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New third rail implementation system for conventional railroad tracks in service

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Research paper

New third rail implementation system for conventional railroad tracks in service

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Abstract

Two track gauges coexist in Spain, the Iberian gauge (1668 mm) and the international gauge (1435 mm). Solutions applied on the track were considered to improve the interoperability in Spain. First solution proposed was tracks with two-gauge-ready sleepers. This solution allows changing the gauge from Iberian to International gauge or from International to Iberian gauge. The need of two gauges evolved this solution to the third rail system with two-simultaneous-gauge sleepers. In this paper, a new third-rail track solution is proposed. This solution is a novel system based on the introduction of special concrete blocks between the existing sleepers to fasten the third rail. Thus, the conventional railroad track becomes in a double-simultaneous-gauge lines without needing change existing sleepers and make long traffic interruptions of the track like in common third-rail systems. In the present study, three-dimensional numerical models are generated using finite-element method. The analysis of the vertical displacements of the top head of the rails, vertical stresses transmitted to the substructure layers and vertical track stiffness, demonstrate the good vertical performance of the new system compared to the conventional lines with dual-gauge-ready sleepers and the common third rail tracks.

Keywords
railroad infrastructures; structural design; simulation; finite element method (FEM); numerical methods

1. Introduction

In Spain, the conventional railroad track has traditionally used the Iberian gauge (1668 mm). The Iberian gauge is different to the International gauge (1435 mm) used in most European networks. Thus, gauge borders are created in the connections between Spain and the rest of Europe. In addition, Spanish high speed network has been designed with the international gauge, as indicated by Santamaría *et al*. (2013). This design criterion has created internal gauge borders.

Some solutions applied on the vehicles were proposed. These solutions are based in special bogies. The bogies are able to change wheels position and adapt them to the new setup on the gauge changers built for this purpose. In Spain, the most applied technologies are Talgo and CAF systems.

These types of solutions applied on the vehicles solved partially problems of the passengers' transportation. Nevertheless, the gauge problem persists at freight traffic due to its complexity. Nowadays, there is no solution in widespread use. In order to solve this problem and improve the interoperability in Spain, track solutions were studied and applied in some railroad networks. The first decisions were based on the concept of dual-gauge tracks. Dual-gauge-ready sleepers were considered to adapt the conventional Iberian tracks to the international gauge (Fig.1(a)). However, this solution only allows circulation of tracks in one gauge. In this way, according to Cuadrado *et al*. (2008) the track solutions evolved to the third rail system. This system, transforms conventional tracks in two-simultaneous-gauge tracks (Fig. 1(b)). The third rail solution allows the circulation of the national (1668 mm) and standard (1435 mm) gauge vehicles over the same railroad track.

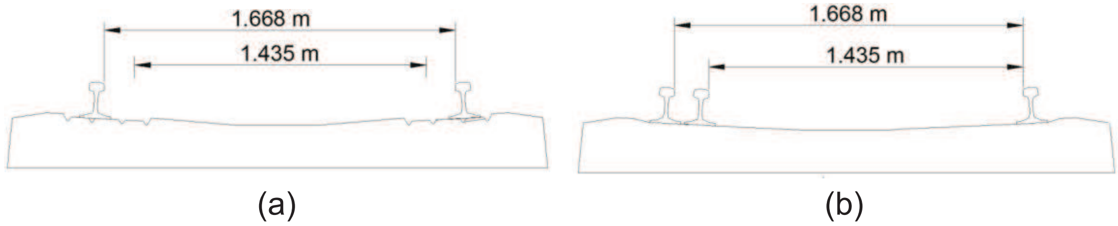


Fig. 1. Dual-gauge-ready sleeper in a two-rail track compared with a three rail sleeper in a third rail track: (a) Dual-gauge-ready sleeper, (b) Three rail sleeper.

Currently, third-rail system implementation needs to change all the existing sleepers by the third-rail sleepers. This need requires large investments and extended traffic interruptions. In order to avoid this, a new system to implement the third rail is presented in this paper. This new third rail implementation system is based on the introduction of a special element between the existing sleepers where the third rail is fasten, as shown in Fig.3. In this way, traffic interruption is not needed since conventional superstructure is not affected. Moreover, the installation of this particular element is simpler and quicker, and it does not require any special equipment. Thus, these two advantages mean third rail implementation cost savings.

The main objective of this study is to check the good vertical behavior of this new proposed system. Three-dimensional numerical models using finite elements method were used for this purpose.

2. New third rail system description and methodology

The proposed system is a novel way to introduce the third rail in a conventional line (Fig.2). This system transforms the conventional line in a double-simultaneous-gauge railroad track. This new system consists in introducing concrete blocks between the existing sleepers, as can be seen from comparing Fig 2 and Fig. 3.

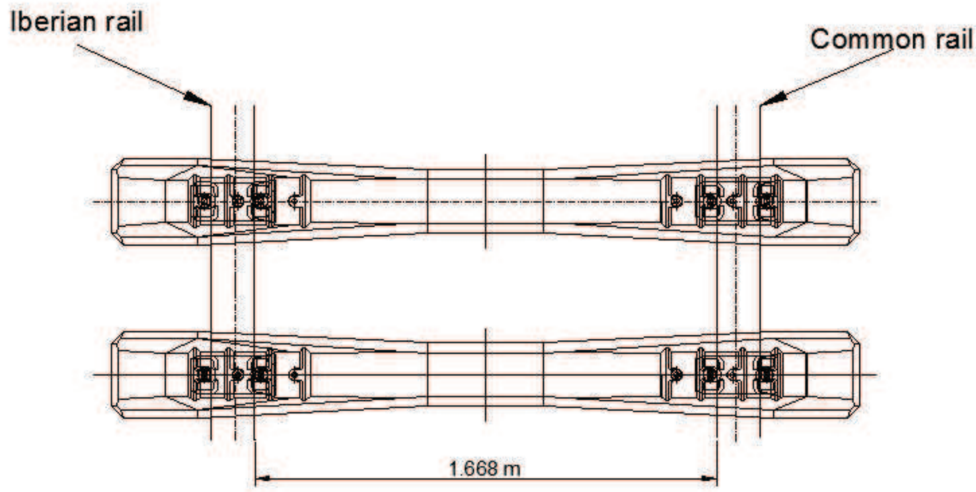


Fig. 2. Conventional railroad track

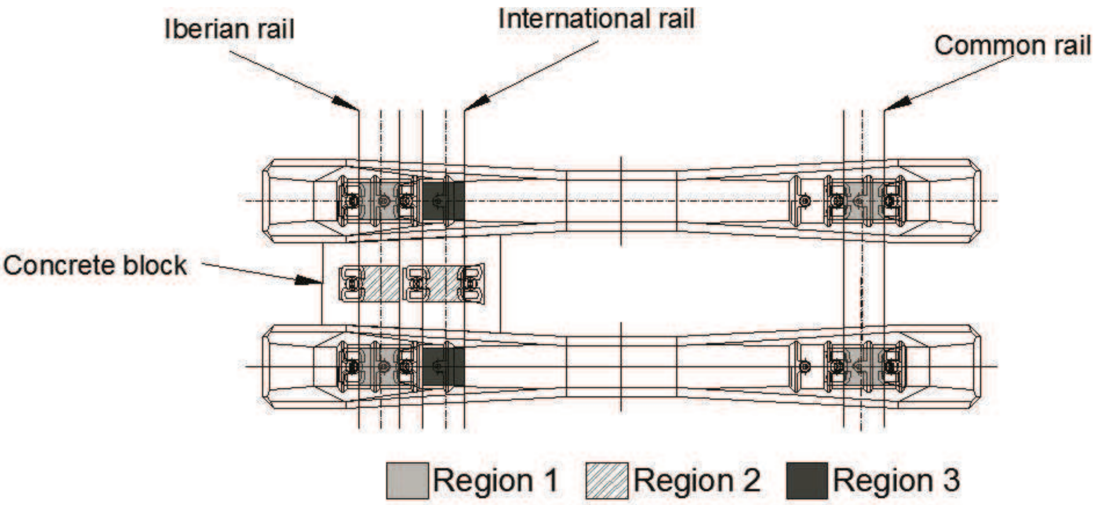


Fig. 3. New third-rail implementation systems for conventional railroads in service

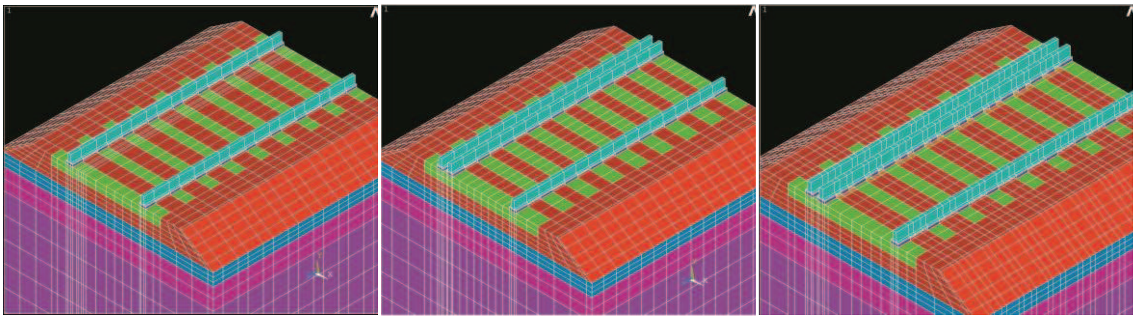
The concrete block fasts the new implemented rail (International rail) and the Iberian rail which is fasten in the special element to both maintain the track gauge and guarantee the correct transverse performance. International rail is free in the conventional pre-existing sleeper section. On the other hand, the maintenance operations, as ballast tamping, were considered in concrete block design.

The initial new third rail implementation system analyzed (Case 1) was a system with 100 kN/mm pads located in Region 1 and in Region 2 (Fig.3). The results obtained shown that, vertical displacements of top head of International rail in Region 3 could produce risk of impact. Moreover, vertical stresses distribution must be improved since it was unbalanced in the sleeper section, as can be seen in the results part.

The first results shown the need to optimize the first new third-rail implementation system proposed. Two cases were analyzed. In both cases, different vertical stiffness pads were implemented in Region 1 and in Region 2 (Fig.3). The pad located in Region 1 had vertical stiffness equal to 100 KN/mm in both new cases and the pad located in Region 2 had vertical stiffness equal to 50 KN/mm in the first new case (Case 2) and 30 KN/mm in the second system analyzed (Case 3). On the other hand, an elastic element was considered between the International rail and the sleepers in Region 3 (Fig.3). This elastic element is not a rail fastening system. The aim of this elastic element is to allow elastic contact between two rigid elements, in case of impact.

Three-dimensional numerical models were developed to study the vertical performance. The finite element method is the most appropriate resolution method since it takes into consideration the track as an entire system which parts work together (Cuadrado, *et al.* 2008, Gallego and Lopez, 2009, Gallego, *et al*. 2011, Gallego, *et a.l* 2012, Real, *et al*. 2012, Banimahd, 2013, Montalbán, 2013).Furthermore, this method appears in the Recommendations by Ministerio de Fomento (1999).

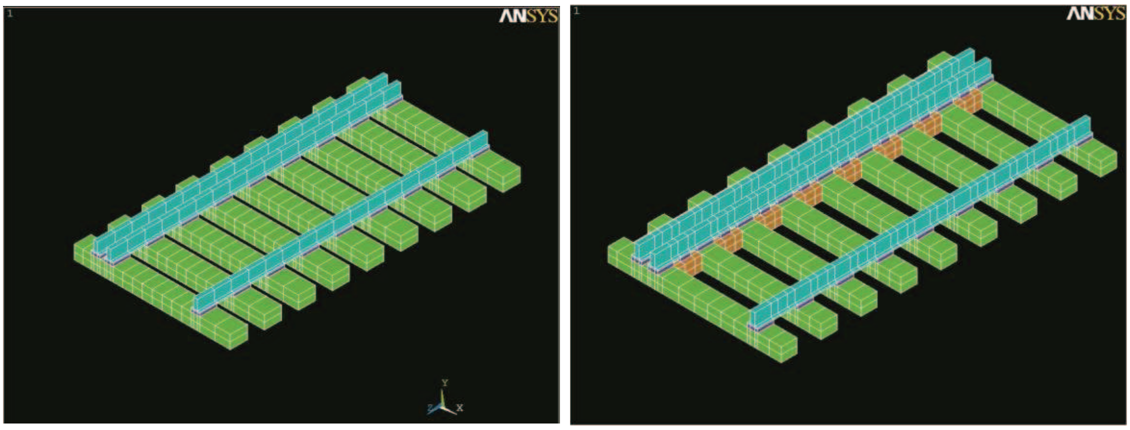
Finite element models of different railroad superstructure configuration were developed: traditional Iberian-gauge railroad track with dual-gauge-ready sleepers (PR track), track with third-rail sleepers (AM track) and track with the new third-rail implementation system proposed. These models can be seen in Fig. 4(a), Fig. 4(b) and Fig. 4(c), respectively.



(a) (b) (c)

Fig. 4. Numerical track models: (a) PR track model, (b) AM track model, (c) Track with the new third-rail implementation system

PR track model was calibrated and validated with real data (Fig.4(a)). After this numerical model calibration, AM track model was obtained changing the sleepers in the calibrated PR track model and introducing the new rail (Fig.5(a)). The introduction of both the special concrete blocks and International rail in the calibrated model implied the model of a track with new third-rail implementation system (Fig.5(b)).



(a) (b)

Fig. 5. Changes in the calibrated model: (a) AM track model, (b) Track with the new third-rail implementation system.

Vertical displacements of the top head of the rail in Region 3 and stresses transmitted to the substructure layers were the analyzed parameters. The results obtained, allowed the comparison with the other two models named, concluding good vertical behavior of the new third-rail implementation system Dahlberg (2010) considered vertical track stiffness as important design parameter since differential settlements; vibrations and track degradation depend on it.

The vertical stiffness of the track is defined by the relation between the vertical applied load on the top head of the rail (Q) and the produced displacement (δ), as indicated by Texeira (2003), Gallego *et al.* (2011), Gallego *et al.* (2012) and Choi (2013).

$$k = \frac{Q}{\delta} \qquad (1)$$

According to Eq. (1), once the vertical applied load on the top head of the rail and the produced displacements are determined, the vertical track stiffness can be calculated in order to assess the good performance of the new proposed system. Maintaining vertical track stiffness into an adequate interval is important in order to optimize the dynamic loads value and track maintenance costs (Pita, Teixeira and Robusté 2004, Puzavac, Popovic and Lazarevic 2012).

3. Numerical model

Regarding to the geometry of the model, x-axis represents the transversal direction, y-axis the vertical direction and the z-axis the longitudinal direction. The cross section was considered as a simple track, (Real, *et al.* 2012, Montalbán, *et al.* 2013).

The ballast layer has been designed with length of shoulders equal to 50 cm and slope equal to 3:2. The height of the other layers and the rest of the material characteristics are indicated in Table 1. All these material characteristics are based on the Recommendations by Ministerio de Fomento (1999).

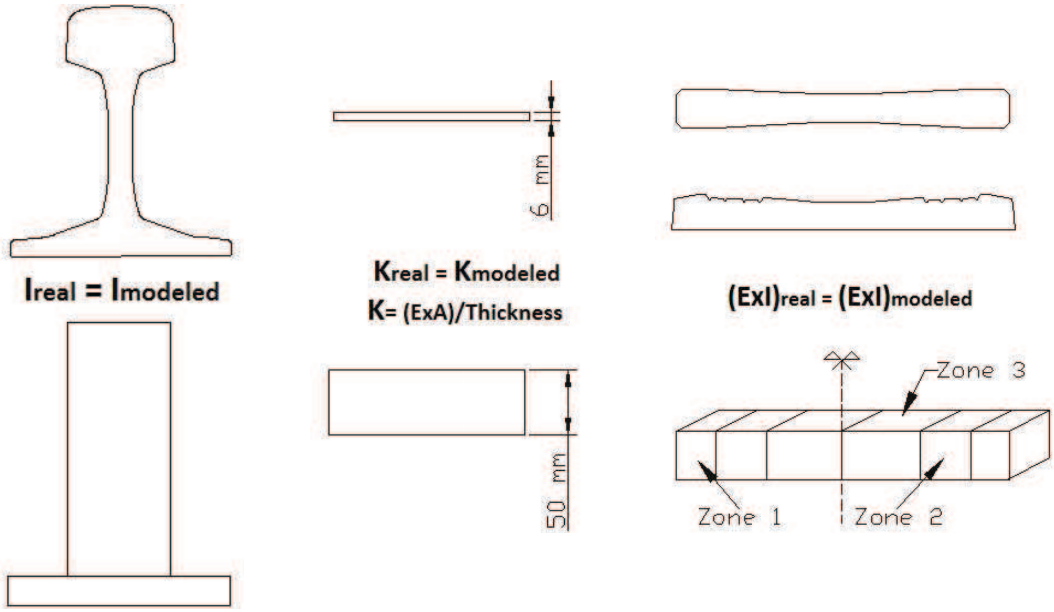
Table 1. Material characteristics

Layer	Thickness (cm)	Young's modulus E (N/m^2)	Poissons ratio	Density (kg/m^3)	Friction angle (deg)	Cohesion
Ballast	30	1.3×10^8	0.2	1900	45	0
Subballast	30	1.2×10^8	0.3	1600	35	0
Embankment (QS3)	350	8×10^7	0.3	2000	35	0

An elastic, isotropic and lineal model was used to characterize the rails, elastic pads and sleepers. A perfect elastoplastic behavior was assumed for granular material. (Gallego and López, 2009, Gallego, *et al*. 2011, Gallego, *et al*. 2012). Drucker-Prager model, which was defined in Drucker and Prager(1952) has been used to model the granular material behavior. This model limits the material behavior for states of hydrostatic stresses, as it is said Gallego and López (2009). This model was validated by ORE Committee D-171 (1983).

In order to assess the correct railroad track behavior, the enough length was adopted in the longitudinal direction according to Real, *et al*. (2012). In accordance with this theory, a length equivalent to nine sleepers with 60cm of distance between them was analyzed.

All the elements components of the track were modeled with the simplifications proposed in the Recommendations by Ministerio de Fomento (1999). The rail modeled has been the UIC 60 kg/m. One simplification like parallelepiped elements was made obtaining an equivalent rail in which the inertia was equal to the real. Then, the rail pad was modeled with an unreal thickness (50 mm) to avoid problems related to the model meshing. Thus, the Young's modulus was modified to maintain the compressive stiffness equal to the real. Finally, the sleeper was modeled as a simplified parallelepiped sleeper. The differences in the real cross sections were taken into account since the parallelepiped sleeper was implemented keeping the real longitudinal bending properties, like in Real, *et al*. (2012). All these elements modeled can be seen in Fig.6.



(a) (b) (c)

Fig. 6. Substructure elements modeled: (a) Rail modeled, (b) Pad modeled, (c) Sleeper modeled

The boundary conditions are, as reported by Ministerio de Fomento (1999) and Montalbán, *et al*. (2013):

- In the delimiting planes, movement perpendicular has been constrained.
- Surfaces of the embankment slopes were considered free.

Additionally, it is not possible to consider the model continuity in the sleeper-ballast contact and in the concrete block-ballast contact due to their grain size which are not comparable, (Real, *et al*. 2012). In this three finite element models proposed, the link between the sleeper-ballast and concrete block-ballast do not share the nodes in contact surfaces. These contact surfaces must mobilize the frictional mechanism were modeled by means of contact elements, as indicated in Montalbán *et al*. (2014). The friction coefficient adopted in the contact surfaces was equal to 0.8. Moreover, the contact surfaces between the concrete block and the pre-existing sleepers were designed by coupling the displacements in the z-axis direction as recommended by Ministerio de Fomento (1999).

Self-weight and train traffic loads were considered in this analysis. Then, two stages were needed in model calculation since the deformational response model depends on the load history due to the non-elastic behavior of the granular materials. The first stage is structure self-weight and the second stage analyses the effect of the vehicle passage over the first step. This procedure studies separately the stresses and the displacements caused by each stage. The purpose of the study is to determine the stresses transmitted to the ballast and the vertical displacement on the top head of the rail caused by the train passage. With this type of two stages analysis, they can be calculated from the difference between the totals obtained after applying the train loads to the first stage, as stated by Gallego *et al*. (2012) and Montalbán *et al*. (2013).

The load value applied considers the static value due to the vehicle passage and the dynamic effects, in accordance with Montalbán *et al*. (2013). The method applied to take into account this overloads has been the Eisenmann's formulation which is indicated in Texeira (2003). This formulation is based on Eq. (2):

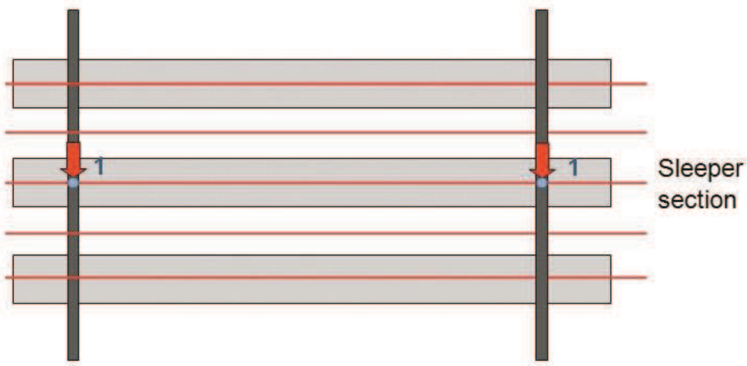
$$Qd = Qn \times \left(1 + t \times \bar{s} \times \left(1 + \frac{v - 60}{380}\right)\right) \tag{2}$$

In the calculation, the speed (v) was 200 km/h, the vehicles static load were 22 t/axe (Qn) , the statistical security coefficient was t=2, corresponding to the percentile 95.5%, the track quality factor was s=0.2, due to the good conditions of the track,. Thus, the amplified load considered value was 34 t/axle.

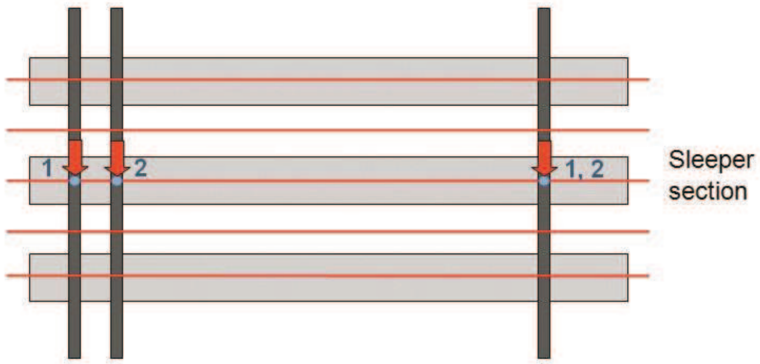
Four load cases were studied to simulate the traffic. These four load cases were needed to obtain the necessary results in the three railroad types analyzed. These four cases are:

- H.1: train load on the Iberian-gauge rails. Sleeper section
- H.2: train load on the international-gauge rails. Sleeper section.
- H.3: train load on the Iberian-gauge rails. Concrete block section.
- H.4: train load on the international-gauge rails. Concrete block section.

In Fig. 7, load hypothesis adopted in each model can be observed.



(a)



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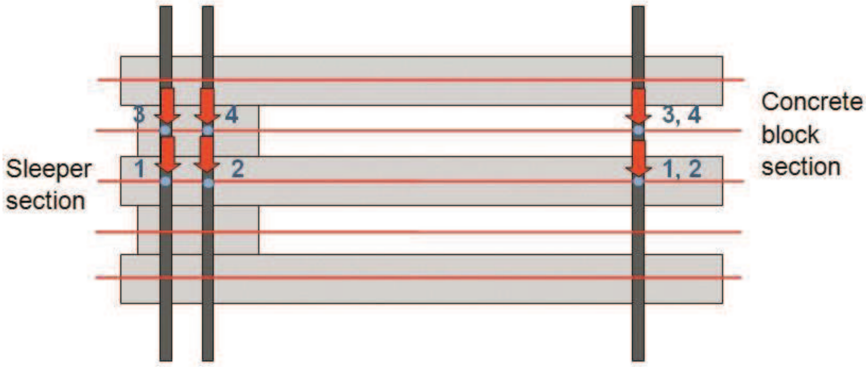

(c)

Fig. 7. Load hypothesis adopted: (a) Load hypothesis considered in a conventional track, (b) Load hypothesis adopted in a third rail track, (c) Load hypothesis considered in a track with the new third-rail implementation system

4. First results

In this section, first new third-rail implementation system results are shown. As explained before, there is no contact between the International rail and the existing sleepers (Case 1).

This new third-rail implementation system was studied in comparison with the PR model and AM model. The results of the stresses transmitted to the ballast layer in each load hypothesis considered are shown in Table 2 for each track type analyzed. These results were obtained in points where the load was applied.

Table 2. Stresses transmitted to the ballast

Stresses transmitted to the ballast (10^4 Pa)						
Load hypothesis	PR model	AM model		Case 1		
	Sleeper section	Sleeper section		Sleeper section		Concrete block section
	On both sides	Double rail side	Common rail side	Double rail side	Common rail side	Double rail side
H.1	7.7	6.7	6	7.90	8.00	6.50
H.2	-	5.7	6.4	4.30	9.00	10.00
H.3	-	-	-	7.00	8.00	8.00
H.4	-	-	-	3.40	7.50	13.00

As can be observed in Table 2, stresses transmitted to the ballast are higher in the new third-rail implementation system than in the others analyzed tracks. In concrete block section, the increase of stresses transmitted to the ballast is more obvious (10 x 10^4 and 13 x 10^4 Pa) due to the reduced area of this element and the support conditions of the International rail. The worst case is H.4 in concrete block section. The maintenance of the ballast layer could be affected. This stresses situation could require more regular maintenance in the ballast layer. On the other hand, a better symmetric stresses distribution must exist in the sleeper section when the International-gauge trains pass over them.

Here, the stresses transmitted to the subballast and the embankment must be tested. The results shown in Table 3 indicate the stresses transmitted to the substructure layers. These values are for the worst load cases in each model.

Table 3. Stresses transmitted to the substructure layers

Stresses transmitted to the substructure layers (10^4 Pa)			
Surface between layers	PR model	AM model	New system model
	On both sides	Under most loaded rail	Under most loaded rail
Sleeper-ballast	7.7	6.7	13
Ballast -subballast	6.6	6	7.0
Subballast-embankment	4.3	4.1	4.1

As can be observed in Table 3, the stresses transmitted to the subballast and to the embankment do not experiment large variations from one model to the others. Nevertheless, this situation does not occur in the ballast layer. Thus, next results only show stresses transmitted to the ballast layer.

Maximum vertical displacement on the top head of the rail in Region 3 was 1.46 mm. This value on the vertical displacement suggests that risk of impacts could appear in this region.

Analyzed the vertical displacement of the top head of the rail, the vertical track stiffness were valued. These values were obtained in the points when the loads were applied. These values are shown in interval form in the AM model and in track with the new third-rail implementation system proposed. In a conventional track with dual-gauge-ready sleepers, the vertical track stiffness was 75.46 KN/mm. In a third-rail track with three rail system sleepers the values of the vertical stiffness were situated between 78.76 KN/mm and 84.97 KN/mm. In Table 4, the vertical track stiffness values for a conventional railroad track with the first new third-rail implementation system are shown.

Table 4. Vertical stiffness in the new third-rail implementation system proposed

Vertical stiffness (KN/mm)				
Rail	Sleeper section		Concrete block section	
Iberian rail	89.63	98.19	88.96	98.19
International rail	74.01	80.95	74.11	88.23
Common rail	71.51	81.81	-	-

As can be seen in Table 4, the results show that all the values in the Iberian rail are higher than the vertical track stiffness in the other cases analyzed.

The new third-rail implementation system initially proposed must be optimized, as the values obtained about the stresses, vertical displacements of the top head of the rail and the vertical track stiffness shown. In this way, an elastic pad implemented between the International rail and the sleeper (Region 3, see Fig.3) was considered to avoid possible impacts and allow a better distribution of stresses in sleeper section, as it was indicated in methodology section. Furthermore, the vertical stiffness of the pads in Region 2 was reduced in the next studied cases in order to decrease the stresses transmitted to the ballast layer in the concrete block section. Pads located in Region 2 were 50 KN/mm for Case 1 and 30 KN/mm for the Case 2. On the other hand, these decreases in pads improve the difference between the vertical track stiffness in both track sides of the concrete block section.

5. Optimized results

In this section, the results about stresses transmitted to the ballast and vertical stiffness in Case 2 and Case 3 are shown. In these cases, the stresses have been studied in load hypothesis H.4, which is the worst load case in the concrete block section. The values can be observed in Table 5.

Table 5. Stresses transmitted to the ballast layer with the load hypothesis H.4

Stresses transmitted to the ballast (10^4 Pa)				
Sections	Case 2		Case 3	
	Double rail side	Common rail side	Double rail side	Common rail side
Sleeper section	5.99	8.86	7.53	8.09
Concrete block section	7.86	0.00	4.53	0.00

As can be seen in Table 5, better stress distribution in sleeper section was obtained with the changes considered in these two cases. Moreover, the stress transmitted to the ballast was significantly decreased. In Case 3, the stresses transmitted to the ballast are practically symmetric in the sleeper section. Additionally, the stresses transmitted in the concrete block are significantly reduced.

On the other hand, vertical track stiffness values were calculated on both cases. Vertical track stiffness intervals of variation are shown in Table 6 for both analyzed cases.

Table 6. Vertical stiffness values

Vertical stiffness (KN/mm)					
Case	Rail	Sleeper section		Concrete block section	
Case 2	Iberian rail	87.16	95.33	86.33	95.33
	International rail	84.90	92.87	84.11	92.87
	Common rail	71.46	81.87	-	-
Case 3	Iberian rail	82.44	89.93	81.44	89.93
	International rail	86.26	87.02	78.25	89.93
	Common rail	71.46	81.87	-	-

Table 6 shows that vertical track stiffness intervals are higher in Case 2 than in PR and AM models. Nevertheless, in the last case studied, the vertical track stiffness values are closer than the intervals obtained for the other two track types analyzed.

Therefore, case 3 is the most optimized case. To complete the results, Table 7 includes the values of the stresses transmitted from sleeper to the ballast layer in each load hypothesis.

Table 7. Stresses transmitted to the ballast layer in the sleeper section in Case 3

Stresses transmitted to the ballast (10^4 Pa)			
Load hypothesis	Sleeper section		Concrete block section
	Double rail side	Common rail side	Double rail side
H.1	9.05	8.85	2.89
H.2	8.52	9.51	4.52

H.3	7.51	7.63	3.27
H.4	7.53	8.09	4.53

From comparing Table 7 with Table 2, the stresses transmitted to the ballast in the sleeper section have better stresses distribution in Case 3 than in Case 1. Moreover, the stresses transmitted to the ballast in the concrete block section are lower than those obtained in the first case.

On the other hand, the stresses transmitted to the ballast are slightly higher in the new third-rail implementation system proposed in Case 3 than in the other track types analyzed. The stresses results in the loaded points of the sleeper section are shown in the Table 8 for each track type. These values are obtained in the worst load case of each model.

Table 8. Stresses transmitted to the ballast layer in the sleeper section

Stresses transmitted to the ballast (10^4 Pa)				
PR model	AM model		New system model	
On both sides	Double rail side	Common rail side	Double rail side	Common rail side
7.7	6.7	6	8.52	9.51

6. Conclusions

This paper presents a new third-rail implementation system which has the aim to transform the conventional railroad tracks in dual-simultaneous-gauge lines with less investments and less traffic interruptions than the existing methods. On the other hand, maintenance conditions are kept like in the conventional railroad tracks.

A finite element method has been used to check an appropriate vertical behavior. Three types of numerical models were carried out to achieve this objective: a numerical model of a track with this novel system, a numerical model of a track with dual-gauge-ready sleepers and a model with the three rails sleepers track. The study has been done by comparison of results.

A set of cases were studied in order to optimize the new third-rail implementation system presented. This analysis has been performed through the vertical displacements of the top head of the rail in Region 3 (Fig.3) and the stresses transmitted to the ballast layer. After the study, some remarks are shown:

- Elastic pads in Region 3 are necessary in order to avoid the possible risk of impacts between the international rail and the sleeper.
- Pads with less stiffness in Region 2 are better since they reduced the stresses transmitted to the ballast layer in concrete block section.

- Less stiffness in pads located in Region 2 and elastic pads in Region 3 improve the stresses distribution in the sleeper section.
- A decrease in the stiffness of the pads located in Region 2 makes more equal the vertical track stiffness in both sides of the concrete block track section.

Finally, it can be concluded that the best new third-rail implementation system is Case 3. In this case, pads located in Region 1 are 100 kN/mm and in Region 2 are 30 kN/mm. Moreover, an elastic element is located between the sleeper and the international rail in order to guarantee an adequate vertical structural behavior.

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