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OPTIMIZATION OF THE PHASED REPLACEMENT OF A BUS SYSTEM BY A RAIL TRANSIT LINE WITH A FEEDER BUS SERVICE

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ABSTRACT

TITLE OF DOCUMENT: **OPTIMIZATION OF THE PHASED REPLACEMENT OF A BUS SYSTEM BY A RAIL TRANSIT LINE WITH A FEEDER BUS SERVICE.**

José de Jesús Reyes Sánchez-Cutillas, M.S., 2018.

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This thesis develops a method for both determining if it is worth replacing an existing conventional bus line with a rail transit line and optimizing the rail line's construction phases if the extension is justified. This optimization is done by minimizing the total cost of the system under different financial constraints (i.e., unconstrained, budget-constrained, politically constrained and physically constrained cases).

The model used for this optimization problem analyzes a corridor with one main transit line and branching feeder lines. It connects the Central Business District (CBD) with a suburban area. Due to the great capital cost required to begin the project, and to demand which may only justify rail development after future growth, a rail transit project is sometimes divided into several phases. Five different options are developed, and the most interesting result obtained with the baseline inputs indicates that the best option for every case is to replace the main bus line with a rail line. Sensitivity analysis examines the effects of input parameters (e.g., unit operating costs, interest rate and value of time) on optimized results. This study provides valuable guidelines to decision-makers and transportation planners in deciding construction phases for projects that consist of replacing of conventional bus lines with rail lines.

KEYWORDS: Phased development, feeder bus system, rail transit line, public transit service optimization, conventional bus system.

RESUMEN

TÍTULO DEL DOCUMENTO: **OPTIMIZACIÓN DEL REEMPLAZO PROGRESIVO DE UN SISTEMA DE AUTOBÚS POR UNA VÍA DE TRÁNSITO FERROVIARIO CON RUTAS DE AUTOBÚS INTEGRADAS.**

José de Jesús Reyes Sánchez-Cutillas, M.S., 2018.

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Este estudio desarrolla un método para determinar no solo si vale la pena el reemplazo de una línea de autobús por una vía ferroviaria, pero también la optimización de las fases de construcción de la línea ferroviaria si la extensión estuviese justificada. Esta optimización se realiza minimizando el coste total del sistema teniendo en cuenta diferentes restricciones (políticas, económicas y físicas).

El modelo utilizado para este problema de optimización analiza un corredor con una línea principal de tránsito y líneas de autobuses integradas. Esta conecta el distrito financiero con el área suburbana. Debido tanto a la gran cantidad de dinero requerido para empezar un proyecto de gran envergadura, como al incremento de la demanda (siempre y cuando se espere un crecimiento futuro de la misma), un proyecto ferroviario a veces se divide en diferentes fases. Se desarrollan cinco opciones diferentes, y el resultado más interesante con los valores introducidos indica que la mejor opción es el reemplazo total de la línea de autobús por una lía ferroviaria. Los análisis de sensibilidad exploran los efectos de diferentes parámetros (por ejemplo, costes de operación, ratio de interés y el valor del tiempo) en los resultados optimizados. Este estudio provee directrices generales útiles para responsables y organizadores de transporte para proyectos que consistan en el remplazo de líneas de autobús por líneas férreas.

PALABRAS CLAVE: Fases de desarrollo, rutas de autobuses integradas, líneas de tránsito ferroviario, optimización del servicio público de tránsito, sistema convencional de autobuses.

RESUM

TÍTOL DEL DOCUMENT: **OPTIMIZACIÓ DEL REEMPLAÇAMENT PROGRESSIU D'UN SISTEMA DE BUS PER UNA VIA DE TRÀNSIT FERROVIARIA AMB RUTES DE BUS INTEGRADES.**

José de Jesús Reyes Sánchez-Cutillas, M.S., 2018.

DIRIGIT PER: Dr. Paul M. Schonfeld
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Aquest estudi desenvolupa un mètode per a determinar no només si val la pena el reemplaçament d'una línia de autobús per una via ferroviària, però també l'optimització de les fases de construcció de la línia ferroviària si l'extensió estiguera justificada. Aquesta optimització es realitza minimitzant el cost total del sistema tenint en compte diferents restriccions (polítiques, econòmiques i físiques). El model utilitzat per a aquest problema d'optimització analitza un corredor amb una línia principal de trànsit i línies de autobusos integrades. Aquesta connecta el districte financer amb l'àrea suburbana. Degut tant a la gran quantitat de diners requerits per començar un projecte de gran envergadura, com a l'increment de la demanda (sempre que s'espere un creixement futur de la mateixa), un projecte ferroviari de vegades es dividix en diferents fases. Cinc options diferents són desenvolupats, i el resultat més interessant amb els valors introduïts indica que la millor opció és el reemplaçament total de la línia de bus per una lia ferroviària. Els anàlisis de sensibilitat exploren els efectes de diferents paràmetres (per exemple, costos d'operació, ràtio d'interès i el valor del temps) en els resultats optimitzats. Aquest estudi proveïx directrius generals útils per a responsables i organitzadors de transport per a projectes que consistisquen en el reemplaçament de línies de autobús per línies fèrries.

PARAULES CLAU: Fases de desenvolupament, rutes d'autobusos integrades, línies de trànsit ferroviari, optimització del servei públic de trànsit, sistema convencional d'autobusos.

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GLOSSARY

HRT: Heavy Rail Transit

LRT: Light Rail Transit

WMATA: Washington Metropolitan Area Transit Authority

AIX: International Airport

AID: Dulles International Airport

ORD: O'Hare International Airport

NPW: Net Present Worth

PSP: Project Scheduling Problems

RCPSP: Resource-Constrained Project Scheduling Problem

CBD: Central Business District

RT: Round Trip

TS: Tabu Search

GA: Genetic Algorithm

SA: Simulated Annealing

SPSA: Simultaneous Perturbation Stochastic Approximation

DCMP: Dulles Corridor Metrorail Project

CNG: Compressed Natural Gas

VBA: Visual Basic

CP: College Park

US: Union Station

EP: Enfant Plaza

SEM: Standard Error of the Mean

Chapter I: INTRODUCTION

I.1. BACKGROUND

Public urban transport plays an important role in densely populated areas, and especially in large metropolitan regions. Since the first horse-drawn omnibus vehicle to the high-speed rail, cities have relied on public transportation for its utility and efficiency. It serves people who cannot afford a car, reduces delays, pollution, accidents and congestion. It also helps to preserve the downtown area, improve access to jobs and enable efficient use of resources. In many cities bus and rail lines operate in mixed traffic. That is why both services should be integrated and coordinated as well as possible, to minimize the wait times of passengers.

Buses have formed the backbone of urban public transport in many cities and towns since the development of the internal combustion engine. The use of bus lines may be declining in large cities for several reasons. This spiral of decline is related to pollution, environment degradation and congestion in the metropolitan areas, along with a decrease of the quality service. However, there are favoring factors that resist the decline of buses as urban public transport such as: low initial cost, low marginal cost per new routes added, ability to avoid obstructions and no requirements for electrification.
Currently, the tendency is to substitute the bus systems with other means of transport, such as rail transit. Rail transit mitigates traffic congestion, decreases energy consumption, has a higher service reliability service than buses, increases mobility, raises land values, and has higher capacity. The problem is that adding new stations and sections requires a great deal of money.
The future of rail lines lies partly on the cost reduction cost of rail systems (HRT and LRT)[1]. Future development would be helped by reductions in their environmental impacts and a quality service comparable to that of light rail.

An important factor of replacing conventional bus systems with rail lines that should be considered is project scheduling, which is an essential component of project management. Each project has a number of constraints that must be satisfied (equipment resources, materials, work force…). That is why the project scheduling phase assigns a start time considering all those restrictions (Martinelli, 1993). If the scheduling is good, many benefits come from it such as facilitating the acquisition of materials on time and the reduction of bottlenecks. On the contrary,

[1] HRT: Heavy Rail Transit (Subway). Used for medium distance trips. Electrical power in rail and platforms are needed.
LRT: Light Rail Transit (Tram). Focused on short trips, its capacity is quite limited, and it emphasizes in acceleration and deceleration.

poor scheduling can lead to equipment waiting for the completion of the previous tasks or to a waste of workforce.
The two types of project scheduling that are more often used are the following ones: time-oriented, and resource-oriented scheduling (Hendrickson, 1989). On the one hand, in the time-oriented approach the stress is, considering the different relations among tasks, to determine the competition time of the project. On the other hand, resource-oriented scheduling is used when resources available are limited, so it is very important to schedule in an effective way the tasks. It is useful to manage multiple projects with fixed resources, both equipment and labor. Nevertheless, both approaches focus on the benefit of the private sector instead of focusing on the users' interest. In particular, for public transportation planning, considering effects on both sides users and operators is necessary for scheduling. That means that the economic feasibility of the project should be assessed from the point of view which includes the whole system, both users and operators. Besides that, the economic side of the project is not the only one considered. In fact there are several factors, such as political ones, that constraint the optimal economical solution. For instance, the best option is usually to keep at the minimum the construction of rail road because the capital cost is really high according to the planners. However, politicians usually want to overextend these constructions even if there is not high utilization rate because they want a system that provides service to as many areas as possible. It is called the social influence on transportation projects. As a consequence of the high capital investment required by transportation projects, a bad scheduling means a waste of resources and money. Thus, decisions should be made carefully.

That is why planners and decision-makers must consider several steps before making a decision. These steps include the evaluation of different alternatives, and several impact studies such as environmental, engineering, traffic or economic. In transportation projects, a small change in the schedule for a high capital cost project could possibly lead to decrease of benefits. That is why for transportation projects a good comprehension of the different analysis regarding construction scheduling and economic feasibility is important.

I.2. WASHINGTON AND CHICAGO EXTENSIONS

The last few decades have witnessed the expansion of rail transit networks across some large metropolitan areas. For instance, since 1965 until 2018 the Metrorail of WMATA (Washington

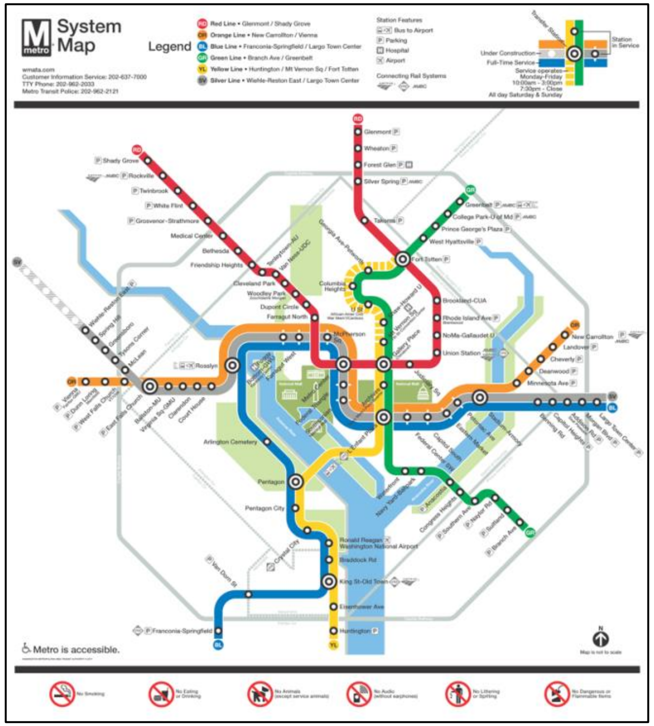


Figure 1.Current Washington DC metro system map. Source:WMATA

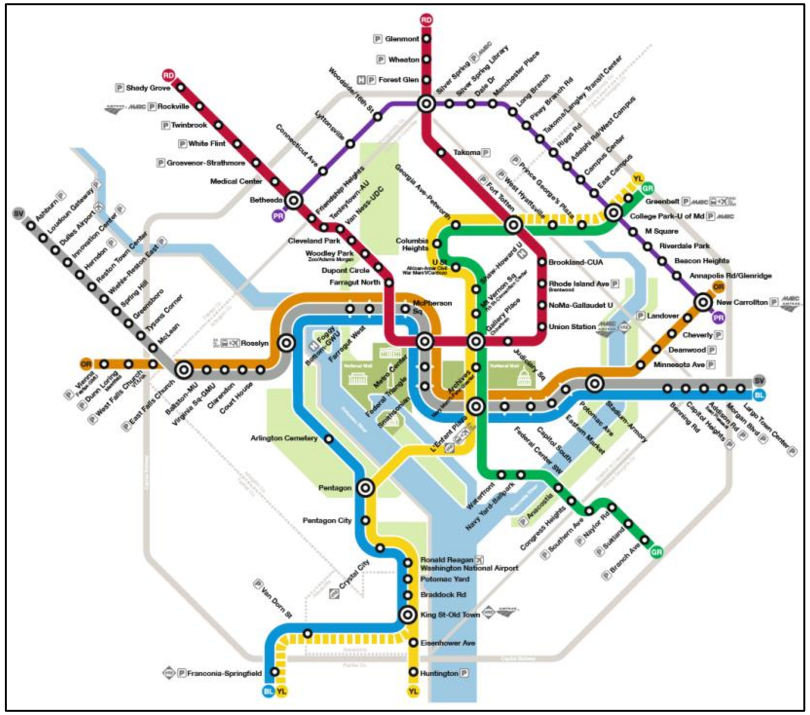


Figure 2. 2022 Washington DC metro system map. Source: Pinterest.com

Metropolitan Area Transit Authority) the rail system has passed from twenty-five miles subway to one hundred nineteen miles of rail road (counting subway, surface and aerial) with ninety-one stations. Moreover, an extension of 23.2 miles of the WMATA's Silver Line (involving the construction of eleven new stations) is expected to connect Washington DC with the Washington Dulles International Airport. This extension is conceived to be done in two phases and should be ready in 2020 (the construction of the Dulles corridor Metrorail project began in 2008, and its planification in 2000). The investment needed to carry on the project is $5.683 billion, but it increased to 6.8 billion, which $2.7 billion corresponds to the first phase, and $4.1 billion to the second phase. With this extension the actual bus service that works between airports and downtown is supposed to be restructured.

Figure 3. (up) Current Chicago's metro service system. Source: Chicago's Transit Authority. (down) Future Airport express line from O'Hare Airport to Downtown. Source: John Greenfield.

Another example of a rail trail extension is the one being deployed in Chicago, a new service to connect O'Hare International Airport (ORD) and Midway with downtown, called *Airport Express*. This growth is valued $3.2 billion dollars for 5.3 miles extension.

It has been a controversial project, because there already is a metro line connecting O'Hare International Airport with downtown, but in fifty-one instead of fifteen minutes.

However, according to the forecasts, there will be enough passengers flow to make the extension justifiable.

I.3. Problem Statement

Although financing public transport is a common practice, it has been always open to question. Due to the present economic situation, the debate about justifying subsides to the public transport is the order of the day. Government policy concerning the subsidies to public transport has changed, decreasing its funds. Meanwhile the users' standards are higher than ever. Users want reasonable time transfers, high frequency and low fares.

Unfortunately, the cost of a rail transit project is considerably high. For that reason, a quite comprehensive evaluation is needed to determine if it is worth to carry them out instead of using a bus network. Figure 4 shows the structure for evaluation of adding new stations to an existing rail transit route. This addition or extension affects many users in different ways: increase mobility, reduce congestion, increase land values, rise land value and reduce emissions among other factors. It also implicates high investments.

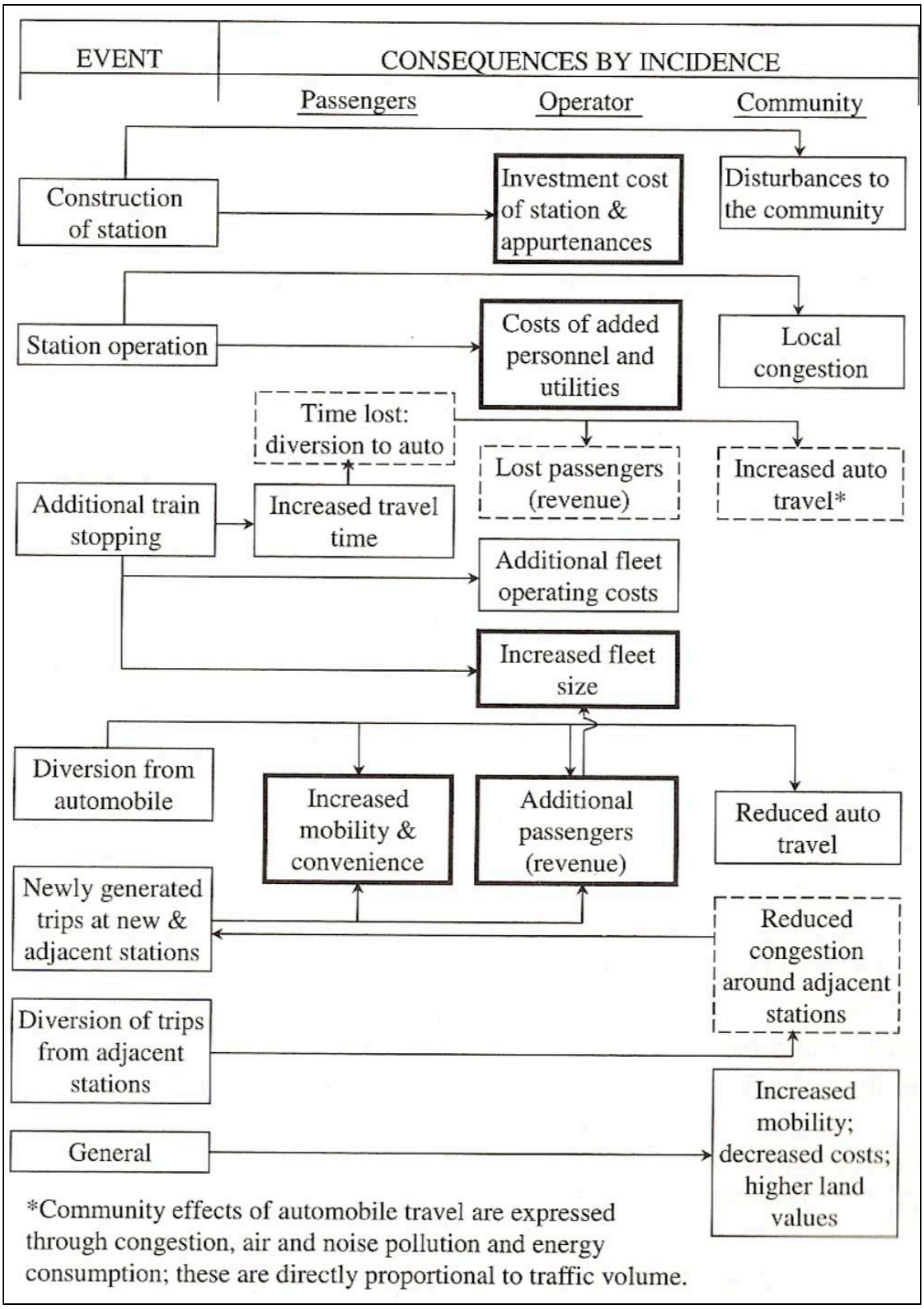


Figure 4. Effects of Rapid Transit Line Extension. Source: Vuchic

The high investments normally required lead to the division of the project in phases. This is not always true. In fact, when economies of scale are considered the best decision is to realize one-time expansion. Nevertheless, if demand growth and the value of money are considered, postponing non-necessary additions to future periods is the best decision, leading to a multiple-phase expansion. Furthermore, the addition of links and stations in the network affect directly the service quality for existing users. That is to say, these additions change the connectivity among stations, leading to a demand redistribution, which is an important factor to be considered in rail transit extension research. That is what makes the phased problem nontrivial. In fact, nowadays budget limitations and demand growth over time are the factors which determine the number of phases of a project. For instance, the WMATA's project (twenty-three miles extension) mentioned before is divided in two phases. These divisions are usually based on budget limitations and demand growth over time.

Presently, despite de fact that both extension and construction of new rail transit lines are critical decisions, no general guidelines are yet available to know neither how many phases are needed nor when is the best timing to implement each phase. That means that insufficient attention has been paid to the effects of route extensions. Scheduling decisions have an influence over the system performance all along the analysis period. That is to say, the result for the entirely analysis is affected by any decision made.

Therefore, a method is proposed to determine whether it is worth to replace a bus network with a rail transit line, and if it is economically feasible, to determine how many phases should be planned to complete a rail extension and when is the best time to implement them.

I.4. RESEARCH OBJECTIVE

The objective of the research is to determine the feasibility of an already planned and designed rail transit line extension, which would replace a conventional bus system, minimizing the costs and considering economies of scale. The already designed and planned 5.9 miles extension corresponds to a hypothetical International Airport Corridor[2]. Here it is studied the feasibility of four new stations, from the *Agua Azul (A)* stop to the *International Airport* (AIX) stop. However, the project remains open if it were to increase the extension further from the Airport. As Figure 5 shows, each new station is given a letter as notation:

A → Archives

B → Bellecour

C → Châtelet

D → Darongers

E → International airport

[2] This Project does not exist, it has been created for the purpose of this study.

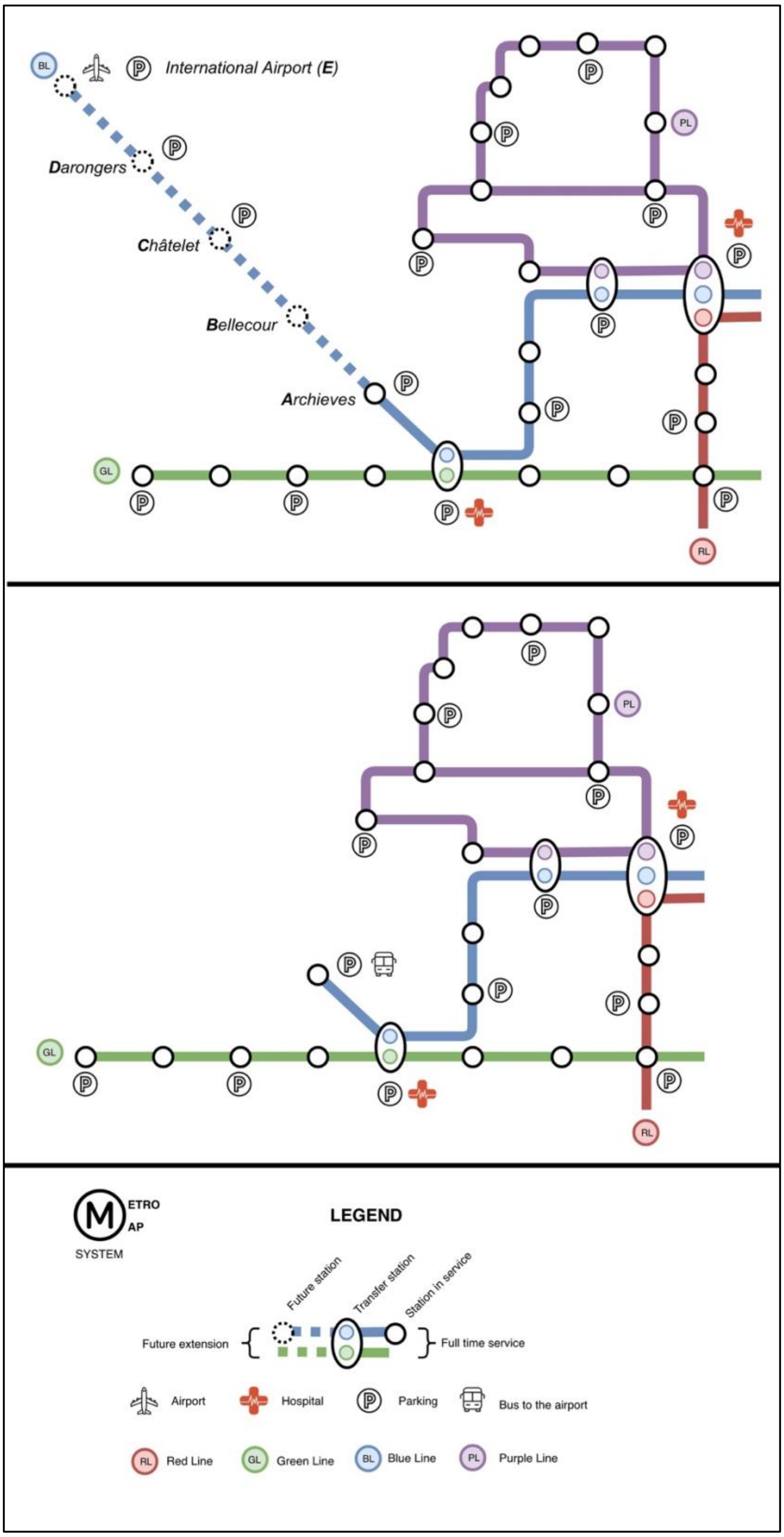


Figure 5. (Up) Future extension of the Blue line. (Down) Present status of the Metro system Source: Own elaboration

It is evaluated if a new trunk rail line with a feeder bus system is justified to be built for replacing the existing bus corridor. Once assessed the feasibility of the replacement, it is studied the possibility of subdividing the project in phases to optimize the costs. Determining the optimal number of phases to implement and how many stations and links should be built in each phase depends on the different constraints. Demand and costs might be notably affected by adding stations. Therefore, the model analyzes which is the best option considering the limitations. There are five different possibilities for the project: no replacement of the bus line, building one, two, three or four links.

The first option studies the current situation (Figure 6). That would let us know if it if a good decision to initiate the replacement of the bus line.

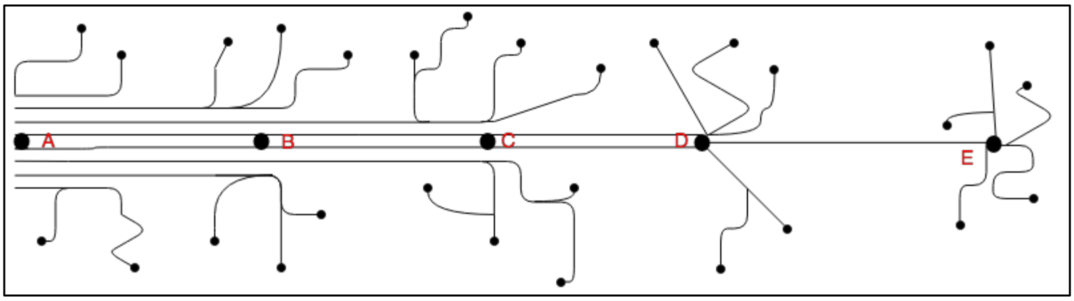


Figure 6. Current conventional bus system from Archieves East to AIX. Source: Own elaboration

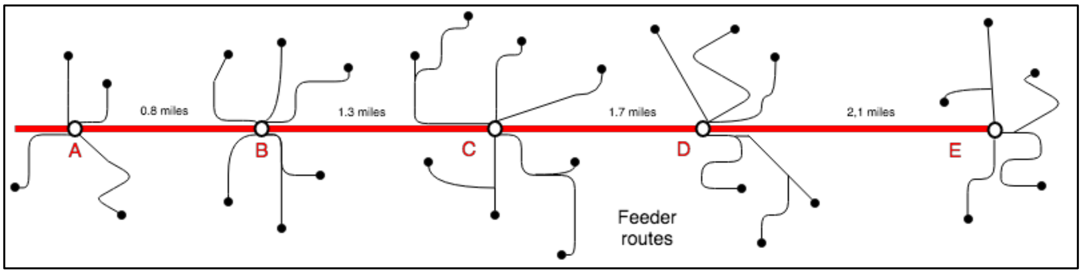


Figure 7. Rail trunk line and feeder bus system from Archieves to AIX. Source: Own elaboration

Figure 7 shows another option, the five point nine miles possible replacement of the bus line. The project consists on the replacing bus with rail transit on the entire main line of the corridor, while keeping the feeder bus system at each mainline station.

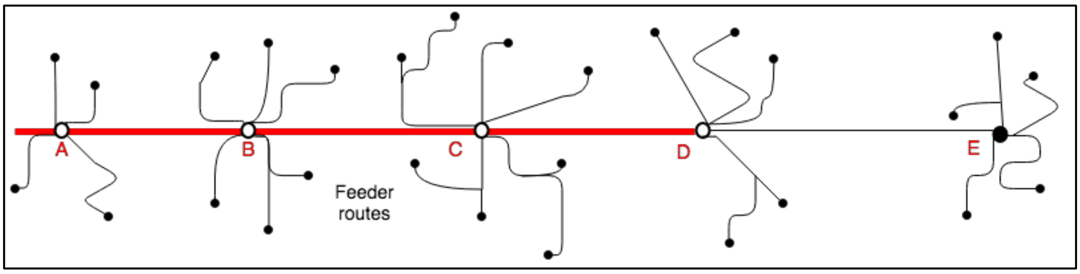


Figure 8. Rail trunk line until D, and conventional bus system from D to E. Source: Own elaboration

The third option (Figure 8) evaluates the addition of three stations with their corresponding bus feeder system, maintaining a bus line in the last section (from Darongers to AIX).

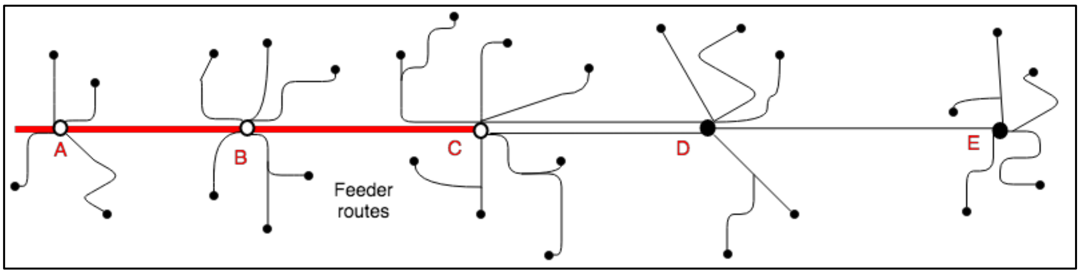


Figure 9. Rail trunk line until C, and conventional bus system from C to E. Source: Own elaboration

Figure 9 represents the replacement of the bus line up to *C* (Châtelet). From this point forward, a conventional bus system is used to connect the other two stations.

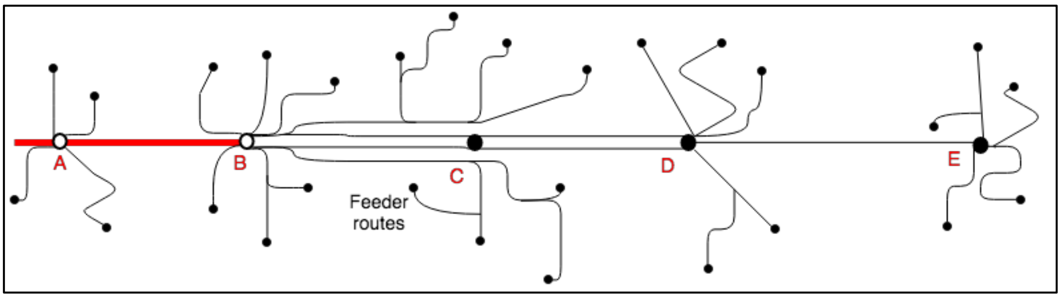


Figure 10. Rail trunk line until B, and conventional bus system from B to E. Source: Own elaboration

Figure 10 corresponds with the fifth option, which assess the costs of a single section replacement (up to *B*).

I.5. THESIS ORGANIZATION

This thesis is organized as follows:

- Chapter 2 first reviews the empirical and theoretical literature not only on models for rail transit systems but also for and bus transit systems.
- Chapter 3 specifies the properties of the model and the formulations for measuring the system performance. Some assumptions are made to simplify the problem.
- Chapter 4 presents the methodology for solving the proposed mathematical model.
- Chapter 5 develops numerical examples and demonstrates the performance of the model proposed. The results between the different options are compared and it the best choice is made.
- Chapter 6 presents the sensitivity analyses, in order to inquire into the effects of several input parameters on the resulting optimized values.
- Chapter 7 summarizes the thesis findings and recommends further research directions

Chapter II: LITERATURE REVIEW

For the past years, scheduling problems and transit optimization models have been considerably interesting topics in urban planning. As a guidance to construct a new model, in this chapter it is summarized the previous studies related to this thesis. Therefore, it is easier to understand the characteristics of rail trail and bus routes, and the methodologies to solve it. The literature reviewed in this section is divided into the following categories: transit service optimization and scheduling problems.

II.1. TRANSIT SERVICE OPTIMIZATION

A fair amount of research has been devoted to this problem, setting optimal fares and service frequencies as well as others service parameters for public transport systems. Newell (1979), Wirasinghe and Ghoneim (1981), Kocur and Hendrickson (1982), Chang and Schonfeld (1991) and Chien and Schonfeld (1998) proposed analytical models for optimizing major bus transit parameters such as headway, stop spacing, bus size or service area. Back then computation power was more limited, so they all have in common a great simplification of the demand pattern and network structure. However, in the recent years great progress has been achieved in computation power and optimization methods. That lead to studies with more realistic characteristics of public transit system.

Matisziw et al. (2006) present an optimization model to determine the route extension network for bus transit systems. The objective is to maximize the covering areas and minimize the extension length under resource constraints. Expanding the network coverage means increasing the ridership. It is important to expand the existing service network to tap into emergent areas of demand not being served. A bi-objective model is used to avoid overextending the bus transit system.

Guan et al. (2006) introduce a model for simultaneous optimization of trasit line configuration and passenger line assignment in a general work. The model is solved by branch and bound method, with a fixed demand and many-to-many pattern.

Lownes and Machemehl (2010) present a new mixed integer model for single-route circulator design problem. They proposed an optimization model to minimize total costs, which decision variable is the stop locations, and demand was considered fixed.

Li et al. (2012) develop a heuristic algorithm to solve the design problem of a rail transit line located in a linear urban transportation corridor. The objective is to maximize the net profit, with an elastic and exponential distributed demand density along the corridor. The service variables designed are a combination of rail line length, number and locations of stations, headway and fare.

DiJoseph and Chien (2013) optimize social and fiscal sustainable operation of a feeder bus system considering realistic network and heterogeneous demand. An optimization algorithm is developed to search for the optimal solution that maximizes the profit. The objective total profit is a non-linear, mixed integer function, which is maximized by optimizing the number of stops, headway and fare.

Kim and Schonfeld (2013) analyze conventional and/or flexible bus service alternatives with mixed fleets. A hybrid method which combines analytic optimization and a genetic algorithm is used to minimize the total cost. The demand is considered fixed, uniform and with many-to one pattern. The decision variables used in this study are bus size, headway and route spacing.

Chen et al. (2016) propose a method for integrating, coordinating, and optimizing bus services while considering financial constraints, many-to-many travel pattern, appropriate service type for various regions, and demand elasticity. The purpose is to maximize welfare, that is to say, the sum of producer and consumer surplus. A genetic algorithm with bounded integer variables is selected to solve this problem. Many variables are jointly optimized such as service types, service zones sizes and headways.

Cheng and Schonfeld (2016) propose a problem similar to the one studied here. A simulated annealing algorithm is used to solve the problem of optimizing the construction phases for rail extension projects. The objective is to maximize the NPW and examine the economic feasibility of such extension projects under different financial constraints. It considers a many-to-many pattern, and an elastic demand.

Sun et al. (2017) explain how the selection of public transit modes can be optimized over a planning horizon. It is proposed a dynamic model which contains both discrete and continuous decision variables. The model combines the difficulties of solving mixed integer program, a nonlinear program and a derivative-free optimization problem, a heuristic is proposed for finding the optimized solution.

Sun et al. (2018) present an optimization of a rail transit line over a planning horizon solve by a bilevel model. On the one hand, in upper-level the focus is on the construction and investment, with the goal of maximizing the NPW. On the other hand, in the lower-level problem where the social welfare is maximized with a train capacity constraint. As it proposes a model to optimize the extension of transit lines, it has similar characteristics to the problem proposed in this study.

Many different demand patterns considered in the previous studies. Some of them consider either many-to-many or many-to one pattern. Other studies analyze the total demand at the stop level instead of analyzing the demand at a route level. Moreover, in some research studies the demand variability over the route or network is ignored and only average link demand is assumed. In fact, the highest link demand along the route determines the fleet size. This practice may underestimate the required capacity. Research results from these analytical optimization models for public transport operations are helpful in capturing both the impact of route extensions or network expansions and the demand variability over time. The most common objective functions are maximizing profits and welfare, and minimizing costs. Many previous studies focus on optimizing operational and design characteristics. The papers listed above show how a transit system can be modelled and what variables should be considered. Nevertheless, there are a few papers about construction phasing for rail transit.

II.2. SCHEDULING PROBLEMS

Optimal schedules under various objectives, different constraints and characteristics of the systems are determined by scheduling problems. This thesis considers a rail transit extension project scheduling problem whose objective function is minimizing the costs. Diverse studies can be found about scheduling transit crews, timetable and maintenance activities. As mentioned earlier, the only study found about rail transit extension is the one published by Cheng and Schonfeld (2016). The key words used in the search process include phased development, rail transit extensions, and transit segmental analysis. Also, all available resources are exhausted. There must be models or criteria used by consultants and contractors, but probably not published in scientific journals.

Many methods are used to solve PSP: enumerative search, calculus, branch and bound, mathematical programming, and other problem-dependent algorithms. Those methods can be used only if the problem is sufficiently small or well behave. However, for more complex problems, heuristic algorithms are often applied to determine solutions that are close to the global optimum, including Tabu search, Genetic Algorithm and Simulated Annealing.

Valadares Tavares (1987) defines a set of interconnected railway projects with the objective of maximizing its total NPW. The proposed method is based on dynamic programming using optimality conditions derived by the calculus of variations. This model is applicable to large sets of expensive and interconnected development projects under tight capital constraints. Construction expenditures and payments are the only items taken into consideration in the NPW, and since it is a renewal project, all items that are affected by the project should be taken into account. For instance, the effects of interrupted demand when the project is under construction are not considered in the model.

Kolish and Padman (2001) summarize and classify previous studies on RCPSP by their objectives and constraints: NPW maximization and makespan minimization, with and without resource constraints. They survey the vast literature in the area of project scheduling and management, with a perspective that integrates models, data, and optimal and heuristic algorithms. They present an overview of web-based decision support systems and discuss the potential of this technology in enabling and facilitating researches and practitioners in identifying new areas of inquiry and application. The results are that when maximizing NPW for the resource-constraint case, generally it is optimal to schedule jobs with associated positive cash flows as early as possible, and those with negative cash flows as late as possible. Nevertheless, for the resource constraint case, at high cost of capital or long project duration, it is important to evaluate bonus/penalty and capital constraints when scheduling activities.

Ahern et al. (2006) define some of the findings of work carried out in the course of developing innovative investment-planning models, which will allow the prioritization of funds for improving intercity rail networks. Both qualitative and quantitative criteria are considered in the model. The user benefits are the most important factor in investment decision-making, followed by safety/accident benefits and the total economic benefits of the project, according to the study results. Among the attributes considered in this survey in railway selection, NPW is rated to be the second least important. The Table 1 shows all the attributes in railway projects priorization for investment and its estimated weightings. Nevertheless, there are some weaknesses in the model. Some of these weak points are the following ones: optimizing some attributes conflicts with optimizing others (if the objective is to minimize capital costs, the other objective that maximizes passengers on train cannot be achieved), mean weightings values are used to get the final decision, and it is difficult to quantify qualitative items (it is not shown a detailed method to calculate those quantitative attributes).

Wang and Schonfeld (2007) present a simulation model to evaluate waterway system performance and optimize the improvement project decisions with demand model incorporated. They maximize the present worth of net benefits for the entire analysis period instead of minimizing total costs, since traffic demand and benefits are significantly affected by the simulated decisions. Different scenarios are tested, and the results reveal that more negative demand elasticity with respect to travel time can significantly reduce traffic during work closures. If considering a renewal project, demand elasticity is a main factor and it should be considered in the model. In this thesis, the extensions will not affect the current users in the network at all, so the demand elasticity can be omitted in this problem.

Cheng and Schonfeld (2016) develop a method for optimizing the construction phases for rail transit line extension projects with the objective of maximizing the NPW and examines the economic feasibility of such extension projects under various financial constraints. For solving this problem, a Simulated Annealing algorithm is used. This method should be useful to transportation planners and decision-makers in optimizing construction phases for rail transit line extension projects. Demand growth is considered over the years and increases at higher rate after a new link is completed.

Attribute/Goal	Weight
User benefits	0.092
Safety/accident benefits	0.091
Total economic benefits	0.088
Capital cost	0.085
To support land use, social and economic policy at local, national and regional level	0.079
Additional passengers on train	0.078
Benefit/cost ratio	0.078
To exploit the particular strengths of rail to provide a highly integrated and competitive public transport service	0.076
Car resource cost saving	0.073
To improve environmental quality and health	0.073
Increase in revenue in railway	0.067
Net present value	0.062
To promote sound project selection measures	0.057

Table 1.Estimated weightings of attributes for railway project selection. Source: Ahern

II.3. SUMMARY

As summarized above, previous studies about rail transit scheduling were limited until recently with the studies of Cheng and Schonfeld (2016), Sun and al. (2017) and Sun and al. (2018).

The main objective of this research is to know if the replacement of the bus line is worth it from a socio-economic point of view. In order to know that it is needed to compare the different alternatives: bus only, bus and rail or rail only. That is why even if the previous study of Sun and Al. (2017) is not about rail transit extension scheduling but about the selection of the most cost-effective mean of transport for a given demand, it has similar characteristics. The main differences between these two studies are the following ones: first, as mentioned before, the objective of the study is only the selection of different means of transport. Second, demand is fixed, that is to mean, it does not change over time. Third, the demand pattern is many-to-one, so all users go from the city boundary to the CBD.

Sun and al. (2018) study present a bilevel model for optimizing the extension of a rail road. In this model nothing is being replaced as in the present study and there is not a budget constraint but a capacity constraint. This study also diverges from the present one in two aspects, there is not a demand redistribution after route extension, and the length of the rail transit line is modelled as a discrete variable instead of using a continuous variable. In order to solve this problem, an exact solution method is proposed based on the model's structure. There are two similarities with the present study: many-to-many demand pattern and elasticity demand are considered.

The Cheng and Schonfeld (2016) research, is the most similar research to the present one. Nevertheless, there are some assumptions that differs between studies. One of the main differences is that in Cheng and Schonfeld (2016) study the extension is not replacing a current conventional bus system, the extension not the rail road in not replacing anything. The optimization is focus on maximizing the NPW instead of minimizing costs as done in the present research. In order to solve the problem a meta-heuristic method is used (Simulated Annealing). Moreover, there is a capital budget constraint and a revenue one, and in the present research only budget constraint is taken into account. Nonetheless, both have in common that stations can only be added in a sequential way (from the CBD to the suburbs) and the many-to-many demand pattern.

Chapter III: Model Formulation

In this chapter a mathematical model is presented for determining whether it is worth to replace the bus line. If it is worth it, the model helps to determine how many phases there should be and when to implement each one to minimize the costs.

III.1. Assumptions

The following assumptions are made in order to simplify the problem:

1. Station locations and transit routes are already predetermined. Therefore, users access costs are omitted from this analysis.
2. Stations can be only added sequentially from the *Archieves* station (point *A* in Figure 11) to *AIX* station (point *E* Figure 11). With a double crossover track at every station, any station can be at least temporally the line's terminal station. Consequently, turnaround time is omitted from this analysis.
3. Economies of scale are considered. Capital costs are reduced if multiple stations are built together.
4. The interest rates are effective rate which already consider the inflation.
5. Effects of development schedules of other routes on the demand of our route are neglected. Hence, we do not need to transform the cash flow from actual dollars to constant dollars.
6. There is a 15 years binding construction time constraint (from 2019 to 2034).
7. Bus headways are uniform.
8. All attractions are not assumed to be located at the AIX station, implying that demand pattern in many-to-many.
9. Coordinated bus and rail operations.
10. Average waiting time is the half of the common headway.
11. The future demand growth is deterministic.

Table 2 defines the notation used in this research.

Variables	Descriptions	Units
α	Unit cost of user in-vehicle time	\$/passenger . hour
β	Unit cost of user waiting time	\$/passenger . hour
b_c	Cost of buses	\$/bus
C_A	Access cost	\$/h
C_C	Capital cost	\$/h
C_I	In-vehicle cost	\$/h

C_M	Maintenance cost	\$/h
C_O	Operating cost	\$/h
C_W	Waiting cost	\$/h
c_1	Bus capacity	Passengers
c_2	Train capacity	passengers
c_t	Cost per train	\$/train
D	Demand	passengers/hour
d_{AB}	Distance between stations A and B	miles
d_{BC}	Distance between stations B and C	miles
d_{CD}	Distance between stations C and D	miles
d_{DE}	Distance between stations D and E	miles
e_b	Economic lifetime of buses	years
e_t	Economic lifetime of trains	years
f	Frequency	min^{-1}
g	Demand growth rate	-
h_0	Common headway	min
h_1	Bus headway	min
h_2	Train headway	min
i	Origin	-
ir	Interest rate	% / years
j	Destination (CBD)	-
λ_0	Fixed cost of extending the rail road	\$
λ_1	Marginal capital cost per mile	\$/mile
N	Fleet size	trains
n_c	Number of cars per train	cars
N_p	Operation hours per year	hours
n_s	Number of stops	stops
η_1	Load factor for buses	-
η_2	Load factor for trains	-
q	Rail through flow	passengers
rc	Reduction coefficient for demand	passengers/mile
rf	Reserve factor	%
R_t	Round trip time	min
s	Construction cost savings	%
S_C	Supplier cost	\$
τ	Maintenance cost	\$/passenger . mile

t	Time interval	years
t_{d1}	Bus dwell time	min
t_{d2}	Train dwell time	min
T_C	Total cost	$
t_{t1}	Bus terminal time	min
t_{t2}	Train terminal time	min
U_C	User cost	$
v_{c1}	Bus cruise speed	mph
v_{c2}	Train cruise speed	mph
ω_1	Hourly operating cost per bus	$/bus . hour
ω_2	Hourly operating cost per vehicle	$/vehicle . hour
x	Miles of extension of the rail road	miles
$z_i^{(t)}$	Binary variable: 1 if rail extended, 0 if not	-

Table 2. Notation

Figure 11 shows the five different replacement options. Each one of these options are studied and it is determined which is the best option for the bus line, maintaining or replacing it. The study time horizon in thirty years, the International Airport Corridor is supposed to be finished in 2034 or earlier. The transit system is 5.9 miles long with 5 stations, one of them is already completed and in service (*Archieves*, station *A*). Our decision variable $z_i^{(t)}$ represents that a link *i* exists in the time period *t*. If $z_i^{(t)}=0$ it means that the link *i* has not been built in the time period *t*. Whereas, if $z_i^{(t)}=1$ then the link *i* has been built in the time period *t*. Here link *i* is represents the section between *i-1* and *i*, and link *i* includes station *i.* The decision variables are:: $z_i^{(t)}=0$ or 1, $i = [1,5]$, $t = [0,20]$, where *i* express links, and *t* indicates time interval.

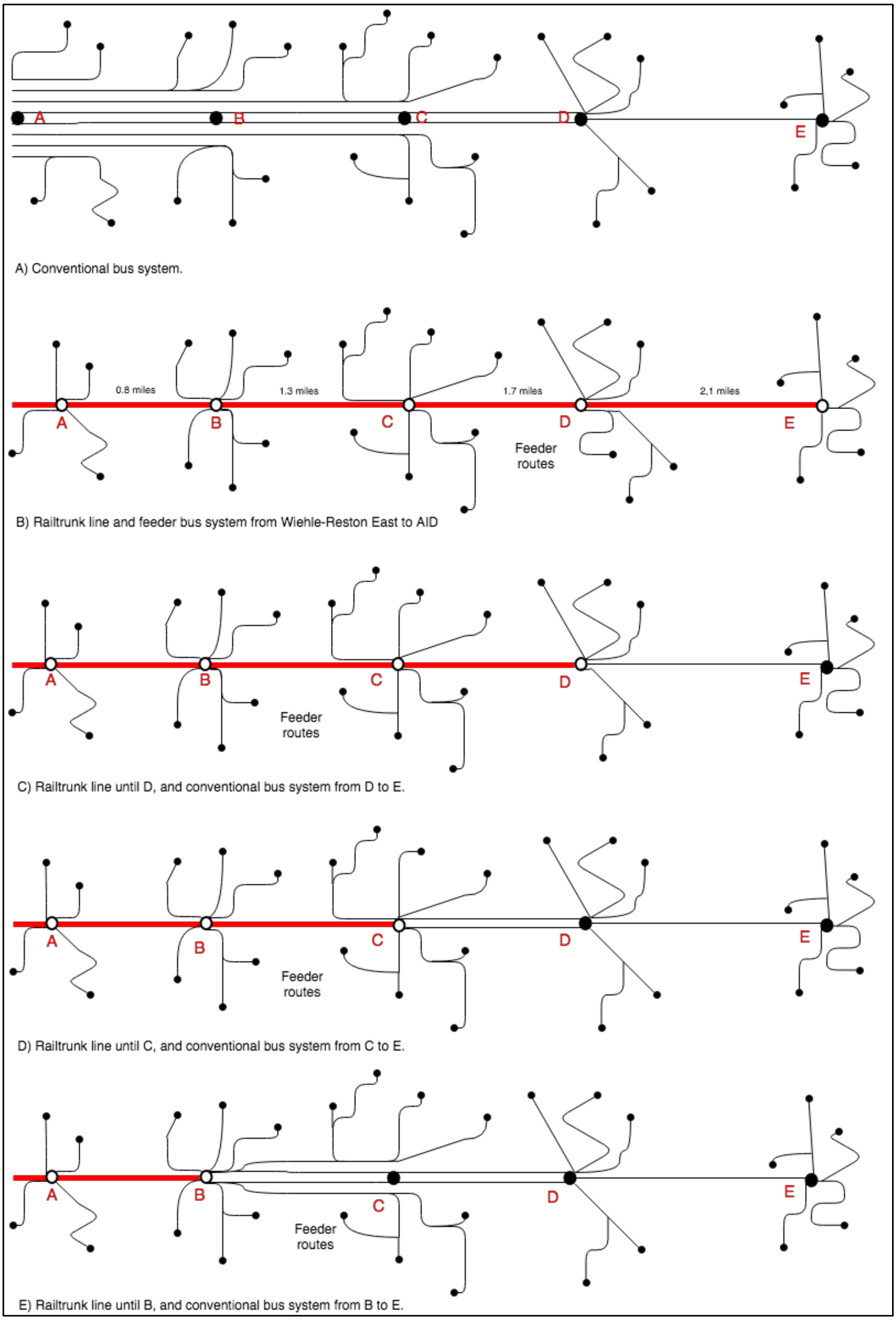


Figure 11.The five different options studied. Source: Own elaboration.

III.2. DEMAND FUNCTION

Demographic and economic growth lead to traffic increases in the long term. That is why demand growth is considered here multiplying the demand relation for the initial period (t=0) with a compound growth rate $(1+ g)^t$. The components of the demand function are the following ones: q is the rail passengers flows from origin i to destination j, t represents the intervals of growth, g is the growth rate per time interval (Figure 12)., k_1, k_2 and k_3 are constants that relate the in-vehicle time, waiting time and the price with the demand respectively. Thus, the demand function is as follows:

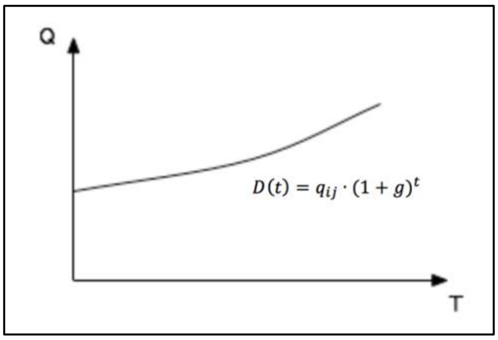


Figure 12. Demand growth over time. Source: Wei-Chen Cheng

$$D(t) = q_{ij}^{(0)} \cdot (1-g)^t \text{ ,} \forall\, i,j \tag{1}$$

There is one station in service at the time interval zero (*Wiehle-Reston East*, point A). So, the *O/D* matrix is:

$$OD^{(t)} = \begin{bmatrix} - & z_2q_{12} & z_3q_{13} & z_4q_{14} & z_5q_{15} \\ z_1q_{12} & - & z_3q_{23} & z_4q_{24} & z_1q_{25} \\ z_1q_{12} & z_2q_{23} & - & z_4q_{34} & z_1q_{35} \\ z_1q_{12} & z_2q_{24} & z_3q_{34} & - & z_1q_{45} \\ z_1q_{12} & z_2q_{25} & z_3q_{35} & z_4q_{54} & - \end{bmatrix}^{(t)}$$

where at t=0, z_1=1 and $z_2= z_3= z_4= z_5= 0$.

III.3. COST FUNCTIONS

The total cost is the addition of the supplier cost (S_C) plus the user costs (U_C):

$$C_T = S_C + U_C \tag{2}$$

III.3.1. Supplier cost

$$S_C = C_O + C_C \tag{3}$$

There supplier cost includes the costs of components such as: links, terminals, control systems and vehicle In this research the components considered are the operation cost (C_O) and the capital cost (C_C).

III.3.1.1. Capital cost

It has been decided to include as well in the capital costs the maintenance cost to make the problem simpler. Capital cost includes land acquisition, design, constructions and rail-track laying cost:

$$C_C = (\lambda_0 z_t + \lambda x_t) \cdot s \tag{4}$$

where λ_0 is the fixed cost of extending the rail road, z_t is a binary variable: 1 if rail extended, 0 if not, x_t corresponds with the miles of extension of the rail road, λ is the marginal capital cost per mile and s is the construction saving percentage (assumption 3 above) that is considered when several stations are built together. In the numerical examples of the study the saving costs of building several stations at the same time are the following ones: 3% for two stations, 6% for three stations and 9% for four stations.

The capital cost, or initial investment, has been divided for the entire life time of the metro construction. That is to mean that the total amount of money calculated for the capital cost ($), has been translated into a yearly cost ($/year). It could not have been possible to make the addition of the other types of cost (operating, in-vehicle…) with the capital cost given that they are not measured with the same units. Finally, the formula used to divide the capital cost into a yearly cost ($/year) was the following one:

$$A = P\left(\frac{i\cdot(1+i)^n}{(1+i)^n-1}\right) \qquad (5)$$

where P is the initial investment (what is considered as capital cost in this study), i is the interest rate per year, n is the economic lifetime, and A is the average cost per year.

III.3.1.2. Operating cost

The operating cost (C_O) is the transit fleet size (N) multiplied both by the hourly operating cost per vehicle (ω) and by the number of cars in each train:

$$C_O = N \cdot \omega \cdot n_c \qquad (6)$$

The fleet size is the round-trip time (R_t) divided by the headway (h):

$$N = \frac{R_t}{h} \qquad (7)$$

The round-trip time (R_t) is determined by the distance between stations (d), the cruise speed (v_c), the number of stops, the delay in each stop (s), and the terminal time:

$$R_t = 2\cdot\left(\frac{d}{v_c} + t_d + t_t\right)$$

Since it is assumed that there is no terminal time, then:

$$R_t = 2\cdot\left(\frac{d}{v_c} + t_d\right) \qquad (8)$$

So, replacing in (8) in (7), and afterwards (7) in (6) :

$$C_O = \frac{2\left(\frac{d}{v_c}+t_d\right)}{h}\cdot \omega \cdot n_c \qquad (9)$$

It is considered that the extension miles coincide with the distance between stops, then d=x. Substituting (9) and (4) in (3) the formula for the supplier cost is the following one:

$$S_C = (\lambda_0 z_t + \lambda d) \cdot s + (\frac{2\left(\frac{d}{v_c}+t_d\right)}{h} \cdot \omega \cdot n_c) \quad (10)$$

III.3.2. User cost

The user cost is (U_C) composed by different variables: access time, accidents, waiting time, in-vehicle time… among others. This example does not consider accidents cost or access time. In this example the variables taken into account are the following: in-vehicle cost (C_I) and the waiting cost (C_W).

$$U_C = C_I + C_W \quad (11)$$

III.3.2.1. In-vehicle cost

The in-vehicle cost (C_I) is the through flow (q) multiplied by the round-trip time (R_t) and the cost of in-vehicle time (α):

$$C_I = q \cdot R_t \cdot \alpha \quad (12)$$

Substituting (8) in (12):

$$C_I = q \cdot (2 \cdot (\frac{d}{v_c} + t_d)) \cdot \alpha$$

Through flow is equal to inflow minus outflow at each link:

$$Through\ flow = 2 \cdot \sum_{m=1}\left[\sum_{i=1}^{m}\left(\sum_{j=i+1} z_i q_{ij} - \sum_{j=1} z_i q_{ij}\right)\right] \quad (13)$$

where m is the row in the O/D matrix, i is the origin in the O/D matrix and j is the destination in the O/D matrix. From now on through flow will be designated as q.

Substituting in (12):

$$C_I = 2 \cdot q \cdot (2 \cdot (\frac{d}{v_c} + t_d)) \cdot \alpha \quad (14)$$

III.3.2.2. Waiting cost

The waiting cost is the accumulated demand multiplied both by the waiting time (approximately half of the headway) and the unit cost of user waiting time (β):

$$C_W = D^t \cdot \frac{h}{2} \cdot \beta \quad (15)$$

Thus, replacing (14) and (12) in (11):

$$U_C = 2 \cdot q \cdot (2 \cdot (\frac{d}{v_c} + t_d)) \cdot \alpha + D^t \cdot \frac{h}{2} \cdot \beta \quad (16)$$

As it is assumed to be a coordinated rail and bus operations, a common headway h is used. The transfer time is negligible. It is assumed that the cost of user waiting time for the bus and the train is the same.

So, all in all, replacing (15) and (10) in (2) the objective function is the following one:

$$Minimize\ C_T(d) = (\lambda_0 z_t + \lambda d) \cdot s + (\frac{2\left(\frac{d}{v_c} + t_d\right)}{h} \cdot \omega \cdot n_c) + q \cdot 2 \cdot (\frac{d}{v_c} + t_d) \cdot \alpha + D^t \cdot \frac{h}{2} \cdot \beta \tag{17}$$

Subject to:

$z^t = 1$ or 0 (18)

$z_i^{(t)} - z_i^{(t-1)} \geq 0, \qquad for\ all\ i, t \geq 1$ (19)

$z_i^{(t)} - z_{i+1}^{(t)} \geq 0, \qquad for\ all\ t, i \geq 1$ (20)

Equation 18 is the binary integer constraint for decision variables. Equation 19 assures that after building link *i*, it remains in operation. Equation 20 represents the constraint that forces any link *i* not to start if any one of its predecessors in the set has not been completed, that is to mean, the stations have to be built sequentially since there are not big benefits if we haphazardly choose any segment to build along the route.

III.4. OPTION A: CONVENTIONAL BUS SYSTEM FROM ARCHIEVES TO AIX

This is the current system that users used daily. In the Figure 11 we can observe the that only buses are used to satisfy the demand, from *A* to *E* (rail line length is zero). The objective is to minimize the sum of supplier costs and user costs, subject to bus capacity constraint. There is not capital cost because it is only applied when the rail road is built, and in the current system no rail trail is built. The parameter n_c is not used because it refers to the number of cars in each train, and in this option only buses are considered.

$$\min_{h>0} C_T = (\frac{2\left(\frac{d_{AE}}{v_{c1}} + t_{d1}\right)}{h} \cdot \omega_1) + \int_A^E (q \cdot 2(\frac{d}{v_{c1}} + t_{d1}) \cdot \alpha) dd + \int_A^E D^t \cdot \frac{\mathrm{h}}{2} \cdot \beta \tag{21}$$

All the parameters with the subscript *1* are refer to the bus. To optimize the function, the optimal headway must be found. In order to find the optimal headway (h^*) we set the derivative of the C_T function depending on the headway equal to zero:

$$\frac{\partial C_T}{\partial h} = 0$$

Then,

$$\frac{\partial C_T}{\partial h} = -\frac{2(d_{AE} + t_{d1} v_{c1}) \cdot \omega_1}{h_1^2\, v_{c1}} + \frac{D^t \cdot \beta}{2} = 0,$$

Operating we obtain:

$$h_1^* = 2 \cdot \sqrt{\frac{(d_{AE} + t_{d1} v_{c1}) \cdot \omega_1}{D^t \beta\, v_{c1}}} \tag{22}$$

Since the closed form solution of headway is available, it can be inserted in the objective function (15), which becomes a function of only one variable d_{AE}, which is the length of the corridor:

$$\min_{0<A<E} C_T = \left(\frac{\omega_1 \cdot (t_{d1} \cdot v_{c1} + d_{AE})}{\sqrt{\frac{\omega_1 \cdot (t_{d1} \cdot v_{c1} + d_{AE})}{\beta\, D^t\, v_{c1}}} \cdot v_{c1}}\right) + \int_A^E \left(q \cdot 2\left(\frac{d}{v_{c1}} + t_{d1}\right) \cdot \alpha\right) dd + \int_A^E D^t \cdot \beta \cdot \sqrt{\frac{\omega_1 \cdot (t_{d1} \cdot v_{c1} + d_{AE})}{\beta\, D^t\, v_{c1}}}\, dd \quad (23)$$

III.5. OPTION B: RAIL TRUNK LINE AND FEEDER BUS SYSTEM FROM ARCHIEVES TO AIX.

This is the system that users would use if it were to be a rail trunk line from *A* to *E*. In the Figure 11 we can observe how the rail line covers all the extension, and there is a feeder bus system in each station. This option is studied and compared with option A to determine if it is better than the current system.

The optimization model and results are the same as the ones in option A but replacing the subscript *1* with *2*, adding the number of cars in each train, and adding the capital cost variable in (23), given that all the length extension is encompassed with rail road:

$$\min_{0<A<E} C_T = (\lambda_0 z_t + \lambda d_{AE}) \cdot s + \left(\frac{\omega_2 \cdot n_c \cdot (t_{d2} \cdot v_{c2} + d_{AE})}{\sqrt{\frac{\omega_2 \cdot n_c \cdot (t_{d2} \cdot v_{c2} + d_{AE})}{\beta\, D^t\, v_{c2}}} \cdot v_{c2}}\right) + \int_A^E \left(q \cdot 2\left(\frac{d}{v_{c2}} + t_{d2}\right) \cdot \alpha\right) dd + \int_A^E D^t \cdot \beta \cdot \sqrt{\frac{\omega_2 \cdot n_c \cdot (t_{d2} \cdot v_{c2} + d_{AE})}{\beta\, D^t\, v_{c2}}}\, dd \quad (24)$$

III.6. OPTION C: RAIL TRUNK LINE UP TO D, AND CONVENTIONAL BUS SYSTEM FROM D TO E.

This system combines a rail trunk line with feeder bus system in each station from *A* to *D*, with a conventional bus system from *D* to *E*. In the Figure 11 we can observe how the rail line covers the extension until *D*, and how it continues with the conventional bus system.

The subscript *1* refers to bus system, and *2* to the rail line. The objective is to minimize the objective function:

$$\min_{h>0} C_T = \left(\frac{2\left(\frac{d_{DE}}{v_{c1}} + t_{d1}\right)}{h} \cdot \omega_1\right) + \int_D^E \left(q \cdot 2\left(\frac{d_{DE}}{v_{c1}} + t_{d1}\right) \cdot \alpha\right) dd + \int_D^E D^t \cdot \frac{h}{2} \cdot \beta + (\lambda_0 z_t + \lambda d_{AD}) \cdot s + \left(\frac{2\left(\frac{d_{AD}}{v_{c2}} + t_{d2}\right)}{h} \cdot \omega_2 \cdot n_c\right) + \int_A^D \left(q \cdot 2\left(\frac{d_{AD}}{v_{c2}} + t_{d2}\right) \cdot \alpha\right) dd + \int_A^D D^t \cdot \frac{h}{2} \cdot \beta \quad (25)$$

The first three elements of the equation correspond to the C_O, C_I and C_W of the conventional system, whereas the forth last ones correspond to C_C, C_O, C_I and C_W of the rail trunk line. It is supposed to be a coordinated system, so the time transfer is not taken into account given that it is negligible. That is why a common headway *h* is assumed.

Similar process as done in option A, the optimal headway is calculated:

$$\frac{\partial C_T}{\partial h} = 0$$

Then,

$$\frac{\partial C_T}{\partial h} = -\frac{2(d_{DE}+t_{d1}v_{c1})\cdot \omega_1}{h^2\, v_{c1}} + \frac{D^t\cdot\beta}{2} - \frac{2(d_{AD}+t_{d2}v_{c2})\cdot \omega_2\cdot n_c}{h^2\, v_{c2}} + \frac{D^t\cdot\beta}{2} = 0,$$

Operating we obtain:

$$h^* = \sqrt{2}\cdot\sqrt{\frac{v_{c2}t_{d1}\omega_1 v_{c1}+(d_{AD}+t_{d2}v_{c2})\cdot\omega_2 v_{c1}\, n_c+d_{DE}v_{c2}\omega_1}{D^t\beta\, v_{c1}v_{c2}}} \tag{26}$$

Since the closed form solution of headway is available, it can be inserted in the objective function (16), which becomes a function of only two variables d_{AD} (length of the rail line) and d_{DE} which is the length of the bus system:

$$\min_{0<A<D<E} C_T = \left(\frac{\sqrt{2}\,\omega_1\cdot(t_{d1}\cdot v_{c1}+d_{DE})}{\sqrt{\frac{v_{c2}t_{d1}\omega_1 v_{c1}+(d_{AD}+t_{d2}v_{c2})\cdot\omega_2 v_{c1}n_c+d_{DE}v_{c2}\omega_1}{D^t\beta\, v_{c1}v_{c2}}}\cdot v_{c1}}\right) + \int_D^E (q\cdot 2(\frac{d}{v_{c1}}+t_{d1})\cdot\alpha)dd +$$

$$\int_D^E D^t\cdot\beta\cdot\sqrt{2}\cdot\sqrt{\frac{v_{c2}t_{d1}\omega_1 v_{c1}+(d_{AD}+t_{d2}v_{c2})\cdot\omega_2 v_{c1}n_c+d_{DE}v_{c2}\omega_1}{D^t\beta\, v_{c1}v_{c2}}}\,dd + (\lambda_0 z_t+\lambda d_{AD})\cdot s +$$

$$\left(\frac{\sqrt{2}\,\omega_2\cdot(t_{d2}\cdot v_{c2}+d_{AE})}{\sqrt{\frac{v_{c2}t_{d1}\omega_1 v_{c1}+(d_{AD}+t_{d2}v_{c2})\cdot\omega_2 v_{c1}\, n_c+d_{DE}v_{c2}\omega_1}{D^t\beta\, v_{c1}v_{c2}}}\cdot v_{c2}}\right) + \int_A^D (q\cdot 2(\frac{d}{v_{c2}}+t_{d2})\cdot\alpha)dd + \int_A^D D^t\cdot\beta\cdot\sqrt{2}\cdot$$

$$\sqrt{\frac{v_{c2}t_{d1}\omega_1 v_{c1}+(d_{AD}+t_{d2}v_{c2})\cdot\omega_2 v_{c1}\, n_c+d_{DE}v_{c2}\omega_1}{D^t\beta\, v_{c1}v_{c2}}}\,dd \tag{27}$$

Which in an easier form corresponds to equation (27) but with the optimal headway (h*) instead of the regular headway (h):

$$\min_{0<A<D<E} C_T = \left(\frac{2\left(\frac{d_{DE}}{v_{c1}}+t_{d1}\right)}{h^*}\cdot\omega_1\right) + \int_D^E (q\cdot 2(\frac{d_{DE}}{v_{c1}}+t_{d1})\cdot\alpha)dd + \int_D^E D^t\cdot\frac{h^*}{2}\cdot\beta + (\lambda_0 z_t +$$

$$\lambda d_{AD})\cdot s + \left(\frac{2\left(\frac{d_{AD}}{v_{c2}}+t_{d2}\right)}{h^*}\cdot\omega_2\cdot n_c\right) + \int_A^D (q\cdot 2(\frac{d_{AD}}{v_{c2}}+t_{d2})\cdot\alpha)dd + \int_A^D D^t\cdot\frac{h^*}{2}\cdot\beta \tag{25-b}$$

III.7. Option D. Rail trunk line up to C, and conventional bus system from C to E.

This system combines both, as option C, a rail trunk line with feeder bus system in each station and a conventional bus system. The difference lies on the rail trail length. In this case, as shown in Figure 11, the trunk line goes from A to C and the conventional bus system from C to E.
The optimization model and results are the same as the ones in option C but replacing the subscript D with C in (25). That is to say, replace d_{DE} with d_{CE} and d_{AD} with d_{AC}. The objective is to minimize the objective function:

$$\min_{h>0} C_T = \left(\frac{2\left(\frac{d_{CE}}{v_{c1}}+t_{d1}\right)}{h}\cdot\omega_1\right) + \int_C^E (q\cdot 2(\frac{d_{CE}}{v_{c1}}+t_{d1})\cdot\alpha)dd + \int_C^E D^t\cdot\frac{h}{2}\cdot\beta + (\lambda_0 z_t+\lambda d_{AC})\cdot$$

$$s + \left(\frac{2\left(\frac{d_{AC}}{v_{c2}}+t_{d2}\right)}{h}\cdot\omega_2\cdot n_c\right) + \int_A^C (q\cdot 2(\frac{d_{AC}}{v_{c2}}+t_{d2})\cdot\alpha)dd + \int_A^C D^t\cdot\frac{h}{2}\cdot\beta \tag{28}$$

The last four elements of the equation correspond to the C_C, C_O, C_I and C_W of the rail trunk line, and the first three to the C_O, C_I and C_W of the conventional system. It is supposed to be a coordinated system, so the time transfer is not taken into account given that it is negligible. That is why a common headway h is assumed.
Similar process as done in option A, the optimal headway is calculated:

$$\frac{\partial C_T}{\partial h} = 0$$

Then,

$$\frac{\partial C_T}{\partial h} = -\frac{2(d_{CE}+t_{d1}v_{c1})\cdot \omega_1}{h^2\, v_{c1}} + \frac{D^t\cdot\beta}{2} - \frac{2(d_{AC}+t_{d2}v_{c2})\cdot \omega_2 n_c}{h^2\, v_{c2}} + \frac{D^t\cdot\beta}{2} = 0,$$

Operating we obtain:

$$h^* = \sqrt{2}\cdot\sqrt{\frac{v_{c2}t_{d1}\omega_1 v_{c1}+(d_{AC}+t_{d2}v_{c2})\cdot \omega_2 v_{c1} n_c + d_{CE}v_{c2}\omega_1}{D^t\beta\, v_{c1}v_{c2}}} \qquad (29)$$

Since the closed form solution of headway is available, it can be inserted in the objective function (16), which becomes a function of only two variables d_{AC} (length of the rail line) and d_{CE} which is the length of the bus system. The easier way to express the objective function depending on the optimal headway (h^*) is the following one:

$$\min_{0<A<C<E} C_T = \left(\frac{2\left(\frac{d_{CE}}{v_{c1}}+t_{d1}\right)}{h^*}\cdot \omega_1\right) + \int_C^E \left(q\cdot 2\left(\frac{d_{CE}}{v_{c1}}+t_{d1}\right)\cdot\alpha\right)dd + \int_C^E D^t\cdot\frac{h^*}{2}\cdot\beta + (\lambda_0 z_t + \lambda d_{AC})\cdot s + \left(\frac{2\left(\frac{d_{AC}}{v_{c2}}+t_{d2}\right)}{h^*}\cdot\omega_2\cdot n_c\right) + \int_A^C \left(q\cdot 2\left(\frac{d_{AC}}{v_{c2}}+t_{d2}\right)\cdot\alpha\right)dd + \int_A^C D^t\cdot\frac{h^*}{2}\cdot\beta \qquad (28\text{-b})$$

III.8. OPTION E. RAIL TRUNK LINE UP TO B, AND CONVENTIONAL BUS SYSTEM FROM B TO E.

As options C and D, this system combines a rail trunk line with feeder bus system in each station and a conventional bus system. The difference lies on the rail trail length. In this case, as shown in Figure 11, the trunk line goes from A to B and the conventional bus system from B to E.
The optimization model and results are the same as the ones in option D but replacing the subscript C with B in (28). That is to say, replace d_{CE} with d_{BE} and d_{AC} with d_{AB}. The objective is to minimize the objective function:

$$\min_{h>0} C_T = \left(\frac{2\left(\frac{d_{BE}}{v_{c1}}+t_{d1}\right)}{h}\cdot \omega_1\right) + \int_B^E \left(q\cdot 2\left(\frac{d_{BE}}{v_{c1}}+t_{d1}\right)\cdot\alpha\right)dd + \int_B^E D^t\cdot\frac{h}{2}\cdot\beta + (\lambda_0 z_t + \lambda d_{AB})\cdot s + \left(\frac{2\left(\frac{d_{AB}}{v_{c2}}+t_{d2}\right)}{h}\cdot\omega_2\cdot n_c\right) + \int_A^B \left(q\cdot 2\left(\frac{d_{AB}}{v_{c2}}+t_{d2}\right)\cdot\alpha\right)dd + \int_A^B D^t\cdot\frac{h}{2}\cdot\beta \qquad (30)$$

The first three elements of the equation correspond to the C_O, C_I and C_W of the conventional system, whereas the forth last ones correspond to C_C, C_O, C_I and C_W of the rail trunk line. It is supposed to be a coordinated system, so the time transfer is not considered given that it is negligible. That is why a common headway h is assumed.

Similar process as done in option A, the optimal headway is calculated:

$$\frac{\partial C_T}{\partial h} = 0$$

Then,

$$\frac{\partial C_T}{\partial h} = -\frac{2(d_{BE}+t_{d1}v_{c1})\cdot \omega_1}{h^2\, v_{c1}} + \frac{D^t\cdot\beta}{2} - \frac{2(d_{AB}+t_{d2}v_{c2})\cdot \omega_2\cdot n_c}{h^2\, v_{c2}} + \frac{D^t\cdot\beta}{2} = 0,$$

Operating we obtain:

$$h^* = \sqrt{2}\cdot\sqrt{\frac{v_{c2}t_{d1}\omega_1 v_{c1}+(d_{AB}+t_{d2}v_{c2})\cdot \omega_2 v_{c1}\cdot n_c+d_{BE}v_{c2}\omega_1}{D^t\beta\, v_{c1}v_{c2}}} \qquad (31)$$

Since the closed form solution of headway is available, it can be inserted in the objective function (16), which becomes a function of only two variables d_{AB} (length of the rail line) and d_{BE} which is the length of the bus system. The easier way to express the objective function depending on the optimal headway (h^*) is the following one:

$$\min_{0<A<C<E} C_T = \left(\frac{2\left(\frac{d_{BE}}{v_{c1}}+t_{d1}\right)}{h^*}\cdot \omega_1\right) + \int_B^E \left(q\cdot 2\left(\frac{d_{BE}}{v_{c1}}+t_{d1}\right)\cdot \alpha\right) dd + \int_B^E D^t\cdot\frac{h^*}{2}\cdot\beta + (\lambda_0 z_t + \lambda d_{AB})\cdot s + \left(\frac{2\left(\frac{d_{AB}}{v_{c2}}+t_{d2}\right)}{h^*}\cdot \omega_2\cdot n_c\right) + \int_A^B \left(q\cdot 2\left(\frac{d_{AB}}{v_{c2}}+t_{d2}\right)\cdot\alpha\right) dd + \int_A^B D^t\cdot\frac{h^*}{2}\cdot\beta \qquad (30\text{-b})$$

Chapter IV: SOLUTION METHOD

In order to find near-optimal solutions to both nonlinear and linear optimization problems there are several widely known methods. Among these different methods the most used are *branch and bound*, *Tabu search*, *Genetic Algorithm*, *Simulated Annealing* or *SPSA*.

Branch and bound is a method for global optimization for nonconvex problems. It is an algorithm which solves discrete and combinatorial optimization problems. It consists on enumerating different solutions by means of state space search[3], these solutions form a rooted tree. So, the algorithm examines each branch of the tree and it is rejected if the solution of that branch is worse than the one found so far. It is a local optimization method.

TS employs a local search method for mathematical optimization, it is a metaheuristic algorithm search method used for solving combinatorial optimization problems. It is based on local neighborhood searches, that is to mean, taking a possible solution and checking its neighbors hoping to find a better solution. *TS* is used in different to solve problems in different areas such as: resource planning, telecommunications, financial analysis, scheduling and logistics. It is a local optimization method.

Genetic Algorithms are called that way because they are inspired in the genetic evolution and they rely on biological operators like mutation, crossover and selection to find high-quality solutions to optimization and search problems. *GA* is a metaheuristic algorithm that belongs to the group of evolutionary algorithms. They are commonly used when it is required to calculate non-derivative functions (or very complex derivative functions). However, if the function has a lot of maximums and minimums a great deal of iterations is necessary, and if there are many points near the optimum value only one is found. It is focus on finding a global optimum.

SA is a metaheuristic optimization technique, based on annealing process to escape from a local optimum to find a near-optimal solution. Its name comes from the annealing process in metallurgy, where a controlled heating and cooling of the material to increase the size of its crystals and reduce the defects is done. This algorithm is used when a global optimum is more important than finding a local optimum in a fixed amount of time.

SPSA is a type of stochastic approximation algorithm. It is used for optimizing systems with multiple unknown parameters. *SPSA* is especially efficient in high-dimensional problems, it provides a reliable solution according to the small number of measurements of the objective function. *SPSA* is used is different areas such as: neural work training, adaptive feedback control, signal processing or pattern recognitions among others.

[3] State space search: is a process used in the field of computer science, in which successive configurations or states of an instance are considered, with the intention of finding a goal state with a desire property (Wikipedia, 2018)

Nevertheless, all these methods are used to solve a very complex models. In this case of study, it is not worth to implement any of these algorithms because it would be counter-productive, given that the complexity is not great. That is why instead of applying one of the methods above, a code in VBA[4] have been developed to compare the different alternatives mentioned before.

IV.1. PROCEDURE

<u>*Step 1:*</u> Set values for the current year iteration to the demand and miles of extension.

<u>*Step 2:*</u> For those values calculate the user cost (in-vehicle cost, waiting cost and access cost), the supplier cost (operating cost, maintenance cost and capital cost) which lead to the **total cost** of each option. In order to calculate the total cost, the headway is optimized in every option ($\boldsymbol{h^*}$). The **cost per passenger mile** is also calculated.

<u>*Step 3*</u>: Compare h^* with h_{max}. If h^* is higher than h_{max} the headway is compromised by a demand constraint (the headway cannot be higher than the h_{max} if the demands is wanted to be satisfied). Therefore, the h^* is no longer taken into account and step 2 is done again with the new headway.

<u>*Step 4*</u>: Compare the headway (optimal or not optimal) with the h_{min}. If the headway is lower than the h_{min} it is not physically feasible to operate with such a headway, it is a physical constraint.

<u>*Step 5*</u>: With the value of the headway obtained, the total cost of each option is calculated and compared. The option which corresponds to the minimum cost is chosen.

<u>*Step 6*</u>: Compare the option selected with the budget constraint. If it were not to keep within the budget limitations, we should come back to step 5 and chose the next minimum total cost option.

<u>*Step 7*</u>: If the simulation has arrived to i=30 years, we stop it. Otherwise a new template is created, and new values are given to the variables of demand D and miles of extension x.

[4] Visual Basic: it is a programming language developed by Alan Cooper for Microsoft.

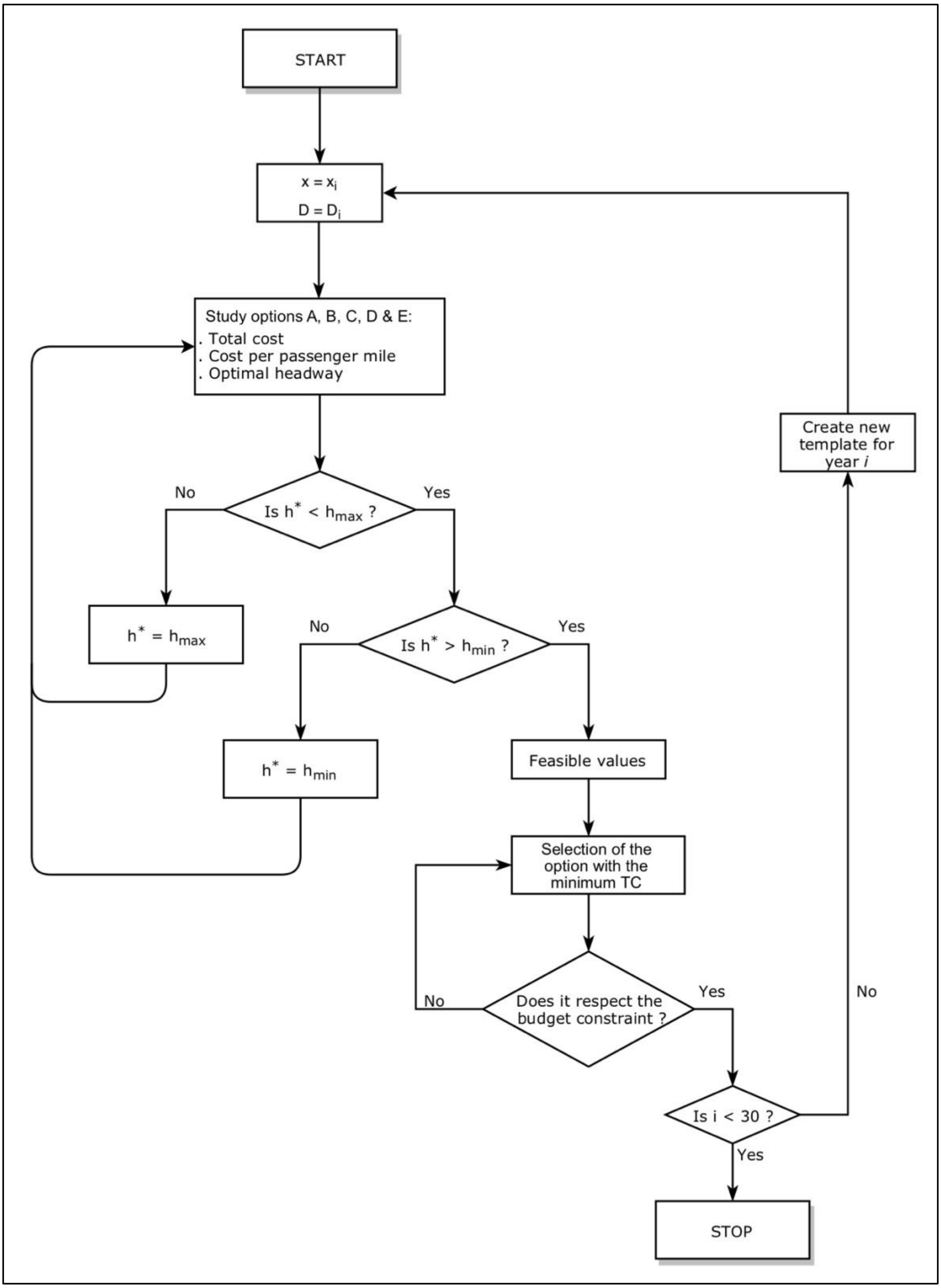


Figure 13. Implementation model. Source: Own elaboration

Chapter V: Numerical Results

The procedure was realized using Excel, programming in VBA. A possible replacement of a bus line is studied over a 30 years analysis period. The problem has been tested both as an unconstrained case and as a budget constrained case, in which the headway was optimized when possible.

V.1. Description of Input Parameter Values

Variables	Descriptions	Units	Value
α	Unit cost of user in-vehicle time	\$/passenger • hour	12
β	Unit cost of user waiting time	\$/passenger • hour	24
b_c	Cost of buses	\$/bus	250,000
c_1	Bus capacity	passengers	62[5]
c_2	Car type A capacity	passengers	64[6]
c_2	Car type B capacity	passengers	68
c_t	Cost per train (7000 series railcar)	\$/train	1,694,915.25[7]
D	Demand at year 0 in station A	passengers/hour	3000[8]
d_{AB}	Distance between stations A and B	miles	0.8[9]
d_{BC}	Distance between stations B and C	miles	1.3
d_{CD}	Distance between stations C and D	miles	1.7
d_{DE}	Distance between stations D and E	miles	2.1
e_b	Economic lifetime of buses	years	13
e_t	Economic lifetime of trains	years	30
g	Demand growth rate	%	12
ir	Interest rate	% / years	4
λ_0	Fixed cost of extending the rail road	\$	230,000
λ_1	Marginal capital cost per mile	\$/mile	379,629,629.63[10]
n_b	Economic lifetime of buses	years	13
n_c	Number of cars per train	cars	8

[5] Capacity of the CNG articulated bus. See Appendix A: vehicles used in the International Airport Corridor
[6] Capacity of the 7000 series railcar. Each train has an ABBAABBA configuration. See Appendix A: vehicles used in the International Airport Corridor.
[7] Data based on the purchase of 1003 trains of the 7000 series worth 1.7 billion dollars, January 2018. See Appendix B: Data assumptions.
[8] Demand depends on the year. See Appendix B: Data assumptions.
[9] See Appendix B: Data assumptions.
[10] See Appendix B: Data assumptions.

N_p	Operation hours per year	Hours	6,500[11]
n_s	Number of stops	Stops	8
η_1	Load factor for buses	-	1.63
η_2	Load factor for trains	-	2.60
rf	Reserve factor	%	10
rc	Reduction coefficient for demand	passengers/mile	300
s	Construction cost savings for two stations	%	3
s	Construction cost savings for three stations	%	6
s	Construction cost savings for four stations	%	9
t_{d1}	Bus dwell time	seconds	40[12]
t_{d2}	Train dwell time	seconds	35
v_{c1}	Bus cruise speed	mph	40
v_{c2}	Train cruise speed	mph	50
ω_1	Hourly operating cost per bus	\$/bus·hour	80
ω_2	Hourly operating cost per vehicle	\$/vehicle·hour	320

Table 3. Simulation inputs. Source: Own elaboration

V.2. UNCONSTRAINED CASE

The optimized solution obtained for the unconstrained is the one shown in the Figure 14. It is a surprise that from year 0 to 8 the optimal solution is not having any station built. That is to mean, that the best solution the ten first years is the conventional bus system already in use. However, in the 9th year the best option is to build the full extension, adding four stations to the extension. Given that there is no physical nor economic constraints, this answer implies that the whole extension should be built form year 8th to 9th if the demand is enough. This coincides with the combination of two options: the conventional bus system (from year 0 to year 8) and all rail road option (from year 9 to year 30).

[11] See Appendix B: Data assumptions.
[12] See Appendix B: Data assumptions.

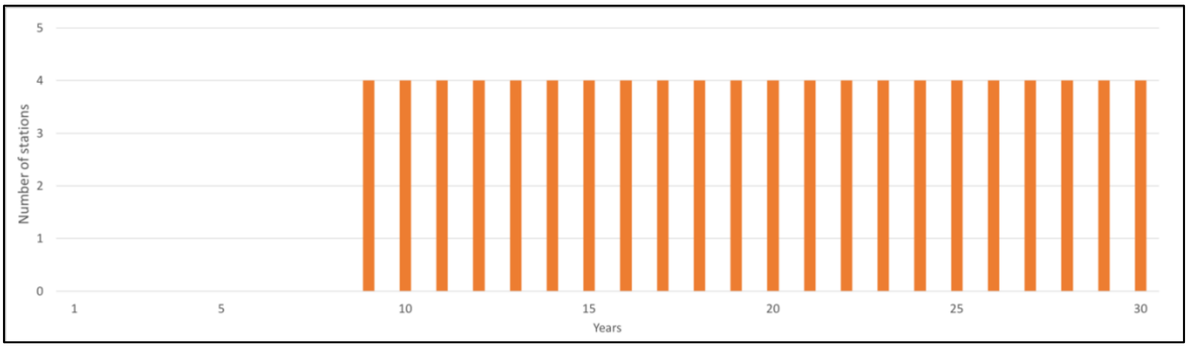


Figure 14. Optimized solution for unconstrained case. Source: Own Elaboration

The Figure 15 shows the evolution (from year 0 to year 15) of the different options: conventional bus system (blue line), 1 link (bright blue line), 2 links (yellow line), 3 links (grey line) or all rail road (orange line). As it can be seen as well in Figure 15, the optimal path is composed by the blue (until year nine) and orange line which corresponds to the conventional bus system and all rail road. It is as well seen other changes:

- The blue line (option A) starts being the best option and it finish being the third optimal option.
- The orange line (option B) starts being the second worst option and at year 15 it turns out to be the best one, in the 9th year there is a cross with option A.
- The grey line (option C) starts being the worst option, and finish being the second-best one.

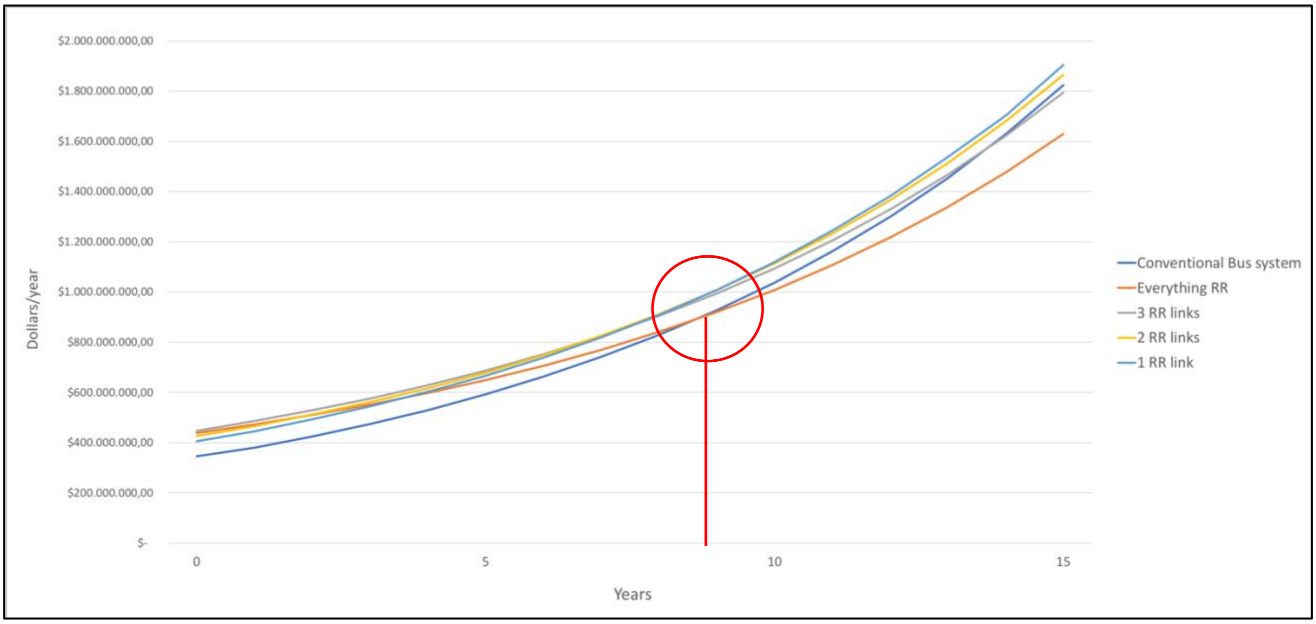


Figure 15. Evolution from year 0 to year 15 of the five different options, for the unconstrained case. Source: Own elaboration

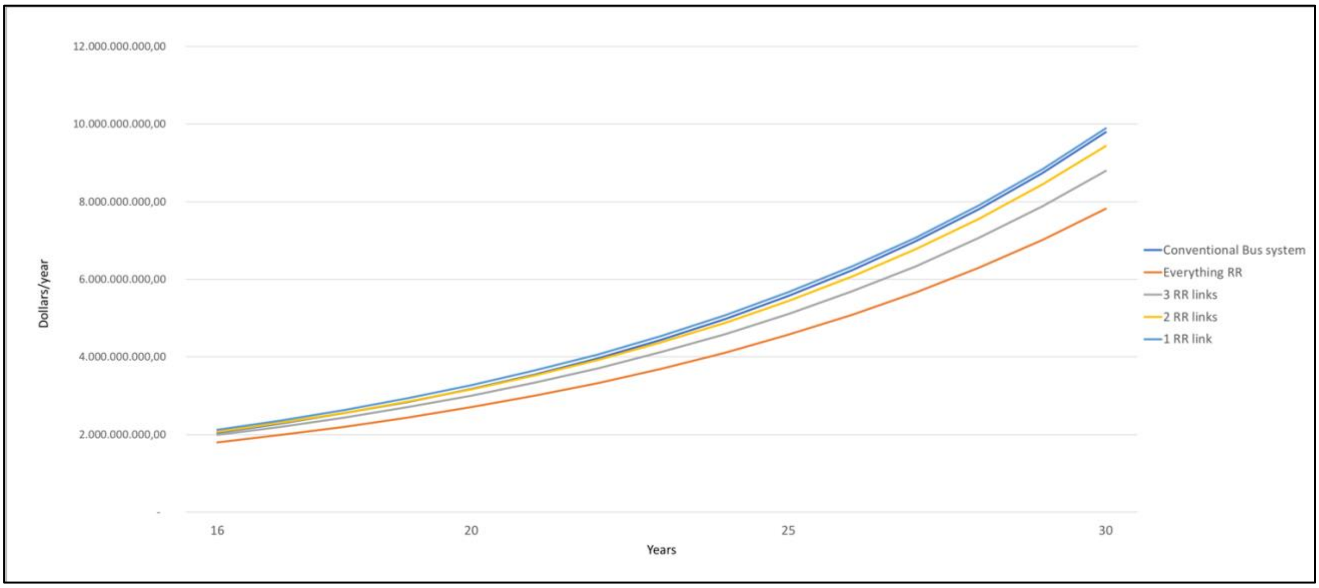


Figure 16. Evolution from year 16 to year 30 of the five different options, for the unconstrained case. Source: Own elaboration

- The yellow line barely changes its position, passing from the 3rd position to the 4th one (second-worst).
- The bright blue line (option E) starts being the second-best option, and finish being the worst one.

We can see the evolution until the 30th year in the Figure 16. In this figure we can see that the evolution continues as it ended before, the only change is that now the option with two links (option D) is better than the one which corresponds with the actual conventional bus system (option A). So, the option A started being the best alternative and ends being second-worst option after the simulations.

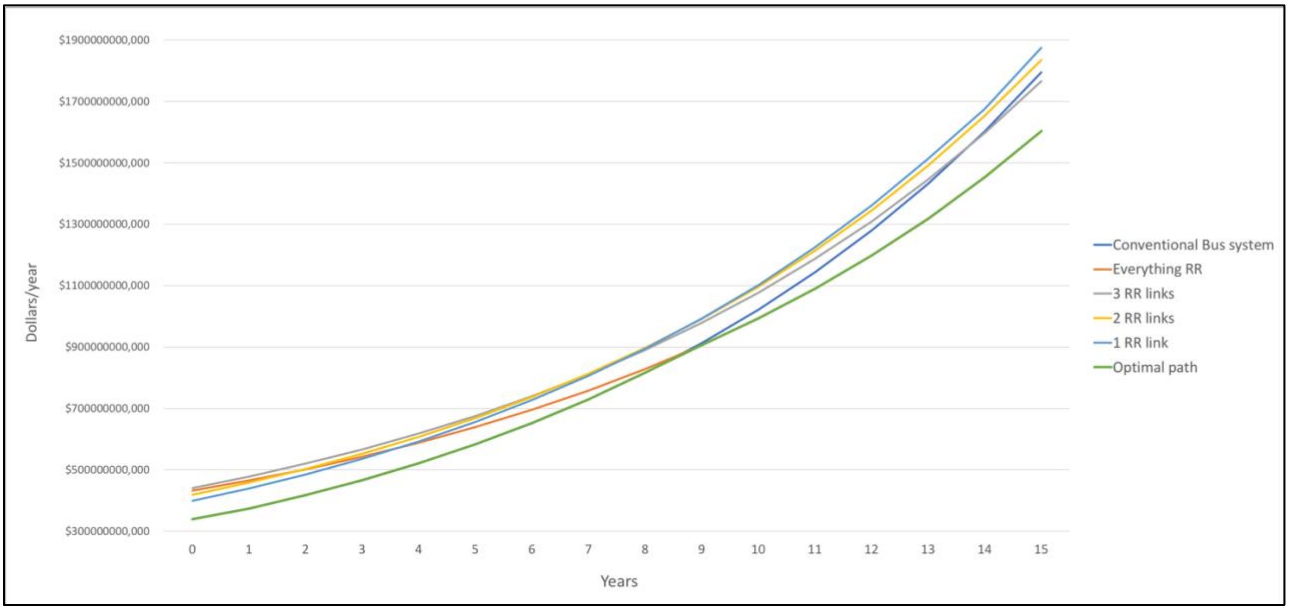


Figure 17. Optimal path during the first 15 years, for the unconstrained case. Source: Own elaboration

So, the optimal path is the one marked in green in Figure 17, composed the blue line (option A) plus the orange line (option B). The optimal path continues being the one that coincides with the orange from the 16th year until the 30th year, as it can be seen in the Figure 16

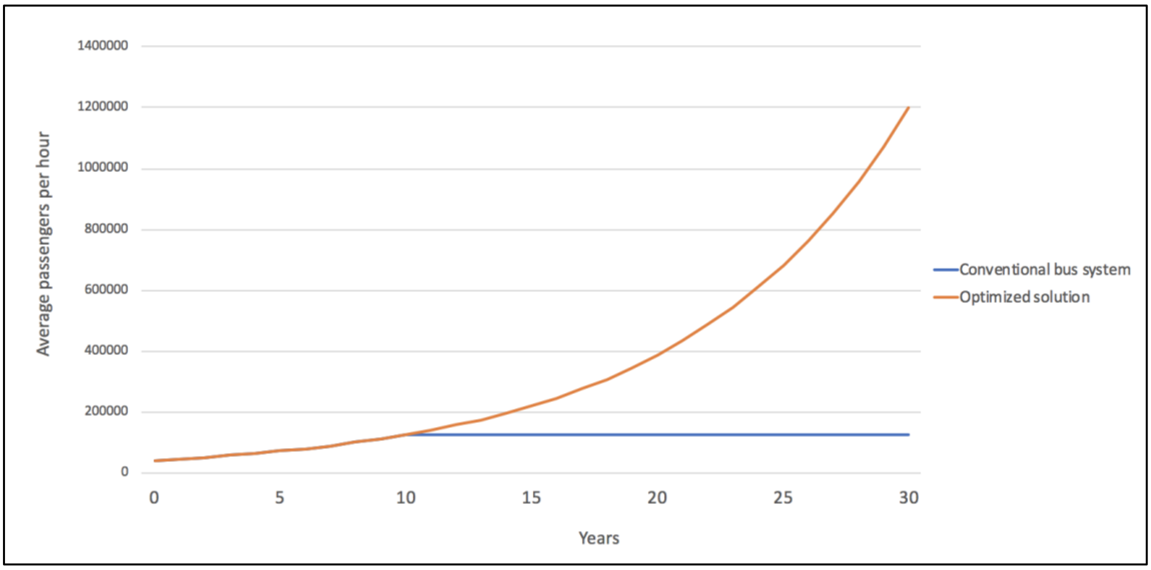


Figure 18. Average passengers per day. Source: Own elaboration

The Figure 18 presents the average ridership per day for two different options: Conventional bus System (option A) and the optimized solution (red line in the Figure 17). Comparing both alternatives, there is a jump in the optimized solution after the 10th year. The flat blue line after the 10th year is due to a physical constraint. The number of passengers per hour is limited by the minimum feasible headway. The headway is the bottleneck, given that the minimum feasible headway limits the amount of buses that can operate in the system. The value of the minimum feasible headways in this study are: 90[13] seconds for trains, and 40 seconds for buses. Therefore, the maximum hourly service frequency is 40 for trains, and 90 for buses. So, the blue line is flat after the 9th year because in this year the headway is lower (39.4 seconds) than the minimum allowed headway of 40 seconds.

Despite the fact that in the first 8 years the conventional bus system and the optimize solution follow the same path, there is a jump in the optimized solution (orange line) because in the 9th year the extension is already built and fully operative. This allows the demand to be satisfies for the following years, which is what causes the exponential growth in the orange line.

Supplier and user costs, of the optimal path[14], are plotted in Figure 19 and Figure 20, and the fraction of both costs as well. In-vehicle cost is the cost that increases the most, since it is related with the

[13] For further details about the minimum headways see Appendix B: Data assumptions.
[14] For further information about the user and supplier costs of the different options, see Appendix C: Results.

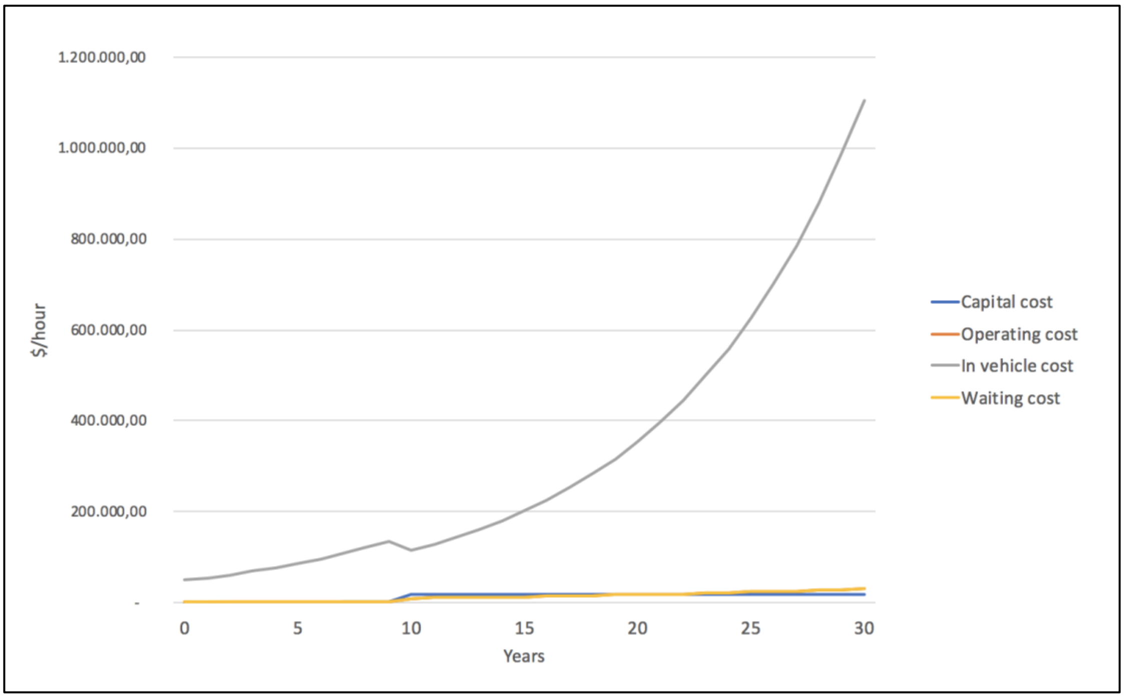


Figure 19. Supplier and user cost of the optimal option for the unconstrained case. Source: Own elaboration

ridership. That is to mean that in-vehicle cost increases as the ridership does. User waiting cost and operating cost are related with the headway. As it can be seen in Figure 19, user waiting cost and operating cost almost overlap between years 0 and 8 because the Y axis units are very large, and they have the same value from year 9 until year 30. The jump from year nine to year ten is due to the change in the optimal solution. At year 9 the optimal path changes from option A to B, causing this little jump.

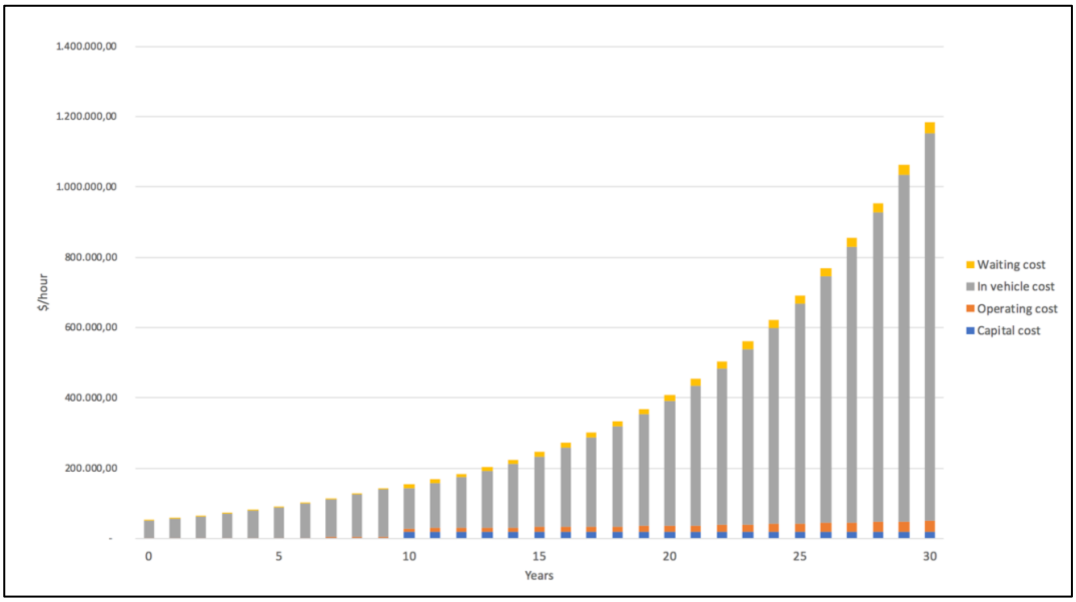


Figure 20. Breakdown of cost of the optimal option for the unconstrained case. Source: Own elaboration

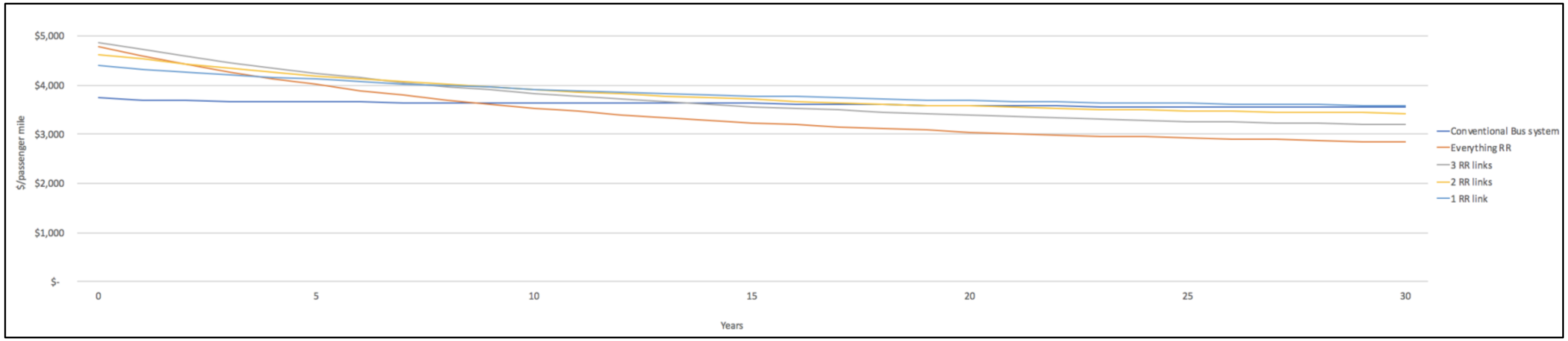


Figure 21. Dollars per passenger mile in years 0-30, for the unconstrained case. Source: Own elaboration

Capital cost varies very little from year to year, due to the indirect relation with the demand.

Figure 21 shows the cost per passenger mile of each option. That is why the less the cost per passenger mile the better. It is another way of analyzing which of the options is the best one, such as in Figure 15 and Figure 16. Due to the fact that this is an unconstrained case and there are no economical either physical constraints, the optimal path according to the cost per passenger mile (Figure 22) and the cost per year (Figure 17) is the same: the combination of option A (from year 0 to year 8) and option B (form year 9 to year 30).

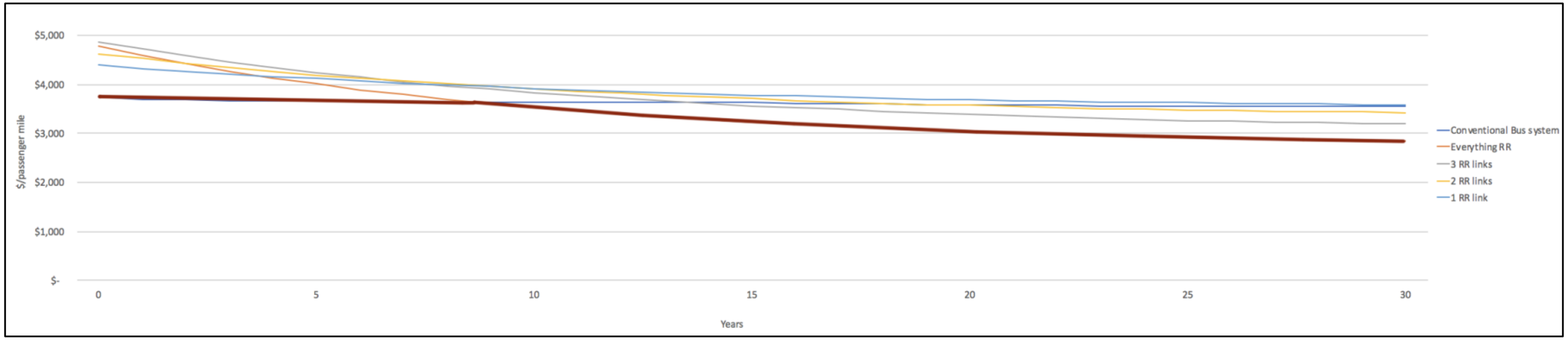


Figure 22. Dollars per passenger mile of the optimal path in years 0-30, for the unconstrained case. Source: Own elaboration

From the analysis already made, without considering any budget constraint nor physical one, the solution would be adding four links (the complete extension) in year 9 as long as the demand is sufficient. In the next point, the sensitivity to the demand is analyzed.

V.3. BUDGET-CONSTRAINED CASE

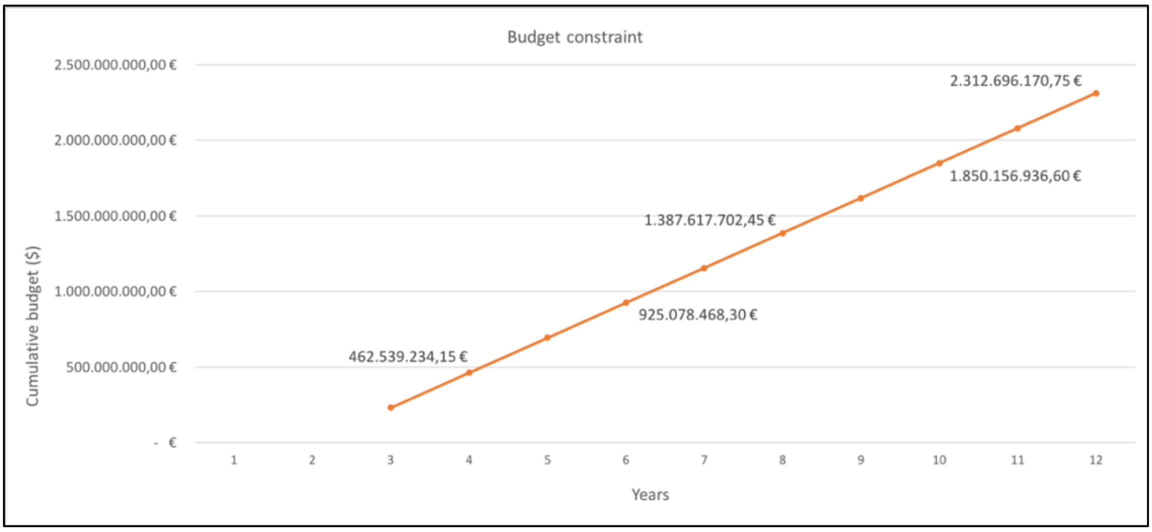


Figure 23. Budget availability over the years. Source: Own elaboration

In this section, a budget constraint is implemented in the model. It is assumed that the whole budget will be unlock at year twelve. The assumption made was that the budget is equally distributed along the first twelve years, excepting years zero, one and two were any budget is available. There is a political constraint that requires to build the whole extension by year fifteen, even if it is not the optimal solution. There is as well a physical constraint, it takes at least 8 years to build the whole extension. That is to say, it takes one year to build 0.74 miles. So, it takes one year and one month to build the first link (station *B*), one year and 9 months to build the second one (station *C*), two years and four months the third one (station *D*), and two years and ten months the last link (station *E*). Considering the three constraints, we can see in Figure 24 that optimized solution (the yellow path) is the following one:

- From year 0 to year 4: Conventional bus system (option A)
- From year 4 to year 6: One link (option E).
- From year 6 to year 9: 2 links (option D).
- From year 9 to year 12: three links (option C).
- > 11 year: the complete extension, four links (option B).

	A: All bus	B: All railroad	C: 3 links RR			D: 2 links RR			E: 1 link RR		
Year	BUS	RR	RR	BUS	Total	RR	BUS	Total	RR	BUS	Total
0	$ 25.000.000,00	$ 2.052.000.103,52	$ 1.374.897.304,83	$ 3.750.000,00	$ 1.378.647.304,83	$ 793.867.638,61	$ 4.500.000,00	$ 798.367.638,61	$ 325.967.602,01	$ 5.250.000,00	$ 331.217.602,01
1	$ 13.250.000,00	$ 2.052.000.103,52	$ 1.376.592.220,09	$ 3.750.000,00	$ 1.380.342.220,09	$ 795.562.553,86	$ 4.750.000,00	$ 800.312.553,86	$ 305.628.632,96	$ 5.500.000,00	$ 311.128.632,96
2	$ 14.750.000,00	$ 2.053.695.018,77	$ 1.378.287.135,34	$ 4.000.000,00	$ 1.382.287.135,34	$ 795.562.553,86	$ 5.000.000,00	$ 800.562.553,86	$ 305.628.633,96	$ 5.750.000,00	$ 311.378.633,96
3	$ 16.500.000,00	$ 2.053.695.018,77	$ 1.378.287.135,34	$ 4.250.000,00	$ 1.382.537.135,34	$ 797.257.469,12	$ 5.250.000,00	$ 802.507.469,12	$ 305.628.633,96	$ 6.250.000,00	$ 311.878.633,96
4	$ 18.250.000,00	$ 2.055.389.934,03	$ 1.379.982.050,60	$ 4.500.000,00	$ 1.384.482.050,60	$ 798.952.384,37	$ 5.500.000,00	$ 804.452.384,37	$ 305.628.634,96	$ 6.500.000,00	$ 312.128.634,96
5	$ 20.500.000,00	$ 2.055.389.934,03	$ 1.381.676.965,85	$ 4.750.000,00	$ 1.386.426.965,85	$ 800.647.299,62	$ 5.750.000,00	$ 806.397.299,62	$ 305.628.635,96	$ 7.000.000,00	$ 312.628.635,96
6	$ 23.000.000,00	$ 2.057.084.849,28	$ 1.383.371.881,11	$ 5.000.000,00	$ 1.388.371.881,11	$ 802.342.214,88	$ 6.000.000,00	$ 808.342.214,88	$ 305.628.636,96	$ 7.250.000,00	$ 312.878.636,96
7	$ 25.750.000,00	$ 2.057.084.849,28	$ 1.385.066.796,36	$ 5.250.000,00	$ 1.390.316.796,36	$ 804.037.130,13	$ 6.500.000,00	$ 810.537.130,13	$ 305.628.637,96	$ 7.750.000,00	$ 313.378.637,96
8	$ 28.750.000,00	$ 2.058.779.764,53	$ 1.385.066.796,36	$ 5.750.000,00	$ 1.390.816.796,36	$ 805.732.045,39	$ 6.750.000,00	$ 812.482.045,39	$ 305.628.638,96	$ 8.250.000,00	$ 313.878.638,96
9	$ 32.250.000,00	$ 2.060.474.679,79	$ 1.386.761.711,61	$ 6.000.000,00	$ 1.392.761.711,61	$ 807.426.960,64	$ 7.250.000,00	$ 814.676.960,64	$ 305.628.639,96	$ 8.750.000,00	$ 314.378.639,96
10	$ 36.250.000,00	$ 2.060.474.679,79	$ 1.390.151.542,12	$ 6.250.000,00	$ 1.396.401.542,12	$ 809.121.875,90	$ 7.750.000,00	$ 816.871.875,90	$ 305.628.641,96	$ 9.000.000,00	$ 314.628.641,96
11	$ 40.500.000,00	$ 2.062.169.595,04	$ 1.391.846.457,38	$ 6.750.000,00	$ 1.398.596.457,38	$ 810.816.791,15	$ 8.000.000,00	$ 818.816.791,15	$ 305.628.642,96	$ 9.750.000,00	$ 315.378.642,96
12	$ 45.250.000,00	$ 2.063.864.510,30	$ 1.393.541.372,63	$ 7.000.000,00	$ 1.400.541.372,63	$ 812.511.706,40	$ 8.500.000,00	$ 821.011.706,40	$ 305.628.643,96	$ 10.250.000,00	$ 315.878.643,96

Figure 24. Capital cost of the five options. Source: Own elaboration

The optimized solution presented before satisfied all the constraints, as it is explained as follows:

1. Political constraint: the whole extension is built by the end of the 14th year, considering that there is a construction time period of almost 3 years and the work starts in the 12th year.
2. Physical constraint: the cell of the 11th year of the option B is green because according to economic constraints it would be possible to start the construction of the forth link in that year. However, it takes more than two years to build the third link, so the fourth one cannot start earlier.
3. Budget constraint: the yellow path has been selected considering the accumulate budget of the Figure 23. That means that every time a new link is decided to be built, there is enough unblock budget to proceed with it.

We can observe in the Figure 25 and Figure 27 the optimal path in green considering all the constraints. The difference is significant when we compare it with the Figure 17, where we can see the optimal path of the unconstrained case. In the unconstrained case there is only one change, between year 9 and 10 when the four links are built at the same time. In the constrained case there are three changes: from year four to five, six to seven, eight to ten and eleven to thirteen. In the constrained case the five options compose the optimal solution, in the unconstrained case only two option (A and B) compose the optimal path. In the Figure 27the green line representing the optimal path, and the orange line representing the option B coincide.

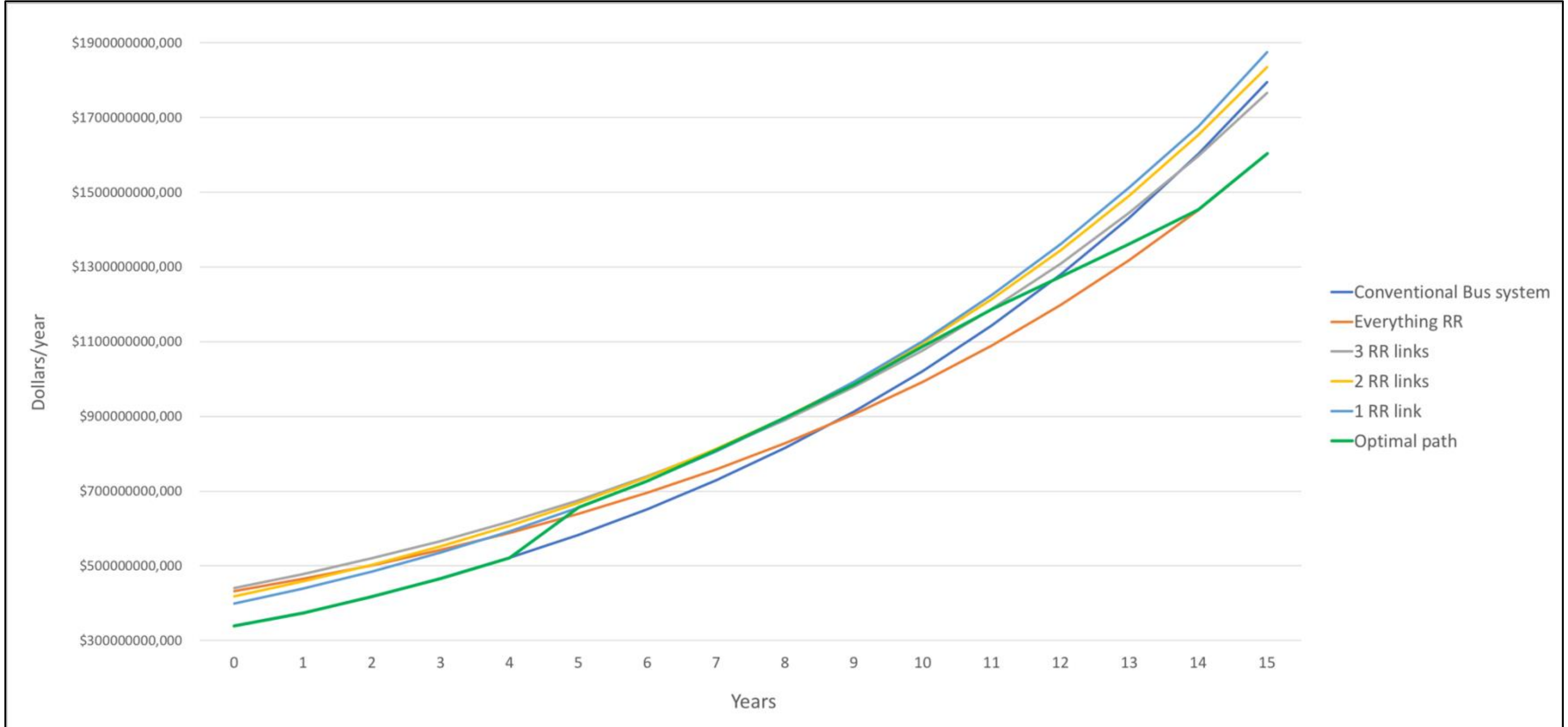


Figure 25. Optimal path during the first 15 years with physical, economic, and political constraints. Source: Own elaboration

Figure 26 shows the stations that are going to be open in each year according to the optimal solution in the constrained case. If we compare these results with the results of the Figure 14, we can see here that the solution is more staggered. In the constrained solution, at the beginning there is not any link construction and the additional links are built one at a time. Unlike that, Figure 14 no link is built until suddenly in the 9th year all four links are built.

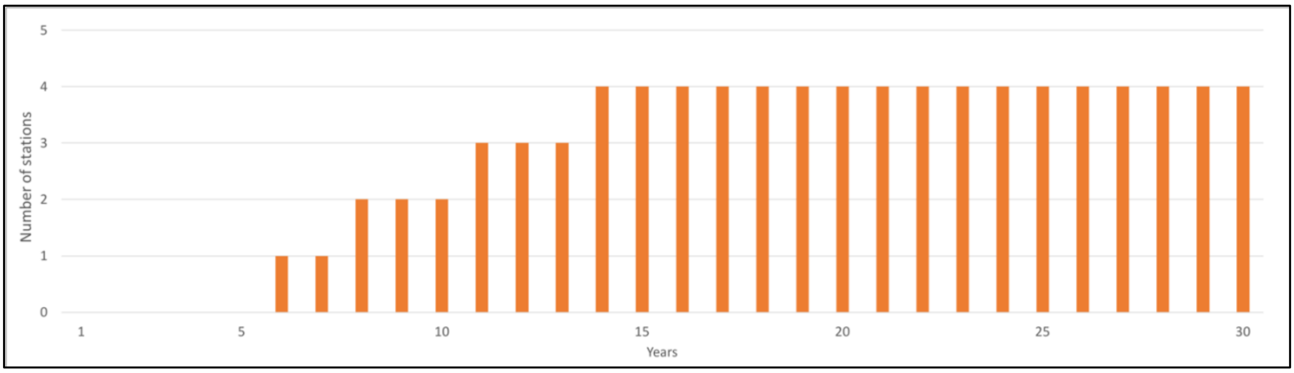


Figure 26. Optimized solution for constrained case. Source: Own Elaboration

How are these constraints translated in economic terms? The total cost of the constrained case is $ 74,200,840,555.97 and for the unconstrained case is $ 75,109,651,304.66. That means that the constrained case has an increment of 1.225 % respect the unconstrained case. It does not seem much, but in financial terms it is $ 908,810,748.69, in the whole-time horizon. [15]

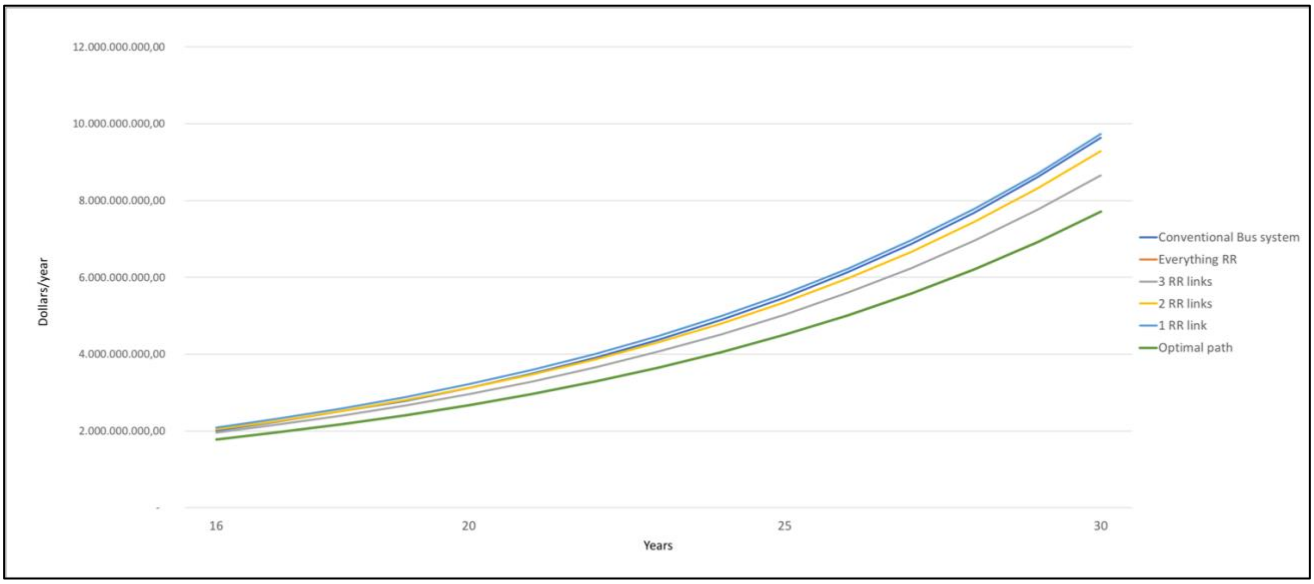


Figure 27. Optimal path from year 16 to year 30 with physical, economic, and political constraints. Source: Own elaboration

[15] More details about the yearly cost of each option in Appendix C: Results

	0			1			2			3			4			5			6			7			8			9			10		
	h (min)	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train
A: All bus	0,95			1,803			1,63			1,45			1,30			1,16			1,03			0,92			0,82			0,74			0,66		
B: All RR	9,47			8,95			8,45			7,99			7,55			7,13			6,74			6,37			6,02			5,69			5,37		
C: 3 links RR	5,93	2,04	27,50	5,60	1,821	24,55	5,29	1,63	21,92	5,00	1,45	19,57	4,73	1,30	17,48	4,46	1,16	15,60	4,22	1,03	13,93	3,99	0,92	12,44	3,77	0,82	11,11	3,56	0,74	9,92	3,36	0,66	8,85
D: 2 links RR	5,17			4,89			4,62			4,37			4,12			3,90			3,68			3,48			3,29			3,11			2,94		
E: 1 link RR	4,51			4,27			4,03			3,81			3,60			3,40			3,21			3,04			2,87			2,71			2,56		

	11			12			13			14			15			16			17			18			19			20		
	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train
A: All bus	0,67			0,67			0,67			0,67			0,67			0,67			0,67			0,67			0,67			0,67		
B: All RR	5,08			4,80			4,53			4,28			4,05			3,82			3,61			3,41			3,23			3,05		
C: 3 links RR	3,18	0,59	7,91	3,00	0,52	7,06	2,84	0,47	6,30	2,68	0,42	5,63	2,53	0,37	5,02	2,39	0,33	4,49	2,26	0,30	4,01	2,14	0,27	3,58	2,02	0,24	3,19	1,91	0,21	2,85
D: 2 links RR	2,77			2,62			2,48			2,34			2,21			2,09			1,97			1,87			1,76			1,67		
E: 1 link RR	2,42			2,29			2,16			2,04			1,93			1,82			1,72			1,63			1,54			1,50		

	21			22			23			24			25			26			27			28			29			30		
	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train
A: All bus	0,67			0,67			0,67			0,67			0,67			0,67			0,67			0,67			0,67			0,67		
B: All RR	2,88			2,72			2,57			2,43			2,30			2,17			2,05			1,94			1,83			1,73		
C: 3 links RR	1,80	0,19	2,55	1,70	0,17	2,27	1,61	0,15	2,03	1,52	0,13	1,81	1,50	0,12	1,62	1,50	0,11	1,44	1,50	0,10	1,29	1,50	0,09	1,15	1,50	0,08	1,03	1,50	0,07	0,92
D: 2 links RR	1,57			1,50			1,50			1,50			1,50			1,50			1,50			1,50			1,50			1,50		
E: 1 link RR	1,50			1,50			1,50			1,50			1,50			1,50			1,50			1,50			1,50			1,50		

Figure 28. Headways for the whole-time horizon for both the constrained and the unconstrained case. Source: Own elaboration

The optimal solution for the constrained case should satisfy the demand. In order to satisfy the demand, it must be considered the maximum headway. Furthermore, there is a physical constrain, the minimum headway. The maximum headway is the maximum time between vehicles in the transit system that allow us to satisfy the demand. Conversely, the minimum headway is the minimum time between vehicles in the transit system in order to prevent the collision of two consecutive vehicles[16]. So, both headways limit the optimal headway calculated for each option.

In Figure 28 we can see that there are optimal headways limited by both the maximum and the minimum headway. The meaning of the colors is the following one:

- Orange: limited by the bus maximum headway. The optimal headway is higher than the maximum headway, and the maximum headway is higher than the minimum headway ($h^*>h_{max}>h_{min}$). With this headway the demand is satisfied.
- Yellow: limited by the bus minimum headway. The optimal headway is higher than both the maximum and the maximum headway, but the maximum headway is lower than the minimum headway. In this case the demand cannot be satisfy because of physical constraints, so the optimal headway is limited by the minimum feasible headway ($h^*>h_{min}>h_{max}$)

[16] For further details about the minimum headways see Appendix B: Data assumptions.

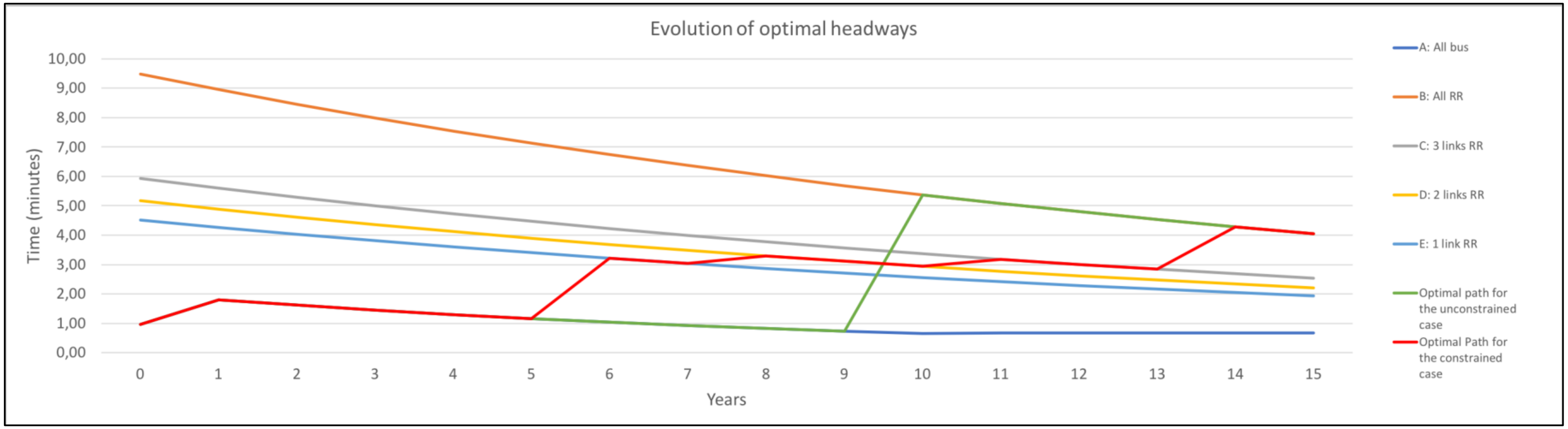


Figure 29. Evolution of the optimal headways for the constrained and unconstrained cases in the first 15 years. Source: Own elaboration

- <u>Blue</u>: limited by the bus minimum headway. The optimal headway is higher than the maximum, but it is lower than the minimum headway ($h_{min}>h^{*}>h_{max}$). As it happens with the yellow ones, the demand is not completely satisfied.
- <u>Green</u>: limited by the train minimum headway. The optimal headway is lower than both the minimum and maximum headway ($h_{max}>h_{min}>h^{*}$). The demand is satisfied but the cost is higher.
- <u>Red</u>: the maximum head is lower than the minimum headway.

Figure 29 and Figure 30 show the headway evolution of each option for the whole-time horizon. It also shows the optimal headways for the constrained and unconstrained case:

- <u>Option</u> <u>A</u> (dark blue line): it stands out a sudden change on the tendency in year 1. Since year two until year nine, the optimal headway is limited by the maximum headway. In year two the optimal headway should be 1.70 minutes, however, the maximum headway is 1.63 minutes. So, the optimal headway of year two is 1.63 minutes, leading to the decrease of the dark blue line (which coincides with the red line the first five years, and with the green line the first 9 years). Moreover, it can be seen that in year ten the headway is constant until year

30 (Figure 30). This is because the optimal headway is higher than both the maximum and the minimum feasible headway, so finally the optimal headway is limited by the minimum headway (0.67 minutes, 40 seconds).

- Option B: The optimal headway decreases as the time goes by, but it is never affected by headway limitations.
- Option C: It follows a constant decrease on the optimal headway until year 24 (Figure 30). In year 25 the optimal headway is lower than the minimum feasible headway, so the optimal headway is limited (h^*=1.50 min= 90 seconds).
- Option D: Same as option C, but the headways start being limited by the minimum headway in year 22.
- Option E: Same as option C and D, but the headways start being limited by the minimum headway in year 20.

In Figure 30, the orange line (option B), red line (optimal headway for the constrained case) and the green line (optimal headway for the unconstrained case) coincide.

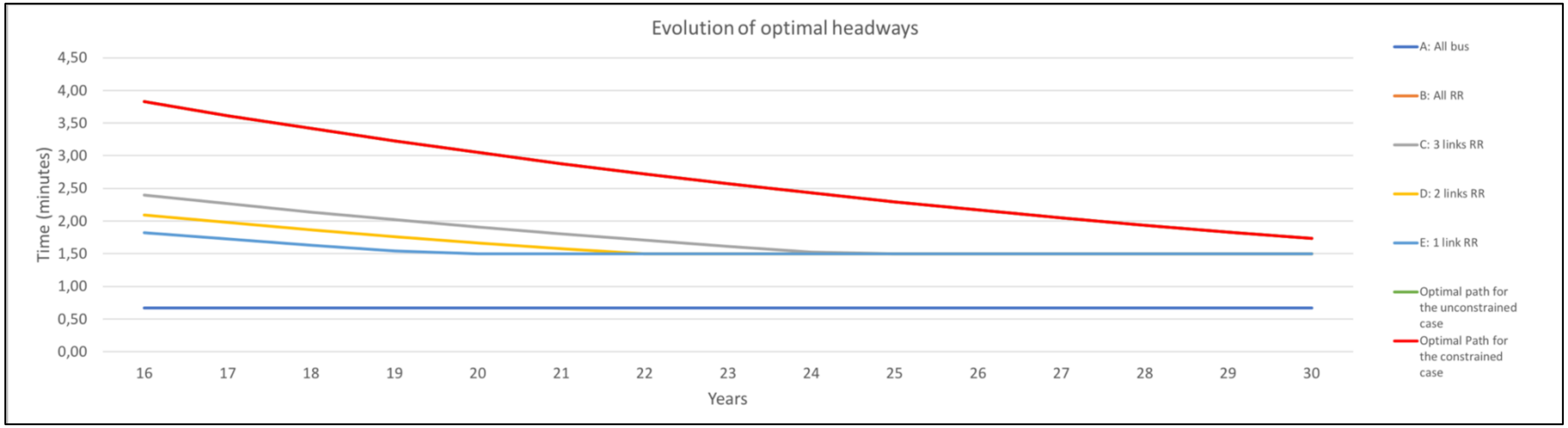


Figure 30. Evolution of the optimal headways for the constrained and unconstrained cases from year 15 to year 30. Source: Own elaboration

This headway analysis is the same for the constraint and unconstrained case because it depends on the miles of extension of the rail road, dwell time, cruise speed, demand, unit cost of user waiting time and the hourly cost per vehicle, which do not vary depending on the three constraints of the constrained case. Therefore, we can see that either way, both cases result in the construction of the four links (full extension) sooner or later.

Chapter VI: Sensitivity Analysis

In order to inquire into the effects of several input parameters (e.g, value of time, interest rate, operating cost....) on the resulting optimized values (e.g., total cost and construction phases) the following sensitivity analysis has been carried out. These analyses are used to determine how sensitive are the solutions of a particular study to the values of the input parameters. The more sensitive the model is to a certain parameter, the more accurately should be the parameter be predicted. If this were to happen, decisions based on those researches should be taken with extreme caution.

VI.1. Sensitivity analysis for the unconstrained case

VI.1.1. Sensitivity to demand

In the Table 4 we can see the sensitivity analysis for different demand levels. In the table we can see the increment/reduction of the demand (being the baseline value 100%), in the second column the main cost per passenger mile of all the options, and the third column represents the optimal solution, that is to say, when should be the extension be built according to the new demand pattern[17].

Demand (%)	Mean cost ($/passenger mile)					Optimized solution
	A	B	C	D	E	
200 %	$ 3.587	$ 3.119	$ 3.451	$ 3.618	$ 3.722	Year 3 – 4 links
175 %	$ 3.590	$ 3.166	$ 3.493	$ 3.648	$ 3.743	Year 4 – 4 links
130 %	$ 3.598	$ 3.289	$ 3.602	$ 3.727	$ 3.797	Year 7 – 4 links
110 %	$ 3.605	$ 3.373	$ 3.675	$ 3.780	$ 3.832	Year 8 – 4 links
100 %	$ 3.609	$ 3.426	$ 3.721	$ 3.813	$ 3.854	Year 9 – 4 links
80 %	$ 3.622	$ 3.568	$ 3.843	$ 3.900	$ 3.912	Year 12 – 4 links
60 %	$ 3.641	$ 3.796	$ 4.035	$ 4.036	$ 4.000	Year 14 – 4 links
30 %	$ 3.704	$ 4.643	$ 4.726	$ 4.512	$ 4.298	Year 20 – 4 links

[17] For further information about the sensitivity demand of the different options, see Appendix C: Results.

10 %	$ 3.882	$ 7.711	$ 7.084	$ 6.063	$ 5.185	Year 29 – 4 links

Table 4. Effects of demand, for the unconstrained case. Source: Own elaboration

Evidently, if the demand grows the total cost grows. That is why the reference cost taken is the cost per passenger mile, instead of considering the total cost. In the Table 4 we can see in grey the lower cost per passenger mile depending on the demand variation.

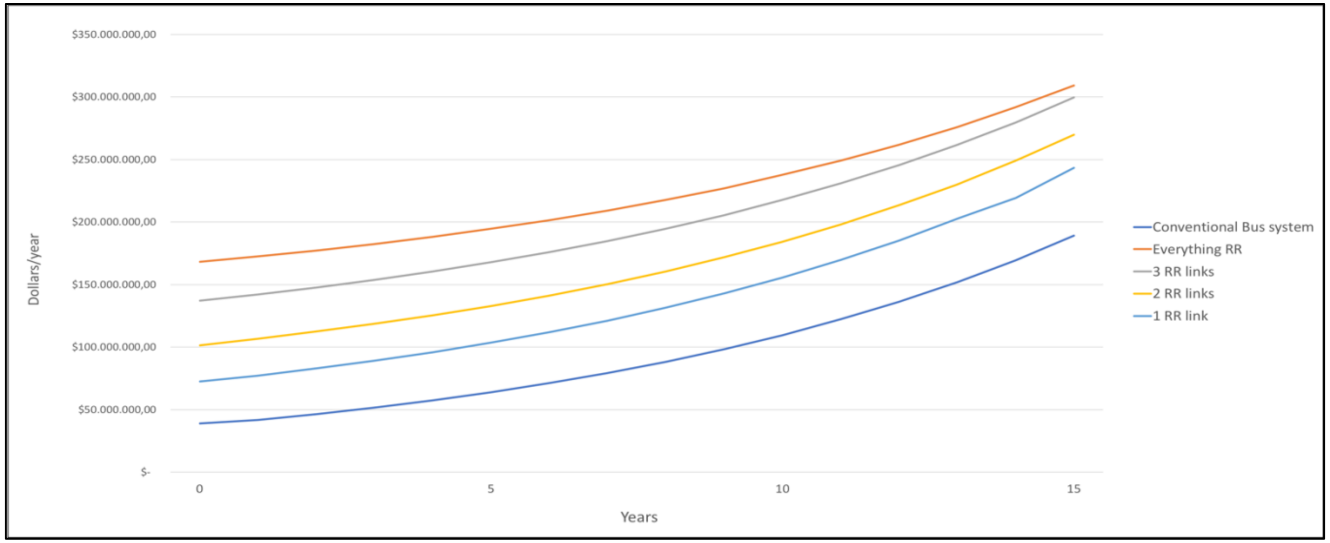


Figure 31. Evolution of every option (from year 0 to year 15) with a reduction of the 90% of the demand, for the unconstrained case. Source: Own elaboration

Even when the demand is reduced up to the 90% of the baseline value, the optimal solution continues being building the full extension (4 links), but now in the 29th year. In the 28th year the cost is almost the same for options A and B, but in the 29th year it is cheaper to build the four links. However, with a horizon of 30 years lifetime, it makes no sense to build the station one for one year.

All in all, what it can be seen is that changes on the demand change the year on which the links must be built, but it does not change the fact that the optimal solution is always composed by options A and B.

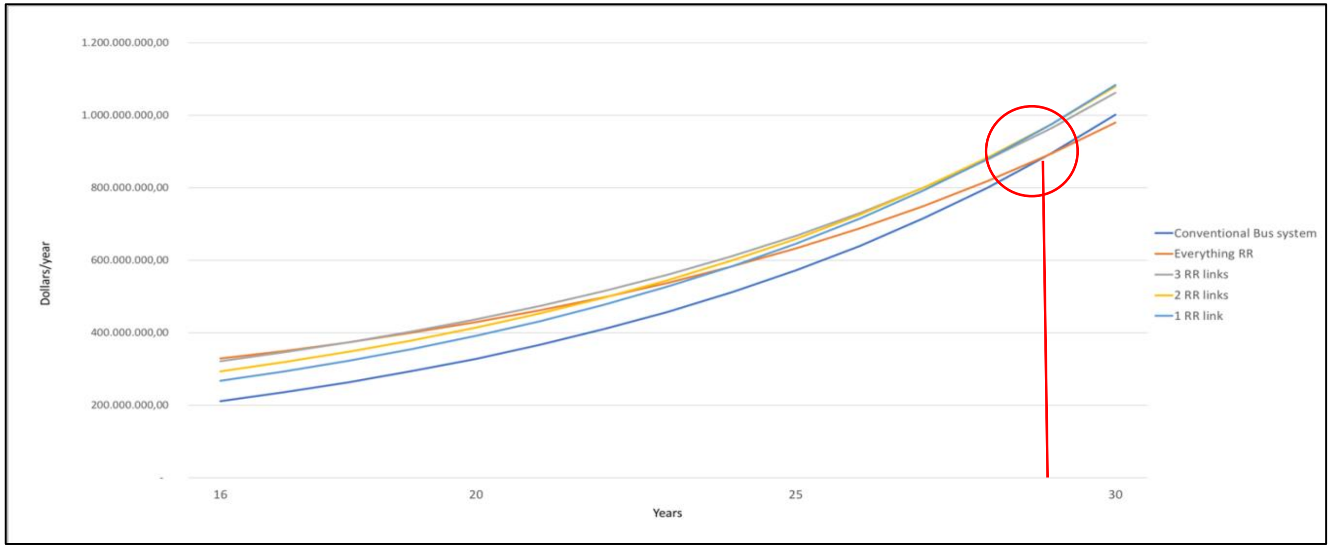


Figure 32. Evolution of every option (from year15 to year 30) with a reduction of the 90% of the demand, for the unconstrained case. Source: Own elaboration

VI.1.2. Sensitivity to analysis period

Analysis Period (year)	**Mean cost ($/passenger mile)**					**Optimized solution**
	A	B	C	D	E	
10	$ 3,668	$ 4,963	$ 4,887	$ 4,572	$ 4,269	No extension
15	$ 3,648	$ 4,196	$ 4,329	$ 4,222	$ 4,093	Year 13 – 4 links
20	$ 3,632	$ 3,812	$ 4,035	$ 4,029	$ 3,986	Year 11 – 4 links
25	$ 3,619	$ 3,580	$ 3,850	$ 3,903	$ 3,910	Year 10 – 4 links
30	$ 3,609	$ 3,426	$ 3,721	$ 3,813	$ 3,854	Year 9 – 4 links
40	$ 3,602	$ 3,337	$ 3,643	$ 3,758	$ 3,817	Year 8 – 4 links
50	$ 3,597	$ 3,309	$ 3,612	$ 3,734	$ 3,799	Year 8 – 4 links

Table 5. Effects of different analysis periods, for the unconstrained case. Source: Own elaboration

In the Table 5 it can be seen the sensitivity solutions to different analysis periods. The longer the analysis period is, the earlier the extension of the four links should be made and the less the cost per passenger mile is, and vice versa. When shortening the analysis horizon to 10 years, the optimized solution has no extension for the entire time horizon. That is because the extension should take place in year 14th as we can see in Figure 33, and the analysis horizon stops in the 10th year. Moreover, it makes no sense to carry out the extension in year 12 if the time horizon is fifteen years. It is surprising that in the 20- and 25-time horizon the lower cost per passenger mile is option A, however, the optimal solution is to implement the full extension by year eleven and ten respectively. This is because we are comparing average values.

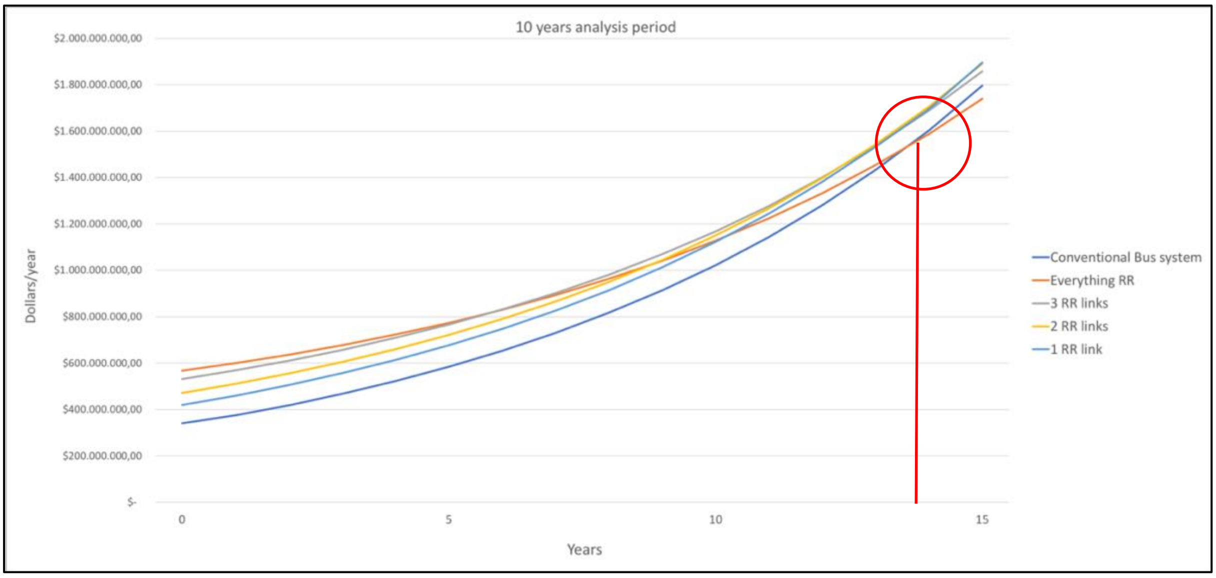


Figure 33. Evolution of a 10 years analysis period, for the unconstrained case. Source: Own elaboration

All the results obtained converge to the same solution sooner or later, combining the current bus system (option A) and after building four links (the completely extension). It is, at least surprising, that having intermediate solutions such as build one, two or three links before building the completely extension, the solution is always passing from zero links to the completely extension. The reasons are, economies of scale, increment of ridership and capital cost divided all along the time horizon. It stands out the increment of ridership, because the option B (building the 4 links) it not the best option at the beginning, in fact, it is the worst or the second-worst option. But thanks to the ridership growth, it ends to be the best option quicker than the other options as it happens in Figure 33 or Figure 15. The higher the analysis period, the higher is going to be de total cost, and therefore the mean cost per year. That is why tin order to compare the different solutions over a time horizon we consider the cost per passenger mile and not the average cost per year.

VI.1.3. Sensitivity to growth rate

One of the reasons given for the sudden construction of the four links at the same time, without any intermediate solution, is the ridership growth. Theoretically, if the demand is very low the extension is postponed. Here are analyzed the changes in the solution once changes in the growth rate are applied. The optimized solution adds 4 links (the completely extension) in year 9. As we can see in the Table 6, every option excepting the increment of 1% of the demand, results in a full extension of 4 links. It seems that it would not make sense to make an extension in the 25th year with a time horizon of 30 years as happens with a 4% growth rate. However, all the other values lead to the same answer: 4 links. To be more accurate, it should have been an option to change the growth rate depending on whether a link in open or not, like that the growth rate changes depending on the links opened and do not stay constant.

Growth rate per year (%)	Optimized solution
100	Year 2 – 4 links
40	Year 3 – 4 links
30	Year 4 – 4 links
20	Year 6 – 4 links
15	Year 8 – 4 links
12	Year 9 – 4 links
10	Year 11 – 4 links
6	Year 16 – 4 links
4	Year 25 – 4 links
1	No extension

Table 6. Ridership for different alternatives, for the unconstrained case. Source: Own elaboration

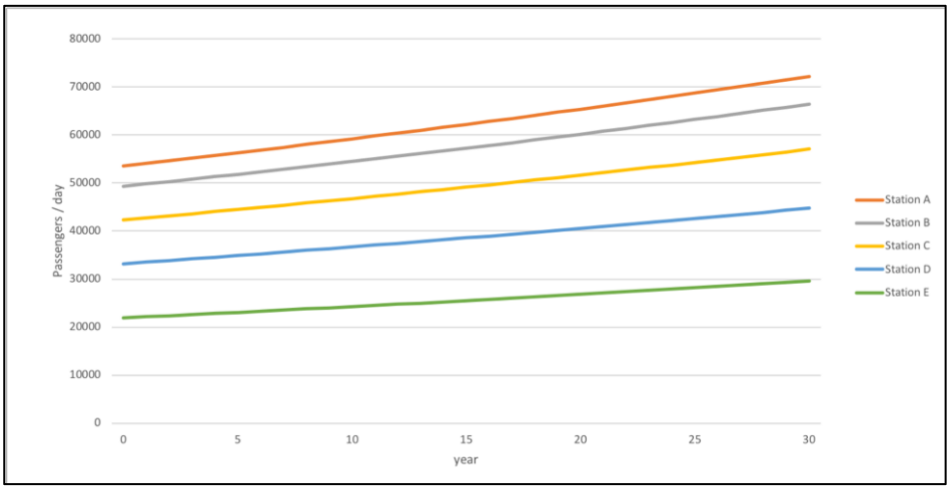


Figure 34. Ridership for a 1% growth. Source: Own elaboration

VI.2. SENSITIVITY ANALYSIS FOR THE CONSTRAINED CASE

VI.2.1. Sensitivity to construction cost savings for the constrained case

It is surprising that even eliminating the economies of scale in the optimal solution of the constrained case, the cross between options A and B continues being in year nine (Figure 25). When four links are

Year	Option B (4 links)	Option C (3 links)	Option D (2 links)
0	2,70%	1,14%	0,33%
1	2,51%	1,05%	0,30%
2	2,32%	0,96%	0,27%
3	2,15%	0,88%	0,25%
4	1,98%	0,81%	0,23%
5	1,82%	0,74%	0,21%
6	1,68%	0,68%	0,19%
7	1,54%	0,62%	0,17%
8	1,41%	0,56%	0,15%
9	1,29%	0,51%	0,14%
10	1,17%	0,46%	0,13%
11	1,07%	0,42%	0,11%
12	0,97%	0,38%	0,10%
13	0,88%	0,35%	0,09%
14	0,80%	0,31%	0,08%
15	0,73%	0,28%	0,08%
16	0,66%	0,26%	0,07%
17	0,60%	0,23%	0,06%
18	0,54%	0,21%	0,05%
19	0,49%	0,19%	0,05%
20	0,44%	0,17%	0,04%
21	0,40%	0,15%	0,04%
22	0,36%	0,14%	0,04%
23	0,32%	0,12%	0,03%
24	0,29%	0,11%	0,03%
25	0,26%	0,10%	0,03%
26	0,23%	0,09%	0,02%
27	0,21%	0,08%	0,02%
28	0,19%	0,07%	0,02%
29	0,17%	0,06%	0,02%
30	0,15%	0,06%	0,01%
Mean	0,98%	0,39%	0,11%

Table 7. Sensitivity of the construction savings. Source: Own elaboration

built at the same time there is a 9% construction savings, 6% if three links are built, and 3% if two are built. As we can see in the Table 7, the economies of scale do not have a great impact on the final cost. Why?

Because the economies of scale are only applied to the construction costs, not to the purchase of vehicles. So, the percentage of saving varies because the number of trains needed varies depending on the demand, and the more the demand increases, the more trains needed, and the less impact has the reduction in the construction costs. That is also the reason why the percentage of cost savings in not the same for all the time horizon for the same option.

Options A (conventional bus system) and E (one link) are not considered in the Table 7 because there is no increment of the cost, given that there are only construction saving if more than one link is built at the same time.

If there are no construction savings, that means that not only the total cost should increase, but also that the cost per passenger mile should too. In the Table 8 we can see the increment between the optimal solution of the unconstrained case and the optimal solution of the constrained case. The increment is almost negligible, being the maximum mean difference lower than 1%. As expected, the maximum difference corresponds to option B, where the construction savings are higher (9%). Options A and E are not studied because there are not economies of scale in either of them.

VI.2.2. Sensitivity to demand for the constrained case

In the Table 9 we can see the sensitivity analysis for different demand levels for the constrained case. In the table we can see the increment/reduction of the demand (being the baseline value 100%), in the second column the main cost per passenger mile of all the options, and the third column represents the optimal solution, that is to mean, when should be the extension be built according to the new demand pattern[18].

Comparing this table with Table 4 we can see that values of options A and E have the same value as in the unconstrained case. This is due to the fact that in the unconstrained case, there are no economies of scale either in option A or E. On the contrary, there is an increment of the cost in the rest of the

Year	Option B (4 links)	Option C (3 links)	Option D (2 links)
0	2,628%	1,123%	0,329%
1	2,445%	1,036%	0,301%
2	2,270%	0,953%	0,274%
3	2,103%	0,876%	0,250%
4	1,943%	0,803%	0,227%
5	1,792%	0,735%	0,206%
6	1,649%	0,672%	0,187%
7	1,514%	0,613%	0,170%
8	1,388%	0,559%	0,154%
9	1,270%	0,509%	0,139%
10	1,160%	0,463%	0,126%
11	1,058%	0,420%	0,114%
12	0,964%	0,381%	0,103%
13	0,876%	0,345%	0,093%
14	0,796%	0,312%	0,084%
15	0,722%	0,283%	0,075%
16	0,654%	0,255%	0,068%
17	0,592%	0,231%	0,061%
18	0,535%	0,208%	0,055%
19	0,484%	0,188%	0,049%
20	0,436%	0,169%	0,044%
21	0,394%	0,152%	0,040%
22	0,355%	0,137%	0,036%
23	0,319%	0,123%	0,032%
24	0,288%	0,111%	0,029%
25	0,259%	0,100%	0,026%
26	0,233%	0,089%	0,023%
27	0,209%	0,080%	0,021%
28	0,188%	0,072%	0,019%
29	0,168%	0,065%	0,017%
30	0,151%	0,058%	0,015%
Mean	0,963%	0,391%	0,109%

Table 8. Sensitivity of the dollars per passenger mile. Source: Own elaboration

[18] For further information about sensitivity to demand for the constrained case, see Appendix C: Results.

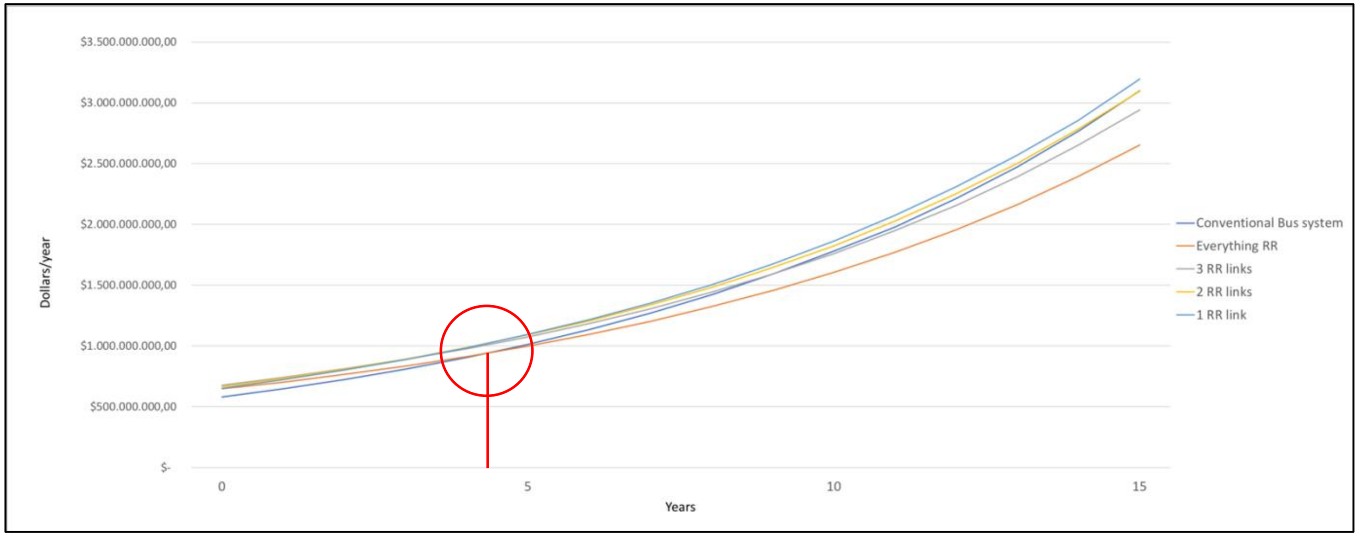
Demand (%)	Mean cost ($/passenger mile)					Optimized solution
	A	B	C	D	E	
200 %	$ 3.587	$ 3.138	$ 3.459	$ 3.620	$ 3.722	Year 4 – 4 links
175 %	$ 3.590	$ 3.187	$ 3.502	$ 3.651	$ 3.743	Year 5 – 4 links
130 %	$ 3.598	$ 3.318	$ 3.614	$ 3.731	$ 3.797	Year 7 – 4 links
110 %	$ 3.605	$ 3.407	$ 3.689	$ 3.784	$ 3.832	Year 9 – 4 links
100 %	$ 3.609	$ 3.463	$ 3.737	$ 3.818	$ 3.854	Year10 – 4 links
80 %	$ 3.622	$ 3.615	$ 3.863	$ 3.906	$ 3.912	Year 12 – 4 links
60 %	$ 3.641	$ 3.858	$ 4.062	$ 4.043	$ 4.000	Year 14 – 4 links
30 %	$ 3.704	$ 4.769	$ 4.780	$ 4.527	$ 4.298	Year 20 – 4 links
10 %	$ 3.882	$ 8.087	$ 7.245	$ 6.108	$ 5.185	Year 30 – 4 links

Table 9. Effects of demand, for the constrained case. Source: Own elaboration

options (B, C and D) due to the same reason. It stands out that at the baseline value (100%) the optimal solution is to have a conventional bus system until year 10 and then to build the four stations at the same time, whereas, in the unconstrained case the optimal solution is maintain the conventional bus system until year nine and then build the four stations. As in the unconstrained case, the optimal path does not have any intermediate solution, it starts with the conventional bus system and at some point, the best option is to build the four stations at once. However, this is not possible in the constrained case due to the budget and physical limitations. When the demand increases 30%, or decreases 20%, 40% or 60% the optimal path is the same. Conversely, in the other hypothesis the links should be build one year later in the constrained case, due to the fact that there are not construction saving anymore.

VI.2.3. Sensitivity to analysis period for the constrained case

In the Table 10 it can be seen the sensitivity solutions to different analysis periods. The longer the analysis period is, the earlier the extension of the four links should be made and the less the cost per passenger mile is, and vice versa. When shortening the analysis horizon to 10 years, the optimized



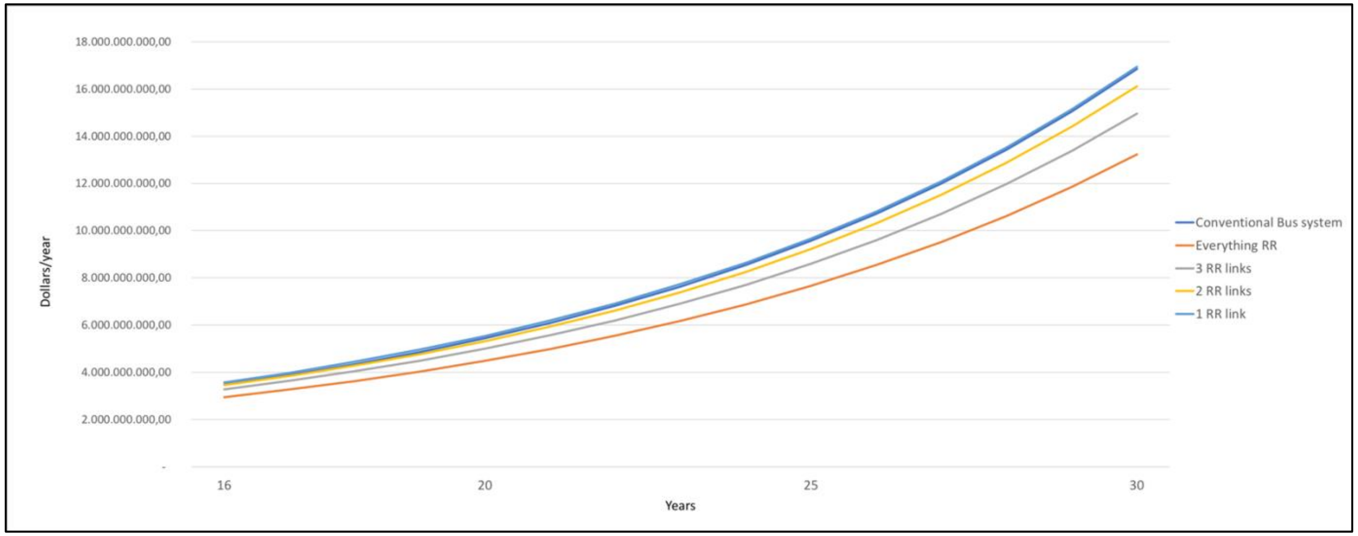


Figure 35. Evolution of every option (from year 0 to year 15, up, and from year 15 to year 30, down) with an increase of the 75% of the demand. Source: Own elaboration

Analysis Period (year)	Mean cost ($/passenger mile)					Optimized solution
	A	B	C	D	E	
10	$ 3.668	$ 5.129	$ 4.958	$ 4.591	$ 4.269	No extension
15	$ 3.648	$ 4.293	$ 4.371	$ 4.233	$ 4.093	Year 13 – 4 links
20	$ 3.632	$ 3.878	$ 4.063	$ 4.037	$ 3.986	Year 11 – 4 links
25	$ 3.619	$ 3.629	$ 3.870	$ 3.909	$ 3.910	Year 10 – 4 links
30	$ 3.609	$ 3.463	$ 3.737	$ 3.818	$ 3.854	Year 10 – 4 links
40	$ 3.602	$ 3.370	$ 3.559	$ 3.696	$ 3.817	Year 9 – 4 links
50	$ 3.597	$ 3.341	$ 3.344	$ 3.610	$ 3.799	Year 9 – 4 links

Table 10. Effects of different analysis periods. Source: Own elaboration

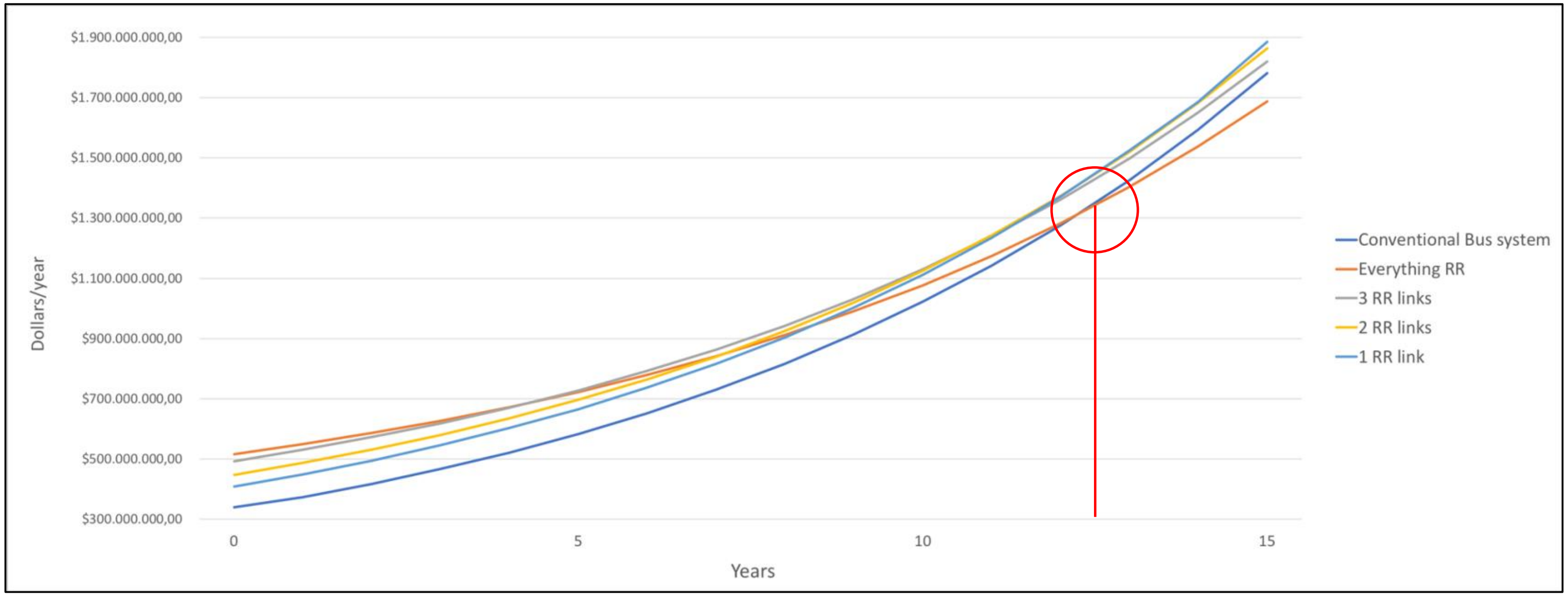


Figure 36. Evolution of a 15 years analysis period. Source: Own elaboratio

solution has no extension for the entire time horizon. That is because the extension should take place in year 15th, and the analysis horizon stops in the 10th year. Moreover, it makes no sense to carry out the extension in year 13 if the time horizon is fifteen years. It is surprising that in the 20- and 25-time horizon the lower cost per passenger mile is option A, however, the optimal solution is to implement the full extension by year eleven and ten respectively. This is because we are comparing average values.

Comparing Table 5 with Table 10, they are quite similar. The average cost per passenger mile is higher in the constrained case because of the construction savings, but it is not a big difference. This difference postpones one year the full extension of the International Airport Corridor in the 25,30,35 and 40 analysis period.

VI.2.4. Sensitivity to growth rate for the constrained case

One of the reasons given for the sudden construction of the four links at the same time, without any intermediate solution, is the ridership growth. Theoretically, if the demand is very low the extension is postponed. Here is going to be analyzed the changes in the solution once changes in the growth rate are applied. As in the unconstrained case (Table 6), the optimized solution adds 4 links (the completely extension) in year 10. As we can see in the Table 11, every option excepting the increment of 1% of the demand, results in a full extension of 4 links. It seems that it would not make sense to make an extension in the 27th year with a time horizon of 30 years as happens with a 4% growth rate. However, all the other values lead to the same answer: 4 links. In fact, comparing this table with the one of the unconstrained case, it just extends one year the decision of adding the four links in the 40% and 12 % (the baseline value) growth rate, and two year concerning the 4% growth rate value.

Growth rate per year (%)	Optimized solution
100	Year 2 – 4 links
40	Year 4 – 4 links
30	Year 4 – 4 links
20	Year 6 – 4 links
15	Year 8 – 4 links
12	Year 10 – 4 links
10	Year 11 – 4 links
6	Year 16 – 4 links
4	Year 27 – 4 links
1	No extension

Table 11. Ridership for different alternatives for the constrained case. Source: Own elaboration

VI.2.5. In-vehicle time values effects for the constrained case

The effects of different values of the unit cost of user in-vehicle time (α) are examined. This value is used to calculate the in-vehicle cost per hour ($/h) in the (12) formula. The Table 12 summarizes the results for different values of the unit cost of user in-vehicle time. The values tested are between 1 and 60 ($/passenger•hour), while the baseline value is 12.

Unit cost of user in-vehicle time ($/pass hour)	Total cumultive Cost ($/year)	Number of stations in service in each year																													
		1	2	3	4	5	6	7	8	19	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	1,61E+10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	2,39E+10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	2	3	3	3	4	4	4	4
5	3,58E+10	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4
7	4,78E+10	0	0	0	0	0	0	0	0	1	1	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
10	6,44E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
12	**7,51E+10**	**0**	**0**	**0**	**0**	**0**	**1**	**1**	**2**	**2**	**2**	**3**	**3**	**3**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**
15	9,12E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
17	1,02E+11	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
20	1,18E+11	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
25	1,45E+11	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
30	1,72E+11	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
35	1,98E+11	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
40	2,25E+11	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
50	2,79E+11	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
60	3,33E+11	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4

Table 12. Effects of the in-vehicle time values on the total cumulative cost and optimized phases. Source: Own elaboration

Some values of the Table 12 are limited by the budget constraint, in particular between 10 and 60. In fact, with a unit cost of user in-vehicle time (α) of 10 the four links should be operating by year 12 without any restriction. With an α of 60 the extension should be completed in year 4.

Figure 37 shows that as α increases, the total cumulative cost increases as well. Moreover, if the α increases the extension takes more time to be completed. The slope of the total cumulative cost is much steeper when α exceeds 20. When α is below 20 the slope of the total cumulative cost is more gradual. As α increases, rail road is preferred over bus because trains run with a higher speed than buses, reducing the in-vehicle time, and therefore the total cumulative costs.

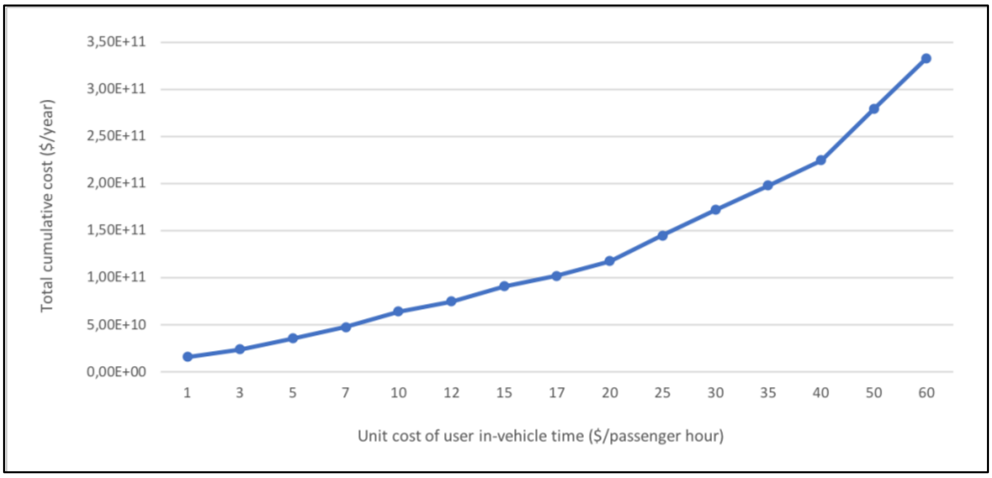


Figure 37. Effects of in-vehicle time values on total cumulative cost. Source: Own elaboration

VI.2.6. User waiting time values effects for the constrained case

This section shows the effects of the unit cost of waiting time ($/passenger hour). The optimized results for the different values of user waiting time (β) are summarized in the Table 13. In order to do the sensitivity analysis of β, values between 1 and 35 are tested, and the baseline value is 24. β is used to calculate the user waiting cost, formula (15).

Unit cost of user waiting time ($/pass hour)	Total cumultive Cost ($/year)	Number of stations in service in each year																													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	7,09E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
5	7,21E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
15	7,39E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
17	7,42E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
22	7,49E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
24	**7,51E+10**	**0**	**0**	**0**	**0**	**0**	**1**	**1**	**2**	**2**	**2**	**3**	**3**	**3**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**
30	7,58E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
35	7,63E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4

Table 13. Effects of waiting time values on the total cumulative cost and optimized phases. Source: Own elaboration

Al the values of the Table 13 are limited by the budget constraint. For instance, when β is 1, the whole extension should be built by year 5, and when it comes to be 35, it should be by year 11. We can see in Figure 38 that the total cumulative cost increases as β increases. The slope between 1 and 15 is steeper than between 15 and 35 where the steep un more continuous. As it happens with the values of the in-vehicle time, the shorter the trip duration is the earlier the construction of the four links is planned. The choice between the different values lead to the same kind of extension, four links by year 14. The model is more sensitive if a low value is chosen than if a high value is, so low values should be avoided. Generally, a higher β leads to construction delays and less construction phases.

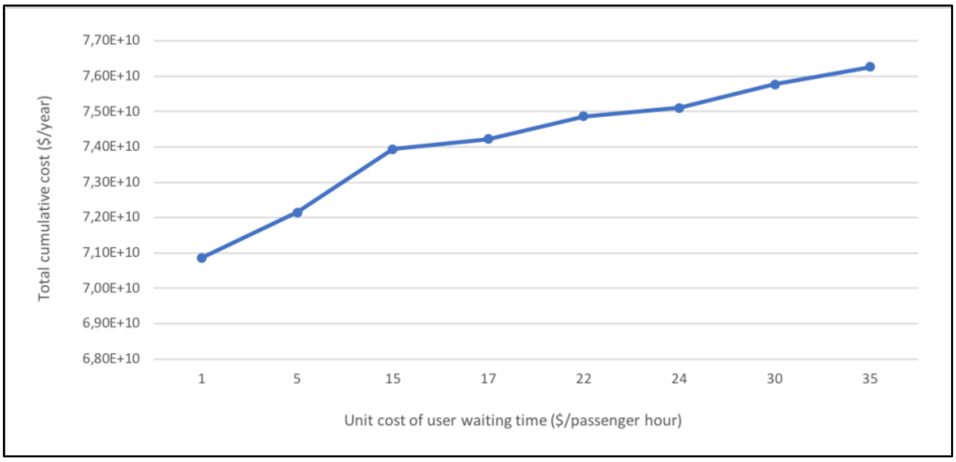


Figure 38. effects of waiting time values on the total cumulative cost. Source: Own elaboration

VI.2.7. interest rate effects for the constrained case

Theoretically, as the interest increase the projects tend to be deferred. That is to mean, the cost of borrowing money is higher if the interest rates are high, so the investment tend to decrease. That is why interest rate plays a key role in project scheduling, particularly in large investment projects. Rarely interest rates are intervened by the government, it is the market that determines its value[19]. This sensitivity analysis has the purpose of showing how can change the extension decision with different values of the interest rate. This parameter is considered in formula (5), in order to calculate the average cost per year.

In the Table 14 we can evaluate the effects of different interest rates on phase decisions and total cumulative costs ($/year). In this section, the values of *ir* (interest rate) chosen are between 1% and 30%.

Interest rate (%)	Total cumultive Cost ($/year)	Number of stations in service in each year																													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
1	7,43E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
2	7,45E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4	**7,51E+10**	**0**	**0**	**0**	**0**	**0**	**1**	**1**	**2**	**2**	**2**	**3**	**3**	**3**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**
6	7,58E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
10	7,73E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
15	7,91E+10	0	0	0	0	0	0	0	0	0	1	1	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4
20	8,08E+10	0	0	0	0	0	0	0	0	0	0	0	1	1	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4
30	8,36E+10	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	2	3	3	3	4	4	4	4	4	4	4	4	4

Table 14. Effects of interest rates on the total cumulative costs and optimized phases. Source: Own elaboration

As expected, the extension is postponed when the interest rate increases. From 1% until 10 % the construction of the links is limited by the budget constraint. In fact, without the budget constraint and considering 1% interest rate, the optimal option is to have operating the four links by year seven. The last percentage limited by the budget constraint is the 10%, which optimal option is to build the four links by year 14. When interest rate increases to 30%, the optimal solution is to build the four links by the 22nd year. Figure 39 shows the effects of the interest rates on the total cumulative costs ($/year). As *ir* increases from 6 to 30, total cumulative cost increases rapidly. Nevertheless, it increases slowly while *ir* keeps increasing from 1 to 6.

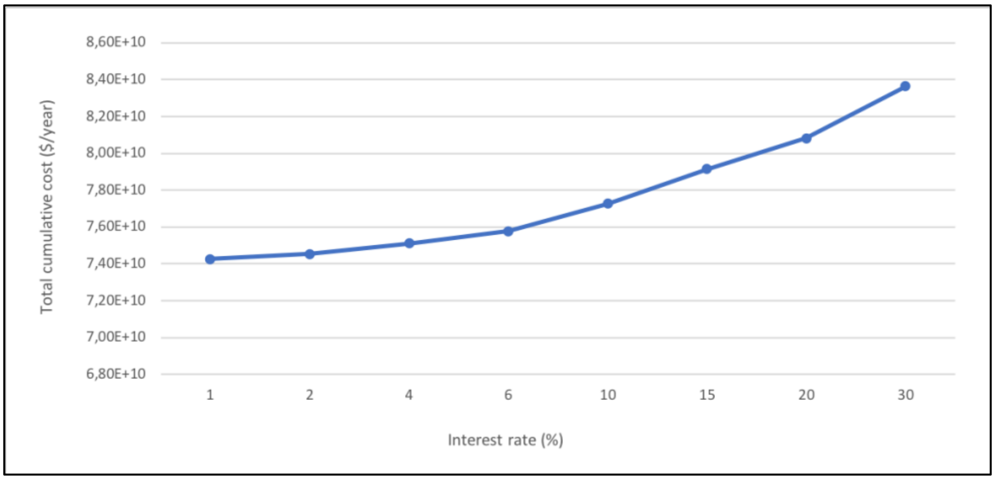


Figure 39. Effects of interest rate on the total cumulative costs. Source: Own elaboration

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[19] Nowadays the interest rate in the USA is around 2%, and before the so-known crisis in 2007 it was around 5 %.

VI.2.8. Hourly operating costs effects for the constrained case

VI.2.8.1. Hourly operating cost per vehicle effects

This section shows the effects of different values of the hourly operating cost per vehicle (ω_2). The values of ω_2 go from 100 until 600 dollars per vehicle hour. The baseline value is 320. Table 15 summarizes the results.

Hourly operating cost per vehicle ($/vehicle hour)	Total cumultive Cost ($/year)	Number of stations in service in each year																													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
100	7,28E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
150	7,34E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
200	7,40E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
250	7,45E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
320	**7,51E+10**	**0**	**0**	**0**	**0**	**0**	**1**	**1**	**2**	**2**	**2**	**3**	**3**	**3**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**
400	7,57E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
500	7,64E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
600	7,71E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4

Table 15. Effects of hourly operating costs per vehicle on total cumulative costs and optimized phases. Source: Own elaboration

As it happened before, all values are limited by the budget constraint. For instance, with a 100 value the four links should be operating by year 7, and for value 600 in the 12th year.

The slope of the total cumulative costs is quite constant in Figure 40. The slope of the total cumulative cost increases as the ω_2 does. Since the operating costs increase as well as the total cost, increasing the value of the hourly operating train cost should delay the construction of the new links.

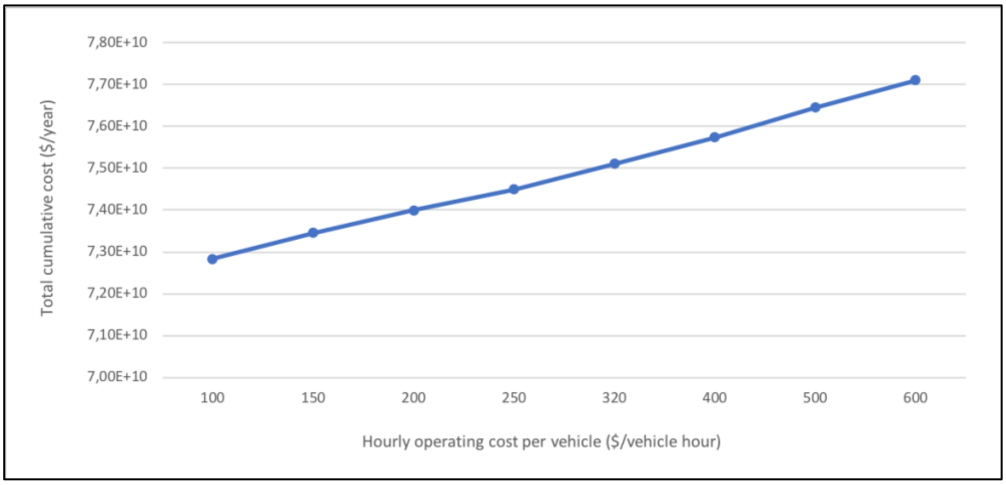

Figure 40. Effects of hourly operating cost per vehicle on total cumulative costs. Source: Own elaboration

VI.2.8.2. Hourly operating cost per bus effects

This section shows the effects of different values of the hourly operating cost per vehicle (ω_l).
The values of ω_l go from 10 until 300 dollars per vehicle hour. The baseline value is 80. Table 16 summarizes the results.

Hourly operating cost per bus ($/bus hour)	Total cumultive Cost ($/year)	Number of stations in service in each year																													
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
10	7,504E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
20	7,506E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
40	7,507E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
60	7,509E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
80	**7,511E+10**	**0**	**0**	**0**	**0**	**0**	**1**	**1**	**2**	**2**	**2**	**3**	**3**	**3**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**	**4**
120	7,514E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
200	7,520E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
300	7,529E+10	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4

Table 16. Effects of hourly operating cost per bus on the total cumulative cost and optimized phases. Source: Own elaboration.

As in the hourly operating cost per vehicle, here all the values are limited by the budget constraint. For example, on the one hand with a value of 10 ($/bus hour) the full extension of the International Airport Corridor should be done by year 7. On the other hand, when the value is 300 the extension should be completed in the 11th year if there was not budget constraint.
In Figure 41 we can see that the operating costs increase as well as the total cost, increasing the value of the hourly operating train cost. Therefore, a higher ω_l should anticipate the construction of the railroad. The slope is quite constant until the value of 120, where it begins to have a steeper slope.

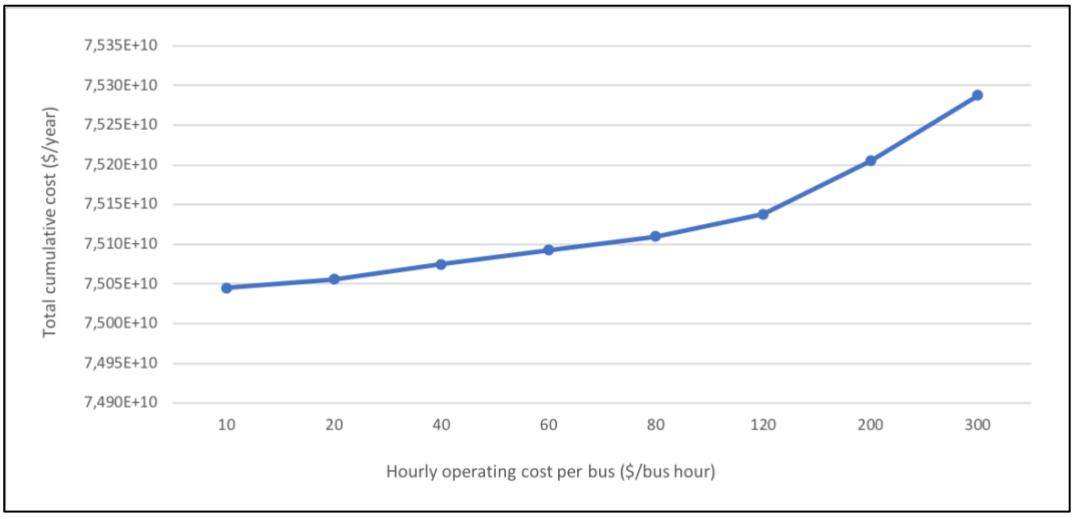

Figure 41. Effects of hourly operating cost per bus on the total cumulative costs. Source: Own elaboration

Chapter VII: CONCLUSIONS

VII.1. SUMMARY OF RESEARCH RESULTS

As cities grow the congestion in them also grows. That is why governments are trying both to mitigate traffic and reduce the carbon footprint in big areas. One solution is to replace current bus lines with rail lines, a problem known by transportation planners for its with combinatorial difficulties. This research assesses five options in order to determine if it is worth to replace an existing conventional bus system by a rail road. and optimize the construction phases. This optimization is done by minimizing the total costs under different constraints (political, physical and economical).

The main contributions of this study include:

- Providing useful guidelines to transportation planners and decision makers in deciding construction phases for projects that consists on the replacement of a conventional bus system by a rail road.
- An optimization model that minimizes the total cost of a project under physical, financial and political constraints for the replacement of a conventional bus system by a rail road.
- Sensitivity analysis of optimized results to different input parameters.
- Comparison and analysis of the effects of different constraints on the optimized phases based on presumed parameters for five different options.

VII.2. CONCLUSIONS

A model for optimizing the phased development of the replacement of a conventional bus system by a rail road is presented. Not only the construction phases have been optimized but also the economic feasibility of additional stations under different constraints. The model is used to minimize the total cost and to determine if it is worth replacing the current system. The different options for the replacement vary on the number of links that will replace the present bus trunk-line. The optimized solution avoids the partial replacement of the bus line, in fact the results tend toward a complete replacing of the bus line with a rail transit line. Based on the numerical results, the major findings are the following ones:

1) The numerical results for the unconstrained case show that the optimal solution is to replace the whole feeder trunk line by year nine. The optimal path is composed by the current system and the complete rail service of the International Airport Corridor. It is strange that there are no intermediate solutions (building firstly one link or two, and then

another one or two links). This is due to economies of scale, high demand, and the absence of constraints. Anticipating the construction of the links when demand grows is what occurs when the objective is minimizing costs without completion constraints. It stands out that at the beginning of the simulation, the conventional bus system is the best option, and at the end of the simulation it ends being the second-worst option.

2) The numerical results for the constrained case (political, physical and financial constraints) show that, contrary to the unconstrained case, the optimal path is the combination of the five different options. First of all, we start with the current system, then continues with one link, after two, afterwards three and finally the whole replacement of the corridor is carried out. Here the cost per passenger mile are higher than in the unconstrained case due to the fact that there are no economies of scale applied, because all links are built individually. Delaying the construction of the links although the demand grows is what happen when the objective is minimizing costs subject to restrictions.

3) In both, the constrained and unconstrained case the feeder trunk service is largely dominated by the rail-only service.

4) The sensitivity analysis conducted leads to obtain more accurate information about the effects of input parameters on the model. Different parameters were tested such as interest rate (ir), unit cost of user in-vehicle time (α), unit cost of user waiting time (β) and operating cost of both buses (ω_1) and trains (ω_2). When any of these parameters increase, the total cost increases. When ir, ω_2 and β increase the replacement of the bus trunk-line is delayed, and vice versa when α and ω_1 increase.

VII.3. RECOMMENDATIONS FOR FUTURE RESEARCH

Despite all the contributions and conclusions this model can be improved, given that it has several limitations. The following aspects are recommended for further studies:

1) To make the model more realistic, a future model may change some assumptions that simplify de model to a point that realism decreases. For instance, in this study links can only be added sequentially and from CBD to the AIX.

2) More variables that would affect the replacement of the bus conventional system could be could be considered such as inflation rate, benefits or costs. For instance, concerning

benefits the revenues coming from the fare collections, employment opportunities, travel time saving, or subsidies given by the government. Concerning costs, instead of including maintaining, acquisition, design, environmental impacts and rail track lying costs in the capital cost, try to estimate them correctly and introduce them into the model.

3) The model presented here is deterministic. Therefore, it could be improved by designing a probabilistic model. For example, in this study the demand growth rate stays constant all along the time horizon, and it is assumed that demand always increases (it could be the opposite over the years). Interest rates may vary as well over the years. So, a probabilistic model is more realistic than a deterministic model.

4) Only the replacement of one trunk bus line with a rail transit line is considered here. So, only a single route is optimized. This may be improved, by adding more lines creating a more complex network.

5) In this study locations and transit routes are already predetermined, considering that the space is continuous. That is why the access time is neglected. However, generally bus and train stops are located discretely. Especially if this model could be extended to optimize stop locations, the access time should be taken into account in the user cost.

6) Some operational values could change over time. In practice, transit agencies adjust variables such as fares or cruise speed over time, instead of fixing these values over a 30-year time horizon.

REFERENCES

[1] Ahern, A., Tapley, N, and Anandarajah, G., *Prioritising Railway Projects for Investments*, Proceedings of the Institute of Civil Engineers, Transportation, Vol. 159, pp. 153-157, 2006.

[2] Chang, S.K., and Schonfeld, P.M., 1991. *Multiple period optimization of bus transit systems.* Transportation Research Part B: Methodological 25(6), 453–478.

[3] Chen, L., Schonfeld, P., & Miller-Hooks, E. (2016). *Welfare maximization for bus transit systems with timed transfers and financial constraints.* Journal of Advanced Transportation, 50(4), 421-433.

[4] Cheng, W-C., Schonfeld, P., 2016. *A Method for Optimizing the phased development of rail transit lines.* Urban Rail Transit (2016) 1(4), 227-237.

[5] Chien, S., and Schonfeld, P., 1998. *Joint optimization of a rail transit line and its feeder bus system.* Journal of Advanced Transportation 32 (3), 253-284.

[6] DiJoseph, P., and Chien, S., 2013. *Optimizing sustainable feeder bus operation considering realistic networks and heterogeneous demand.* Journal of Advanced Transportation. 47(5), 483-497.

[7] Guan, J.F., Yang, H., and Wirasinghe, S.C., 2006. *Simultaneous optimization of transit line configuration and passenger line assignment.* Transportation Research Part B: Methodological 40 (10), 885-902.

[8] Hendrickson, C., *Project Management for Construction.* Prentice Hall, 1989.

[9] Kim, M., and Schonfeld, P., 2013. *Integrating bus services with mixed fleets.* Transportation Research Part B: Methodological 55(4), 227–244.

[10] Kocur, G., Hendrickson, C., 1982. *Design of local bus service with demand equilibrium.* Transportation Science 16(2), 149-170.

[11] Kolisch, R. and Padman, R., *An Integrated Survey of Deterministic Project Scheduling*, OMEGA, The International Journal of Management Science, Vol. 29, pp. 249-272, 2001.

[12] Li, Z.-C., Lam, W.H.K, Wong, S.C., and Sumalee, A., 2012. *Design of a rail transit line for profit maximization in a linear transportation corridor.* Transportation Research Part E: Logistics and Transportation Review 48(1), 50-70.

[13] Lownes, N. E., and Machemehl, R. B., 2010. *Exact and heuristic methods for public transit circulator design.* Transportation Research Part B: Methodological 44(2), 309-318.

[14] Martinelli, D. and Schonfeld, P., *Prioritizing and Scheduling Interdependent Lock Improvement Projects.* Compendium on waterway transportation reliability, Ins of water resources report 93-R-9, Fort Belvoir, April 1993.

[15] Matisziw, T. C., Murray, A. T. and Kim C., *Strategic Route Extension in Transit Networks*. European Journal of Operational Research, Vol. 171, pp. 661-673, 2006.

[16] Newell, G. F., 1979. *Some issues relating to the optimal design of bus routes.* Transportation Science 13(1), 20-35.

[17] Sun, Y., Guo, Q., Schonfeld, P., & Li, Z. (2017*). Evolution of public transit modes in a commuter corridor*. Transportation Research Part C: Emerging Technologies, v75, Feb. 2017, pp 84-102.

[18] Sun, Y., Guo, Q. & Schonfeld, P. (2018*). Optimal extension of rail transit lines.* International J. of Sustainable Transportation, 12-10, published online Feb. 2018: https://www.tandfonline.com/doi/full/10.1080/15568318.2018.1436730.

[19] Valadares Tavares, L., *Optimal Resource Profiles for Program Scheduling*, European Journal of Operational Research, Vol. 29, pp. 83-90, 1987.

[20] Wang, S. and Schonfeld, P., *Demand Elasticity and Benefit Measurement in a Waterway Simulation Model*, TRB 86th Annual Meeting Compendium of Papers, 2007

[21] Wirasinghe, S.C., and Ghoneim, N. S., 1987. *Spacing of bus-stops for many-to-many travel demand.* Transportation Science 15(3), 210-221

[22] U.S Department of transportation, Federal Highway Administration. https://www.fhwa.dot.gov/ipd/project_profiles/va_dulles_corridor.aspx

[23] Wikipedia. https://en.wikipedia.org/wiki/Chicago_%22L%22

[24] Washington Metropolitan Area Transit Authority, *Metro facts*, 2016. https://www.wmata.com/about/upload/Metro-Facts-2017-FINAL.pdf

[25] Washington Metropolitan Area Transit Authority, July 2, 2008. *7000 series Railcar RAC Overview.* https://www.wmata.com/about/riders-advisory-council/minutes/upload/7000-series%20overview,%20July%202,%202008.pdf

[26] The Federal Transit Administration, June 2017. *Historic context report for transit rail system development*.

[27] Jefferson, C., and Skinner J., 2005. *The evolution of urban public transport*. WIT Transactions on the built environment, Vol 77.

[28] Levy, D.N.L., March 1991. *Heuristic programming in artificial intelligence: the second computer Olympiad.*

[29] Spall, J., March 2001. *Simultaneous Perturbation Stochastic Approximation*. Johns Hopkins University. Applied physics laboratory, http://www.jhuapl.edu/SPSA/.

[30] Smith, M., January 2018. *Metro's $3.1 billion Budget: tasers, fare system upgrades and more trains.* https://wtop.com/tracking-metro-24-7/2018/01/metro-budget-tasers-fare-system-trains/

[31] Stanford University. March 2018: https://stanford.edu/class/ee364b/lectures/bb_slides.pdf

[32] Dulles Corridor Metrorail Project. Feb. 2018: www.dullesmetro.com

[33] *Madison BRT Transit Corridor Study. Proposed BRT Travel Time Estimation Approach*. March 2018: http://www.madisonareampo.org/documents/DBRTTravelTimeEstimationApproach.pdf

[33] Trading Economics. Feb. 2018: https://es.tradingeconomics.com/united-states/interest-rate

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INDEX OF THE QUOTATION

Chapter I: INTERNATIONAL AIRPORT CORRIDOR QUOTATION

In the following chapter it is shown the three different quotations for the optimal solution of the constrained case: budget for project execution, budget for provision of the contracted services and the total budget of the International Airport Project.

I.1. BUDGET FOR PROJECT EXECUTION

The Budget for project execution is the result of the addition of the products of each work unit multiply by his unitary price and costs overrun.

CHAPTER	SUMMARY		DOLLARS
EM	ELECTRO MECHANIC EQUIPMENT		450,525,228.07
-EM01	- DRAINAGE SYSTEM	98,990,375.81	
-EM02	- CHEMICAL PROPORTIONING	2,644,195.92	
-EM03	- SAND FILTERS	34,938,413.44	
-EM04	- HIGH PRESSURE PUMPS	269,182,610.13	
-EM05	- WASHING AND MOVEMENT SYSTEMS	964,457.62	
-EM06	- AUXILIARY EQUIPMENT	1,877,746.68	
-EM07	- REPLACEMENTS	31,845,950.40	
-EM08	- EQUIPMENT IMPROVEMENTS	10,081,478.06	
CO	CIVIL WORKS		1,041,571,571.91
-CO1	- CHANNELING	73,822,390.19	
-CO2	- RAIL CONSTRUCTION	967,749,181.72	
EYC	ELECTRICITY AND CONTROL		22,646,722.69
-EYC01	- PROCESSING CENTERS	4,214,042.63	
-EYC02	- ELECTRICAL ENCLOSURES	4,270,499,40	
-EYC03	- FREQUENCY CONVERTERS AND STARTERS OF MV AND LV	5,147,978,48	
-EYC04	- ELECTRICAL INSTALLATION	5,080,508.92	
-EYC05	- LIGHTING	76,566.30	
-EYC06	- AUTOMATION AND CONTROL	3,857,126.97	
TF	TRAINS		67,796,610.00
SYH	SECURITY AND HEALTH		6,103,372.07
WM	WASTLE MANAGEMENT		803,075.27
QC	QUALITY CONTROL		15,419,045.24
RIR	REGISTRATION IN THE INDUSTRIAL REGISTRY		1,284,920.44
		BUDGET FOR PROJECT EXECUTION	**1,606,150,545.70**

The budget for project execution comes to the quantity of ONE BILLION SIX HUNDRED SIX MILLION ONE HUNDRED FIFTY THOUSAND FIVE HUNDRED FORTY-FIVE DOLLARS and SEVENTY CENTS.

I.2. BUDGET FOR PROVISION OF THE CONTRACTED SERVICES

The Budget for provision of the contracted services is the addition of the Budget for Project execution, general expenses (between the 13% and 17%) and the industrial profit of the contractor (usually 6%).

CODE	SUMMARY		DOLLARS
EM	ELECTRO MECHANIC EQUIPMENT		450,525,228.07
CO	CIVIL WORKS		1,041,571,571.91
EYC	ELECTRICITY AND CONTROL		22,646,722.69
TR	TRAINS		67,796,610.00
SYH	SECURITY AND HEALTH		6,103,372.07
WM	WASTLE MANAGEMENT		803,075.27
QC	QUALITY CONTROL		15,419,045.24
RIR	REGISTRATION IN THE INDUSTRIAL REGISTRY		1,284,920.44
	BUDGET FOR PROJECT EXECUTION		**1,606,150,545.70**
	13.00% General expenses	208,799,570,94	
	6.00% Industrial profits	96,369,032.74	
	Addition of G.E and I.P.		305,168,603.68
	TOTAL BUDGET FOR PROVISION OF THE CONTRACTED SERVICES		**1,911,319,149.38**

The budget for budget provision of the contracted services comes to the quantity of ONE BILLION NINE HUNDRED ELEVEN MILLION THREE HUNDRED NINETEEN THOUSAND ONE HUNDRED FORTY-NINE DOLLARS and THIRTY-EIGHT CENTS.

I.3. TOTAL BUDGET

The total budget includes the tax, in particular for Spain the tax is 21%.

TOTAL BUDGET		DOLLARS
BUDGET FOR PROJECT EXECUTION		1,911,319,149.38
21.00% Taxes	401,377,021.37	
TOTAL BUDGET FOR PROVISION OF THE CONTRACTED SERVICES		2,312,696,170.75

The budget for budget provision of the contracted services comes to the quantity of TWO BILLION THREE HUNDRED TWELVE MILLION SIX HUNDRED NINETY-SIX THOUSAND ONE HUNDRED SEVENTY DOLLARS and SEVENTY-FIVE CENTS.

A P P E N D I C E S

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Appendix A: Vehicles used in the International Airport Corridor

A.1. Buses

The choice of bus type for the extension was based on three main reasons. The first one, was that the bus should have a high capacity and approved to run in the USA. At the Figure 1 we can see the different bus types.

Types of buses in fleet

Authorized

Received 21 new diesel-electric hybrid articulated buses
Ongoing procurement of additional 274 buses for FY16-17

In service

Total	Fuel Type	Size	Seating	Capacity
27	Diesel	30 feet	27	56
179	Diesel	40 feet	38-43	59-77
35	CNG*	30 feet	29	56
404	CNG*	40 feet	40-41	60-77
22	CNG* Articulated	60 feet	61	103
45	Hybrid Electric	30/37 feet	27-29	51-53
840	Hybrid Electric	40/42 feet	39-42	56-63
43	Hybrid Electric Articulated	60/62 feet	61-62	112-113

* Compressed natural gas

Figure 1. Types of buses approved to run in the USA (among others). Source: WMATA.

The second criteria was based on the demand. The demand[1] assumed is pretty high according to the forecast, so it was needed the biggest bus to satisfy the demand.

The third criteria was based on the environment. Nowadays, the society is very concern about our environmental footprint. That is why the *greener* the bus is, the better.

Considering all these parameters, according to the Figure 1 the best possible bus that satisfied both the demand and environmentally friendly conditions was the Hybrid Electrical Articulated. The **load factor** of this type of bus is **1.62**, taking into account that we apply a reduction factor

[1] According to the forecast, a demand of 80,000 passengers/hour needs to be satisfied. See Appendix B: Data assumptions

(also known as safety factor) of 10%. Theoretically, without the safety factor, the load factor would be 1,82 and the total capacity 113 passengers instead of 1.62 and 102 passengers respectively.

Figure 2. Hybrid electric articulated bus. Source: Mario Roberto Durán Ortiz

A.2. RAILCARS

The railcar chosen for the International Airport Corridor is the 7000-series railcar. It is the upcoming railcar used in the Washington metro system and in the Chicago L.
Table 1 it can be observed the characteristics of the 7000-series cars.

	7000-series
In service	2015-present
Manufacturer	Kawasaki Heavy Industries
Built at	Yonkers, New York Lincoln, Nebraska Kobe, Japan.
Replaced	1000-series and 4000-series
Constructed	2012-present
Entered in service	April 14, 2015.
Formation	2 cars (A-B) per trainset

Capacity	A-car: 64 (transverse), 58 (longitudinal). B-car: 68 (transverse), 64 (longitudinal).
Train length	600 feet (182.88 m) (8-car train).
Car length	75 feet (22.86 m).
Width	10 feet 1 ¾ inches (3.09 m)
Height	10 feet 10 inches (3.30 m)
Maximum speed	**75 mph (121 km/h)**
Weight	80,000 lbs (36,000 kg)
Traction system	Toshiba SEA-430 IGBT-VVVF
Power output	140 KW (190 HP) per motor
Electric system	750 V DC third rail

Table 1. Characteristics of the 7000 series railcars. Source: WTOP

In the Figure 5 we can see the difference between the exterior and interior of 7000-serie.

Figure 3. Seats distribution of both car types A (up) and B (down), of the 7000 series railcar. Source: WMATA

As it can be seen in the Figure 3 the floor plan of the A-car has 64 seats, whereas the B-car plan has 68 seats.

According to the Figure 4 each train (talking about an 8-cars train) has a capacity of 1528 passengers with the unit configuration ABBAABBA. That is to mean that the **load factor** of the whole train is **2.60**. However, a reserve factor of 10% has been applied as a security factor.That is to say that the total capacity of an 8-car train with ABBAABBA configuration is 1375 passengers.

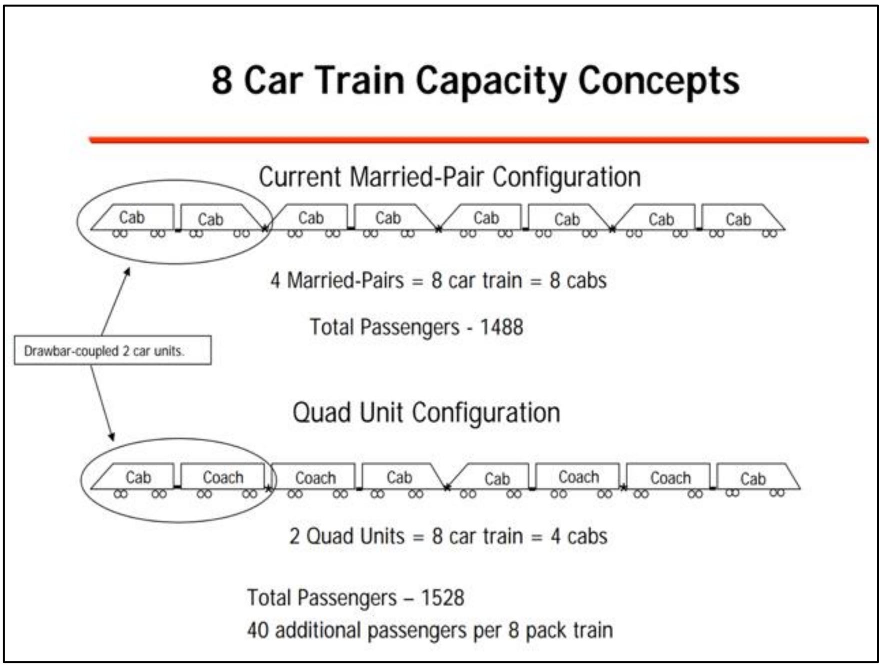


Figure 4. 8-cars distribution (ABBAABBA) of the 7000 series railcar. Source: WMATA

Figure 5. Exterior and interior of the and 7000-series railcar. Source: Railway Age

Appendix B: DATA ASSUMPTIONS

B.1. 7000-SERIES RAILCAR PRICE

The price assumed for the 7000-series railcars is based on a recent purchase of 1,003 eight-cars of the 7000-series model, scheduled to be implemented in the Washington metro system, beginning on July 1, 2018.

The total cost of the 7000-series railcar order was $1.7 billion. This makes a total cost per train of $**1,694.915,25** dollars. The characteristics of this train can be consulted in the Appendix A: Vehicles used in the International Airport Corridor.

B.2. DISTANCES BETWEEN STATIONS, AND NUMBER OF STOPS

The distance between stations of the International Airport Corridor have been based on the new extension of the Washington metro system until the airport. Thanks to the responsible of the communication, media and information: Marcia McAllister, the distances between stations are the ones in the Table 2 below.

Station			Distance in miles between stations (miles)
Name in the WMATA	Name in the study	Shortening	
Wiehle-Reston East	Archieves	A	-
Reston Town Center	Bellecour	B	0.8
Herndon	Châtelet	C	1.3
Innovation Center	Darongers	D	1.7
Dulles Airport	International Airport	E	2.1
Loudoun Gateway	-	-	2.9
Ashburn	-	-	2.0
	Average		1.8

Table 2. Distances between stations. Source: Marcia McAllister

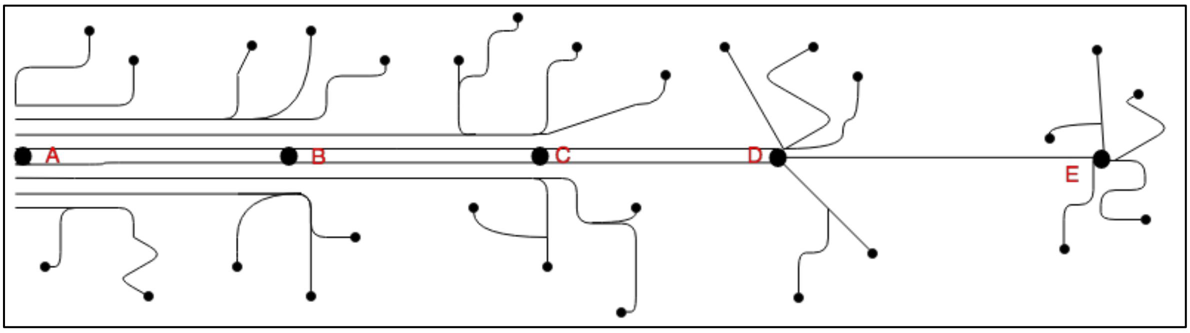


Figure 6. Current conventional bus system from Archieves to AIX. Source: Own elaboration

B.3. MARGINAL COST PER MILE

The marginal cost per mile is based on the cost per mile of the DCMP. It has been calculated dividing the total cost by the total extension miles of the DCMP. The total cost of the second phase of the extension project is $ 4.1 billion dollars, and as it can be seen in the Table 2, the total miles of the second phase of the project are 10.8 miles. So, all in all, the marginal cost per mile of **$ 379.629.629,63** dollars, the cost used in this study.

B.4. OPERATION HOURS OF THE WMATA

The operating hours per year of the metro system has been calculated assuming the following facts:

- A year has 52 weeks
- There is no difference between days (festivity or not festivity day)
- Vacation period is not considered

So, the assumption taken was that the metro service operates 125h per week. That is to mean that the total operating hours in a year of the metro system are 6,500h.

Monday-Thursday: 18.5h (5 am to 11:30 pm)

Friday: 20 h (from 5 am to 1 am)

Saturday: 18h (from 7 am to 1 am)

Sunday: 13h (from 8 am to 11 am)

B.5. DWELL TIME

The dwell tike is the time spent by a vehicle at a scheduled stop without moving. In order to calculate the dwell time a field work has been done. A sampling composed by 70 samples of different dwell times and metro lines, and a sampling of 10 samples of a bus line have been timed. All the metro dwell times correspond to the 7000-series railcar.

B.5.1. Metro

B.5.1.1. Green Line

The following tables contain the dwell time values of different stops in the green line. Trips one and two have been measured on Saturday between 10:30 pm and 11:00 pm.

Stop	Time (seconds)
College Park (CP)	41.78
Prince George Plaza	32.30
West Hyattsville	34.60
Fort Totten	55.90
Georgia Ave-Petworth	28.34
Columbia Heights	27.70
Union Street (US)	30.35
Standard deviation	9.29
Mean	35.85

Table 3. Trip one: from CP to US. Source: Own elaboration

Stop	Time (seconds)
Union Street	33.79
Columbia Heights	50.95
Georgia Ave-Petworth	35.06
Fort Totten	50.62
West Hyattsville	22.91
Prince George Plaza	25.30
College Park	48.78
Standard deviation	11.07
Mean	38.20

Table 4. Trip two: from US to CP. Source: Own elaboration

Trips three and four have been timed on Monday between 12:00 pm and 01:30 pm.

Stop	Time (seconds)
College Park	34.80
Prince George Plaza	26.38
West Hyattsville	24.24
Fort Totten	37.61

Georgia Ave-Petworth	32.81
Columbia Heights	56.62
Union Street	43.90
Shaw-Howard Uniom	47.38
Mt. Vernon Square	44.08
Gallery Palace-Chinatown	44.04
Achives	33.70
L'enfant Plaza (EP)	36.30
Standard deviation	8.77
Mean	38.49

Table 5. Trip three: from CP to l'EF. Source: Own elaboration

Stop	Time (seconds)
L'enfant Plaza	50.45
Achives	30.25
26Gallery Palace-Chinatown	25.60
Mt. Vernon Square	34.44
Shaw-Howard Uniom	35,67
Union Street	37.78
Columbia Heights	50.12
Georgia Ave-Petworth	53.57
Fort Totten	40.37
West Hyattsville	36.4
Prince George Plaza	41.55
College Park	32.98
Standard deviation	8.20
Mean	39.10

Table 6. Trip four: from l'EF to CP. Source: Own elaboration

Trips five and six have been measured on Saturday between 06:00 pm and 07:00 pm.

Stop	Time (seconds)
Gallery Palace-Chinatown (Capital one arena)	37.55
Mt. Vernon Square	31.62
Shaw-Howard Uniom	37.10

Union Street	46.15
Columbia Heights	27.56
Georgia Ave-Petworth	38.36
Fort Totten	47.13
West Hyattsville	25.76
Prince George Plaza	27.56
College Park	34.45
Standard deviation	7.08
Mean	35.32

Table 7. Trip five: from Capital One Arena to CP. Source: Own elaboration

Stop	Time (seconds)
College Park	32.86
Prince George Plaza	33.88
West Hyattsville	40.15
Fort Totten	35.37
Georgia Ave-Petworth	30.43
Columbia Heights	41.20
Union Street	45.16
Shaw-Howard Uniom	28.34
Mt. Vernon Square	31.23
Gallery Palace-Chinatown (Capital one arena)	33.52
Standard deviation	8.77
Mean	38.49

Table 8.Trip six: from CP to Capital One Arena. Source: Own elaboration

B.5.1.2. Silver Line

The following tables contain the dwell time values of different stops in the silver line. They were measured between 01:30 pm and 02:00pm on Monday.

Stop	Time (seconds)
Smithsonian	25.69
Federal Triangle	29.32
Metro center	35.93
McPherson Square	19.60

Farragut Square	55.66
Foggy-Bottom GWU	25.51
Standard deviation	11.68
Mean	31.95

Table 9. Trip seven: from Smithsonian to GWU. Source: Own elaboration

Stop	**Time (seconds)**
Foggy-Bottom GWU	27.45
Farragut Square	31.23
McPherson Square	32.67
Metro center	22.3
Federal Triangle	35.67
Smithsonian	28.11
Standard deviation	4.26
Mean	29.57

Table 10. Trip seven: from GWU to Smithsonian. Source: Own elaboration

Considering the seventy different samples, these are the final results:

Standard deviation	36.05
Mean	8.83
SEM	1.05

Table 11. Data summary. Source: Own elaboration

One thing that has been noticed during the data collection is that the WMATA metro system has a surprisingly high dwell time. This is because it usually takes between 6 and 10 seconds to open/close the doors once the metro has already stopped. There were stops where this time was even higher, going up to 20 seconds. Moreover, there were times when the doors could not be closed because people were trying to enter when the door were closing, increasing the normal dwell time.

B.5.2. Bus

It has not been possible to carry on a work field with a bus going from the city to the airport. That is why all the values have been increased in 20% because it is supposed that almost everyone taking this bus, spend more time than in a usual bus to get out due to the fact that in each stop they need to move one or several suitcases which increases the dwell time. For the bus dwell time it has been only used 10 sample, even if they are not many samples it is enough to have an idea of the bus dwell time.

B.5.2.1. Line D6

Stop	Time (seconds)	Time with 20% luggage factor (seconds)
Q St Nw & Macarthur Blvd Nw	15.30	22.95
Q St Nw & Foxhall Rd Nw	17.70	26.55
Foxhall Rd Nw & Greenwich Pky Nw	16.61	24.92
Reservoir Rd Nw & 44th St Nw	20,35	30.52
Reservoir Rd Nw & French Embassy	15.70	23.55
Standard deviation		2.71
Mean		25.70

Table 12. Bus trip number 1. Source: Own elaboration

Stop	Time (seconds)	Time with 20% luggage factor (seconds)
Reservoir Rd Nw & French Embassy	13.50	20.25
Reservoir Rd Nw & 44th St Nw	18.87	28.31
Foxhall Rd Nw & Greenwich Pky Nw	17.40	26.10
Q St Nw & Foxhall Rd Nw	17.20	25.80
Q St Nw & Macarthur Blvd Nw	18.67	28.01
Standard deviation		2.90
Mean		25.68

Table 13. Bus trip number 2. Source: Own elaboration

Considering the ten samples, the final results are the following ones:

Standard deviation	2.80
Mean	25.70

B.6. MINIMUM HEADWAY

In order to calculate de minimum headway between vehicles, to avoid crashes between trains, it is necessary to know the max dwell time, and the time needed to accelerate over a distance. So, the formula used to calculate this minimum headway is the following one:

$$Min\ headway = Max\ dwell\ time + 2 \cdot time\ needed\ to\ accelerate\ over\ a\ distance \qquad (1)$$

Knowing that the time needed to accelerate over a distance is composed by the train length (L) and the acceleration/deceleration rate (a):

$$Min\ headway = \max dwell\ time + 2 \cdot (\sqrt{\frac{2 \cdot (\frac{L}{2})}{a}}) \quad (2)$$

B.6.1. Minimum feasible headway trains

In this case, as shown before, the train length is 75 feet per car and there are 8 cars per train, so the total length is 600 feet. The acceleration/deceleration rate is 4.8 ft/s^2. The maximum headway measured was 55.9 seconds, with a 20% of security factor the result is 67.08 seconds.

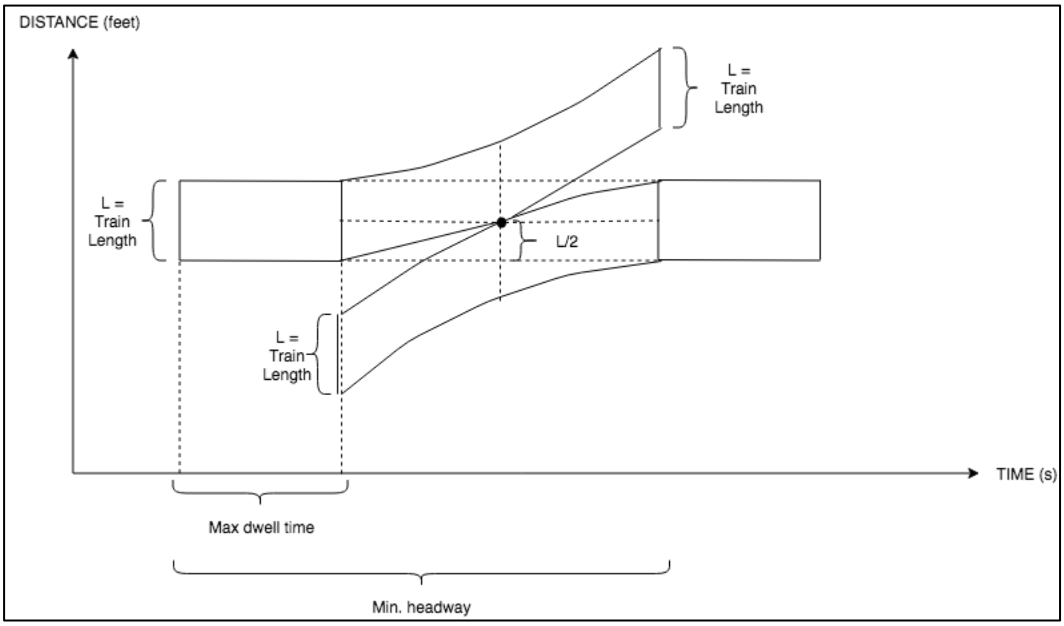


Figure 7. Calculation of the minimum headway for trains. Source: Own elaboration

Replacing in (2):

$$Min\ headway\ for\ trains = 67.08 + 2 \cdot (\sqrt{\frac{2 \cdot (\frac{8 \cdot 75}{2})}{4.8}}) = \mathbf{90}\ seconds$$

With this data, the minimum headway is 90 seconds. That is to mean a maximum of 40 trains per hour.

B.6.2. Minimum feasible headway bus

In this case, as shown before, the bus length is 62 feet per vehicle. Research literature suggests that the acceleration/deceleration rate for buses is 2.0 ft/s^2. The maximum headway measured was 28.01 seconds. So, all in all, substituting in (2):

$$Min\ headway\ for\ buses = 28.01 + 2 \cdot (\sqrt{\frac{2 \cdot (\frac{62}{2})}{2.0}}) = \mathbf{40}\ seconds$$

With this data, the minimum headway is 40 seconds. That is to mean a maximum of 90 buses per hour.

B.7. MAXIMUM HEADWAY

In order to calculate de maximum headway between vehicles, to satisfy the demand, it is necessary to know the max dwell time, and the time needed to accelerate over a distance. So, the formula used to calculate this minimum headway is the following one:

$$Max\ headway = \frac{S \cdot l_f}{D} \quad (3)$$

Knowing that S is the number of seats of the vehicle, l_f is the load factor of the vehicle and D is the demand.

B.7.1. Maximum headway for trains

In this case of study, the 7000 series railcar has 64 seats in the type A car, and 68 seats in the type B car. There are four cars of each type in one train, so the total amount of seats is 528. The load factor for the 7000 series railcar is calculated in the Appendix A, A.2 Railcars section, and it is 2.60. The demand depends on the year of the simulation, so the maximum head depends on the year it is studied.

B.7.2. Maximum headway for buses

In this case of study, the hybrid electric articulated bus has 62 seats. The load factor for the hybrid electric articulated bus is calculated in the Appendix A, A.1 Buses section, and it is 1.65. The demand depends on the year of the simulation, so the maximum head depends on the year it is studied.

In the Figure 8 we can see both the evolution of the maximum and minimum headways in each year. Moreover, we can see the evolution of the optimal headways for both the constrained and unconstrained solutions.

B.8. DEMAND

The demand was based on the actual ridership of the WMATA metro system. Nowadays, according to WMATA data there was a 179,693,126-annual ridership in 2016. With 52 weeks per year and the 125 operating hours per week considered in this study, it leads to a 27,645 passengers per hour. It has been considered that our metro system could have between nine and ten lines, so diving this ridership per nine, the result is 3,071 passengers per hour per line, rounding down in this study to **3,000** passengers per hour.

	0			1			2			3			4			5			6			7			8			9			10		
	h (min)	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train
A: All bus	0,95			1,803			1,63			1,45			1,30			1,16			1,03			0,92			0,82			0,74			0,66		
B: All RR	9,47			8,95			8,45			7,99			7,55			7,13			6,74			6,37			6,02			5,69			5,37		
C: 3 links RR	5,93	2,04	27,50	5,60	1,821	24,55	5,29	1,63	21,92	5,00	1,45	19,57	4,73	1,30	17,48	4,46	1,16	15,60	4,22	1,03	13,93	3,99	0,92	12,44	3,77	0,82	11,11	3,56	0,74	9,92	3,36	0,66	8,85
D: 2 links RR	5,17			4,89			4,62			4,37			4,12			3,90			3,68			3,48			3,29			3,11			2,94		
E: 1 link RR	4,51			4,27			4,03			3,81			3,60			3,40			3,21			3,04			2,87			2,71			2,56		

	11			12			13			14			15			16			17			18			19			20		
	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train
A: All bus	0,67			0,67			0,67			0,67			0,67			0,67			0,67			0,67			0,67			0,67		
B: All RR	5,08			4,80			4,53			4,28			4,05			3,82			3,61			3,41			3,23			3,05		
C: 3 links RR	3,18	0,59	7,91	3,00	0,52	7,06	2,84	0,47	6,30	2,68	0,42	5,63	2,53	0,37	5,02	2,39	0,33	4,49	2,26	0,30	4,01	2,14	0,27	3,58	2,02	0,24	3,19	1,91	0,21	2,85
D: 2 links RR	2,77			2,62			2,48			2,34			2,21			2,09			1,97			1,87			1,76			1,67		
E: 1 link RR	2,42			2,29			2,16			2,04			1,93			1,82			1,72			1,63			1,54			1,50		

	21			22			23			24			25			26			27			28			29			30		
	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train	h	max h bus	max h train
A: All bus	0,67			0,67			0,67			0,67			0,67			0,67			0,67			0,67			0,67			0,67		
B: All RR	2,88			2,72			2,57			2,43			2,30			2,17			2,05			1,94			1,83			1,73		
C: 3 links RR	1,80	0,19	2,55	1,70	0,17	2,27	1,61	0,15	2,03	1,52	0,13	1,81	1,50	0,12	1,62	1,50	0,11	1,44	1,50	0,10	1,29	1,50	0,09	1,15	1,50	0,08	1,03	1,50	0,07	0,92
D: 2 links RR	1,57			1,50			1,50			1,50			1,50			1,50			1,50			1,50			1,50			1,50		
E: 1 link RR	1,50			1,50			1,50			1,50			1,50			1,50			1,50			1,50			1,50			1,50		

Figure 8. Headways for the whole-time horizon for both the constrained and the unconstrained case. Source: Own elaboration

B.9. OTHER VALUES

Other values assumed have been based both on other studies, and the experience and knowledge of experts on the field.

Appendix C: Results

C.1. Unconstrained case

C.1.1. Supplier and users cost of all options for the unconstrained case

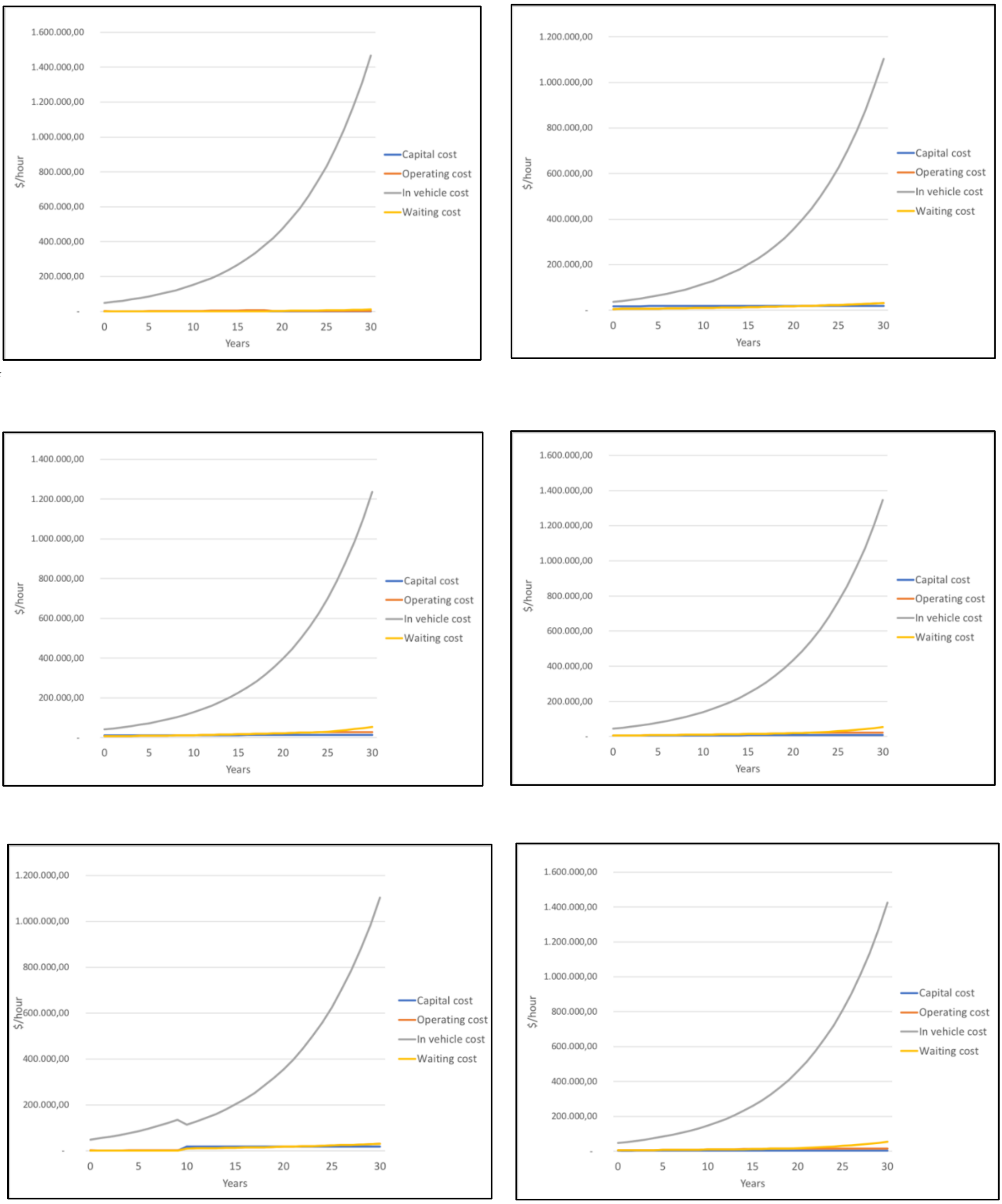


Figure 9. Supplier and User cost of all the options A, B, C, D, E and optimal option (from right to left, and up to down). Source: Own elaboration

Supplier and user costs of all options are plotted in Figure 9, and the fraction of both costs as well. In-vehicle cost is the cost that increases the most, since it is related with the ridership. That is to mean that in-vehicle cost increases as the ridership does. User waiting cost and operating cost are related with the headway. As it can be seen in all figures, user waiting cost and operating cost almost overlap between because the Y axis units are very large, excepting in the Figure 11 where both costs are equal. The jump from year nine to year ten in Figure 13 is caused by the change of options in the optimal path (from option A to option B).

C.1.2. Supplier and User cost Breakdown for the unconstrained case

The following six figures show the supplier and user cost breakdown of all options, including the optimal solution.

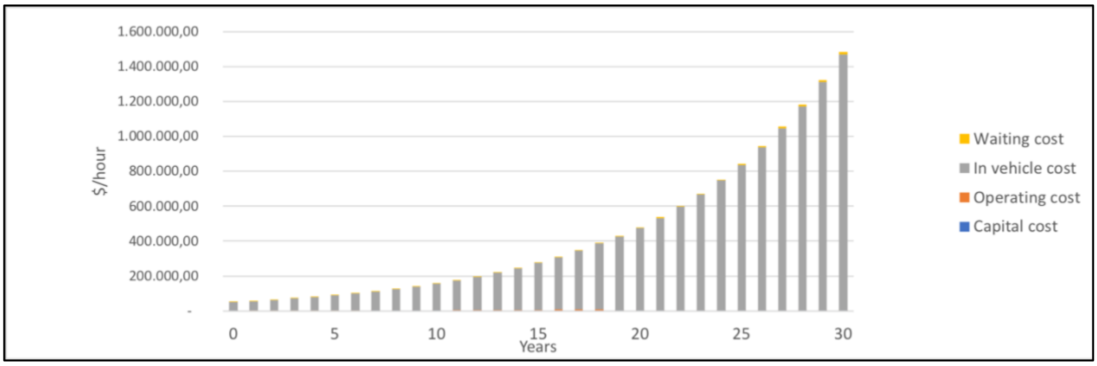


Figure 10. Supplier and User cost breakdown of option A. Source: Own elaboration

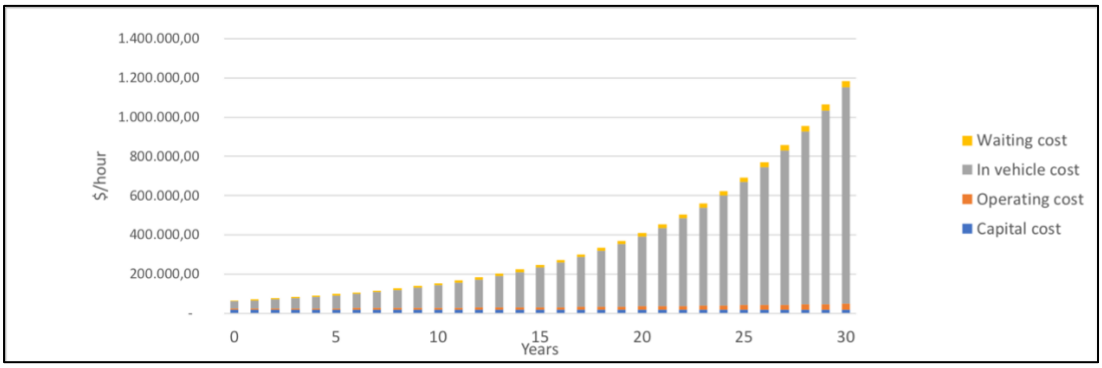


Figure 11. Supplier and User cost breakdown of option B. Source: Own elaboration

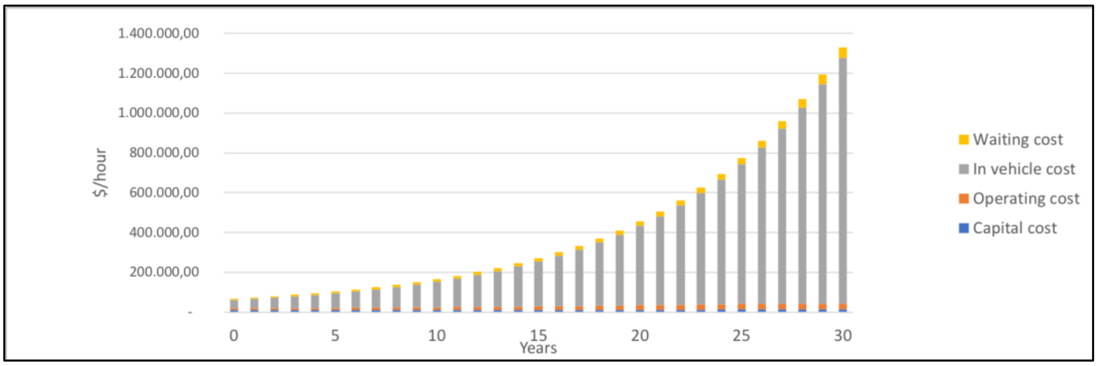


Figure 12. Supplier and User cost breakdown of option C. Source: Own elaboration

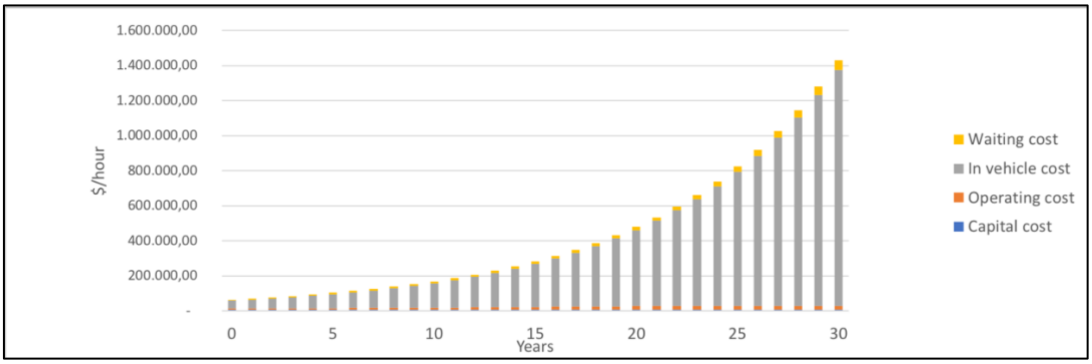


Figure 13. Supplier and User cost breakdown of option D. Source: Own elaboration

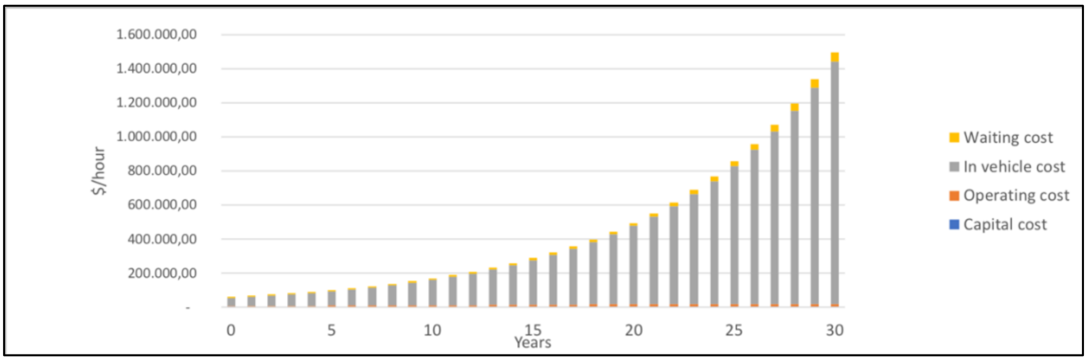


Figure 14. Supplier and User cost breakdown of option E. Source: Own elaboration

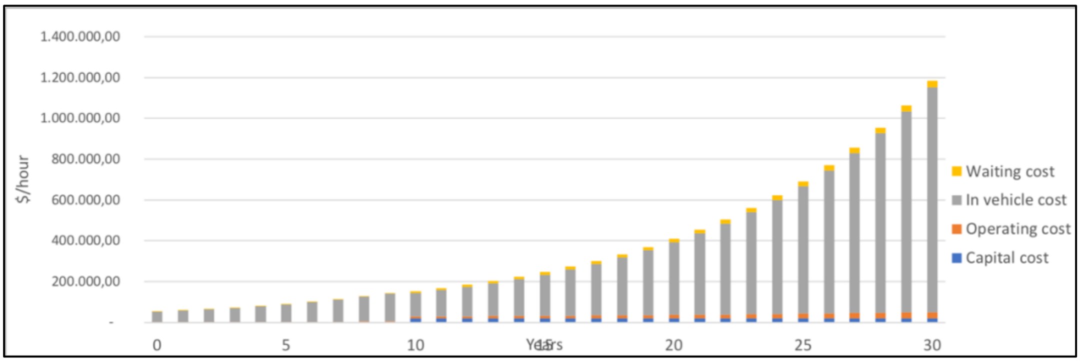


Figure 15. Supplier and User cost breakdown of the optimal path. Source: Own elaboration

C.1.3. Sensitivity to demand for the unconstrained case

Figure 16. Evolution of every option with a 90% reduction (up), 70% reduction (middle) and 40 % reduction (down). Source: Own Elaboration

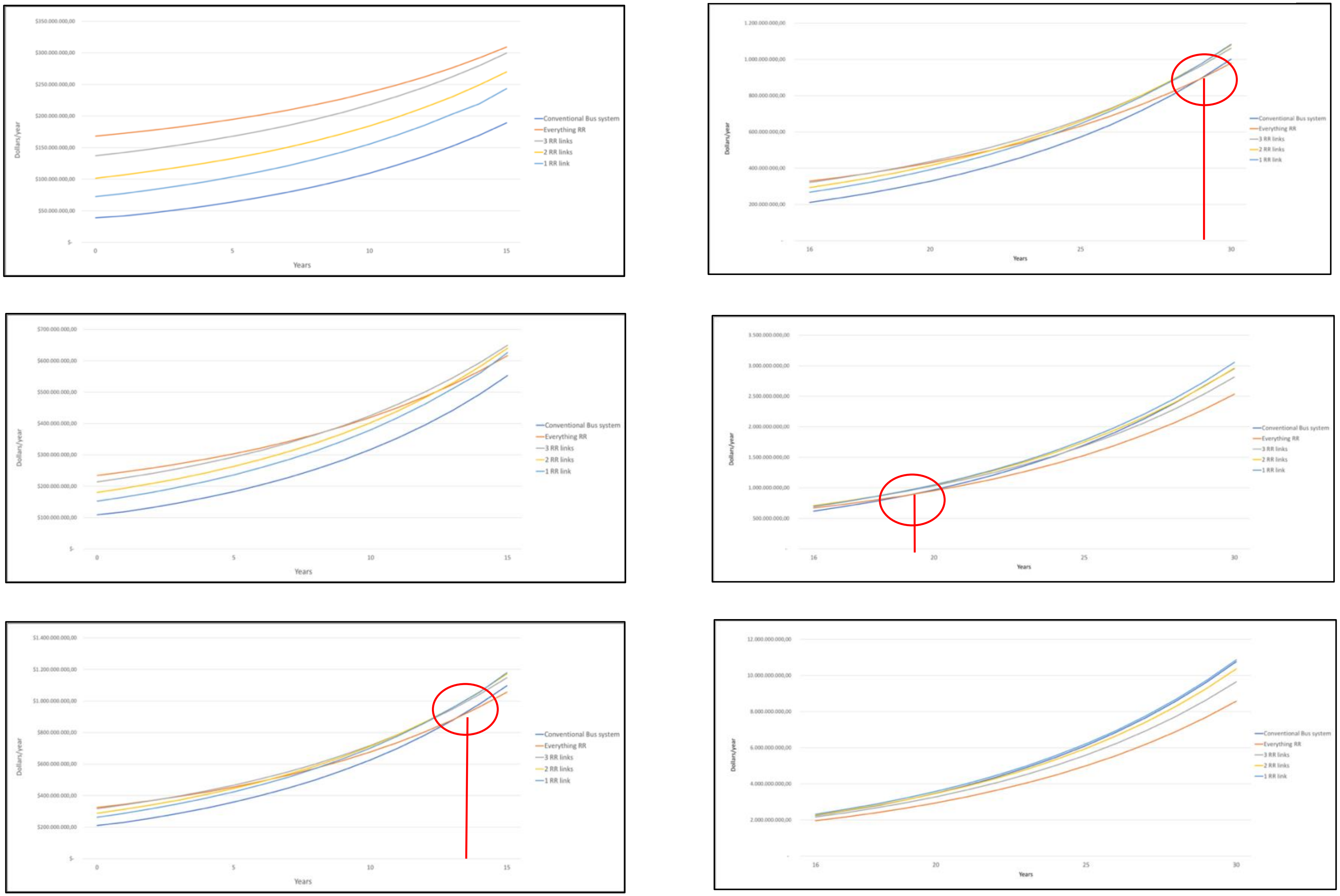
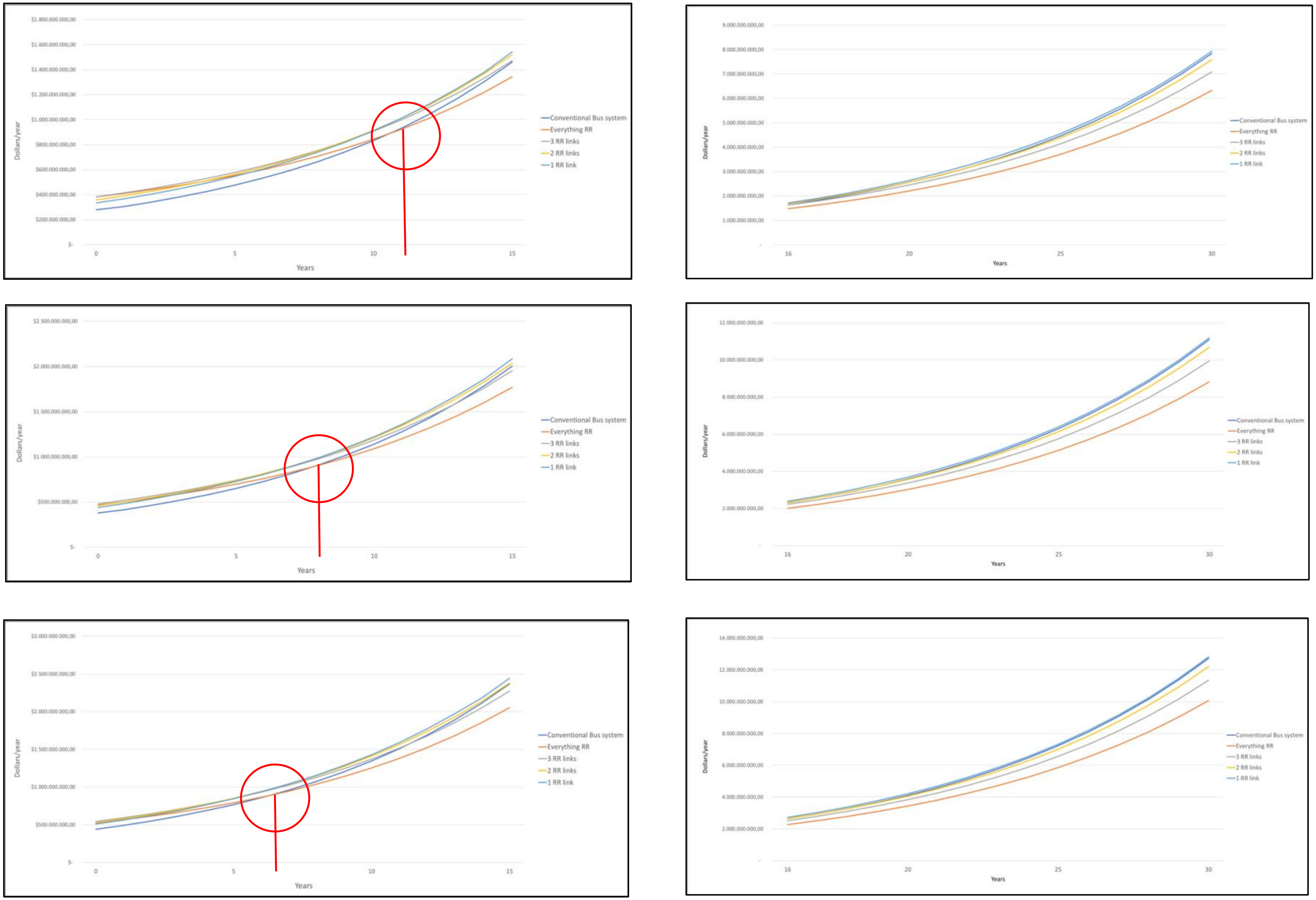


Figure 17. Evolution of every option with a 20% reduction (up), an increment of 10% (middle) and increment of 30 % (down). Source: Own Elaboration



Appendix C: Results

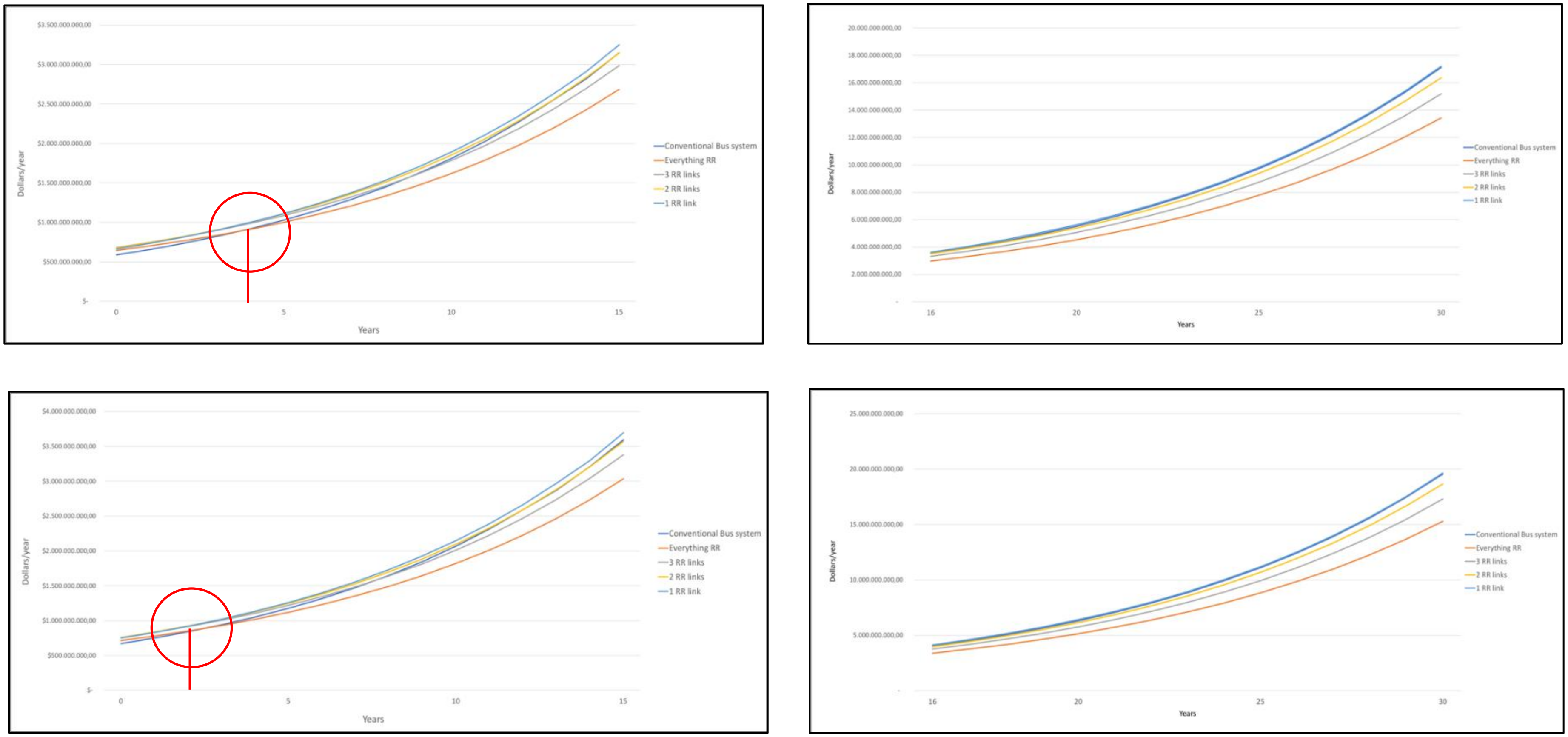


Figure 18. Evolution of every option with an increment of 75% (up) and an increment of 100% (middle. Source: Own Elaboration

C.1.4. Sensitivity to Analysis period for the unconstrained case

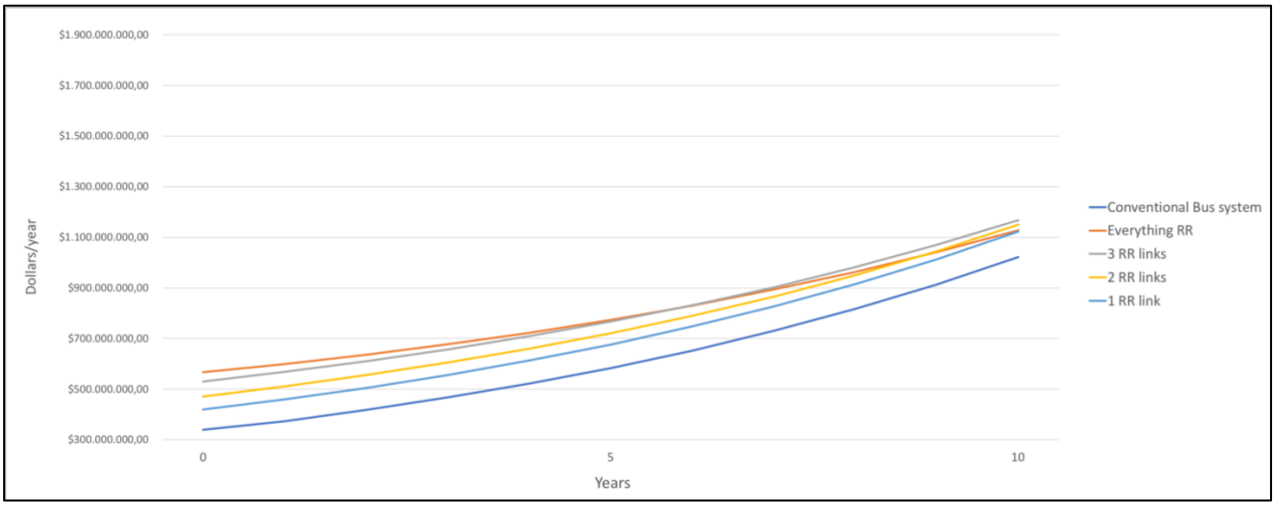


Figure 19. Total cost of each option in a 10 years' time horizon. Source: Own elaboration

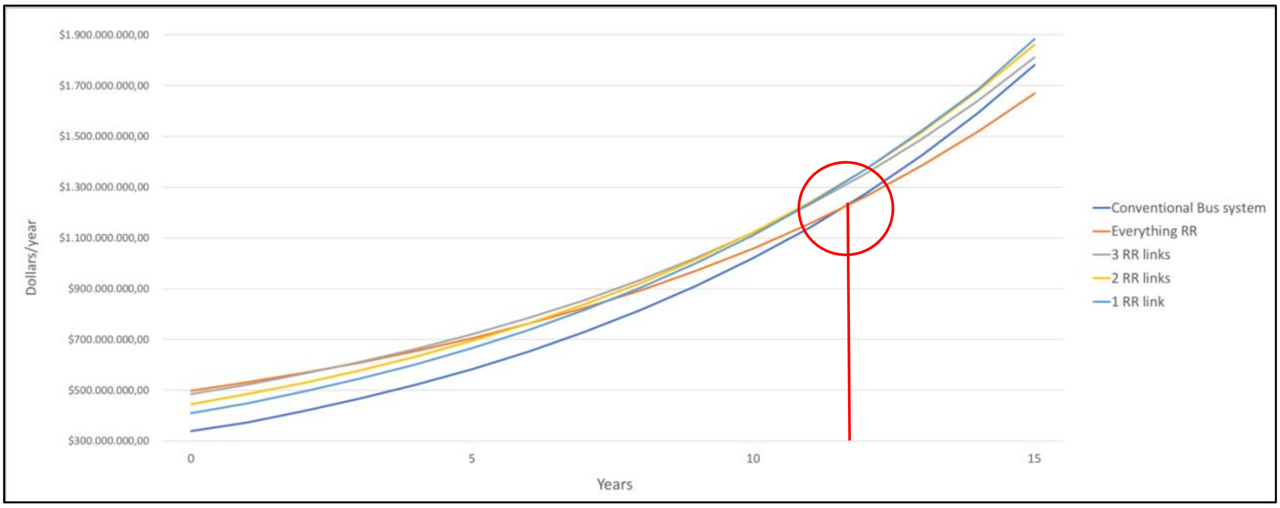
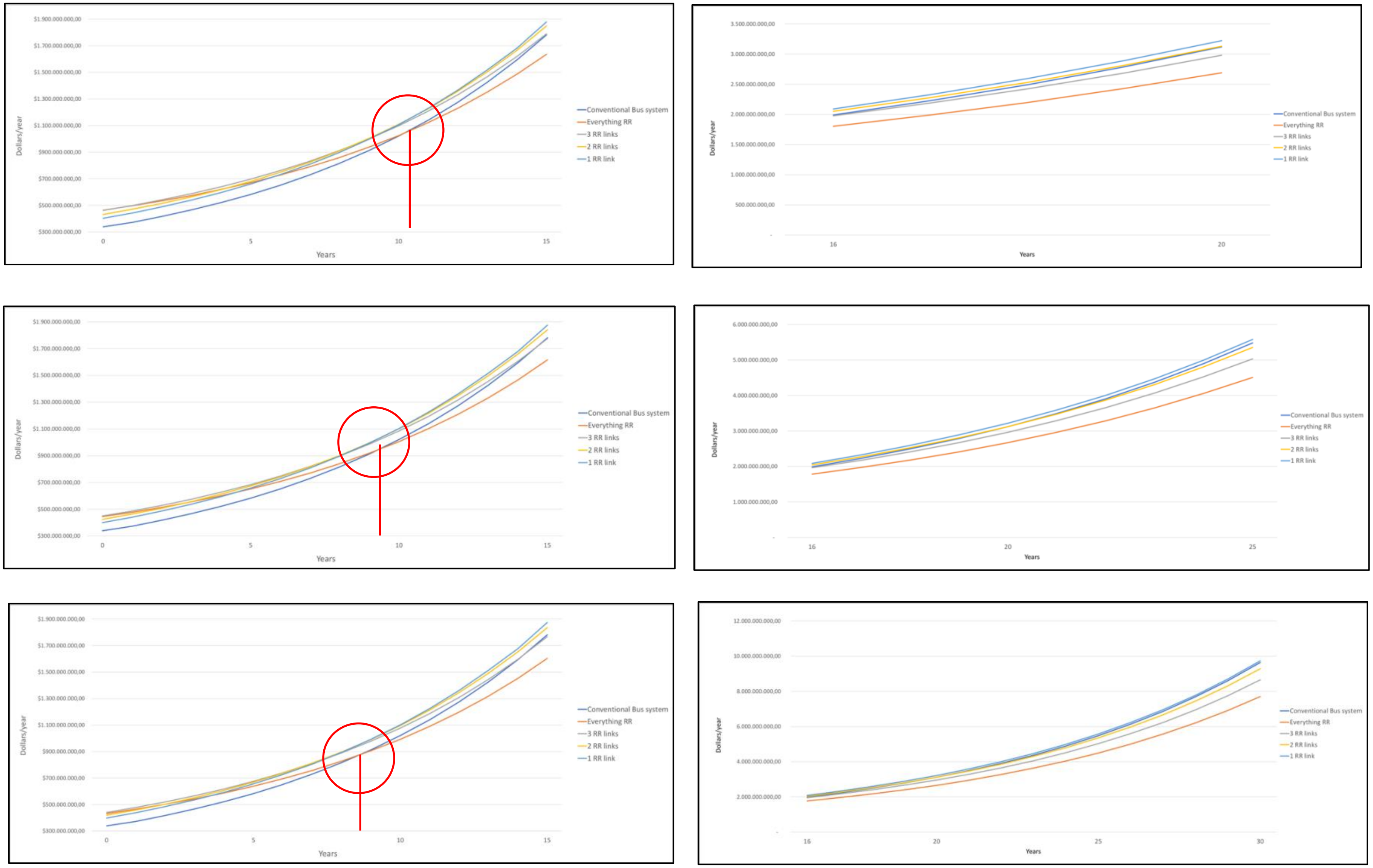


Figure 20. Total cost of each option in a 10 years' time horizon. Source: Own elaboration

Figure 21. Evolution of a 20 years analysis period (up), evolution of a 25 years analysis period (middle), evolution of a 30 years analysis period (down). Source: Own elaboration



C.2. CONSTRAINED CASE

C.2.1. Supplier and users cost of all options for the constrained case

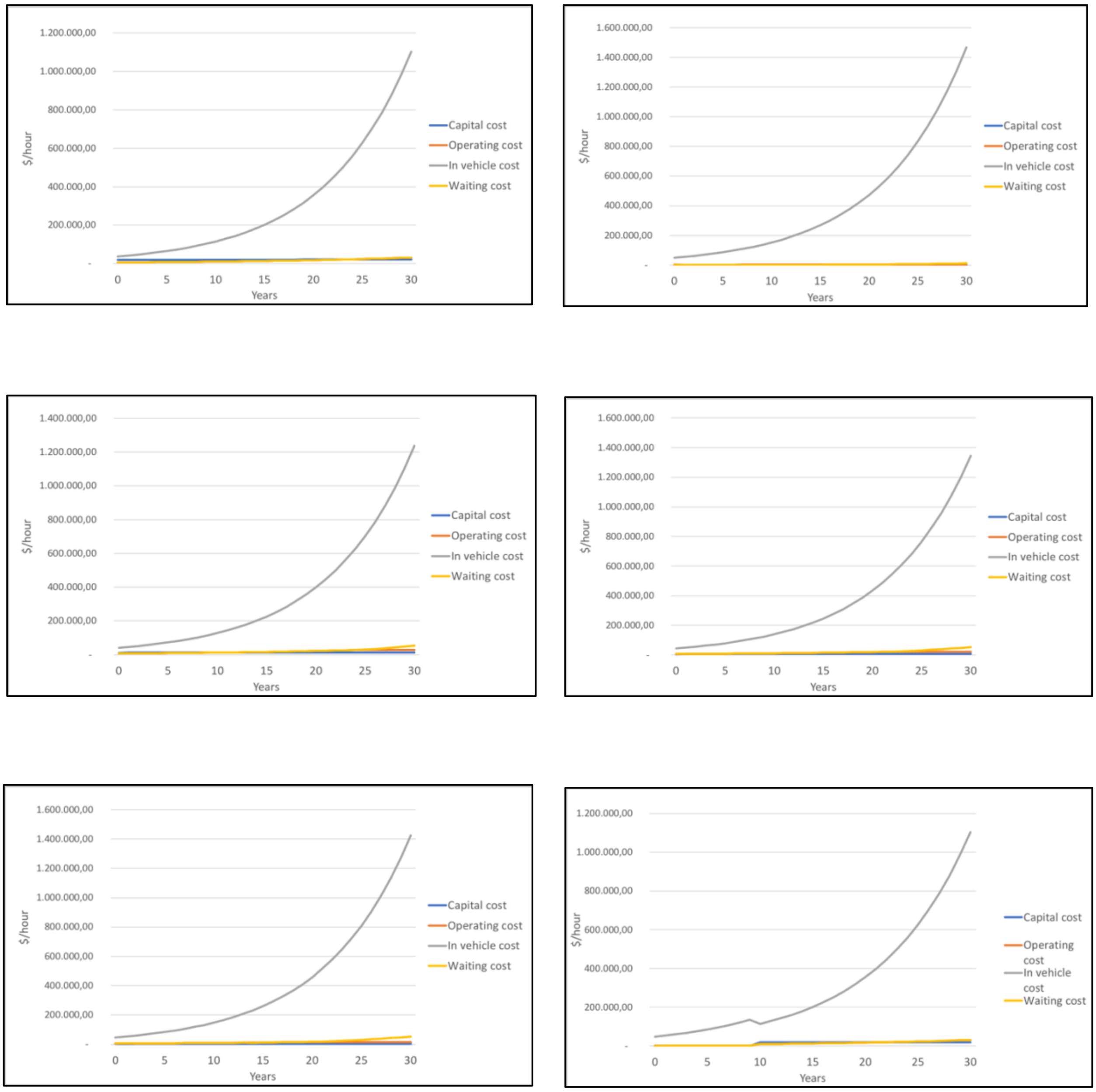


Figure 22. Supplier and User cost of all the options A, B, C, D, E and optimal option (from right to left, and up to down). Source: Own elaboration

Supplier and user costs of all options are plotted from Figure 22, and the fraction of both costs as well. In-vehicle cost is the cost that increases the most, since it is related with the ridership. That is to mean that in-vehicle cost increases as the ridership does. User waiting cost and operating cost are related with the headway. As it can be seen in all figures, user waiting cost and operating cost almost overlap between because the Y axis units are very large, excepting in the Figure 22 where both costs are equal. The jump from year nine to year ten in the Figure 22 is caused by the change of options in the optimal path (from option A to option B).

C.2.2. Supplier and User cost Breakdown for the constrained case

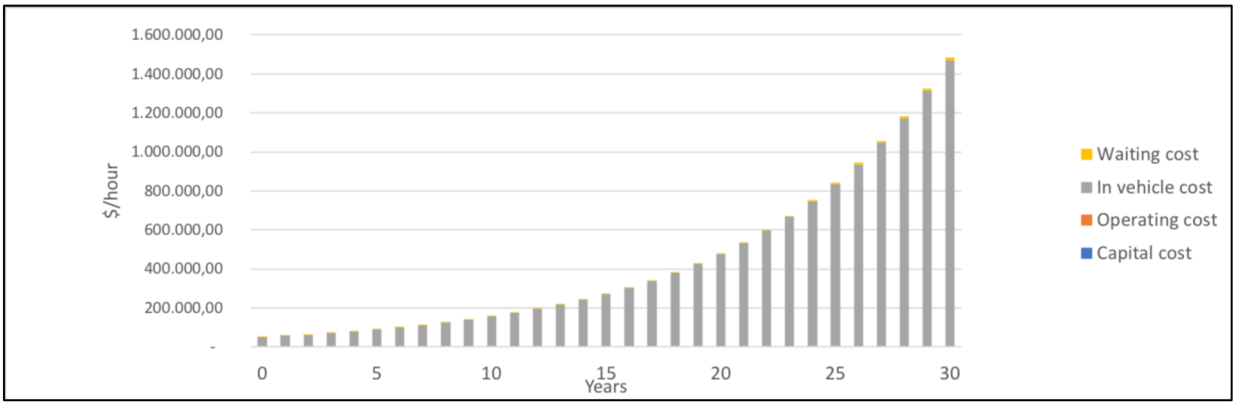


Figure 23. Supplier and User cost breakdown of option A, constrained case. Source: Own elaboration

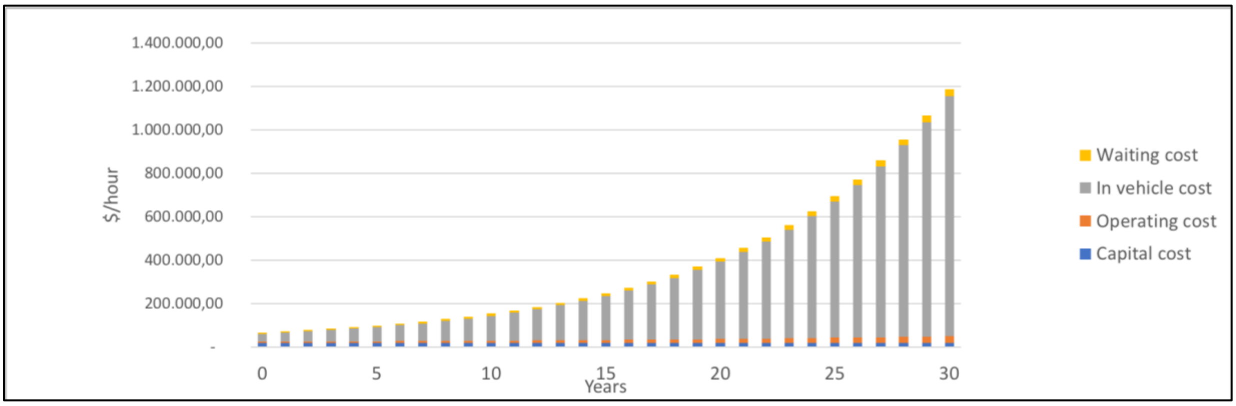


Figure 24. Supplier and User cost breakdown of option B, constrained case. Source: Own elaboration

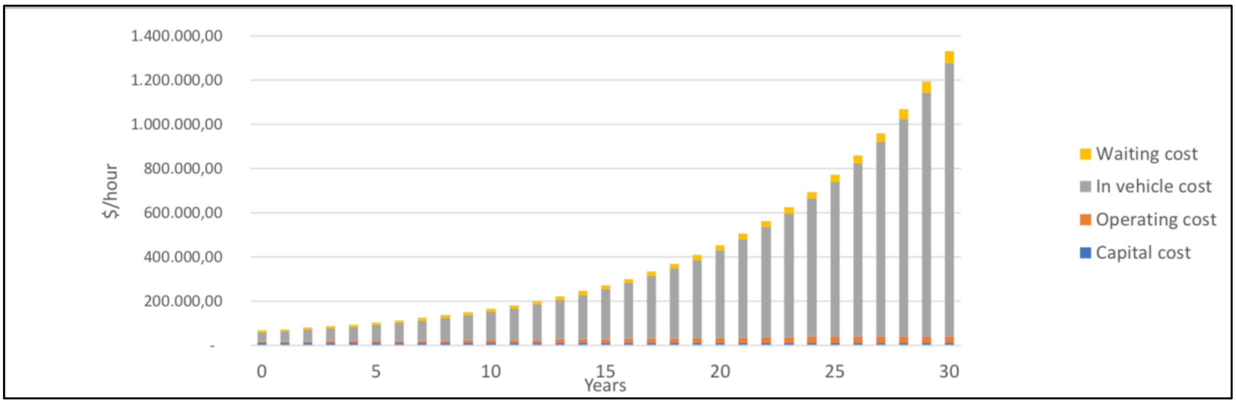


Figure 25. Supplier and User cost breakdown of option C, constrained case. Source: Own elaboration

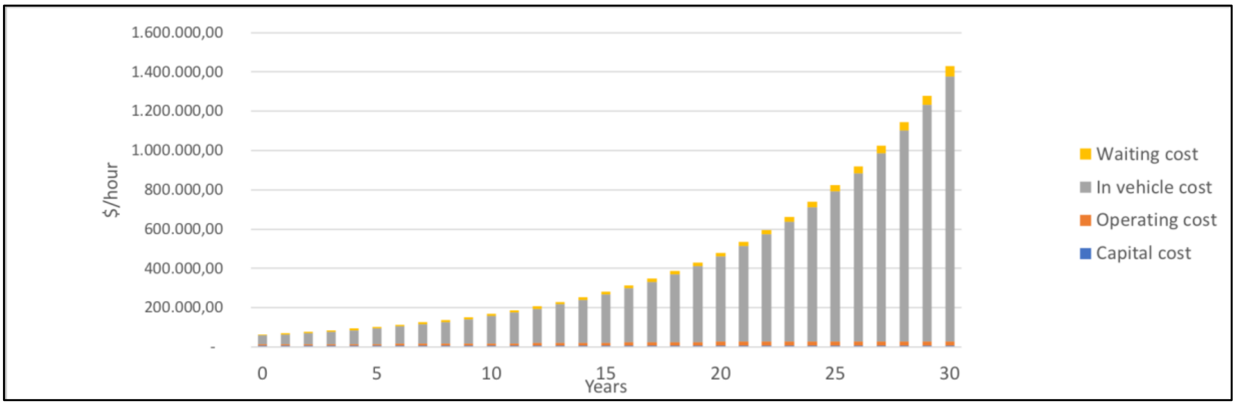


Figure 26. Supplier and User cost breakdown of option D, constrained case. Source: Own elaboration

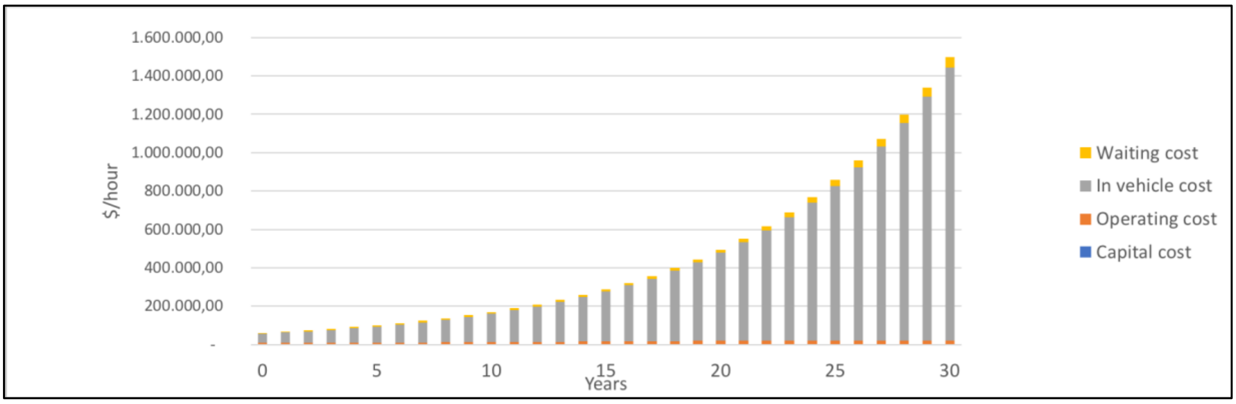


Figure 27. Supplier and User cost breakdown of option E, constrained case. Source: Own elaboration

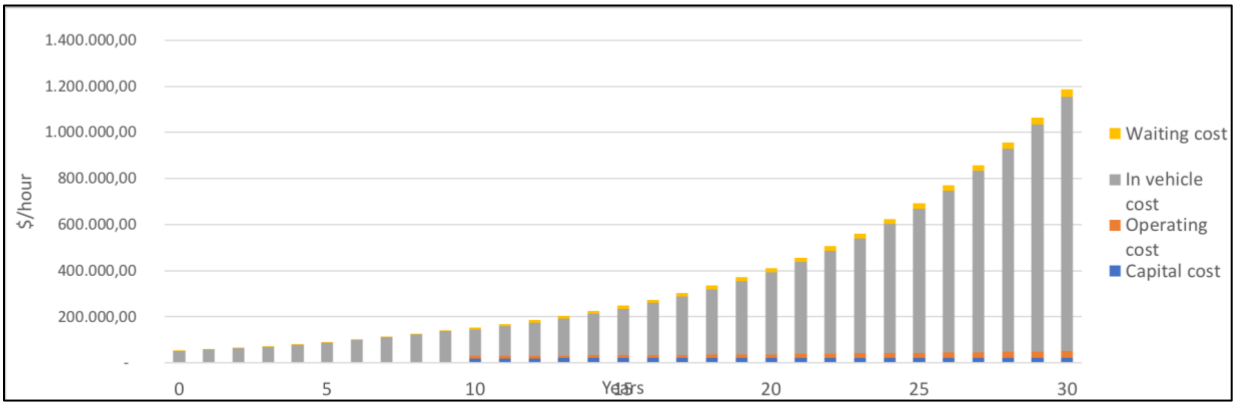


Figure 28. Supplier and User cost breakdown of optimal path, constrained case. Source: Own elaboration

The previous six figures show the supplier and user cost breakdown of all the options, including the optimal solution.

C.2.3. Sensitivity to demand for the constrained case

Figure 29. Evolution of every option with a 90% reduction (up), 70% reduction (middle) and 40 % reduction (down), for the constrained case Source: Own Elaboration

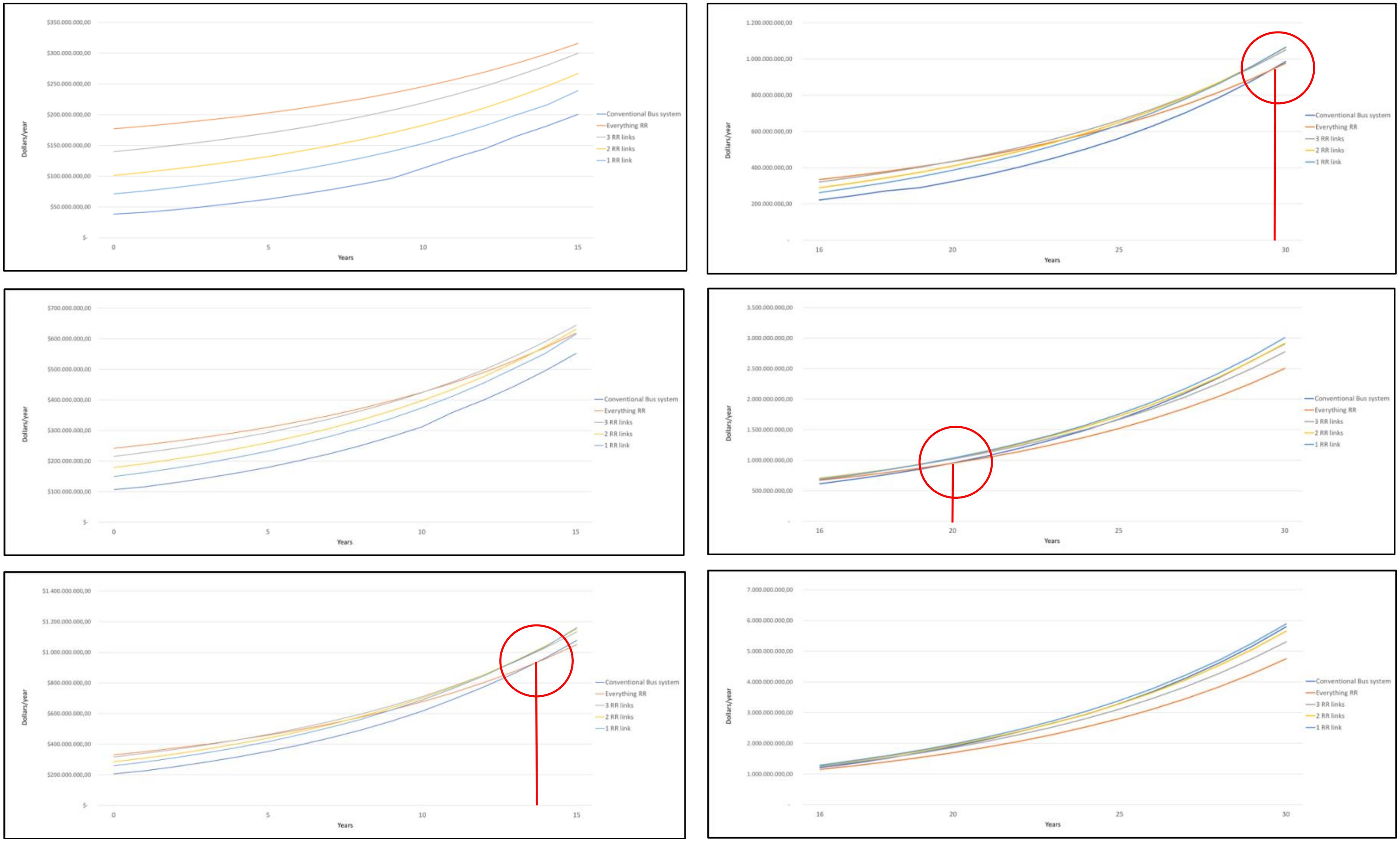
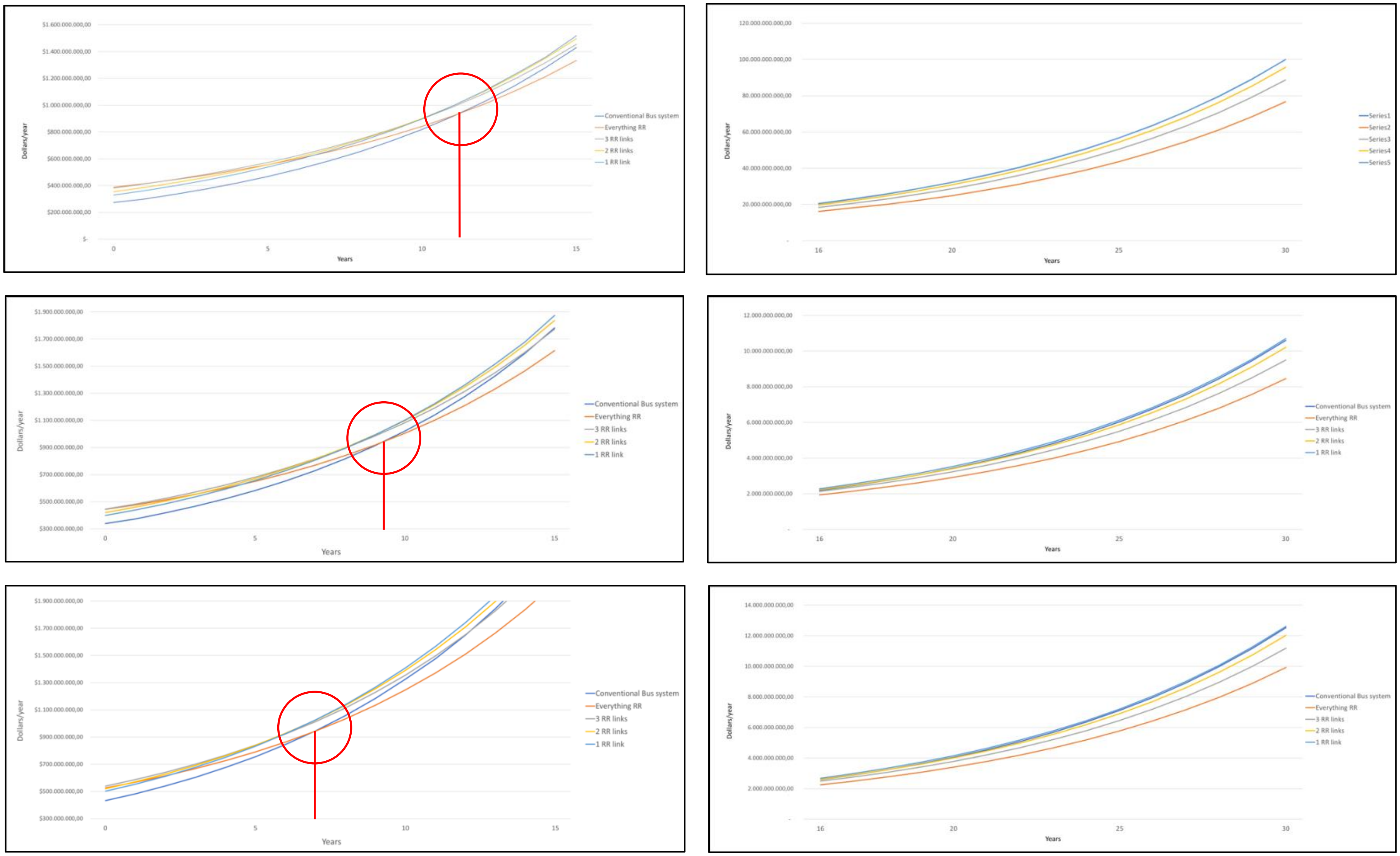


Figure 30. Evolution of every option with a 20% reduction (up), an increment of 10% (middle) and increment of 30 % (down). Source: Own Elaboration



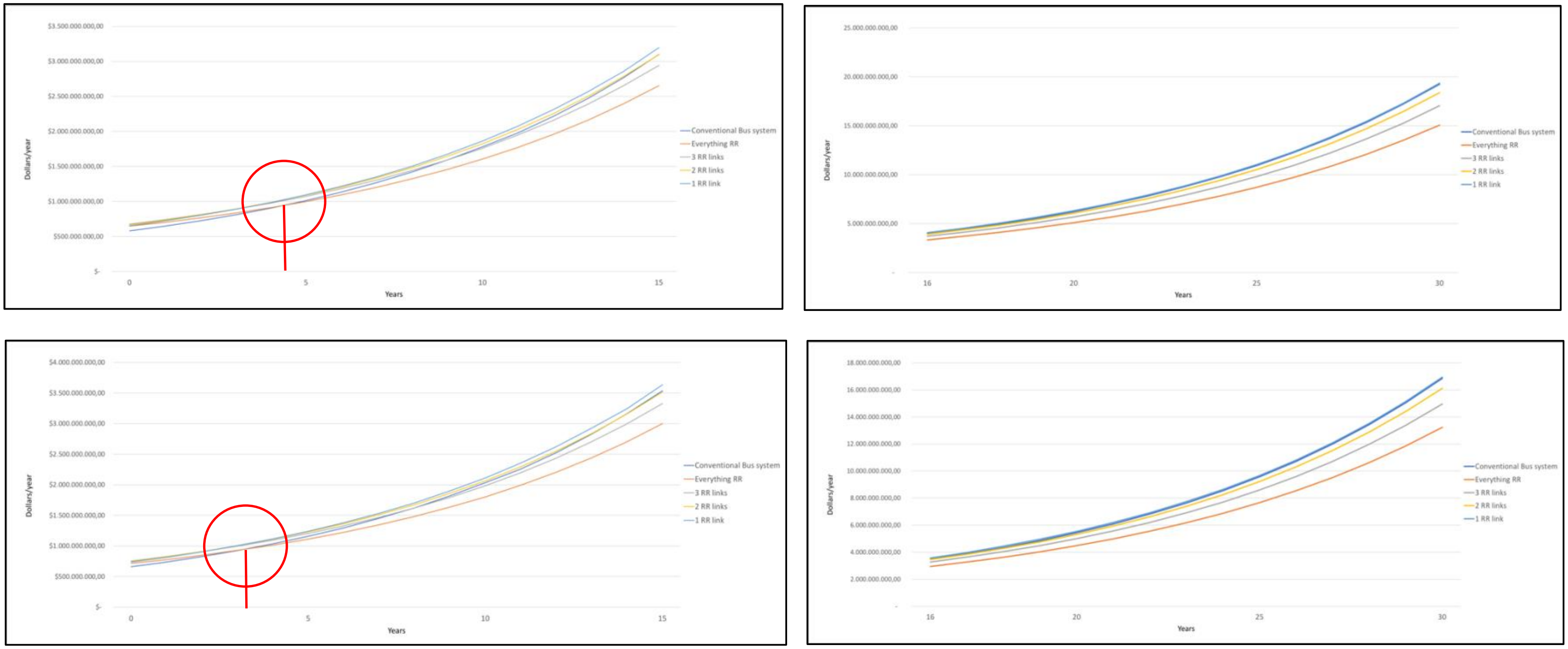


Figure 31. Evolution of every option with an increment of 75% (up) and an increment of 100%, for the constrained case(middle. Source: Own Elaboration

C.2.4. Sensitivity to Analysis period for the unconstrained case

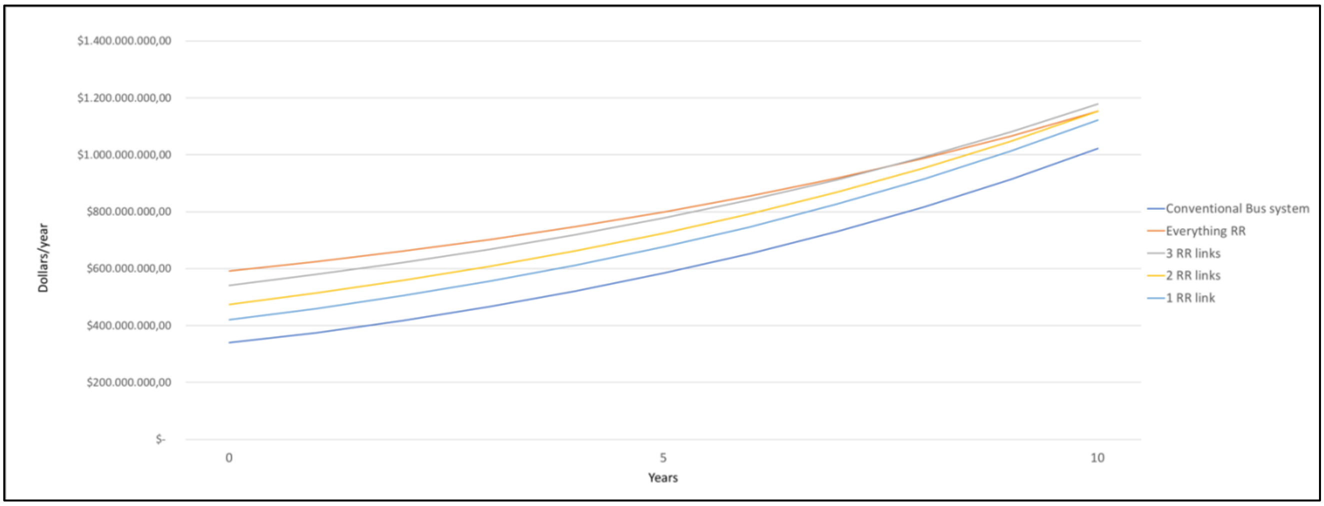


Figure 33. Total cost of each option in a 10 years' time horizon. Source: Own elaboration

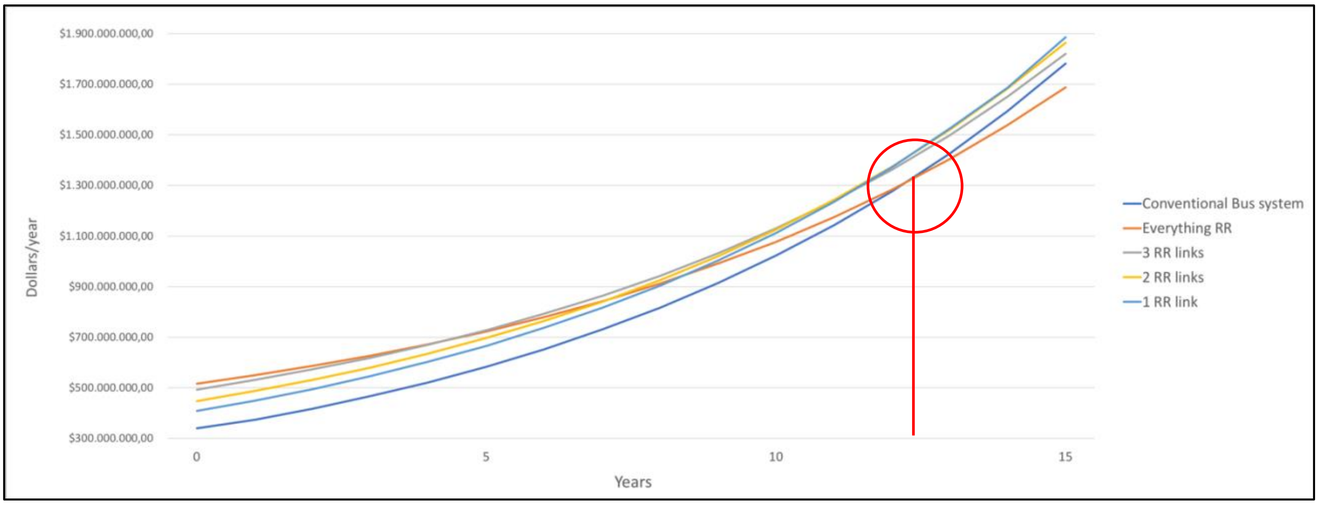
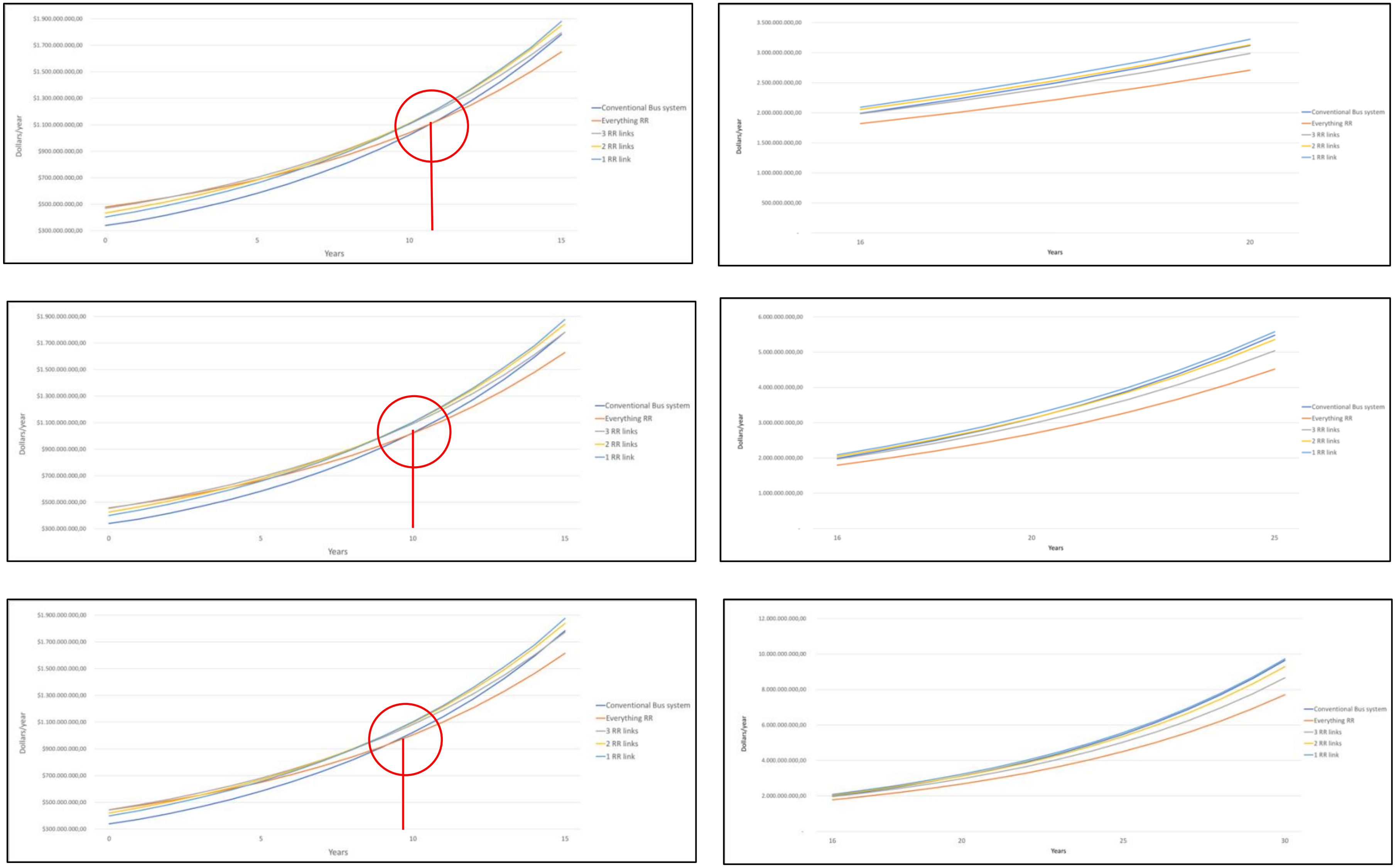


Figure 32. Total cost of each option in a 15 years' time horizon. Source: Own elaboration

Figure 34. Evolution of a 20 years analysis period (up), evolution of a 25 years analysis period (middle), evolution of a 30 years analysis period (down). Source: Own elaboration



LIST OF TABLES

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