

Mathematical modeling of moisture evaporation in co-current foam spray drying

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Abstract

Two models of foam drying are presented in the paper: single droplet drying and perfect mixing of phases spray drying models to describe mechanism of drying of droplet containing bubble.

Analysis of drying curves shows that in constant drying rate period and in the falling drying rate period, evaporation rate decreases due to particle shrinkage and increasing of resistance of moisture diffusion inside the solid crust. Increase of gas pressure in the bubble might cause particle breakage.

Slight differences between theoretical and experimental results caused by disregarding broken particles in the simulations proves accuracy of the developed model.

Keywords: *spray drying, modeling, foamed materials, particle morphology*

1. Introduction

Foaming of slurry in spray drying processes is a method to control rheological properties of feed like viscosity and density and morphology of powder. Powders obtained in foam spray drying process are characterized by lower bulk density, high porosity and particle sizes, enhanced solubility and wettability in relation to conventional spray drying. However, to control product quality, foam spray drying process must to be carried out in specific window of the process parameters selected individually for dried material.

In the literature there is lack of mathematical description of foamed spray drying process; existing models refer only to the constant drying rate period [1].

The aim of this work was to develop and validate mathematical model of foam spray drying process to determine morphological changes of particles and to estimate the quality of the product.

2. Foam spray drying model

Two models of foam drying were developed in the frame of the work: single droplet drying (SDD) model to check correctness of applied correlations and perfect phase mixing foam spray drying model to simulate drying process. All calculations were performed in Matlab and validated on a base of data obtained from the foam spray drying experiments performed at Lodz University of Technology [2].

2.1. Single droplet drying model

SDD model describes moisture evaporation of maltodextrin solution (DE12) from stationary droplet which contains a single saturated nitrogen bubble in constant ambient air temperature T_G , and humidity Y_G . In constant drying rate period, droplet shrinks due to water evaporation whereas in falling drying rate period, particle shrinkage stops because of crust solidification. In this period pressure in the gas bubble increases and might cause particle brakeage. The model allows to determine drying curves, particle density, porosity, crust thickness and pressure in the internal bubble.

2.1.1. Heat transfer

Moisture from the particle evaporates to surrounding air in temperature which, taking into account heat capacity of air and internal bubble, can be calculated from equation (1):

$$\frac{dT_p}{dt} = \frac{A_p \alpha_p (T_p - T_G) + h \left(\frac{dm_{p,a}}{dt} + \frac{dm_{p,b}}{dt} \right)}{m_p c_p + m_b c_b} \quad (1)$$

2.1.2. Mass transfer

Mass transfer model was based on the concept of characteristic drying curves [3]. Overall evaporation from the droplet/particle is a sum of moisture evaporation to the surrounding air and to the internal bubble, eq. (2):

$$\frac{dm_p}{dt} = \frac{dm_{p,a}}{dt} + \frac{dm_{p,b}}{dt} \quad (2)$$

Evaporation rate to drying air is calculated from the equation:

$$\frac{dm_{p,a}}{dt} = f M_w A_p \beta_p (C_s - C_G) \quad (3)$$

where heat and mass transfer coefficients (α_p and β_p) were calculated from McAdams correlations. Amount of water transferred to the bubble was determined from eq. (4):

$$\frac{dm_{p,b}}{dt} = m_N \frac{M_w}{M_N} \left(\frac{P_{st}}{P_a - P_{st}} \right) \frac{1}{dt} \quad (4)$$

To take into account decrease of drying rate (eq. (3)) in the falling drying rate period, coefficient f defined by relation (5) was used [5]. In constant drying rate period coefficient f is equal $f = 1$. After critical moisture content, due to increase in internal mass transfer resistance, f decreases to $f = 0$ at the equilibrium point. According to Woo et al. [4] coefficient f can be expressed as a function of moisture content:

$$f = \left(\frac{X - X_{eq}}{X_{cr} - X_{eq}} \right)^{3.22} \quad (5)$$

Critical moisture content for maltodextrin was determined experimentally in a frame of this work and calculated from eq.(5), (T_G in °C):

$$X_{cr} = 21555 \cdot T_G^{-2.106} \quad R^2 = 0.976 \quad (6)$$

Equilibrium moisture content was calculated from GAB equation [5]:

$$X_{eq} = \frac{X_{mo} c k a_w}{(1 - k a_w)[1 + (c - 1) k a_w]} \quad (7)$$

Where constants $c = 10.866$ and $k = 0.971$. Monolayer moisture content (X_{mo}) for maltodextrin is equal to 0.0518 kg/kg.

2.1.3. Particle morphology

Particle diameter was calculated from the sum of the volumes of liquid and internal gas bubble:

$$d_p = \sqrt[3]{\frac{6(V_L + V_b)}{\pi}} \quad (8)$$

In the constant drying rate period, change of liquid shell volume resulting from evaporation of moisture can be calculated from equation (9):

$$V_L = \frac{\rho_{L,0}(1+X_{cr})\pi}{\rho_L(1+X_0)} \frac{\pi}{6} d_{p,0}^3 \quad (9)$$

In the falling drying rate period, particle can shrink due to thermal deformations of solidified crust. Particle shrinkage can be determined using modified equation proposed by Chen [6]:

$$V_L = \frac{\pi}{6} \left[d_{p,cr} \left(b + (1-b) \frac{X}{X_{cr}} \right) \right]^3 \quad (10)$$

Shrinkage coefficient b was calculated as a ratio of particle diameter when particle moisture content is equal to critical moisture content, to the final particle diameter measured experimentally. Equation (11) describes b as a function of drying air temperature (T_G in °C):

$$b = 0.41 + 0.0038T_G - 6.97 \cdot 10^{-6}T_G^2 \quad R^2 = 0.971 \quad (11)$$

Volume of the internal bubble was calculated from the current bubble mass and density (12):

$$V_b = \frac{m_b}{\rho_b} \quad (12)$$

Density of the bubble depends also on the gas humidity and can be determined from equation (13):

$$\rho_b = \frac{P_b}{\frac{R}{M_N} T_p} (1 + Y_b^*) \frac{1}{1 + Y_b^* \frac{M_N}{M_w}} \quad (13)$$

Equation (12) can be transformed to equation (14) to calculate bubble diameter:

$$d_b = \sqrt[3]{\frac{6V_b}{\pi}} \quad (14)$$

Having diameter of particles (d_p) and bubbles (d_b), crust thickness (particle wall thickness) can be determined.

In constant drying rate period, pressure inside the particle was equal to the ambient pressure $P_b = P_a$. To calculate gas pressure inside the bubble in the falling drying rate period, we assumed proportional grow of pressure with particle temperature in isochoric process according to the equation (15):

$$P_b = P_a \frac{T_p}{T_{p,cr}} \quad (15)$$

Selected results of calculations are presented in Fig. 1. Fig. 1 shows grow of solid concentration and drying curve for the same initial bubble diameter and different initial droplet diameters.

Fig. 1 Solid concentration and evaporation rate from SDD model.

We may observe that as bigger droplet contains more liquid to be evaporated (initial bubble diameter is the same), particle solid concentration grows slower. Analysis of drying curves shows that in constant drying rate period evaporation rate slightly decreases due to particle shrinkage; in the falling drying rate period ($X < X_{cr} = 0.5 \text{ kg/kg}$), drying rate falls down due to increasing of internal resistance of moisture diffusion inside the solid crust. Shape of drying curves is characteristic for evaporation from single droplets which confirms correctness of physical model of the process.

2.2. Co-current foam spray drying model

In co-current foam spray drying model air temperatures, particle velocities and air humidity were calculated from the heat, mass and momentum balance, equations (17) - (22) [7]. Foam spray drying calculations were carried out for perfect mixing of phases, axisymmetrical flow of air and particles and monodispersed atomization. Momentum transfer between drying air and particles was determined from classical equations:

$$\frac{dv_{p,x}}{dt} = g \left(1 - \frac{\rho_G}{\rho_p} \right) - \frac{3}{4} C_D \frac{v_p(v_{p,x} - v_{G,x})\rho_G}{\rho_G d_p} \quad (17)$$

$$\frac{dv_{p,r}}{dt} = -\frac{3}{4} C_D \frac{v_p(v_{p,r}-v_{G,r})\rho_G}{\rho_G d_p} \quad (18)$$

$$\frac{dv_{p,z}}{dt} = -\frac{3}{4} C_D \frac{v_p(v_{p,z}-v_{G,z})\rho_G}{\rho_G d_p} \quad (19)$$

where v_p is relative particle velocity. Air humidity was calculated from mass balance:

$$\frac{dY_G}{dt} = \frac{-\dot{M}_L}{\dot{M}_G \rho_G} \left(\frac{\frac{dm_p}{dt}(1+X)}{m_{p,0}c_0 + m_{p,w}} \right) \quad (20)$$

Particle moisture content was determined from equation (21):

$$\frac{dX}{dt} = \frac{\frac{dm_p}{dt} + (Y_{eq} \cdot m_N - m_{G,W})(1+X)}{m_{p,0}c_0 + m_{p,w}} \quad (21)$$

Air temperature was determined from heat balance (22):

$$\frac{dT_G}{dt} = \frac{1}{\dot{M}_G c_G} \left(-\dot{M}_G (c_v T_G + h) \frac{dY_G}{dt} - \dot{M}_L (X c_w + c_s) \frac{dT_p}{dt} - \dot{M}_L c_w T_p \frac{dX}{dt} \right) \quad (22)$$

Selected results of calculations are shown in Fig.2. Figure 2 presents comparison of theoretically and experimentally determined wall thickness and particle density for different air temperatures and foaming gas rate. Final density of the material determined from the mathematical model decreased from 1300 kg/m³ to 600 kg /m³, which is in line with the experimental results. Differences in relation to the experiments are caused by disregarding broken particles in the simulations. Decrease in the wall thickness depends on the amount of gas introduced into the interior of the droplet. The higher degree of foaming, the larger gas bubble is trapped in the particle and the less material forms the particle.

Fig.2 Wall thickness and particle apparent density for a different air temperatures and different foaming gas rate.

3. Conclusions

Model of co-current foamed spray drying was developed to determine mechanism of droplet drying containing a single bubble.

Analysis of theoretical and experimental results shows that particle solid concentration grows slower for bigger droplet as it contains more liquid to be evaporated as initial bubble diameter is the same. In constant drying rate period evaporation rate slightly decreases due to particle shrinkage which typical for evaporation from droplets and confirms correctness of physical model of the process.

Density of the material determined from the mathematical model decreased twofold in relation to the initial which was in line with the experimental results. Decrease in the wall thickness is a function of the amount of gas introduced into the interior of the droplet. Particle wall thickness and particle density for different air temperatures and foaming gas rate are in line with the experimental results. Differences in relation to the experiments are caused by disregarding broken particles in the simulations.

4. Nomenclature

A	surface	m^3	T	temperature	$^{\circ}C$
c	specific heat	$J\ kg^{-1}\ K^{-1}$	t	time	s
C	vapor concentration	$kg\ mol\ m^{-3}$	v	velocity	$m\ s^{-1}$
h	heat of evaporation	$J\ kg^{-1}$	V	volume	m^3
m	mass	kg	X	moisture content	$kg\ kg^{-1}$
M	molar weight	$kg\ mol^{-1}$	Y^*	saturated humidity	$kg\ kg^{-1}$
P	pressure	Pa	R	universal gas constant	$J\ mol^{-1}K^{-1}$

Greek letters

α	heat transfer coefficient	$Wm^{-2}K^{-1}$
β	mass transfer coefficient	ms^{-1}
σ	stress	Pa

Subscripts

0	initial	L	liquid
a	ambient	N	nitrogen
b	bubble	p	particle

cr	critical	s	surface
eq	equilibrium	st	saturation
G	gas	w	water

5. References

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