lower side of
211 the spray contour, making it possible to calculate the angle among both. While the spray
212 contour exhibits a conical shape, the error when performing both linear fits will
213 diminish as the number of segments increases, since more points will be available.
214 However, when the spray appearance deviates from this linear trend, the associated
215 error to the linear fits will increase. Thus, the spray angle is taken as the one defined by
216 the segment that leads to a lower error when performing the linear fit.

217 The evolution of spray cone angle against chamber density (ρ_a) is represented in Fig. 7
218 for all the nozzles. Measurements are displayed with the standard deviation. As
219 expected, the higher the air density in the chamber, the higher the spray cone angle. This
220 is due to aerodynamic interaction between the fuel and the air in the chamber.
221 Regarding the comparison between the different nozzles, neither significant nor clear
222 influence of the nozzle diameter on the spray cone angle can be confirmed.

223 **3.2. Non-perturbed length and transitional length**

224 As it has been mentioned, the spray shows an initial region at which the spray width is
225 constant, which has been defined as non-perturbed length (L_{np}). The values of non-
226 perturbed length for all the nozzles and the different backpressures tested are displayed
227 in the bottom part of Fig. 8. A decrease on this parameter when chamber density
228 increases can be noted, due to the effect of the aerodynamic forces on the primary
229 atomization process. A significant and clear influence of the nozzle diameter on the
230 non-perturbed length is noticed, as opposed to the spray cone angle results previously
231 exposed: the higher the nozzle diameter, the higher the non-perturbed length. Therefore,
232 the highest values of L_{np} are seen for Nozzle C, followed by Nozzle B, whereas the
233 lowest values are observed for Nozzle A. As far as the transitional length is concerned,
234 the values obtained from the analysis of the images are depicted against chamber
235 density in the upper part of Fig. 8. As it can be observed, it exhibits a similar trend
236 against density as the one observed for the non-perturbed length. Thus, the transitional
237 length decreases as chamber density increases as a consequence of its effect on
238 atomization and air-entrainment processes. Additionally, it is noticeable that the nozzle
239 outlet diameter has a strong influence on transitional length, showing a similar trend as
240 the one seen for the non-perturbed length. If both parameters are compared for a given

241 density (chamber pressure) it can be seen that the transitional length values are higher
242 than the non-perturbed length ones, although they have the same order of magnitude.

243 **4. MODEL FOR LIQUID CORE LENGTH AND RELATION WITH PREVIOUS** 244 **EXPERIMENTALLY DETERMINED PARAMETERS.**

245 **4.1 Theoretical derivation.**

246 The model is obtained under the hypothesis of momentum flux conservation along the
247 spray axis. This hypothesis was validated using momentum measurements [1], and it
248 implies that:

$$\dot{M}_o = \dot{M}(x) \quad (4)$$

249 where $\dot{M}(x)$ and \dot{M}_o are the momentum flux at a section at a distance x from the
250 nozzle tip in the axial direction and the momentum flux at the orifice outlet,
251 respectively. Momentum flux at the nozzle outlet is defined as:

$$252 \quad \dot{M}_o = \dot{m}_f \cdot U_o \quad (5)$$

253 If Eq. (4) is integrated over the whole spray section, it can be written as:

$$\dot{M}_o = \dot{M}(x) = \int_0^{\infty} 2\pi\rho(x,r)rU^2(x,r)dr \quad (6)$$

254 where the x -coordinate follows the axial direction and the r -coordinate is perpendicular
255 to the spray axis. In Eq. (6), $U(x,r)$ is the local spray velocity and $\rho(x,r)$ is the local
256 density. If a Gaussian profile is assumed for both fuel concentration and axial velocity,
257 the integration of Eq. (6) leads to Eq. (7). It is important to remark at this point that the

258 Gaussian profile has proved to be suitable to explain the radial distributions of
 259 concentration and velocity in Diesel sprays [1, 5, 7, 8, 31, 34].

$$\dot{M}_o = \frac{\pi}{2\alpha} \rho_a \tan^2\left(\frac{\theta_u}{2}\right) x^2 U_{axis}^2 \sum_{i=0}^{\infty} \frac{1}{\left(1+i\frac{Sc}{2}\right)} \left[C_{axis}(x) \left(\frac{\rho_f - \rho_a}{\rho_f} \right) \right]^i \quad (7)$$

260 All the steps followed in the integration of Eq. (6) can be found in Desantes et al. [35].

261 In Eq. (7), ρ_a and ρ_f are the air density and the fuel density, respectively, whereas α is
 262 the shape parameter of the Gaussian profile and Sc is the effective Schmidt number,
 263 which is defined as the ratio of momentum diffusivity to mass diffusivity:

$$264 \quad Sc = \frac{\nu}{D} \quad (8)$$

265 being D the mass diffusivity and ν the kinematic viscosity.

266 The spray velocity angle θ_u is defined by the points in the border of the spray at which
 267 velocity drops 1% of its value at the spray axis for the same axial coordinate.

268 This model was extensively validated in previous studies [31, 35], both in terms of local
 269 velocity and local mass concentration, by means of spray momentum flux
 270 measurements and PDA (phase doppler anemometry) measurements, among others.

271 The momentum flux at the nozzle outlet can also be defined as:

$$\dot{M}_o = \rho_f A U_o^2 \quad (9)$$

272 where A is the area of the nozzle orifice at the outlet and U_o is the effective injection
 273 velocity at this location. Substituting Eq. (9) in Eq. (7), it can be easily transformed into:

$$\rho_f A = \frac{\pi}{2\alpha} \rho_a \tan^2\left(\frac{\theta_u}{2}\right) x^2 \left(\frac{U_{axis}}{U_o}\right)^2 \sum_{i=0}^N \frac{1}{\left(1+i\frac{Sc}{2}\right)} \left[C_{axis}(x) \left(\frac{\rho_f - \rho_a}{\rho_f}\right) \right]^i \quad (10)$$

274 where N is the number of terms used in series truncation. Previous studies show that
 275 axial concentration and velocity can be related in terms of Schmidt number [36] as:

$$\left(\frac{U_{axis}(x)}{U_o}\right) = C_{axis}(x)^{Sc} \quad (11)$$

276 If Eq. (11) is introduced in Eq. (10), an implicit equation for C_{axis} as a function of Sc can
 277 be obtained:

$$1 = \frac{\pi}{2\alpha} \frac{\rho_a}{\rho_f} \frac{1}{A} \tan^2\left(\frac{\theta_u}{2}\right) x^2 (C_{axis}(x))^{2Sc} \sum_{i=0}^N \frac{1}{\left(1+i\frac{Sc}{2}\right)} \left[C_{axis}(x) \frac{\rho_f - \rho_a}{\rho_f} \right]^i \quad (12)$$

278 Finally, considering that the spray mass angle (θ_m) is related to the spray velocity angle
 279 (θ_u) through the following equation:

$$\tan\left(\frac{\theta_u}{2}\right) = \sqrt{Sc} \tan\left(\frac{\theta_m}{2}\right) \quad (13)$$

281 and introducing Eq. (13) in Eq. (12), the following expression is obtained:

$$1 = \frac{\pi}{2\alpha} \frac{\rho_a}{\rho_f} \frac{1}{A} Sc \tan^2\left(\frac{\theta_m}{2}\right) x^2 (C_{axis}(x))^{2Sc} \sum_{i=0}^N \frac{1}{\left(1+i\frac{Sc}{2}\right)} \left[C_{axis}(x) \frac{\rho_f - \rho_a}{\rho_f} \right]^i \quad (14)$$

282 In previous works by the authors [7, 8], projected mass distributions obtained from
 283 experiments based on X-ray absorption were transformed into mass concentration in the
 284 axis for different nozzles and conditions and compared to the results provided by this
 285 model (Eq. 14). As a sample of this procedure, Fig. 9 shows a comparison among the

286 mass concentration in the axis and the mass concentration predicted by the model for
 287 different Sc numbers between 0.5 and 1. The filled circles represent the values
 288 reconstructed from X-ray measurements following the procedure explained in [7, 8].
 289 This measurement belongs to a nozzle with orifices of 131 μm of diameter and $P_{inj} = 80$
 290 MPa and $P_b = 1.85$ MPa ($\rho_a = 21$ kg/m³).

291 As it can be noted, Sc has an important influence on C_{axis} evolution until an axial
 292 position of 75-80 D_o (~10-12 mm), where the difference between the curves becomes
 293 almost indiscernible. As can be seen, attending to the behavior of C_{axis} , two different
 294 zones can be defined. From 30 D_o (i.e. ~4 mm) onwards, the axial concentration is well
 295 reproduced by the theoretical model for $Sc = 0.5$. On the contrary, for positions up to
 296 nearly 30 D_o , C_{axis} does not follow any specific theoretical curve. This is mainly due to
 297 fact that, as reported in Section 3, the spray cone angle near the nozzle outlet (in the
 298 zone close to the intact core length) is not well established (see results depicted in Fig.
 299 6). Despite this limitation of the model, as shown in Fig. 9, a very good estimation of
 300 the intact core length (further point in the axis with $C_{axis} = 1$) can be obtained using the
 301 0-D model when Sc number equals the unity. This result was also observed for other
 302 different nozzles and conditions in previous investigations [7, 8] using X-ray
 303 measurements. Thus, in this situation ($Sc = 1$), it is possible to obtain an explicit
 304 expression for the intact liquid core length (L_c). Indeed, particularizing Eq. (14) for $Sc =$
 305 1 and $C_{axis}(x) = 1$, the following expression is obtained for $x = L_c$:

$$306 \quad L_c = \frac{\left(\frac{\alpha}{2}\right)^{1/2} \cdot D_{eq}}{\tan\left(\frac{\theta_m}{2}\right)} \cdot \frac{I}{\left[\sum_{i=0}^N \frac{1}{\left(I + \frac{i}{2}\right)} \left[\left(\frac{\rho_f - \rho_a}{\rho_f}\right)^i \right] \right]^{1/2}} \quad (15)$$

307 where the definition of equivalent diameter ($D_{eq} = D_o \sqrt{\rho_f / \rho_a}$) has been used, and the
308 area A in Eq. (14) has been substituted by:

$$309 \quad A = \frac{\pi \cdot D_o^2}{4} \quad (16)$$

310 Chehroudi et al. [37] found the following expression for the normalized liquid core
311 length:

$$312 \quad L_c / D_o = C_c \cdot (\rho_f / \rho_a)^{0.5} \quad (17)$$

313 being C_c an empirical constant in the range from 7 to 16. This expression, although
314 simpler than Eq. (14), keeps the same dependency with the density ratio as the one
315 expressed by Eq. (14) (if the definition of the equivalent diameter is taken into account).

316 In the next Section, the evaluation of the intact core length will be addressed for all the
317 nozzles and operating conditions and it will be correlated with the previously examined
318 external parameters (non-perturbed length and transitional length).

319 **4.2 Evaluation of the intact core length and comparison with previous** 320 **experimentally determined parameters.**

321 The liquid core length can now be evaluated for the three nozzles and different injection
322 conditions. In Fig. 10, the liquid core length evaluated by means of Eq. (15) has been
323 depicted for all nozzles and densities in the chamber. As expected according to Eq. (15),
324 there is a great influence of the chamber air density on the liquid core length: the higher
325 the density, the higher the spray angle (as shown in Fig. 7), and therefore the shorter the
326 liquid core length due to the enhanced air entrainment. As far as the influence of the
327 orifice diameter is concerned, the higher the diameter, the longer the liquid core length,

328 as can be clearly seen for all the conditions displayed in Fig. 10. The same conclusion is
329 reached if the equivalent diameter is considered. It should be noted that, although the
330 difference in the liquid core length between the three nozzles is reduced in absolute
331 terms when increasing the chamber density, their differences in relative terms remain
332 similar. This result was expected in the light of Eq. (15), due the differences in D_{eq}
333 among nozzles and the fact that there is no clear influence of the nozzle on the spray
334 angle, as pointed out in Section 3.1.

335 If results of intact core length are compared to the previous results of non-perturbed
336 length and transitional length, it can be concluded that even though the intact core
337 length is quite higher than both of them in overall terms, the intact core and the
338 transitional length come closer for high densities. For instance, whereas the values for
339 the liquid core, transitional length and non-perturbed length for nozzle 156 at 5.8 kg/m^3
340 of density are 12.5 mm, 2.8 mm and 0.85 mm, respectively, the values encountered for a
341 density of around 60 kg/m^3 are 2 mm, 1.7 mm and 0.2 mm.

342 A non-dimensional intact liquid core length can be obtained by dividing this parameter
343 by the equivalent diameter. This non-dimensional intact length has been depicted in the
344 upper part of Fig. 11 against the normalized transitional length. As can be noted, there is
345 a clear quadratic correlation among both parameters. The mathematical expression that
346 better fits this relation is:

$$347 \quad \frac{L_c}{D_{eq}} = 0.0199 \cdot \left(\frac{L_t}{D_{eq}} \right)^2 \quad (18)$$

348 with a coefficient of determination R^2 equal to 0.99.

349 If the values of non-dimensional liquid core length are compared to the corresponding
 350 non-dimensional non-perturbed length, the results displayed in the bottom part of Fig.
 351 11 are obtained. As can be noted, the dependency between both parameters is linear in
 352 this case, obtaining the following equation that relates them:

$$353 \quad \frac{L_c}{D_{eq}} = 12.9245 \cdot \frac{L_{np}}{D_{eq}} \quad (19)$$

354 with a coefficient of determination R^2 equal to 0.97.

355 The correlations obtained for the transitional length and non-perturbed length make it
 356 possible to determine the dependencies of those parameters with the equivalent diameter
 357 (including geometrical diameter, D_o , the fuel properties, ρ_f , and the density in the
 358 chamber, ρ_a) and the spray cone angle, $\tan\left(\frac{\theta_m}{2}\right)$. Indeed, as established by Eq. (15), the
 359 liquid core length depends on the equivalent diameter and the spray cone angle as
 360 follows:

$$361 \quad L_c \propto \frac{D_{eq}}{\tan\left(\frac{\theta_m}{2}\right)} \quad (20)$$

362 where the term involving the series in the denominator in Eq. (15) has been neglected as
 363 a first approximation and for simplicity reasons:

364 Taking into account Eq. (18), the transitional length can be written as:

$$365 \quad \frac{L_t}{D_{eq}} \propto \sqrt{\frac{L_c}{D_{eq}}} \quad (21)$$

366 and therefore:

367 $L_t \propto \sqrt{D_{eq} \cdot L_c}$ (22)

368 Introducing Eq. (20) into Eq. (22) yields:

369 $\frac{L_c}{L_t} \propto \frac{1}{\sqrt{\tan\left(\frac{\theta_m}{2}\right)}}$ (23)

370 This last equation helps explaining the quadratic correlation observed in Fig. 11 (upper
 371 part): for lower chamber densities (and therefore smaller spray cone angles), differences
 372 between both parameters become higher, whereas for higher chamber densities (and
 373 therefore bigger spray cone angles), the differences become smaller. Table 6 shows a
 374 comparison among the values of $1/\sqrt{\tan\left(\frac{\theta_m}{2}\right)}$ and the values of the ratio L_c/L_t for all the
 375 nozzles and density conditions, using the previously experimentally (L_t) and
 376 theoretically derived (L_c) values. As can be noted, even though both values differ
 377 because Eq. (23) only shows a proportional relationship, they show a similar trend when
 378 moving from low densities to high densities.

379 With regard to the non-perturbed length, a linear correlation was found between L_{np} and
 380 L_c (recall Eq. (19)). Thus, carrying out a similar procedure to the one previously
 381 described for the transitional length would lead to the conclusion that the dependencies
 382 of this parameter with the equivalent diameter and spray cone angle are the same as in
 383 the case of the liquid core length, i.e.:

384 $L_{np} \propto \frac{D_{eq}}{\tan\left(\frac{\theta_m}{2}\right)}$

385

386 **5. CONCLUSIONS**

387 In the current paper, a visualization technique has been used to study the stationary
388 spray structure in the vicinity of the nozzle. Two different levels of image resolution
389 have been obtained: a visualization window of around 5 mm from which the axial
390 evolution of spray width has been characterized, and a window of 1.5 mm that has made
391 it possible to evaluate the external non-perturbed length. A qualitative analysis of the
392 spray contour has shown the existence of three different zones in the spray attending to
393 the axial evolution of the spray width: a first zone, where spray width is equal to the
394 nozzle outlet diameter; a second zone, called transitional zone, at which air-entrainment
395 has already begun but where the evolution of spray width is not linear; and a third zone
396 (or steady-state region) characterized by a linear spray width evolution defined by a
397 steady spray cone angle value.

398 Spray cone angle has shown to be similar for the three nozzles tested, with a significant
399 influence of the density. No clear dependency with the nozzle outlet diameter has been
400 observed. With regard to the non-perturbed length, it has been seen that it decreases
401 when chamber density increases due to the effect of aerodynamic forces on the primary
402 atomization process. In this case, there is a significant and clear influence of the
403 diameter on the non-perturbed length: the higher the diameter, the higher the non-
404 perturbed length. The transitional length (axial distance from the nozzle outlet at which
405 the spray width starts its linear evolution) has shown a similar behavior as the non-
406 perturbed length, but with higher values.

407 An equation for the liquid core length has been derived using a previously validated
408 model. According to this model, the liquid core length depends mainly on the air
409 density (or more generally the fuel/air density ratio) and the nozzle diameter. For all the

410 nozzles and conditions, the liquid core length has exhibited the highest values for the
411 nozzle with the highest diameter for the lowest air density in the chamber, whereas it
412 has shown the lowest values for the nozzle with the lowest diameter for the highest air
413 density in the chamber. The estimated values of liquid core length have been compared
414 with the experimentally obtained transitional length and non-perturbed length values.
415 As a result of the comparison, the non-dimensional liquid core length (normalized using
416 the equivalent diameter) has shown to correlate with a very high level of confidence
417 ($R^2= 0.99$) with the non-dimensional transitional length. In this case, a quadratic
418 equation has been found to be the best approach to describe the relationship among both
419 parameters. On the other hand, when comparing the non-dimensional liquid core length
420 with the external non-perturbed length, a linear relationship between them has been
421 found. Again, the coefficient of determination found is close to 1, highlighting the
422 potential of the correlation for predicting the liquid core length from the non-perturbed
423 length, or vice versa.

424 The analysis of the obtained correlations allows to conclude that the ratio among the
425 transitional length and the liquid core length is proportional to the square root of the
426 half-angle tangent. This result would explain the quadratic correlation found among
427 both parameters. In the case of the non-perturbed length, the dependency with the
428 equivalent diameter and the angle is exactly the same as the one for the liquid core
429 length.

430 **Acknowledgements**

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- 544

545 **TABLES AND FIGURE CAPTIONS**

546 Table 1: Physical and chemical properties of Diesel fuel used in the experiments.

Test	Unit	Result	Uncertainty
Density at 15°C	Kg/m ³	843	±0.2
Viscosity at 40°C	mm ² /s	2.847	±0.42
Volatility			
65% distilled at	°C	294.5	±3.7
85% distilled at	°C	329.2	±3.7
95% distilled at	°C	357.0	±3.7
Average fuel molecular composition		C ₁₃ H ₂₈	

547

548 Table 2: Results for nozzles geometry by silicone moulding technique

Nozzle	D_i [μm]	D_o [μm]	k -factor
Nozzle A	140	112	2.8
Nozzle B	167	138	2.9
Nozzle C	195	156	3.9

549

550 Table 3: Biconvex lens characteristics

Focal length (FL)	100 mm
Lens diameter	50 mm
Material	BK7
Refractive index	1.52

551

552 Table 4: Distances between elements for the two optical configurations used

Visualization window[mm]	d_1 [mm]	d_2 [mm]
1.2 x 1.5	131	566
4.2 x 5.5	188	227

553

554 Table 5: Values of chamber pressure tested and their associated chamber densities.

Chamber pressure [MPa]	Chamber density [kg/m ³]
0.5	5.77
1.0	12.00
1.5	16.79
2.5	28.76
3.5	40.25
5.0	57.49

555

556

557 Table 6: Values of different spray parameters for all the nozzles.

Nozzle A		
ρ_a [kg/m ³]	$\frac{1}{\sqrt{\tan(\theta/2)}}$	$\frac{L_c}{L_r}$
5.77	4.31	5.38
11.99	3.34	2.74
16.79	3.20	2.34
28.76	3.04	1.98
40.25	2.72	1.72
Nozzle B		
ρ_a [kg/m ³]	$\frac{1}{\sqrt{\tan(\theta/2)}}$	$\frac{L_c}{L_r}$
5.77	3.86	4.47
11.99	3.42	3.09
16.79	3.21	2.43
28.76	2.81	1.61
40.25	2.62	1.37
Nozzle C		
ρ_a [kg/m ³]	$\frac{1}{\sqrt{\tan(\theta/2)}}$	$\frac{L_c}{L_r}$
5.77	3.84	4.69
11.99	3.39	3.02
16.79	3.22	2.51
28.76	2.81	1.67
40.25	2.71	1.59

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561

562 **FIGURE CAPTIONS**

563 Figure 1: Experimental setup for near-nozzle field visualization.

564 Figure 2: Near-nozzle field visualization test rig.

565 Figure 3: Samples of images obtained from nozzles A and C with image resolution of
566 1.2 x 1.5 mm at an injection pressure of 50 MPa and backpressure of 1 MPa.

567 Figure 4: Samples of images obtained from nozzle A with image resolution of 4.2 x
568 4.5 mm at an injection pressure of 50 MPa and backpressures of 1MPa (left) and 2.5
569 MPa (right).

570 Figure 5: Parameters evaluated from the images of the spray: Non-perturbed length
571 (L_{np}) and transitional length (L_t).

572 Figure 6: Spray angle determination method.

573 Figure 7: Spray angle as a function of the chamber density for the different nozzles.

574 Figure 8: Transitional length and non-perturbed length for different nozzles and
575 density conditions.

576 Figure 9: Mass concentration in the axis of the spray: experimental and modeled for
577 different Schmidt numbers. Liquid core length determination.

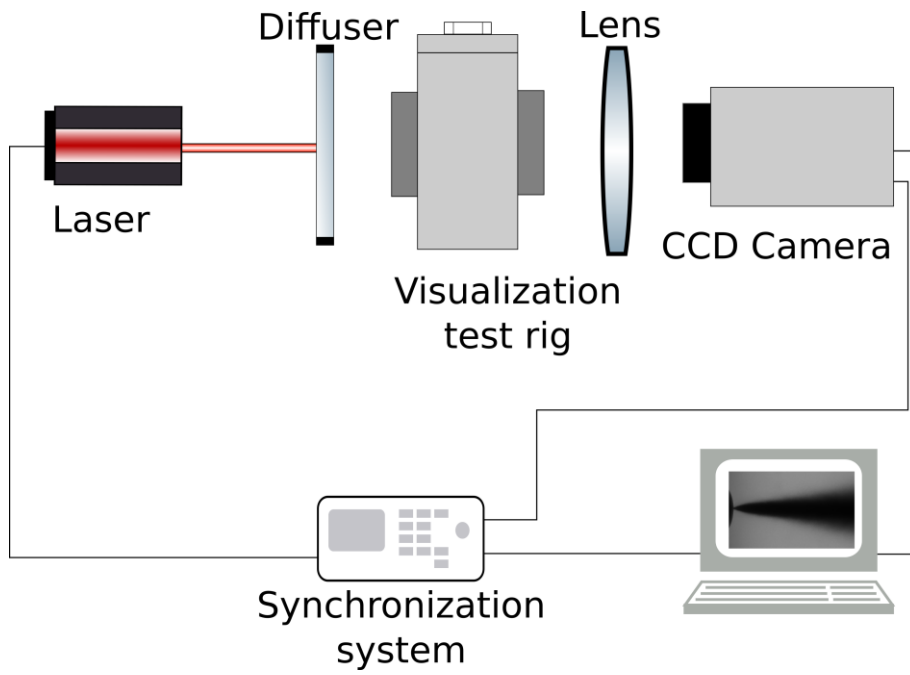
578 Figure 10: Intact core length as a function of density in the chamber.

579 Figure 11: Non-dimensional intact core length vs transitional length (upper part) and
580 Non-dimensional intact core length vs external non-perturbed length (bottom part).

581

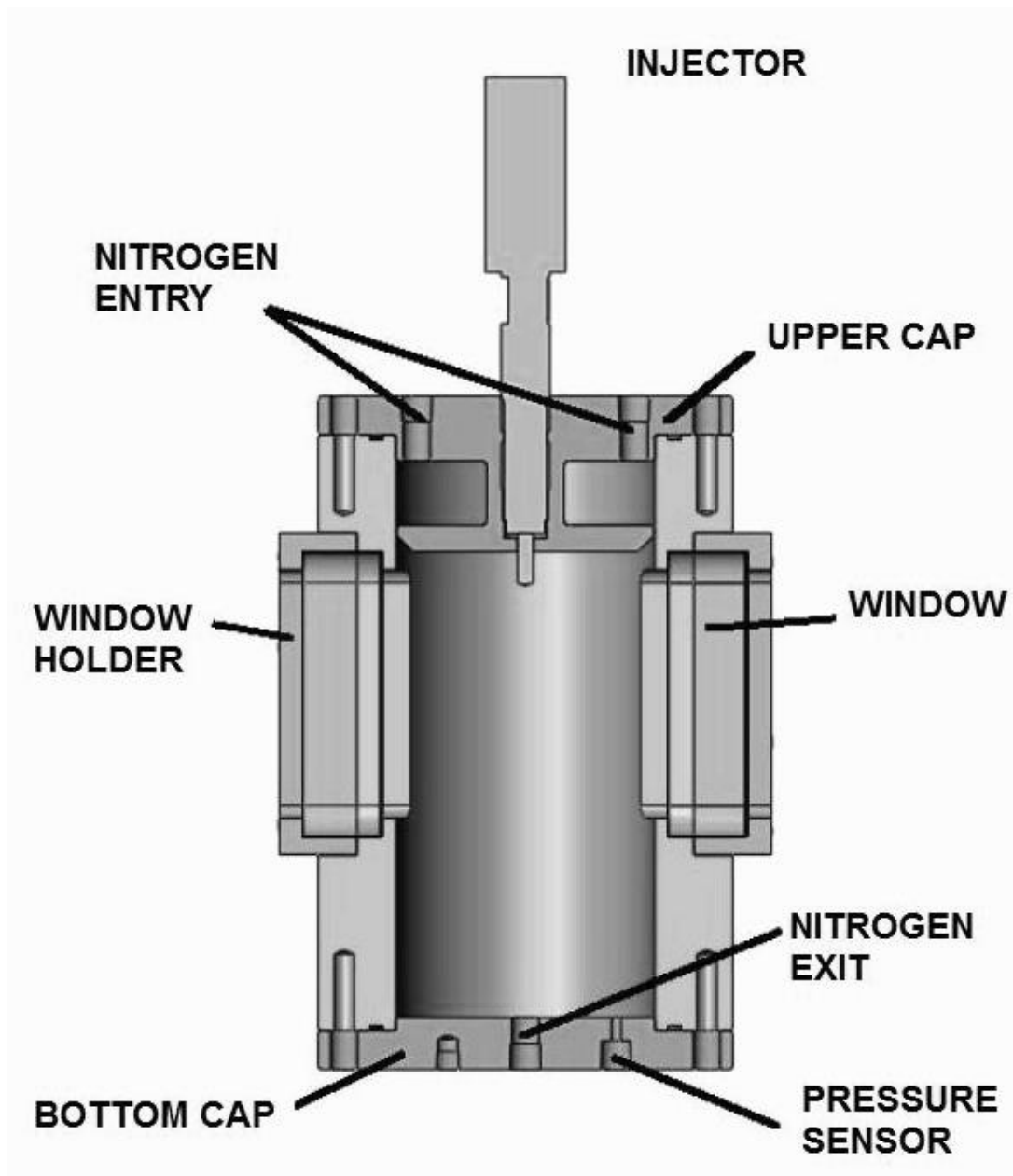
582 **FIGURES**

583



584

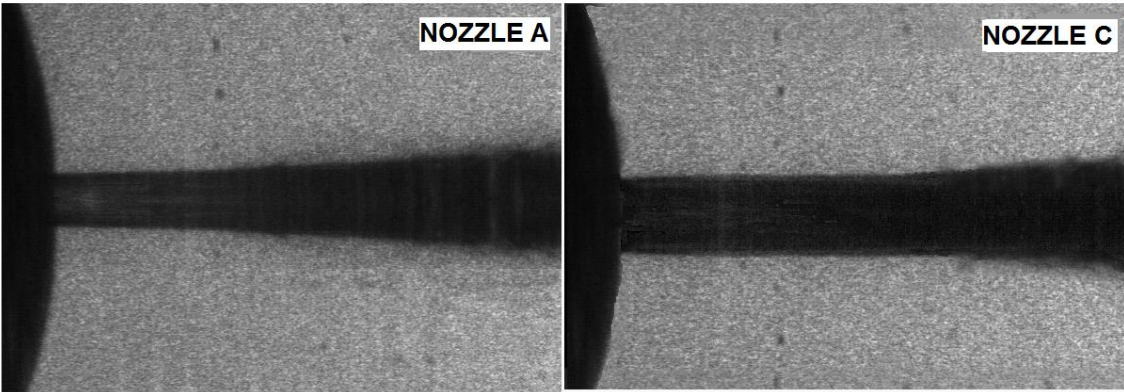
585 Figure 1: Experimental setup for near-nozzle field visualization



586

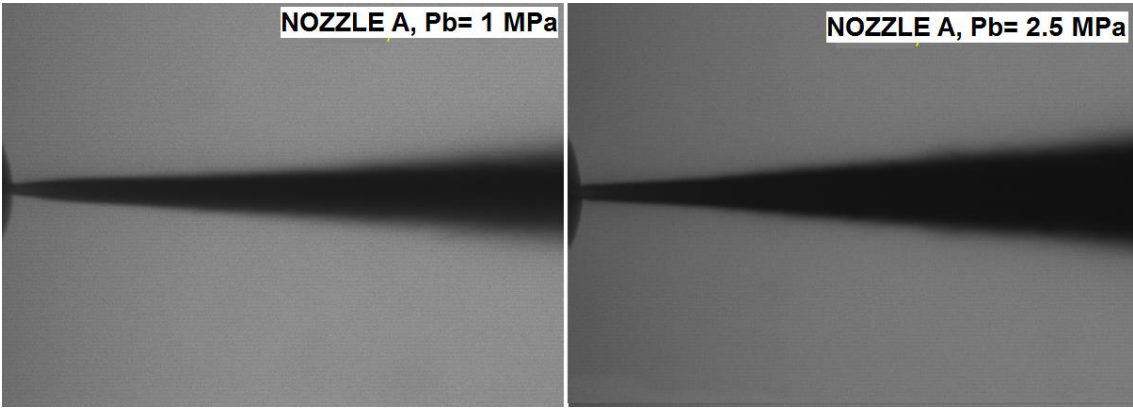
587 Figure 2: Near-nozzle field visualization test rig

$P_{inj}=50\text{MPa}$, $P_b=1\text{ MPa}$



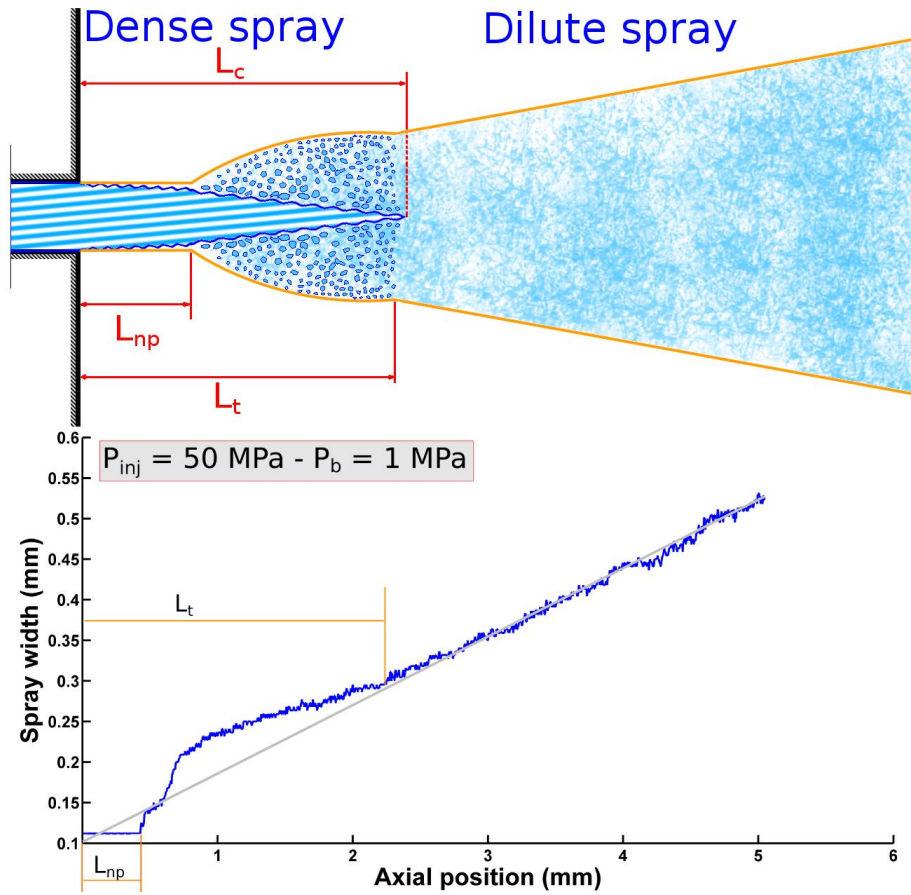
588

589 Figure 3: Samples of images obtained from nozzles A and C with image resolution of
590 1.2 x 1.5 mm at an injection pressure of 50 MPa and backpressure of 1 MPa.



591

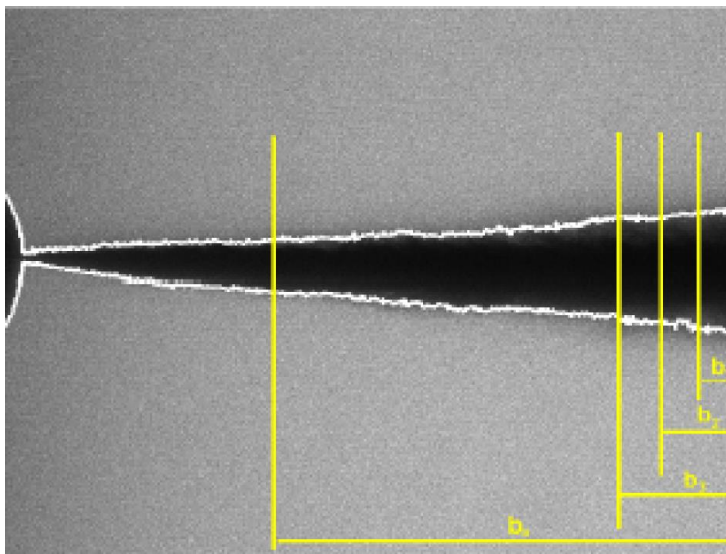
592 Figure 4: Samples of images obtained from nozzle A with image resolution of 4.2 x
593 4.5 mm at an injection pressure of 50 MPa and backpressures of 1MPa (left) and 2.5
594 MPa (right).



595

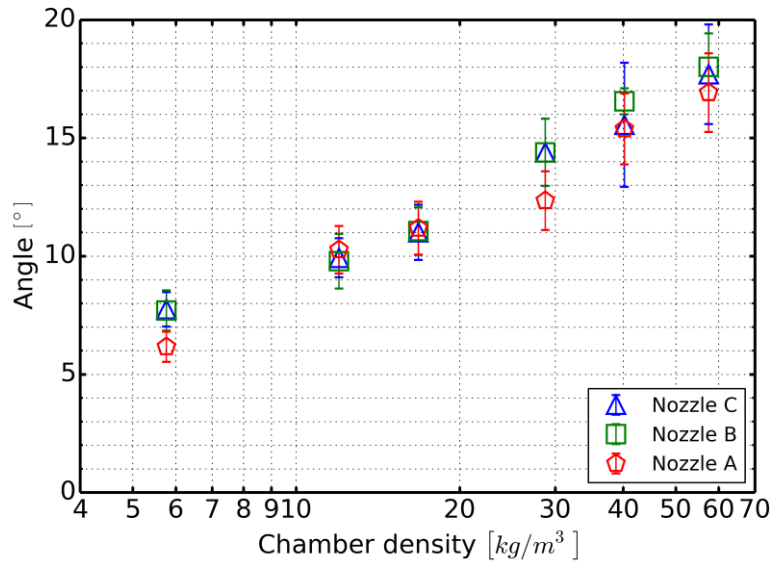
596 Figure 5: Parameters evaluated from the images of the spray: Non-perturbed length

597 (L_{np}) and transitional length (L_t).



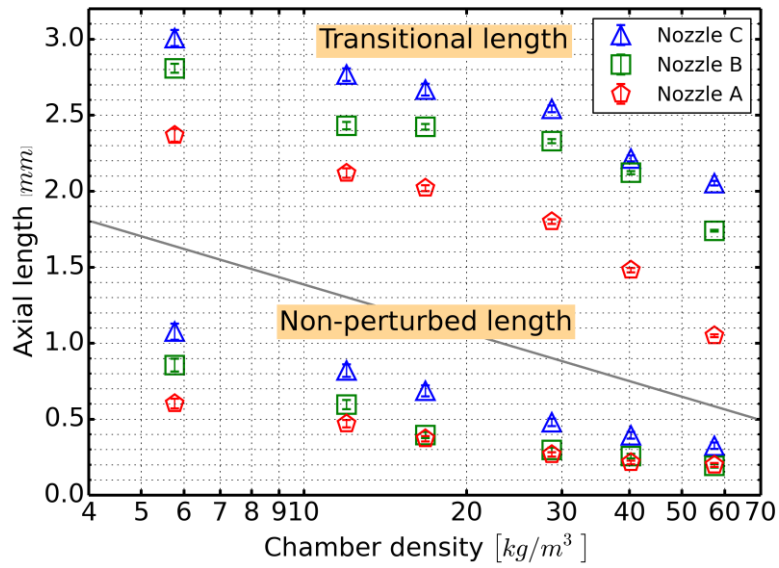
598

599 Figure 6: Spray angle determination method.



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601 Figure 7: Spray angle as a function of the chamber density for the different nozzles.



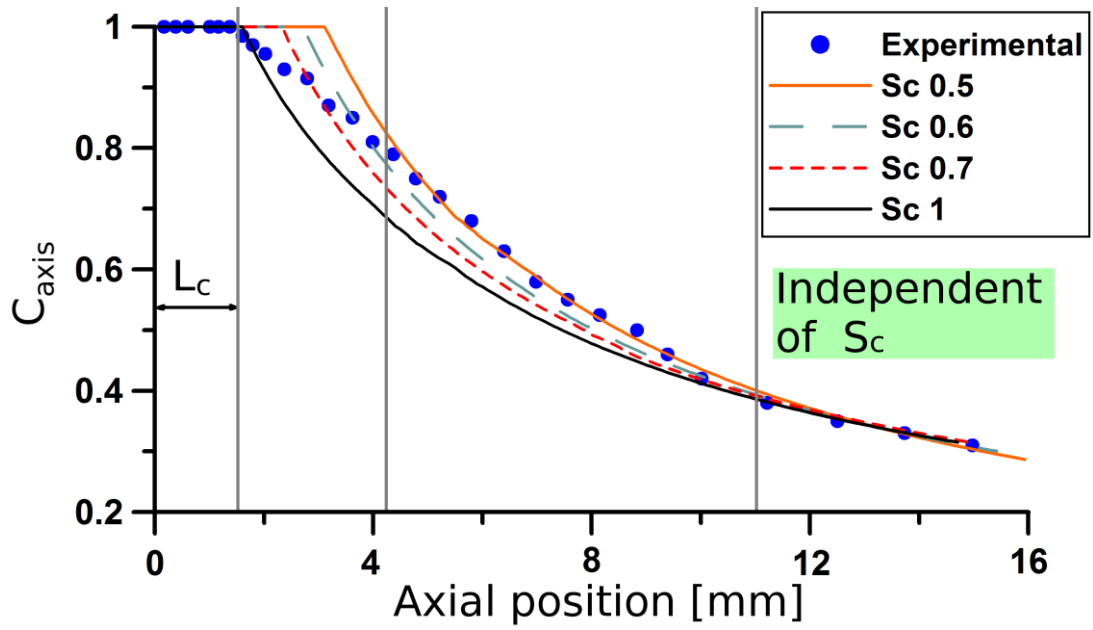
602

603 Figure 8: Non-perturbed length and transitional length for all the nozzles and density

604

conditions.

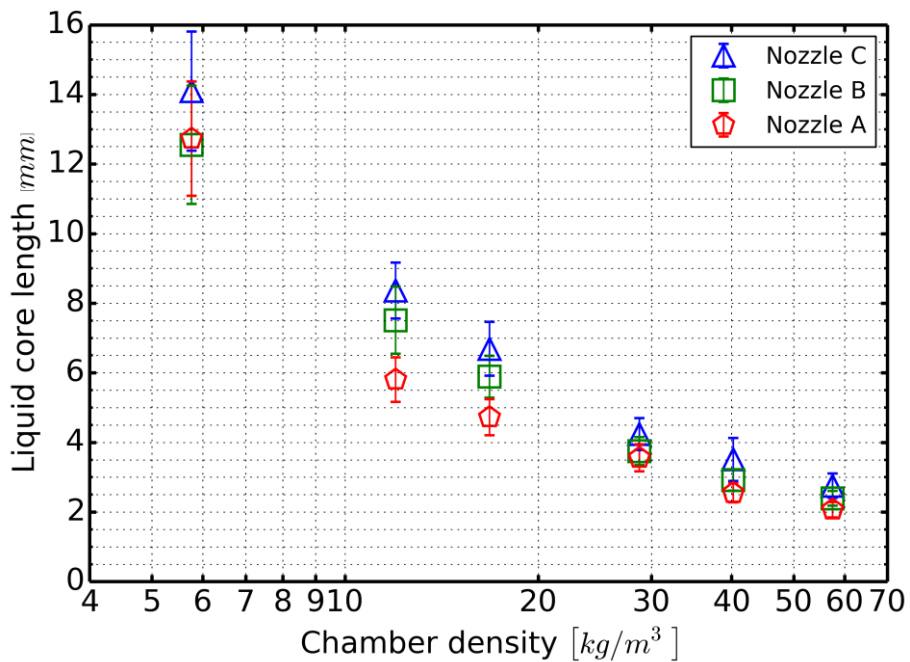
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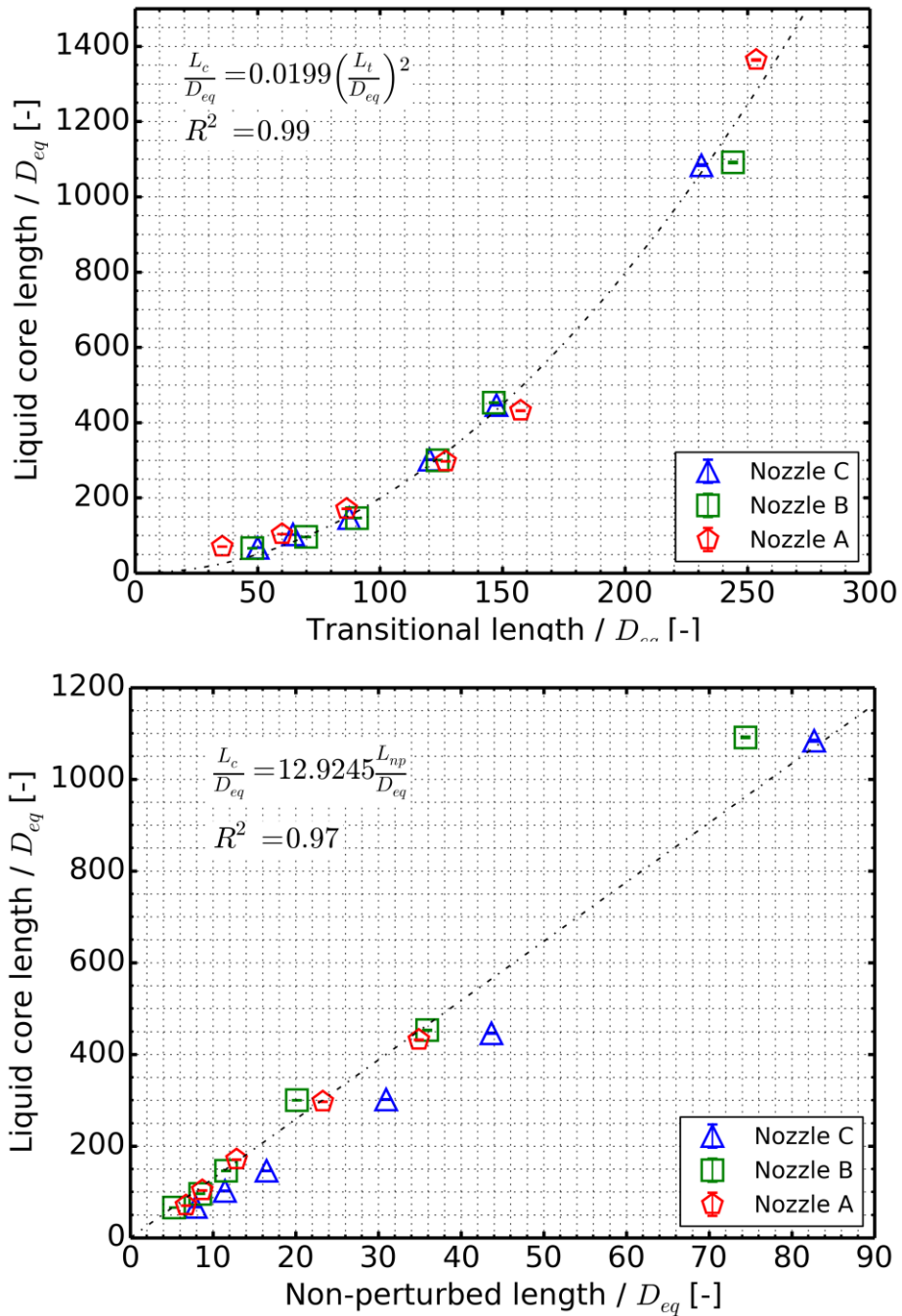
607 Figure 9: Mass concentration in the axis of the spray: experimental and modeled for
 608 different Schmidt numbers. Intact core length determination.

609



610

611 Figure 10: Intact liquid core length as a function of density in the chamber.



612

613 Figure 11: Non-dimensional intact liquid core length vs transitional length (upper

614 part). Non-dimensional liquid intact core length vs external non-perturbed length.

615 (bottom part).

A	Orifice outlet area
b_i	Vectors for the spray angle determination
C	Local fuel concentration
C_c	Empirical constant for the normalized liquid core length
D_{eq}	Equivalent diameter
D	Mass diffusivity
D_i	Diameter at the nozzle orifice inlet
D_o	Diameter at the nozzle orifice outlet
d_1	Distance from the spray axis to the lens
d_2	Distance from the lens to the camera sensor
FL	Lens focal length
h_s	CCD camera sensor length
h_w	Visualization window length
i	Counter for the series in the 0-D model
$k\text{-factor}$	Nozzle orifice conicity factor
L_c	Liquid core length
L_{np}	Non-perturbed length
L_t	Transitional length
\dot{m}_f	Mass flow rate
\dot{M}_f	Spray momentum flux
M	Magnification ratio for the visualization tests

P_{inj}	Injection pressure
P_b	Discharge pressure
Sc	Schmidt number
U_{axis}	Velocity in the axis of the spray
U_o	Effective velocity at the orifice outlet
Greek symbols:	
α	Shape parameter used in Gaussian distributions
θ_m	Spray angle from point of view of mass
θ_u	Spray angle from point of view of velocity
ρ_a	Density of air
ρ_f	Density of fuel
ν_f	Fuel kinematic viscosity

617