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La aritmética matricial como modelo del entorno cotidiano The matrix arithmetic as a model of the everyday environment

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Abstract

This article is a proposition for the teaching / learning of some matrix calculation elements from mathematical modeling. As a matter of fact, some daily situations are established showing how we can create models to illustrate the matrix concept and also by introducing basic operations of difference and product of matrices. Firstly, a matrix is shown as a mathematical model of an image and then how the matrix difference becomes a model for image comparison is discussed. However, to do this task software such as Octave (or similar software) is necessary. This tool allows the research of a numerical model of a black and white image represented by a matrix. Furthermore, we see how the product of matrices is a model which can be naturally deduced from the grocery shopping routine. The main idea is to underline the matrix calculation epistemology in order to reinforce the students? cognitive character, bringing a contextual view of daily matters in real life at the same time, enriching the heuristic, thus allowing the visualization of the connection among the mathematical symbolism (introduced on the model) and the real situations.

Este artículo es una propuesta para la enseñanza/aprendizaje de algunos elementos de cálculo de matrices a partir del modelado matemático. De hecho, algunas situaciones cotidianas se establecen teniendo también las matrices y sus operaciones como modelo matemático, en particular mostrando cómo podemos crear modelos para ilustrar el concepto de matriz y también introduciendo operaciones básicas de diferencia y producto de matrices. En primer lugar, una matriz se muestra como un modelo matemático de una imagen y luego se discute cómo la diferencia de la matriz se convierte en un modelo para la comparación de imágenes.Sin embargo, para realizar esta tarea es necesario un software como Octave (o similar). Esta herramienta permite la búsqueda de un modelo numérico de una imagen en blanco y negro representada por una matriz. Además, vemos cómo el producto matriz es un modelo que puede deducirse naturalmente de la rutina de la compra de comestibles. La idea principal es subrayar la epistemología del cálculo matricial para reforzar el carácter cognitivo del alumno, aportando al mismo tiempo una visión contextual de lo cotidiano en la vida real, enriqueciendo lo heurístico, permitiendo así la visualización de la conexión entre el simbolismo matemático (introducido en el modelo) y las situaciones reales.

Keywords: teaching / learning, mathematical model, matrix concept Palabras clave: enseñanza/aprendizaje, modelo matemático, matriz

1. Introduction

The concept of matrix is present in countless mathematical models of different situations ranging from Applied Sciences and Engineering to everyday life. However, its introduction into mathematical studies at the secondary level is often anecdotal and, at the tertiary level, it is almost always linked to the notion of linear mappings between vector spaces. This situation has two very negative effects. The first one is that students perceive matrices as abstract constructions that are alien to reality. The second one is that the understanding of the operations with matrices and their use in the various contexts of application where they appear becomes obscure for the students.

The attempts to contextualize mathematics in the field of tertiary education have been diverse, mainly in universities of the Catalan language area (see for example Sánchez Pérez, E.A., García-Raffi, L.M., Sánchez Pérez, J.V., 1999 and Joan Gómez Urgellés, 2007) and in the same manner the attempts to introduce matrices to students in applied contexts (see for example Jose M. Calabuig, Lluís M. García Raffi, Enrique A. Sánchez-Pérez 2013 and 2015). In this work different real situations are presented that provide frameworks where not only to apply the matrices as a mathematical model but to introduce in a natural way operations with them. Some of them have been applied to students of the first course of the Computer Science degree at EPSEVG University

2. Working with Images: The Matrices Difference as a Mathematical Model

2.1. A Matrix as a Mathematical Model of a Black and White image

When we talk about images, mathematics has an important role. Actually, technically each image can be seen as a table of numbers (formally known as a matrix). Then, defining an image as "composed by M per N pixels", it means that it can be represented by a matrix with M rows and N columns, generally with values between 0 and 255 (256 elements). The number of pixels is called "resolution". The procedure to obtain the matrix has been done through very sophisticated mathematical algorithms implemented by the software (as MatLabTM with an expense of 50\$ for students or 150\$ for home users). When we say 15 per 13 pixels, we mean something similar to the figure below —see Fig. 1—.

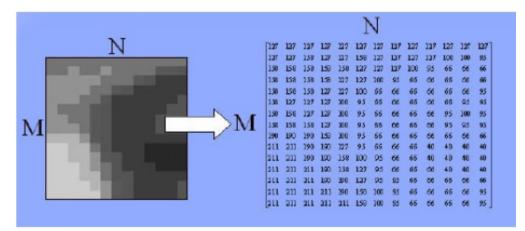


Figure 1 – The pixels and their correspondent array.

When we talk about "5 megapixels", we are really talking about 5 million pixels. However, if we read "640 x 480", that means a matrix of 640 columns per 480 rows Now lets go to analyze a real situation. Consider these violin pictures —see Fig. 2—:

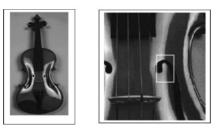


Figure 2 – Violin and Violin detail.

This violin picture image matches the Matrix below —see Fig. 3—:

170	170	180	185	184	188	185	184	178	175	144	10.002			87			8.6	40	**	**	*1	45	-	-	- 47
148	175	177	177	188	104	187	184	179	149	242	100	80		85	40	80	49	47	40	- 14	44	**	45	40	- 10
140	175	1.812	180	180	104	100	187	170	174	1.58	10.046	- 90	40	8.5	8.6	80	40		40	48	-	40	40	40	45
175	175	185	192	195	194	199	187	174	170	1.64	100	87		84	81	40	47	40	47	44	42	40	42	43	40
176	181	185	189	191	192	195	190	178	274	348	340	-	54	48	47	44	48	4.0		42	45	41	42	40	40
178	180	189	194	187	197	194	1.000	181	174	104	1.446	73			48	44	45	4.5	50		4.6	40	45	43	40
180	184	192	197	200	204	1.94	190	104	104	176	154	74		48	45	45	2.0	41	42	40	-04	42	40	45	-40
160	189	1.915	206	0.04	217	218	204	1.845	148	141	114			40	40	40	40	4.6	47		40	45	**	10	
180	187	2:00	233	234	223	140	0.4	28	14	11.0		12	12.0	2.9	38	49	40	4.6	44	45	41	40	28	40	40
191	195	220	214	143	43			+							12	25	33		40	-	42	42	40	40	-40
1.9-6	227	190	45	6	6										6		11	24	47	37	19	41	**	40	41
115	180	2.5	. 4								- 15	- 64							1.0	21	20	40	42	48	41
180									1.0	100	178	1.64								1.5			1.0	**	
45	- 2							- 22	142	280	178	140	124	80		6				6	14	10	18	2.8	45
8.8							1.0	141	100	187	10.00	1.68	1477	112	15							1.4		-	-40
							117	171	148	187	182	140	1.04	101	63	21				6			18	17	27
					7		+12	174	170	180	180	148	140	127	85	35	5.2							23	27
10							1.2	1.1-4	1.616	181	174	1.40	1.000	1.2-8		41.	2.4								1.7
				. 7				68	180	176	170	1.64	1.54	1.1/9	92	41	3.5	17							14
								2.4	100	174	1.48	140	151	524	87	40	1.0	2.0							
								18	117	180	200	1.80	147	120	15	3.8	2.9	3.6	1.6						
10						- 7			108	177	147	139	181	104	62	40	39	37	27						
10								11	1.1.1	172	140	1.00	1.02	1.17	80	40	41	17	10	1.8					
30	10		10					24	246	248	340	190	3.44	1.10	62	44	3.8	39	34	1.0					
	10	10							247	187	10.00	140	147	1002		40	42	24	2.4	1.5					
3.6							42	184	244	142	140	180	1.00	144	- 12	40	40	3.6	28	34					
115	80	24	12	1.5	24	44	142	187	185	141	140	1/52	140	107	67	48	40	18	34	3.6	12				
140	1.0.0	117	95	14	129	104	1.69	118	1.09	140	187	181	1.40	10.04	62		10	18	14	14	1.7				
134	124	145	146	141	152	151	187	264	247	166	140	153	1477	112	78	40	29	41	36	23	23	. 7	7		
158	141	1.64	244	150	1.8.9	104	100	148	175	171	187	140	1.00	100	61		42	40	27	24	24				
140	141	145	149	183	159	179	187	147	174	179	147	140	1.54	1.5/0	61	48	43	10	35	14	10	14			
148	150	145	151	154	159	178	3.62	140	172	347	142	156	1.50	114	59	55	45	38	35	30	23	127			
180	1.50	1.80	187	1.64	144	144	170	174	175	174	100	1.55	1.011	10.04	74	64	47	41	3.6	24	3.8	25			
140	140	153	158	1.60	148	148	171	178	178	170	140	141	1.60	104	66	47	43	40	37	24	33	25			
1.6.1	185	184	187	141	144	148	174	174	179	170	1.418	140	1.04	1.08	47	45	41	40	38	18	14	18			
147	182	156	141	147	149	147	175	177	180	180	247	154	1.55	128	80.	84	2.8	37	37	24	24	2.6	10		
1.84	155	158	144	170	249	171	172	185	181	170	171	179	1.54	128	01	40	43	42	40	24	33	27	10		
1.80	1.6.6	1.412	147	1.08	1/14	1/10	174	178	100	178	140	1.00	1.00	100	-979	10.00		6.6		1.0	10	1.0			

Figure 3 – Matrix of the Violin.

Take a glance at this unbelievable numerical table, even the density and the placement of the numbers drawing show the violin profile. Next, the procedure in order to get the model is fully explained.

Actually, we can choose any image in our computer, by selecting with the cursor over the image, doing right-clicking on it, and then open a *Properties Window* that shows all of the information about the image size. Depending on the resolution and the available space on the disk, it is possible to save the image in different formats such as BMP, TIFF, or JPEG as well—see Fig. 4—:

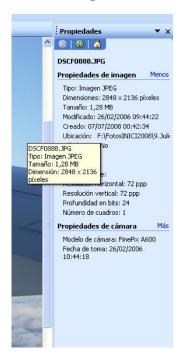


Figure 4 – Octave Properties Window.

In order to get a matrix model of an image, one needs software such as Octave. Formally Octave works with sophisticated numeric methods for obtaining the Matrix Model. The procedures are explained below.

2.2. Octave: Generating the Pixels Matrix of an image

Octave is a free program available for Windows, Mac, and Linux, developed for Numeric Calculations. It is available on the web https://www.gnu.org/software/octave/ It was developed around 1988, created by Chemical Engineering students from Texas University and Wisconsin-Madison University to be applied to support Chemical Reactors drawing. Actually, Octave is a free option to the well-known MatLabTM. Octave has a wide kit of tools to solve algebra, calculation, and statistics problems. It is also able to process digital images. A Smartphone version is available. We are working with Black and White images because the associated Matrix is bi-dimensional, which means that it is a Numbering Table with rows and columns. On the other hand, in the case of colored images, a "three-dimensional Matrix" would be generated, and each color would be obtained from the basic RGB (Red, Green, Blue). With Octave, we can generate the Pixel Matrix of any image. How can we do that? By following the steps below:

- 1. To install the program, go to the link: https://www.gnu.org/software/octave/ or ftp://ftp.gnu. org/gnu/octave/windows/octave-4.0.0_0-installer.exe
- 2. Once the Octave installation is completed, run the program by opening a similar window such as:

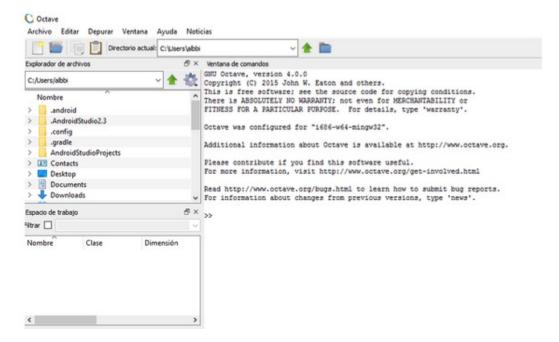


Figure 5 – Running Octave.

3. Choose a previously saved image in the directory.



Figure 6 – Saving in the directory.

4. And then, select the image whose Pixel Matrix Associated we want to know.

Remember that it is necessary to save the image in the folder created by Octave, which in our case is C:\Users\albbi. Literally, in the Octave window we should go to the directory in which the images have been

SUMMARIZING: We can introduce the concept of Matrix as a model of a Black and White image.

3. Modeling Experience

Now, we are doing an experience which has been explained during the first course of the computer Science degree at the Universitat Politècnica de Catalunya (UPC) by showing the Matrix difference as a Mathematical Model of an image. In our example, we are considering two different black and white images, previously saved in our computer, and comparing both. In our computer, these images have been saved as "imatge1.jpg" and "imatge2.jpg" —see Fig. 7—.



Figure 7 - "imatge1.jpg" and "imatge2.jpg".

Apparently, these pictures seem identical, but this is not true; the second picture has a small different dot in yellow in order to be easily seen. Now we are introducing the following lines in octave command Variable Name :

```
>>I = imread (''fotografia1.jpg'')
```

where "I" has been chosen as a Variable Name. In this line, we are saving the Variable "I", the "imatge 1". However, we are really saving the Matrix of picture 1. Next, we are introducing the command Variable Name

```
>> I2= imread (''fotografia2.jpg'')
```

saving the variable "I2" ("picture 2").

By pressing "Enter" after each instruction, the window shows respectively the Matrix as a Mathematical Model of each image respectively obtaining the result shown in Fig. 8.

I	- 1																														
	Colum	ns 1	throu	gh 21	1																										
Ŀ	161 161	161 161 159	161 161 159 158	161 161		161 161 159 158	161 161 159 158	161 161 159 158	159 157 157 158	15! 15! 15!	158 158 157 155 155 156		150 150 150 157 155 155		158 158 157 155 155 156	150 150 150 157 155	158 158 157 155 155 155	150 150 157 155 155 156	150 156 156 154 155 155	161 159 150 150 150 156 156 156 156 156 156 156	159 159 157 155 156 156	159 159 157 155 156 156	160 150 150 159 157 155 155	161 159 157 159 157 156	162 161 160 150 150 150 156 156	162 160 150 150 150 157	165 164 161 159 164 162 159 156	165 164 162 159 164 162 159	166 165 160 164 162 159	167 166 163 161 164 160 159	169 167 166 167 165 165 165 165 165 166 165 165 165 157
	158 157 155 155 155 156	158 157 155 155 156 155	158 157 155 155 156 155	158 157 155 155 156 155	158 157 155	158 157 155 155 156 155	158 157 155 155 156 155	158 157 155 155 156 155	156 154 157 155 154 153	15: 15: 15: 15: 15:	154 250 1497 1497 1496 1497 1496 1497 1496 1499 1499 1499 1499	154 152 150 147 146 147 146 146 146 146 146 146 146 146 146 146	154 1550 1447 1466 1466 1466 1468 1468 1468 1468 1468	25420 2450 244776 2446776 244677777 2477777 2477777	154 152 148 148 148 147 146 147 146 147 146 146 146 146	154 1550 1499 1499 1497 1496 1497 1496 1497 1496 1496 1496	154 150 148 148 149 1447 1447 1447 1447 1445 145	15420 1489 1489 1487 1487 1487 1487 1485 1485	153 151 140 145 148 146 147 146 147 146 143 143 143	153 150 146 146 146 146 146 146 146 146 146 146	154 159 1467 1465 1465 1465 1465 1465 1465 1463 1463 1463 1463 1463 1463 1463 1463	1553077554455653333 14556653333 1455653333	1554 1551 1497 1495 1495 1495 1495 1495 1495 1495 1493 1493 1493 1493 1493 1493 1493 1493	156 155 151 149 147 146 146 146 146 146 146 146 143 143	156 155 149 147 146 145 146 146 146 146 143 143	157 155 150 148 147 146 147 146 143 143 143	155 153 159 148 148 148 148 148 148 148 148 148 148	155 154 151 148 140 147 146 145 145	155 154 150 140 140 140 146 146 146 146 146 146 146	154 150 149 149 149 149 146 147 146 147	151 149 149 149 149 149 147 147 147 147 147

Figure 8 – Left: Image Partial view (imatge 1). Right: Full Capture of the "imatge 1" Matrix.

Similarly for the second image:

>> 12 12 =																														
ans(:, Colum			gh 18	:																										
159 161 161 159 158 158 158 158 155 155 155 156 155 154 152	159 161 159 158 158 158 155 155 155 155 155	159 161 159 158 158 158 155 155 155 155 155 155	159 161 159 150 150 150 155 155 155 156 155 154 152	159 161 159 158 158 158 155 155 155 155 155 155	159 161 159 150 150 150 155 155 155 156 155 154 152	159 161 159 158 158 158 158 155 155 155 155 155	159 161 159 158 158 158 158 155 155 155 155 155	161 159 157 158 158 158 158 157 155 154 153 153 151	161 159 157 158 158 158 158 158 157 156 154 153 153 153	159 161 161 159 1588 1588 1588 1585 1585 1585 1585	161 159 159 159 155 155 155 155 155 155 15	161 159 150 155 155 155 155 155 156 150 147 146 146	154	159 150 150 155 155 155 155 155 155 155 155	161 159 150 150 155 155 155 155 155 155 155 149 149 149 146 146	161 159 150 150 155 155 155 155 155 155 155 156 155 150 149 149 149 147 146	1611 1500 1550 1555 1555 1555 1555 1555	159 157 157 150 150 156 156 155 155 155 153 153	159 157 158 158 158 158 158 158 158 158 158 158	150 150 159 159 155 155 155 155 155 155 155 155	160 159 150 150 150 150 150 155 155 155	160 159 160 150 157 155 155 155 155 155 154 151 140 147 145 145	160 161 161 159 157 159 157 156 156 156 155 151	161 162 160 150 150 150 150 156 156	161 162 160 150 150 150 157 156 157 155	165 164 159 164 162 159 156 155 153	164 165 164 162 159 164 162 159 157 155 153 151 149 140 140 147 146 145	157 155 154 151 150 148 148 148 148 147 146	16667631121155976420998766678	165 161 164 157 156 155 151 149 149 149 149 147 140 147 147

Figure 9 – Left: Image Partial view (imatge 2). Right: Full Capture of the 'imatge 2" Matrix.

As you can see the differences between the images are clear because the values of the matrix models are different. Strictly Speaking: we can get the differences between images by subtracting these matrices and concluding that the regions with zeros do not have changes. On the other hand, the regions with values different from zero mean that they do have changes.

SUMMARIZING: The difference between Matrices would be a Mathematical Modeling to compare images.

The Model has been applied in the class to the First Course of Computer Science EPSEVG University, as a group work developed by students, despite the fact that they had never worked with Matrices before. However, they were able to explain the work in the classroom to their other classmates, as shown in Fig 10.



Figure 10 – Left: Computer Science University class. Right: The Matrix obtained by the Computer Science students.

Octave also allows subtracting the obtained matrices. It's even possible to select the Matrices by attaching them to the Excel database and then subtracting them. The results of our experience can be seen in Fig 11.

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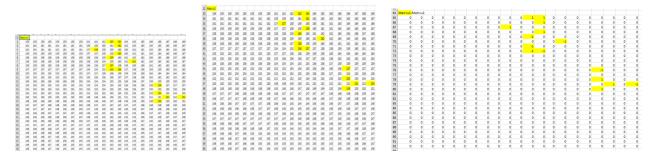


Figure 11 – Result of the difference between Matrix 1 and Matrix 2.

The resultant matrix clearly shows the regions of the image in which all differences have been observed. Another example has been gathered from the Written Press. It refers to finding / spotting the differences —see Fig. 12—. In detail, we realized that as a model, they have respectively their Numerical Matrix —see Fig. 13—.



Figure 12 – The Game of Differences.

101	20	150	77	***	78	***	26	100	101	76	100	117	204	86V	10		17		~**	- 14	10
105	113	98	103	112	99	99	108	113	115	139	116	74	52	29	38	37	43	54	68	72	70
.03	105	92	99	111	106	111	123	128	91	46	32	56	91	114	121	130	128	126	122	122	123
88	86	97	108	99	115	124	77	22	44	72	100	121	130	132	129	128	128	130	131	129	128
98	92	114	96	135	137	27	6	71	94	115	125	125	125	132	140	134	136	137	138	139	138
96	102	86	130	113	19	26	86	118	125	129	128	126	125	128	131	133	131	130	130	130	132
96	99	120	119	23	11	118	135	122	129	133	139	142	138	129	122	128	129	129	129	132	136
108	129	132	28	30	126	118	118	149	151	149	141	133	127	126	125	113	115	118	123	127	129
87	107	38	26	103	117	115	163	141	147	147	142	131	123	118	117	121	118	117	118	123	126
59	0	20	94	110	121	149	131	151	149	145	140	134	129	125	119	91	81	73	66	66	73
237	60	65	111	123	119	135	136	144	122	149	130	117	132	111	47	20	19	25	41	40	32
133		114	126	123	127	115	133	132	120	128	130	126	121	35	50	139	193	213	204	199	212
64	90	142	120	110	120	122	192	120	120	112	110	115	46	5.4	210	240	240	220	247	252	245
01	20	195	107	110	107	100	136	130	200	110	11.7	110	10	101	224	240	017	244	0.45	200	010
76	142	135	126	118	124	133	94	75	77	114	134	112	28	181	236	249	255	244	245	255	252
103	144	120	127	121	127	97	45	44	22	22	49	107	28	198	250	236	255	254	253	254	252
127	122	129	127	120	137	54	65	180	225	176	53	31	42	204	248	255	255	254	255	254	253
130	117	131	123	119	132	34	136	248	212	244	219	57	47	233	236	234	255	255	245	243	255
110	123	117	121	125	114	29	177	231	112	108	226	144	70	233	243	247	231	237	253	243	242
78	126	121	128	115	124	46	177	247	233	148	92	215	168	239	247	244	249	245	234	234	255
41	117	115	115	132	127	35	167	250	170	72	107	243	236	255	214	147	123	120	159	254	243
108	51	130	120	127	130	66	94	255	93	195	214	228	2.53	163	78	79	93	47	49	243	250
223	37	68	128	118	115	108	33	195	147	120	243	239	154	36	138	191	94	52	66	176	259

Figure 13 – Searching the differences.

As previously shown, the differences between images have been found in row 3, column 10. Over there, number 91 has been converted to 2. Also in row 21, in column 20 number 49 has been converted to 240. This means that by subtracting both matrices, zero is obtained in all operations except in row 3, column 10, and in row 21, column 20 as well.

To our students, we can comment on another daily example when the difference between matrices reaches a remarkable role in the security field.

3.1. The difference between matrices as a Model of Security System.

According to our previous results, we can compare Images. Think about a hypothetical frame series (in black and white) captured by surveillance camera inside a bank. Now, considering the models of two consecutive images, the "intelligent camera" processes the difference between two matrices. If a "zero" matrix (all the elements null) is obtained, we realize that no movement has been made, in this case it is not necessary to record all associated images to those matrices. On the other hand, all the matrices with a difference not null will be recorded registering the movement inside the bank. This simplified example explains how the "intelligent surveillance cameras" do a night surveillance.

4. Shopping at a Supermarket: A Model of the Product between Matrices

In the following situation, the students, naturally discovering how to make the product of matrices into a Model, link the quantities of buying products at a Supermarket to their prices with total expenses. Now we will do an easy study in order to clarify which one between two supermarkets is the least expensive merchant. We are shopping twice a week, buying the articles and quantities according to the Table 1.

	Pork Loin(kg)	Oranges (kg)	Lettuce (3 units \setminus tray)
1 st Day	1	3	1
2 nd Day	3	2	2

Table 1 – A Daily	Consumption.
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This table can be written in a different way:

$$\left(\begin{array}{rrrr}1&3&1\\3&2&2\end{array}\right)$$

And then for each supermarket we can calculate:

- What are the expenses for the first day?
- What are the expenses for the second day?

We can also do a global calculation:

• What is the least expensive option for each day?

These questions could be proposed to our students so that they can calculate and achieve their own conclusions.



Figure 14 – Expenses at Supermarket 1

In the case of Supermarket 1 we have the price list in the Table 2, and for Supermarket 2, in Table 3:

Supermarket 1	Pork Loin(kg)	Oranges (kg)	Lettuce (3 units \setminus tray)
Price	6.49 €	1.25 €	0.75 €

Table 2 – Articles and Prices in Supermarket 1

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Figure 15 – Expenses at Supermarket 2

Supermarket 1	Pork Loin(kg)	Oranges (kg)	Lettuce (3 units \setminus tray)
Price	5.90 €	0.85 €	1.25 €

Table 3 – Articles and Prices in Supermarket 2

Then, we can perform the daily calculation of our expenses in each supermarket:

- Supermarket 1
 - 1st Day \longrightarrow 1 · 6.49 + 3 · 1.25 + 1 · 0.75 = 10.99 \in
 - 2^{nd} Day $\longrightarrow 3 \cdot 6.49 + 2 \cdot 1.25 + 2 \cdot 0.75 = 23.47 \in$
- Supermarket 2
 - 1st Day \longrightarrow 1 · 5.90 + 3 · 0.85 + 1 · 1.25 = 9.70 \in
 - 2^{nd} Day $\longrightarrow 3 \cdot 5.90 + 2 \cdot 0.85 + 2 \cdot 0.75 = 21.90 \in$

Mathematically, we can write the expenses in each supermarket as:

- Supermarket 1
 - 1^{st} Day $\longrightarrow (1,3,1) \cdot (6.49, 1.25, 0.75) = 10.99 \in$
 - 2^{nd} Day $\longrightarrow (3, 2, 2) \cdot (6.49, 1.25, 0.75) = 23.47 \in$
- Supermarket 2
 - 1st Day \longrightarrow (1,3,1) · (5.90, 0.85, 1.25) = 9.70 \in
 - 2^{nd} Day $\longrightarrow (3, 2, 2) \cdot (5.90, 0.85, 1.25) = 21.90 \in$

You can see that what we apply is the so called Scalar Euclidian Product. It is remarkable that he students had been building the scalar product in an intuitive manner.

SUMMARIZING: A daily matter such as shopping at a Supermarket has a Mathematical Model, the "Scalar Euclidean Product".

Globally, we can write the expenses in each supermarket as:

$$\begin{pmatrix} 1 & 3 & 1 \\ 3 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 6.49 \\ 1.25 \\ 0.75 \end{pmatrix} = \begin{pmatrix} 10.99 \\ 23.47 \end{pmatrix}$$
(1)

$$\begin{pmatrix} 1 & 3 & 1 \\ 3 & 2 & 2 \end{pmatrix} \cdot \begin{pmatrix} 5.90 \\ 0.85 \\ 1.25 \end{pmatrix} = \begin{pmatrix} 9.70 \\ 21.90 \end{pmatrix}$$
 (2)

Here we are naturally building the product of a matrix by a vector column. Then the Matrix Model can be represented as the next matrices:

• Q: Quantity of Products

$$Q = \left(\begin{array}{rrrr} 1 & 3 & 1 \\ 3 & 2 & 2 \end{array}\right)$$

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- *P*: Price in each supermarket
- $P = \begin{pmatrix} 6.49 & 5.90\\ 1.25 & 0.85\\ 0.75 & 1.25 \end{pmatrix}$ $\begin{pmatrix} 10.90 & 9.70\\ 23.47 & 21.90 \end{pmatrix}$

D: Expenses

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By doing it as previously mentioned, it is possible to introduce the matrix product. The following Mathematical Model links Quantities (Q), Price (P), and Expenses (D) by Eq. 3.

$$Q \cdot P = D \tag{3}$$

SUMMARIZING: We realize that by linking purchased quantities and prices, it is possible to naturally obtain the algorithm to multiply matrices.

Going back to the previous supermarket comparison, the Supermarket 2 has the best deals. The total expenses in Supermarket 2 mean significant money saving when compared with those in Supermarket1. The students who did this exercise discovered how to multiply Matrices naturally. Now, the professor feels free to propose situations by introducing the inverse matrix concept and other elements of Matrix calculation.

5. Conclusions

These examples are useful to show how the use of real life situations makes it feasible to find patterns (models) giving information about the proposed situations.

Obviously, the reader can translate the involvement of the Mathematical Modeling in the Academic Curricula, at the same time realizing that competent teaching is taking place.

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