Contents

Declaration of Authorship v

Abstract ix

1 Introduction 1
- 1.1 Objectives . . . 2
- 1.2 Preliminaries and Basic Concepts . . . 4
- 1.3 Overview on Iterative Methods . . . 7
  - 1.3.1 Basic Iterative Methods . . . 8
  - 1.3.2 Krylov Subspace Methods . . . 9
  - 1.3.3 Iterative Methods for General Matrices . . . 11
  - 1.3.4 Iterative Methods for Least Squares Problems . . . 13
- 1.4 Preconditioning . . . 15
  - 1.4.1 Generalities . . . 15
  - 1.4.2 Jacobi, GS and SOR preconditioners . . . 18
  - 1.4.3 Incomplete LU and Cholesky preconditioners . . . 18
  - 1.4.4 Convergence Criteria . . . 19
- 1.5 Background and objectives of the thesis . . . 19
  - 1.5.1 Almost Symmetric Problems . . . 20
  - 1.5.2 Modified and Rank Deficient Least Squares Problems . . . 22
    - Modified Least Squares Problems . . . 23
    - Rank Deficient Least Squares Problems . . . 24

2 General Updated Preconditioner Method 27
- 2.1 Sherman-Morrison and Woodbury formulas . . . 27
- 2.2 Schur Complement . . . 28
- 2.3 Preconditioner Computation and Application . . . 30
- 2.4 Equivalent Augmented System . . . 32

3 Preconditioners for Almost Symmetric Linear Systems 35
- 3.1 Introduction . . . 35
- 3.2 Updated preconditioner method . . . 36
- 3.3 Approximation properties of the updated preconditioner . . . 38
- 3.4 Low-rank approximation of the skew-symmetric part . . . 45
- 3.5 Numerical Experiments . . . 50
  - 3.5.1 A class of simple examples . . . 51
  - 3.5.2 The Bratu problem and Love's equation . . . 55

3.5.3 Problems from the University of Florida sparse matrix collection 57
3.6 Conclusions . . . 59

4 Updating Preconditioners for Modified Least Squares Problems 63
4.1 Introduction . . . 63
4.2 Preconditioner update computation and application . . . 65
4.3 Updating preconditioners: Overdetermined case . . . 67
4.3.1 Adding equations to an overdetermined system . . . 67
4.3.2 Removing equations from an overdetermined system . . . 68
4.3.3 Adding and removing equations from an overdetermined system . . . 69
4.3.4 Adding columns to an overdetermined system . . . 70
4.3.5 Removing columns from an overdetermined system . . . 71
4.3.6 Adding and removing columns from an overdetermined system 72
4.4 Updating preconditioners: Underdetermined case . . . 72
4.4.1 Adding equations . . . 73
4.4.2 Removing equations . . . 73
4.4.3 Adding and removing equations . . . 74
4.4.4 Adding columns . . . 74
4.4.5 Removing columns . . . 74
4.4.6 Adding and removing columns . . . 75
4.5 Numerical experiments . . . 76
4.6 Conclusions . . . 87

5 Preconditioners for rank deficient least squares problems 91
5.1 Introduction . . . 91
5.2 Updated preconditioner method . . . 92
5.2.1 Preconditioner computation . . . 94
5.2.2 Preconditioner application . . . 94
5.3 Numerical environment . . . 95
5.4 Numerical experiments . . . 97
5.4.1 Study of the convergence criteria and choice of α . . . 97
5.4.2 Results . . . 102
5.5 Conclusions . . . 103

General conclusions 105

Bibliography 107