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# The flyby anomaly: A multivariate analysis approach

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## Abstract

The flyby anomaly is the unexpected variation of the asymptotic post-encounter velocity of a spacecraft with respect to the pre-encounter velocity as it performs a slingshot manoeuvre. This effect has been detected in, at least, six flybys of the Earth but it has not appeared in other recent flybys. In order to find a pattern in these, apparently contradictory, data several phenomenological formulas have been proposed but all have failed to predict a new result in agreement with the observations. In this paper we use a multivariate dimensional analysis approach to propose a fitting of the data in terms of the local parameters at perigee, as it would occur if this anomaly comes from an unknown fifth force with latitude dependence. Under this assumption, we estimate the range of this force around 300 km.

**Keywords:** Flyby anomaly, multivariate analysis, Juno spacecraft, fifth force

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# 1 Introduction

In the last quarter-century there have been important advances in the high-accuracy measurements of spacecraft, planets and moons's ephemerides in the Solar system through Doppler and laser ranging [18, 41]. On parallel with these developments, faster computers and improved numerical methods have allowed to test the predictions of standard orbit determination models with a precision never achieved before [21]. As a consequence of these improvements several discrepancies among the theoretical models and the observations have been disclosed [29]. Among them we can enumerate: (i) The Pioneer anomaly [38, 39] (ii) The flyby anomaly [8] (iii) The anomalous secular increase of the astronomical unit [31] (iv) An unexplained secular increase in the eccentricity of the orbit of the Moon [27, 26] (v) The Faint Young Sun Paradox [28] and other [29]. In many of these anomalies we cannot exclude that further research should render them statistically not significant. But this is certainly not the case for the Pioneer anomaly (and also for the flyby anomaly) which has also revealed very clearly in the fitting of orbital data.

The Pioneer 10 and Pioneer 11 are the first spacecraft within the context of the "Grand Tour" program whose objective was to send robotic spacecraft to all the planets of outer Solar system from Jupiter to Neptune and, more recently, also Pluto [14]. These spacecraft are provided with a wide antenna designed for sending downlink signals and receiving uplinks from the Earth. Thanks to the monitoring of the spacecraft throughout the years it was found an anomalous constant drift of the redshifted signal [38, 9], to be interpreted as an acceleration directed, approximately, towards the Sun with magnitude  $a_P = (8.74 \pm 1.33) \times 10^{-8} \text{ cm/s}^2$ . Many conventional and unconventional proposals were proposed to explain away this anomaly to no avail until the whole dataset for the mission was retrieved [38]. The analysis of this data showed that this acceleration diminishes with time with the same time-scale that the thermal recoil force arising from the anisotropic emission of thermal radiation off the spacecraft [39, 36]. The spacecraft heat is diffusing from the radioisotope thermoelectric generators filled with Plutonium 238, whose half-life is 87.74 years. This correlation eventually lead to a complete explanation of the anomaly in terms of a thermal radiation effect. For this reason, it is useful to look for this kind of systematic behaviour of correlations in the case of the flyby anomaly as well, because it could also provide a clue about its origin.

On December, 8th, 1990 the Galileo spacecraft performed a flyby of the

Earth at a minimum altitude at perigee of 960 km. After processing Doppler tracking data for obtaining a fit to the trajectory, NASA engineers found that the post-encounter and pre-encounter trajectories cannot be accommodated into the same model and that a small unexplainable residue remained in the Doppler data corresponding to a difference of the post-encounter and pre-encounter asymptotic velocities of 3.92 mm/s [8, 32]. Later on, a second flyby was performed on December, 8th, 1992 in which a total residual velocity decrease of  $-8$  mm/s was found. However, it has been estimated that  $-3.4$  mm/s should correspond to atmospheric friction because in this flyby the spacecraft crossed through the middle of the thermosphere. Anyway, it was concluded that an anomalous velocity decrease of  $-4.6$  mm/s remains to be explained. The maximum anomaly was found in the NEAR flyby of January, 23th, 1998 in which an increase of 13.46 mm/s has been unexplained to date. Similar anomalies were also detected at the Cassini and Rosetta flybys but they were not detected (or were below the threshold of measurement errors) in the Messenger flyby [8], the second and third Rosetta flybys [30] and, more recently, in the Juno flyby of October, 9th, 2013 [37].

Nowadays, Doppler ranging and Delta-Differential one-way ranging have achieved an impressive nanosecond accuracy. It is estimated that random effects, such as clock instability or fluctuations in the atmosphere, could account for delay errors up to 0.053 nanoseconds [11]. Consequently, the results for the flyby anomaly are sufficiently precise to be attributed only to measurement errors.

In their seminal paper about this problem, Anderson et al. [8] proposed a phenomenological formula which fitted rather well the six flybys whose data was available at the time. According to Anderson and his team the anomalous velocity variation is given by:

$$\Delta V_\infty = V_\infty K (\cos \delta_i - \cos \delta_o) , \quad (1)$$

where  $V_\infty$  is the osculating asymptotic velocity at perigee,  $\delta_i$ ,  $\delta_o$  are the declinations for the incoming and outgoing velocity vectors and  $K$  is a constant. These authors also ventured to postulate that  $K$  is related to the quotient of the tangential velocity of the Earth at the Equator and the speed of light as follows:

$$K = \frac{2 \Omega R_E}{c} = 3.099 \times 10^{-6} \text{ s}^{-1} . \quad (2)$$

Here  $\Omega = 2\pi/86400 \text{ s}^{-1}$  is the angular velocity for the Earth's rotation around its axis,  $R_E = 6371 \text{ km}$  is the average Earth's radius and  $c$  is the speed of light in vacuum. The fit provided by Eq. (1) was good for the six flybys studied in their paper but it fails to provide a prediction for the null results of the Rosetta II and III, and the Juno flybys. By proposing a model such as that in Eqs. (1) and (2) these authors are also hinting at an explanation in terms of an unknown axisymmetric interaction arising from the rotating Earth, which they called an enhanced Lense-Thirring effect [8].

The point of view for an ignored classical effect has also been discussed in detail [32]. In particular, general relativistic effects [25, 23], Lorentz's charge acceleration [10] and thermal radiation [36]. Other studies have looked for an explanation beyond standard physics: an halo of dark matter around the Earth [6, 7], extensions of general relativity [24, 1, 3, 34, 35] or phenomenological formulas [30, 13], alternative to that of Eq. (1). But none of these approaches have provided a satisfactory fitting of all the data available and, even more, they have not explained the null results for the anomalies of subsequent flybys (with the exception of the work of Pinheiro [35]). The difficulty to find a pattern in the anomalies, allowing them to be fitted by a single phenomenological formula, comes from two facts: (i) the anomaly corresponds either to an increase (Galileo I, NEAR and Rosetta I flybys) or a decrease (Galileo II and Cassini) of the asymptotic velocity of the spacecraft (ii) in some cases no anomalous increase or decrease of the asymptotic velocity has been detected (Messenger, Rosetta II, III and Juno flybys). On the other hand, this null effect has been anticipated by Pinheiro [35] for flybys in the prograde direction in the context of a topological torsion current model. But, the problem persists if we assume that the origin of the anomaly is a velocity-independent field of force as assumed in this paper.

Moreover, point (ii) suggest the irreproducibility of the results in point (i) as announced by Anderson et al. [8]. This is even more evident for the Juno and NEAR flybys whose perigee's altitudes were similar. But, in this paper we will pursue the case for a flyby anomaly originating for an unknown field (or fifth force) as it has been proposed before. Our approach will be a general multivariate statistical analysis to test if the data for the nine flybys, whose results have been reported until now, can be encompassed by an expression in terms of the local parameters at perigee (as expected in the effect arises from an interaction with the rotating Earth). These parameters are the geocentric latitude of the perigee,  $\phi$ , the orbital inclination,  $I$ , and the perigee's altitude,  $h$ . We will show that expressions for  $\Delta V_\infty/V_\infty$ , in terms of

these parameters, with a coefficient of determination  $R^2 > 0.9$  are possible. Anyway, a very fast decay of the predicted ratio  $\Delta V_\infty/V_\infty$  with altitude is necessary for obtaining a reasonably good fit. This implies that, in case the flyby anomalies originate in an unknown force field, this field extends only to the Earth's exosphere and it is almost undetectable at higher altitudes.

The paper is organized as follows: In section 2 we analyze the available data and their correlations including the azimuthal and polar components of the velocity at perigee. A multivariate fitting model is discussed in section 3 and we apply it to the recent Juno flyby of Jupiter. The paper ends with section 4 with a brief discussion, and the implications of our model for the study of the anomalies, in the near future.

## 2 Flyby data

In this section we will collect the data for the nine flybys enumerated in the previous section. Most of these parameters can be retrieved directly from the NASA's ephemeris website: such as the altitude at perigee and the latitude of the point on the surface of the Earth lying at the vertical of the perigee [16]. Notice that if we use equatorial celestial coordinates the declination of the spacecraft coincides with the geocentric latitude [40]. The asymptotic velocity,  $V_\infty$ , in Anderson et al. [8] phenomenological model corresponds to the asymptotic value of the velocity for the osculating hyperbolic at perigee. This idealized orbit is defined in terms of parameters at perigee as the orbit that the spacecraft would follow if all the perturbations were switched off (mainly the perturbations from the Sun and the Moon). These perturbations give rise to variations of  $V_\infty$  over the whole data interval of the flyby manoeuvre in the range of a few meters per second. Anyway, the average of the incoming velocities a day before and after the closest approach is a good approximation for  $V_\infty$  if our purpose is to obtain a phenomenological formula for the flyby anomalies.

Alternatively, we can obtain the parameters for the osculating orbit at perigee by considering the celestial coordinates and the altitude of the perigee and another point of the trajectory a minute (or a few minutes) after the closest approach. If we denote by  $r_t$  the distance to the center of the Earth at  $t = 1$  min and by  $r_P$  the distance at perigee, we get that the orbital eccentricity  $\epsilon$  at perigee is found as the solution of the system [17, 12]

$$r_t = r_P \frac{\epsilon \cosh \eta - 1}{\epsilon - 1}, \quad (3)$$

$$t = \sqrt{\frac{r_P^3}{\mu}} (\epsilon \sinh \eta - \eta), \quad (4)$$

where  $\eta$  is the eccentric anomaly for the osculating orbit at time  $t$  and  $\mu = GM_E = 398600.4 \text{ km}^3/\text{s}^2$  is the product of the gravitational constant and the mass of the Earth. This way we find also the semi-major axis,  $a$ , of the osculating keplerian orbit from the relation:

$$r_P = a(1 - \epsilon). \quad (5)$$

From here we can define the time-scale,  $T = \sqrt{-a^3/\mu}$ . The osculating hyperbolic asymptotic velocity at perigee is then defined as  $V_\infty = -a/T$  and the velocity at perigee is given by:

$$V_{\text{perigee}} = V_\infty \sqrt{\frac{\epsilon + 1}{\epsilon - 1}}. \quad (6)$$

The direction of this vector can also be found from an orbital frame of reference defined by the inclination vector, the unit position vector at perigee and the tangential vector obtained as the cross product of the other two vectors [1, 3].

Another important parameter is the orbital inclination that we found from the cross product of the position vectors at perigee and time  $t = 1 \text{ min}$  as follows:

$$\cos I = \left( \frac{\mathbf{r}_P \times \mathbf{r}_t}{|\mathbf{r}_P \times \mathbf{r}_t|} \right)_z. \quad (7)$$

Here we divide by the magnitude of the cross product to obtain a unit vector and  $z$  denotes the third component of the cross product vector. The definition in Eq. (7) is ambiguous because for any angle  $I$  we can also choose  $180^\circ - I$ . This ambiguity is solved by defining the inclination vector according to the right-hand rule.

By following these criteria we obtain the orbital inclination in Table 1 and also the rest of parameters we need for our model: the perigee's altitude and latitude, the anomalous velocity increase and the asymptotic velocity. In this table we also list the azimuthal and the polar components of the

Table 1: Parameters for the nine flybys mentioned in the main text: orbital inclination,  $I$ , and geocentric latitude,  $\phi$ , in sexagesimal degrees, perigee's altitude,  $h$ , in km, asymptotic velocity,  $V_\infty$ , in km/s, anomalous velocity increase  $\Delta V_\infty$  in mm/s, azimuthal,  $V_a$ , and polar,  $V_p$ , components of the velocity at perigee in km/s.

Spacecraft	Date	Inclination	Latitude	h (km)	$V_\infty$	$\Delta V_\infty$	$V_a$ (km/s)	$V_p$ (km/s)
Galileo I	12/8/1990	142.9°	25.2°	960	8.949	3.92	-11.993	6.707
Galileo II	12/8/1992	138.7°	-33.8°	303	8.877	-4.60	-12.729	6.018
NEAR	1/23/1998	108.0°	33.0°	539	6.851	13.46	-4.694	11.843
Cassini	8/18/1999	25.4°	-23.5°	1175	16.010	-2	18.740	-3.279
Rosetta I	3/4/2005	144.9°	20.20°	1956	3.863	1.8	-9.170	5.154
Messenger	8/2/2005	43.05°	46.95°	2347	4.056	0.02	-10.387	-0.353
Juno	9/10/2013	47.13°	-33.39°	559	10.389	0	11.845	-8.429
Rosetta II	13/11/2007	25.08°	-64.76°	5322	5.064	0	-12.463	-1.356
Rosetta III	13/11/2009	65.63°	-7.44°	2483	9.393	0	-12.263	-5.274

velocity at perigee obtained by projecting the velocity vector at perigee for the osculating orbit onto the unit azimuthal vector and the unit polar vector at that point.

Some correlations are manifested from simple inspection of the data in Table 1. In particular we find that for retrograde orbits (those in which the spacecraft moves opposite to the rotation of the Earth) the anomalous velocity increase is positive, being negative for prograde orbits (for the Cassini flyby). Anyway, this correlation is broken by the Galileo II flyby in which a retrograde orbit was accompanied by an anomalous decrease of  $V_\infty$  [5].

A better correlation appears among the latitude of the perigee and the sign of the anomalous variation of the asymptotic velocity. The cases in which an anomalous decrease of the asymptotic velocity was found are the Galileo II and the Cassini flybys in which the vertical of the perigee was located in the southern hemisphere. It is also clear that the anomalies tend to decrease, in absolute value, for higher perigee's altitudes and, in particular, there are null results reported for the Rosetta II and Rosetta III flybys. This is also evident for the EPOXI flybys of 2007, 2008 and 2009 performed at even higher altitudes [30]. But the most conflicting data is the one corresponding to the Juno flyby of October, 9th, 2013: its altitude was similar to that of the NEAR flyby with a geocentric latitude at perigee almost coincident with that of the Galileo II flyby. If we notice that in these two flybys the anomalies with the largest magnitudes were found, it is difficult to justify,



within a model searching for a new local interaction, the reason for the null result in the Juno flyby. However, the Juno shares with the Messenger flyby that the orbit was inclined almost  $45^\circ$  with respect to the equatorial plane. In the next section, we will propose a fitting multivariate formula taking into account this fact.

### 3 Multivariate fitting analysis

In this section we will propose a phenomenological fitting formula for the flyby anomalies listed in Table 1. Later on, we will apply this formula to the recent Juno flyby of Jupiter on August, 27th, 2016 in order to estimate the magnitude of the anomalies that could be found in these flybys after processing the data for the trajectories.

The formula for the Earth flybys will take into account the following correlations found in the qualitative analysis in the previous section and also dimensional analysis consistency:

- The anomalous velocity change,  $\Delta V_\infty$ , should be proportional to the asymptotic velocity for the osculating orbit at perigee,  $V_\infty$ .
- The anomalies decrease with the altitude of the perigee. So, the expression should include a power of  $R_E/(R_E + h)$ ,  $R_E = 6371$  km being the average radius of the Earth.
- The anomaly cancels for orbits with a  $45^\circ$  inclination angle at perigee. For this reason, we will include a factor  $\cos 2I$  in the phenomenological formula for the anomalous velocity increase. A possible field configuration giving rise to this cancellation of the anomalous velocity variation, for this particular orbit inclination, is obtained by assuming two components of the field (equal in magnitude) along the polar and the azimuthal directions ( $|\mathbf{F}_\theta| = |\mathbf{F}_\phi|$ ). If the spacecraft moves along  $\mathbf{F}_\theta$  and opposite to  $\mathbf{F}_\phi$ , or viceversa, we have:

$$\Delta V(t) = \sqrt{(\mathbf{V}_p + \delta\mathbf{V}_\theta(t))^2 + (\mathbf{V}_p - \delta\mathbf{V}_\phi(t))^2} - V_p, \quad (8)$$

where  $\mathbf{V}_p$  is the velocity vector at perigee,  $\Delta V(t)$  is the anomalous velocity variation at time  $t$  after perigee and  $\delta\mathbf{V}_\theta(t)$ ,  $\delta\mathbf{V}_\phi(t)$  are the velocity perturbations induced by the polar and azimuthal components

of the field, respectively. If  $\mathbf{V}_p$  is oriented at a  $45^\circ$  inclination angle the scalar products in Eq. (8) cancel out and we have:

$$\Delta V(t) \simeq \delta V \left( \frac{\delta V}{V_p} \right) \ll \delta V, \quad (9)$$

where  $\delta V = |\delta \mathbf{V}_\theta(t)| = |\delta \mathbf{V}_\phi(t)|$ . Then, Eq. (8) implies that the anomaly is almost undetectable in this particular configuration in contrast with flybys along the celestial parallels or meridians (in which case the variation  $\Delta V(t)$  would not be suppressed by the small ratio  $\delta V/V_p$ ).

- A null result is also expected for orbits whose perigee is attained at the poles. This is predicted by Anderson et al. [8] phenomenological formula and it is a reasonable assumption for interactions arising from the Earth's rotation. In our model this is achieved by including a term  $\sin 2\phi$ , where  $\phi$  is the latitude of the perigee. The latitude,  $\phi$ , is coincident with the declination,  $\delta$ , in the celestial equatorial coordinate system.
- The anomaly is positive (negative) for latitudes  $\phi > 0$  ( $\phi < 0$ ), respectively. This condition and the previous one are verified by a term  $\sin 2\phi$ . This condition suggests that the underlying force, originating the anomaly, has a quadrupolar distribution around the Earth.

Under these assumptions our proposal for a phenomenological formula relating  $\Delta V_\infty$  and the aforementioned parameters is:

$$\Delta V_\infty = \kappa V_\infty \left( \frac{R_E}{R_E + h} \right)^\beta |\cos 2I|^\gamma \sin 2\phi. \quad (10)$$

The fit to the data in Table 1 gives us the coefficients listed in Table 2. The corresponding coefficient of determination is  $R^2 = 0.9429$ , denoting that most of the variability of the data can be explained in term of these three parameters,  $h$ ,  $I$  and  $\phi$ . A consequence of Eq. (10) is that the escape velocity of a spacecraft from a putative fifth force field goes as  $1/(R_E + h)^\beta$  in comparison with the  $1/(R_E + h)^{1/2}$  dependence for standard Newtonian gravity. This suggests that the fifth force decrease much faster (with the distance to the center of the Earth) than Newtonian gravity or gravitomagnetic effects in General Relativity. Nonetheless, it is difficult to find a rationale for

Table 2: Parameters  $\kappa$ ,  $\gamma$  and  $\delta$  obtained after fitting the flyby data in Table 1 to the expression in Eq. (10). The standard error and the  $t$ -statistics ratio and  $p$ -values are also shown.

Parameter	Estimated value	Standard Error	t-statistics	p-value
$\kappa$	$16.2247 \times 10^{-6}$	$7.9816 \times 10^{-6}$	2.0327	0.0883
$\beta$	21.8739	5.4244	4.0324	0.0068
$\gamma$	1.1537	0.1922	6.00187	0.00096

this distance dependence at this stage of the research on the flyby anomaly but it could point towards the existence of a very low mass boson mediating the interaction. One of these X bosons in the dark sector could have been already detected [19].

We also notice that the  $p$ -values are statistically significant for the exponents  $\beta$  and  $\gamma$  ( $p < 0.05$ ) but it is also close to be significant for the prefactor  $\kappa$ . We can also notice that  $\kappa$  can be written as follows:

$$\kappa = \kappa' \frac{\Omega R_E}{c}, \text{ with } \kappa' = 10.47 \pm 5.15. \quad (11)$$

The predictions of this model in Eq. (10) and the standard error of these predictions are displayed in Table 3.

We see that according to Table 3 only the predictions for the Galileo II, NEAR and Cassini flybys are different from zero within their error bars. The other, including the Messenger, Juno and the second and third Rosetta flybys are compatible with a null result in the search for a flyby anomaly as it has been reported.

On the other hand, the predictions of Anderson et al. phenomenological formula in Eq. (1) can be obtained for Juno by taking into account and incoming declination angle,  $\delta_i = -14.308^\circ$ , and an outgoing angle,  $\delta_o = 39.409^\circ$ . This gives  $\Delta V_\infty = 6.3355$  mm/s in disagreement with the data obtained after the orbit reconstruction. Similarly, we have the predictions  $\Delta V_\infty = 0.523$  mm/s and  $\Delta V_\infty = 1.099$  mm/s for the Rosetta II and the Rosetta III flybys, respectively. These are smaller and could still agree with the predictions of the alternative formula we have proposed in this paper.

The main problem for a further understanding of this puzzling anomaly

Table 3: Prediction and standard error vs the observed anomalies for the nine flybys in Table 1. Velocity anomalies are measured in mm/s.

Flyby	Prediction	Error	Observation
Galileo I	1.157	1.546	3.92
Galileo II	-4.529	2.015	-4.6
NEAR	13.457	2.066	13.46
Cassini	-2.759	1.992	-2
Rosetta	0.033	1.463	1.8
Messenger	0.003	1.463	0.02
Juno	-1.228	1.558	0
Rosetta II	0.000	1.463	0
Rosetta III	-0.018	1.463	0

is the lack of data. But, at present the Juno mission is being carried out with a total of 36 close flybys of Jupiter in the mission program. The first of these flybys took place on August, 27th, 2016 with the Juno spacecraft approaching at a minimum distance around 4200 km over the the top clouds of the giant planet (with radius  $R_J = 71492$  km). The tangential velocity at periapsis was estimated as 57.77 km/s. We can also obtain the orbital inclination at periapsis and the declination from the Horizons web-interface proceeding as we discussed in the previous section. This way we find:  $\delta_P = 28.56^\circ$  and  $I = 90.31^\circ$ , but Jupiter's rotation axis is only slightly tilted with respect to the celestial equator so we can assume that the declination is similar to the latitude with respect to Jupiter's equator ( $\phi_P \simeq \delta_P$ ). To apply our phenomenological formula we should also take into account that Jupiter's rotational period around its axis is 9.925 hours and this changes Anderson's ratio  $K$  to  $K_J = 8.115 \times 10^{-5} \text{ s}^{-1}$  and, consequently, also the value of the parameter  $\kappa$  in Eq. (11). We must also mention that Juno's trajectory is now elliptical because it is trapped by Jupiter's gravitational field, so it is meaningless to define the osculating hyperbolic orbit at periapsis or  $V_\infty$ . Anyway, we should use  $V_P$  instead to estimate the perturbation of the putative unknown field generated by Jupiter's rotation on Juno as it passed through the periapsis. This way we obtain a prediction of  $\Delta V = 6$  m/s for

the post-encounter velocity with respect to the pre-encounter velocity. This difference is three orders of magnitude larger than the typical anomaly for Earth flybys and it should be detected in the orbit reconstruction of the Juno flybys of Jupiter.

If this is not the case, and no anomalies are found at these close flybys of Juno over Jupiter, we should cast a serious doubt on the reality of the phenomenon and the statistical significance of the previous results on Earth flybys.

## 4 Conclusions and discussion

In this paper we have considered the data for the flyby anomaly as reported in nine gravity assist manoeuvres from 1990 to 2013. In five of these flybys significant anomalies were reported after trying to find a single fit for the post-encounter and pre-encounter Doppler tracked trajectories. This motivated a team at JPL lead by Anderson to propose a daring hypothesis on the connection of this phenomenon with an unknown field produced by the rotating Earth and to find a phenomenological formula which fitted reasonably well the data known until 2008 [8]. Other authors have also tried to fit the anomalies [1, 15, 33, 34, 35]. On the other hand, other flybys of the Earth by spacecraft developed at ESA, such as the second and third Rosetta flybys, and NASA, the Juno flyby of the Earth in 2013, have failed to detect any anomaly in the difference among the post-encounter and the pre-encounter velocities [37]. The impasse in this problem is enhanced by the fact that the Juno flyby was performed at altitudes and latitudes similar to previous flybys in which the anomaly has been detected. Apart from conventional explanations: such as atmospheric drag [32], ocean and solid tides [4], electric charge or magnetic moment of the spacecraft [10], or relativistic effects [25, 23], there have also been some attempts for modified models of gravity in order to make some sense of the flyby anomaly data [24, 3].

In this paper we have suggested an alternative phenomenological formula to that of Anderson et al. under the hypothesis that the effect arises as a short-ranged field from the rotating Earth which decays very fast with altitude and whose effect depends on the orbital orientation at perigee. In the fitting formula, the null results for the Juno and Messenger flybys are obtained, in part, as a result of its particular inclination, around  $45^\circ$ , over the celestial equator. We also consider a latitude dependence in which the

sign of the anomaly depends on the hemisphere flybyed by the spacecraft and we implement also the condition of symmetry under which the anomaly is null for flybys whose perigee is attained at the Earth poles.

The resulting formula in Eq. (10) is adjusted using multivariate analysis techniques and a fitting with a  $R^2 > 0.9$  is obtained. The moral of this approach is that we can still find a fitting for a phenomenological formula encompassing both the anomalies prior to 2005 and the recent null results using only local parameters at perigee, but this comes at the cost of proposing a complicate dependence on the parameter characterizing the orbital orientation (the inclination angle,  $I$ ) and the parameters corresponding to the location of the perigee (the latitude,  $\phi$ , and the altitude,  $h$ , at the point of closest approach to the Earth).

If we assume that a new force field (a fifth force) is causing the anomalies, this would have a range of  $R_E/\beta = 291.2$  km but, instead of depending only of the distance, as the standard fifth force proposals [22, 20], it would also vary with latitude. So, our main conclusion in this paper is that, from a phenomenological point of view, there are good reasons to propose that the flyby anomaly may be caused by a velocity-independent field of force generated by the rotating Earth decaying very fast with the distance to the surface of the Earth and with a complex angular structure, still to be determined. This force should depend on the mass of the object and it would be, fundamentally, gravitational in origin. It could also be connected with torsion or other extensions of General Relativity [2]. But the only way to solve the riddle posed by the flyby anomaly is to obtain more data as it could be provided by the ongoing Juno mission or future Earth flybys.

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