Pérez-Sánchez, M.; López Jiménez, PA.; Ramos, HM. (2018). Modified Affinity Laws in Hydraulic Machines towards the Best Efficiency Line. Water Resources Management. 32(3):829-844. https://doi.org/10.1007/s11269-017-1841-0

Modified Affinity Laws in Hydraulic Machines towards the Best Efficiency Line

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ABSTRACT

The development of hydraulic and optimization models in water networks analyses to improve the sustainability and efficiency through the installation of micro or pico hydropower is swelling. Hydraulic machines involved in these models have to operate with different rotational speed, in order that in each instant to maximize the recovered energy. When the changes of rotational speed are determined using affinity laws, the errors can be significant. Detailed analyses are developed in this research through experimental tests to validate and propose new affinity laws in different reaction turbomachines. Once the errors have been analyzed, a methodology to modify the affinity laws is applied to radial and axial turbines. An empirical method to obtain the Best Efficiency Line (*BEL*) in proposed (*i.e.*, based on all the Best Efficiency Points (*BEPs*) for different flows). When the experimental measurements and the calculated values by the empirical method are compared, the mean errors are reduced 81.81 %, 50%, and 86.67% for flow, head, and efficiency parameters, respectively. The knowledge of *BEL* allows managers to define the operation rules to reach the *BEP* for each flow, improving the energy efficiency in the optimization strategies to be adopted.

KEYWORDS: affinity laws, best efficiency line (BEL), variation operating strategies, energy recovery

1 Introduction

Nowadays, the development of mathematical models to analyse the behavior of hydraulic systems requires theoretical and experimental laws. In some cases, the direct application of these equations can lead to erroneous results (Simpson and Marchi 2013), being necessary to correct them in order to consider the necessary simplifications in the initial assumptions (e.g. viscosity effects, friction losses, turbulence, vortex). In water distribution networks, the model simulation with installed hydraulic machines is very common, and the energy analyses have a great significance due to the increasing price of the energy (Corominas 2010) and the need to reduce the energy consumption and the system efficiency to satisfy the European standards requirements (Pasten and Santamarina 2012).

Traditionally, these optimizations of energy consumption in water systems have been focused on the reduction of consumed power by installed pumps. Hence, some authors have worked in the development of pumped systems to adapt the rotational speed of the machine to reduce the energy consumption (Sarbu and Borza 1998; Moreno et al. 2010; Simpson and Marchi 2013; Jiménez-Bello et al. 2015) and the pressure in the system (Kevin 1990; Giustolisi et al. 2008; Cabrera et al. 2014). These reductions are of paramount importance in economic and environmental savings, which have been analyzed through different algorithms and software such as *EPANET* (Rossman 2000) or WaterGEMS (Nazari and Meisami 2008), providing significant tools for water management in pipe systems.

In the last years, different authors have developed researches using new technologies to leverage the pressure reduction in water distribution networks, increasing the global efficiency in the water system (Abbott and Cohen 2009; Dannier et al. 2015; Pérez-Sánchez et al., 2017). Araujo et al. (2006) and Giugni et al. (2014) enumerate different algorithms to optimize the location the optimal place of the Pressure Reduction Valves (PRVs) in a network for leakages control. The initial study of Pump Working as Turbines (*PATs*) that are installed in water

52 systems to replace *PRVs* (Ramos and Borga, 1999) allowed modellers to analyze the turbine
53 behaviour according to different aspects such as:

54 • The morphology of the machine (Yang et al. 2012; Carravetta et al. 2013a; Shi et al.
55 2015).
56 • The design of installation schemes and operation strategies (Carravetta et al. 2012,
57 2014b; Fontana et al., 2012).
58 • The design of machines, adapted to specific work conditions (i.e. low head and high
59 range of flows) in these water drinking network such as the tubular propeller (Ramos et
60 al. 2013; Samora et al. 2016b).
61 • The analysis of potential recovered energy according to circulating flow along the time
62 through of simulated annealing techniques (Pérez-Sánchez et al. 2016; Samora et al.
63 2016a; Pérez-Sánchez et al., 2018).
64 • The economic and feasibility analyses of these installations showing the sustainability
65 and environmental profit of these solutions (Ramos et al. 2010; McNabola et al. 2014).

66 Furthermore, the study of the performance behavior of these machines, Singh (2005) and
67 Derakhshan and Nourbakhsh (2008) proposed the efficiency and head curves as function of
68 flow according to the specific rotational speed of the machine. These curves can be used in the
69 simulations of energy analyses with good results when the hydraulic machine operates in its
70 nominal rotational speed. However, if the energy studies consider operation strategies with
71 variation of the rotational speed, the use of affinity laws in the simulations can bring erroneous
72 results of recovered energy in the system (Sarbu and Borza 1998; Gulich 2003) since the
73 turbines do not behave as the similarity described. Therefore, considering the need to know the
74 efficiency of the machine as function of rotational speed, the first aim of this research is to
75 obtain the errors between measured and calculated efficiency through the application of affinity

laws. This analysis was developed for two machines (with axial and radial impellers) based on experimentation. The second objective is to develop modified affinity laws to establish the best efficiency line (*BEL*) and the best efficiency head (*BEH*) of each machine as function of flow. *BEL* and *BEH* establish the rotational speed of the machine when the Variable Operating Strategies (*VOS*) are used (Carravetta et al. 2013b), claiming for each flow the best operation point (*BEP*). The development of both objectives presents the main novelty of this research that obtains the *BEL* and *BEH*. These lines can be used on optimization techniques to maximize the recovered energy in water distribution system when the rotational speed changes, obtaining result more exactly than the values when similarity laws are used.

2 Material and Methods

2.1 Type of Hydraulic machines

A general typification of hydraulic machines refers to action or reaction according to the exchange of energy between fluid and impeller at atmospheric pressure or not. Inside the group of the reaction machines, the impeller shape and the specific speed establish a second classification. This parameter is defined as the rotational speed of a similarity turbine (geometrically) to generate one kW when the head is equal to one. The specific speed establishes the impellers' typology and it is defined by equation (1):

$$n_s = n\frac{P^{1/2}}{H^{5/4}} \quad (1)$$

where n_s is the specific speed of the machine in (m, kW); n is the rotational speed of the machine in rpm; P is the power in shaft, which is measured in (kW); and H is the recovered head in (m w.c.).

Based on the specific speed, the impellers can be: radial, semi-axial, or axial according to Figure 1a. The radial or centrifugal impeller are those in which the fluid enters in the machine in radial direction and exists in axial direction. This type of machines has a specific speed number

between 36 and 93 rpm. For n_s between 93 and 176 rpm, the machine is called diagonal or semi-axial. In this sort of machines, the inlet of the fluid is diagonal while the outlet has an axial direction. Finally, the axial machines present the fluid direction both inlet and outlet with axial direction and the specific speed is greater than 176 rpm.

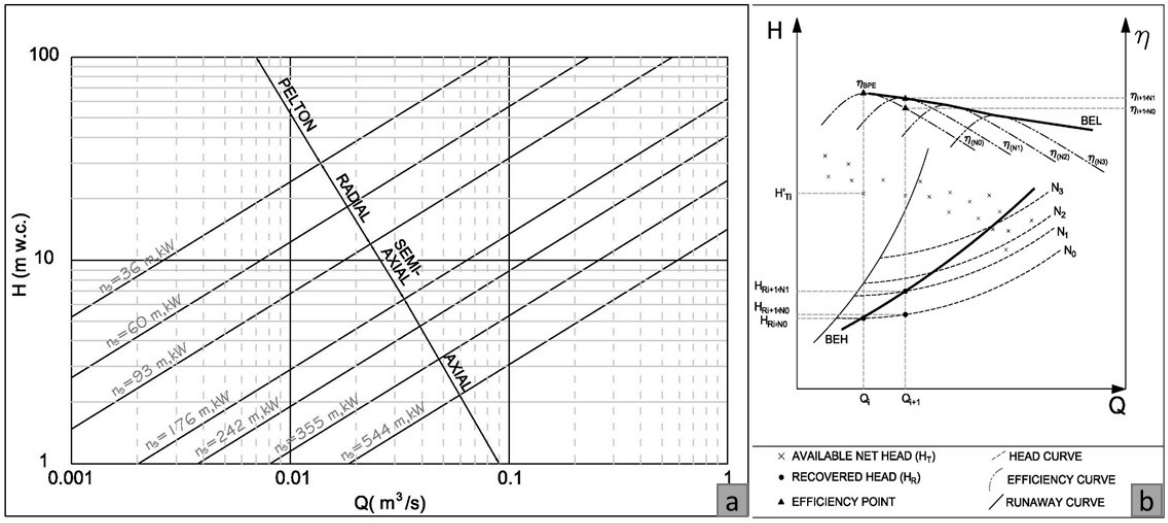


Fig. 1 (a) Type of impeller according to specific rotational speed (adapted from Alexander et al. 2009) and (b) scheme of operating points

2.2 Theoretical approximation: affinity laws

The study of the behavior of turbomachines with equal specific rotational speed can be tackled if the conditions of similarity (geometrical, kinematic and dynamic) are entailed. The first condition is satisfied, when the study is focused on analyzing the behaviour of a machine with different rotational speed. Kinematic condition establishes that in the inlet and outlet impeller, the triangles are similar. These two conditions are defined by equations (2) to (4) (Mataix, 2009):

$$\frac{Q_1}{Q_0} = \left(\frac{D_1}{D_0}\right)^3 \frac{N_1}{N_0} \qquad (2)$$

$$\frac{H_1}{H_0} = \left(\frac{D_1}{D_0}\right)^2 \left(\frac{N_1}{N_0}\right)^2 \quad (3)$$

$$\frac{P_1}{P_0} = \left(\frac{D_1}{D_0}\right)^2 \left(\frac{N_1}{N_0}\right)^3 \quad (4)$$

where Q_1 is the flow in the new conditions of rotational speed in m^3/s; Q_0 is the flow in nominal rotational speed in m^3/s for the *BEP*; D_1 is the diameter of the impeller in new situation of rotational speed in m; D_0 is the nominal diameter of the impeller in m; N_1 is the new rotational speed in rpm; N_0 is the nominal rotational speed of the impeller in rpm; H_1 is the head in new condition in m w.c.; H_0 is the head in the nominal conditions in m w.c.; P_1 is the shaft power in new conditions in kW; and P_0 is the shaft power in the nominal condition in kW.

Therefore, if hydraulic parameters of the turbomachine (flow, head, and power) can be related through affinity laws and the efficiency of the machine can be indirectly determined for different rotational speeds, keeping the impeller's size. Based on this assumption, equations (2) to (4) can be simplified into equations (5) to (7) (Mataix 2009):

$$\frac{Q_1}{Q_0} = \frac{N_1}{N_0} = \alpha \quad (5)$$

$$\frac{H_1}{H_0} = \left(\frac{N_1}{N_0}\right)^2 = \alpha^2 \quad (6)$$

$$\frac{P_1}{P_0} = \left(\frac{N_1}{N_0}\right)^3 = \alpha^3 \quad (7)$$

where α is the ratio between N_1 and N_0.

The characteristic curve of a turbomachine (as a second-degree polynomial) can be written by equation (8) (Mataix 2009):

$$H_0 = AQ_0^2 + BQ_0 + C \quad (8)$$

where *A*, *B*, and *C* are coefficients of the characteristic curve.

The efficiency curve can be also established by a second (Mataix 2009) around the nominal point, or by third-degree polynomial if a discretized range of flows is considered (Ulanicki et al. 2008). The nominal efficiency curve is then defined by equation (9):

$$\eta_0 = EQ_0^3 + FQ_0^2 + GQ_0 + I \quad (9)$$

where: η_0 is the efficiency of the machine for a flow equal to Q_0; *E*, *F*, *G*, and *I* are coefficients of efficiency curve.

When the affinity laws are applied to equations (8) and (9), the new curves of the turbomachines are defined by equations (10) and (11):

$$H_1 = AQ_1^2 + \alpha Q_1 + \alpha^2 C \quad (10)$$

$$\eta_1 = \frac{E}{\alpha^3} Q_1^3 + \frac{F}{\alpha^2} Q_1^2 + \frac{G}{\alpha} Q_1 + I \quad (11)$$

where: η_1 is the efficiency of the turbine for a flow equal to Q_1.

2.3 Experimental approximation: efficiency curves

The prediction of head and flow values in turbines working with different rotational speeds has a reasonable approximation. Nevertheless, as the affinity laws do not consider the viscosity

effects of the fluid inside of the impeller (third condition of similarity), the use of these laws is limited (Simpson and Marchi 2013). Hence, these equations cannot be applied in all flow range to predict the performance, obtaining good results in turbines for ranges between +/- 20% around of the best efficiency point. As the viscosity effect has to be considered, the dynamic similarity is important in these cases. This effect must be considered together with geometrical and kinematic similarity. To consider the dynamic condition, different researchers (Sarbu and Borza 1998; Gulich 2003; Simpson and Marchi 2013) have proposed equations to define the performance as function of the rotational speed variation. Gulich (2003) proposed the equation (12), which predicts the performance according to the variation of Reynolds number between both rotational speeds of the machine.

$$\frac{1-\eta_1}{1-\eta_0} = K + (1-K)\left(\frac{Re_1}{Re_0}\right)^2 \quad (12)$$

where Re_1 is the Reynolds number for the rotational speed N_1; Re_0 is the Reynolds number for the rotational speed N_0; K is the loss coefficient in the impeller as function of the Reynolds number.

Similar equation was proposed by Sarbu and Borza (1998), who also related the Reynold number as function of the rotational speed, defined in equation (13):

$$\frac{1-\eta_1}{1-\eta_0} = \left(\frac{N_1}{N_0}\right)^{0.1} \quad (13)$$

Equations (12) and (13) were tested by Simpson and Marchi (2013), obtaining good results in the prediction of the efficiency, if the rotational speed is not reduced under to 70% of the nominal speed. This is due to the empirical expression exponent changes with viscosity and friction effects in the impeller.

When variable operating strategy (VOS) is applied, the final objective is to determine the best efficiency line (BEL) as function of the rotational speed for each flow (Figure 1b). The operation point should be fixed in the available maximum point to maximize the efficiency of the recovery system. The knowledge of *BEP* for each flow along the time (*i.e.*, Q_i, Q_{i+1}) as well as the available net head (H_{Ti}) will allow researchers to know the best efficiency head line (BEH) of the installed machine ($H_{Ri,N0}$, $H_{Ri+1,N1}$) for the recovery system. Both lines allow to recover the maximum energy, helping to define the VOS in the system for the necessary rotational speed (*i.e.*, N_0, N_2, …, N_i) in each time.

Traditionally, these variations have been predicted by affinity laws, but only near the *BEP*. In this *BEL*, different authors (Carravetta et al. 2014a, 2014b; Fecarotta et al. 2016) proposed modifications in the affinity laws to improve the prediction of the BEP depending on the rotational speed. This proposal was developed for some semi-axial machines with specific rotational speed between 120 and 162 (m, kW). As proposed, the modification of the affinity laws is developed according to the equations (5) to (7) as well as the use of the Suter Parameters (*SP*), which were defined by Suter (1966) by equations (14) and (15):

- Head SP; $WH = \frac{h}{n^2+q^2}$ (14)

- Torque SP; $WT = \frac{b}{n^2+q^2}$ (15)

where *h*, *q*, *n*, and *b* are the head, discharge, velocity, and torque coefficients defined by equations (16) to (19), respectively.

- Head coefficient: $h = \frac{H}{H_0}$ (16)

- Discharge coefficient: $q = \frac{Q}{Q_0}$ (17)

- Velocity coefficient: $n = \frac{N}{N_0}$ (18)

- Torque coefficient: $b = \frac{T}{T_0}$ (19)

Finally, the performance coefficient (e) (which is also called efficiency ratio) is defined by equation (20):

$$e = \frac{\eta}{\eta_B} = \frac{bn}{qh} = \tan\varphi \frac{WT}{WH} \quad (20)$$

where $\tan\varphi$ is the ratio between q and n, according to the third quadrant ($p<\varphi<3p/2$), when the machine is working as turbine (*PAT*). In this case, h and b are positive, while n and q are negative. According to expert references (Carravetta et al. 2014a, 2014b; Fecarotta et al. 2016), the affinity laws can be modified as:

$$q = \frac{Q_1}{Q_0} = f_1(\alpha) \quad (21)$$

$$h = \frac{H_1}{H_0} = f_2(\alpha) \quad (22)$$

$$p = \frac{P_1}{P_0} = f_3(\alpha) \quad (23)$$

where f_1, f_2, and f_3 are fitted functions that depend on the experimental data according to α.

2.4 Theoretical vs. Experimental: error definition

The determination of the errors is evaluated by equations (24) and (25). Three comparisons have been done: a) experimental data versus classic affinity laws; b) experimental data versus modified affinity laws; and experimental data versus empirical method. Equation (24) defines the absolute relative error between the experimental data and estimated measurements for the same flow value (head or efficiency) and equation (25) defines the mean square error, which is determined according to the number of measured data:

$$Absolute\ error\ (\varepsilon_i) = \left|\frac{X_{est} - X_{exp}}{X_{exp}}\right| \quad (24)$$

$$Mean\ square\ error\ (\sigma_i) = \frac{1}{m}\sqrt{\sum_{i=1}^{m}\left(\frac{X_{est} - X_{exp}}{X_{exp}}\right)^2} \quad (25)$$

where i is the tested parameter, which can be q, h, p, or e; X_{est} is the estimated value through affinity laws or empirical method; X_{exp} is the measured value; and m is the number of measurements.

3 Developed Test

3.1 Tested Impellers

The experimental tests have been carried out in CERIS-Hydraulic Lab of Instituto Superior Técnico from the University of Lisbon with two different machines. In both cases the discharge

was measured by an electromagnetic flowmeter, the pressure was registered by pressure transducers, and the power by digital wattmeter FLUKE which was connected to the generator. The digital multimeter is able to measure active power, reactive power and power factor. Finally, the rotational speed was measured by a frequency meter.

Two different machines were tested (Figure 2). On the one hand, an axial machine with an impeller's size of 85 mm with five blades. This machine has the best efficiency point for a flow of 4.44 l/s and head of 0.24 m w.c.. when the nominal rotational speed is 750 rpm. For this machine, the specific speed number is 283 (m, kW) (Eq. (1)). On the other hand, a radial machine that has an impeller size of 139 mm. For this machine, when the rotational speed is 1020 rpm, the best efficiency point is located for 3.36 l/s and 4 m w.c. and the specific rotational speed is 51 (m, kW). The generator connected to both machines has a maximum efficiency of 62% when the rotational speed is 1350 rpm. The characteristics of the generator are: the rated frequency 50 Hz, the rated power 550 W, the rated current 1.6/2.8 A, the rated phase voltage 230/400 V, the power factor 0.74 and the rated speed 1020 rpm.

3.2 Rotational speed variation

In Figures 2a and 2b, the efficiency and head curve for different rotational speed are shown according to the performed tests in the axial machine. In this case, these figures show the head values and the efficiency as function of the flow for different rotational speeds (500, 750, 1000, 1250, and 1500 rpm) as well as the runaway curve.

These characteristic curves for a rotational speed of 750 rpm can be fitted by polynomial equations (26) and (27):

- Head curve (m w.c.):

$$H = 0.047Q^2 - 0.219Q + 0.272 \qquad (R^2 = 0.998)\ [4.40<Q<13.30\ l/s] \tag{26}$$

- Efficiency (%):

$E = -0.058Q^4 + 1.942Q^3 - 23.690Q^2 + 118.381Q - 143.852$ ($R^2 = 0.995$). [4.40 < Q <13.30 l/s] (27)

When the machine is operating in its nominal rotational speed, the flow range oscillates between 4.40 l/s and 13.30 l/s, varying the recovered head between 0.26 and 5.88 m w.c.. For this range of flows, the efficiency of the machine oscillates between 63.65 % and 29.62%, being the maximum efficiency of 63.65%, when the flow is 4.44 l/s.

Figures 2c and 2d show similar results according to tests carried out with the radial machine. In this case, the machine was tested for 600, 900, 1020, and 1200 rpm. The characteristic curve is presented for a rotational speed of 1020 rpm, being defined by the following equations (28) and (29):

- Head curve (m w.c.):

$H = 0.431Q^2 - 1.310Q + 3.553$ ($R^2 = 0.995$) [1.80<Q<4.40 l/s] (28)

- Efficiency (%):

$E = -11.145Q^2 + 80.698Q - 84.535$ ($R^2 = 0.995$) [4.80 < Q <13.30 l/s] (29)

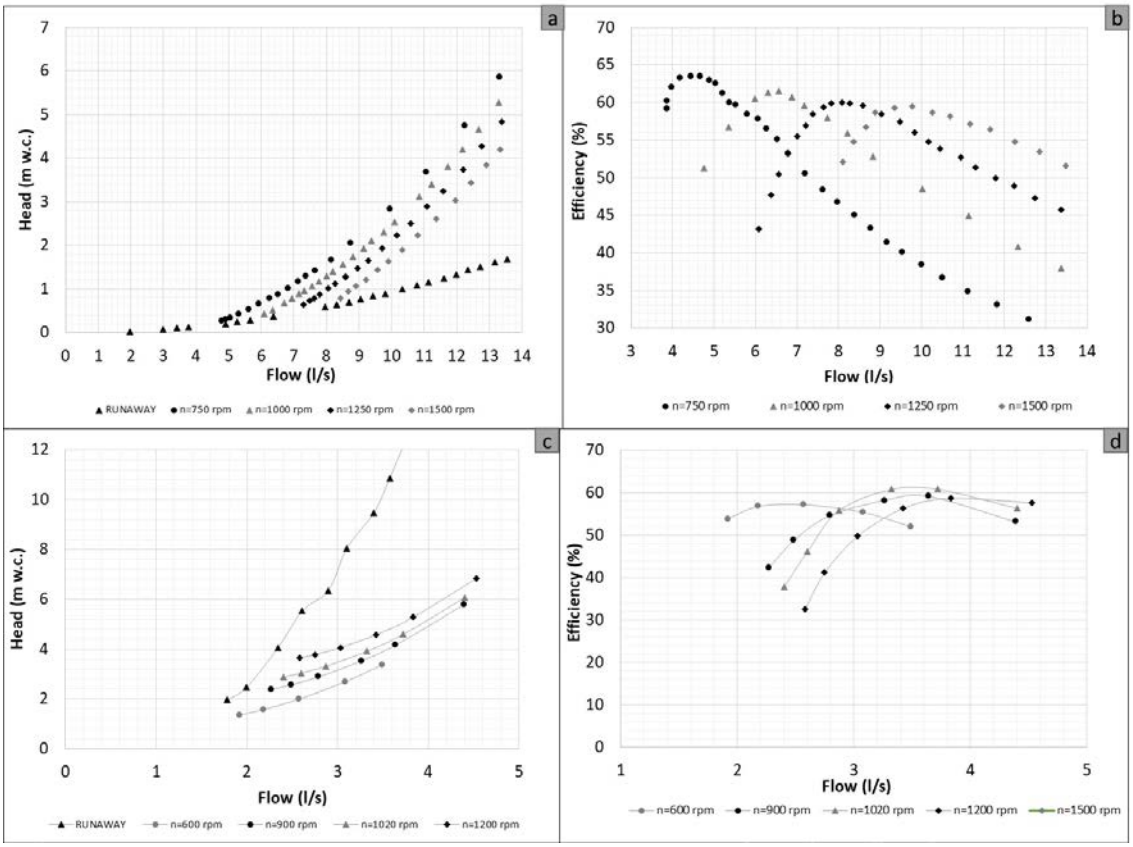


Fig. 2 Experimental data (Head and Efficiency) for machines: (a and b: axial; c and d, radial).

For this machine, the maximum efficiency is 60.5% for a flow of 3.36 l/s.

The runaway curves are also shown in Figures 2a and 2c for both machines (axial and radial). When these curves are analyzed, a significant difference can be determined taking into account the VOS. If a same value of the recovered head is established, a reduction of the flow with a high rotational speed can lead to cause the runaway condition in radial machines. In the case of the axial machine, an increasing of the flow keeping the recovered head can be attained the runaway conditions.

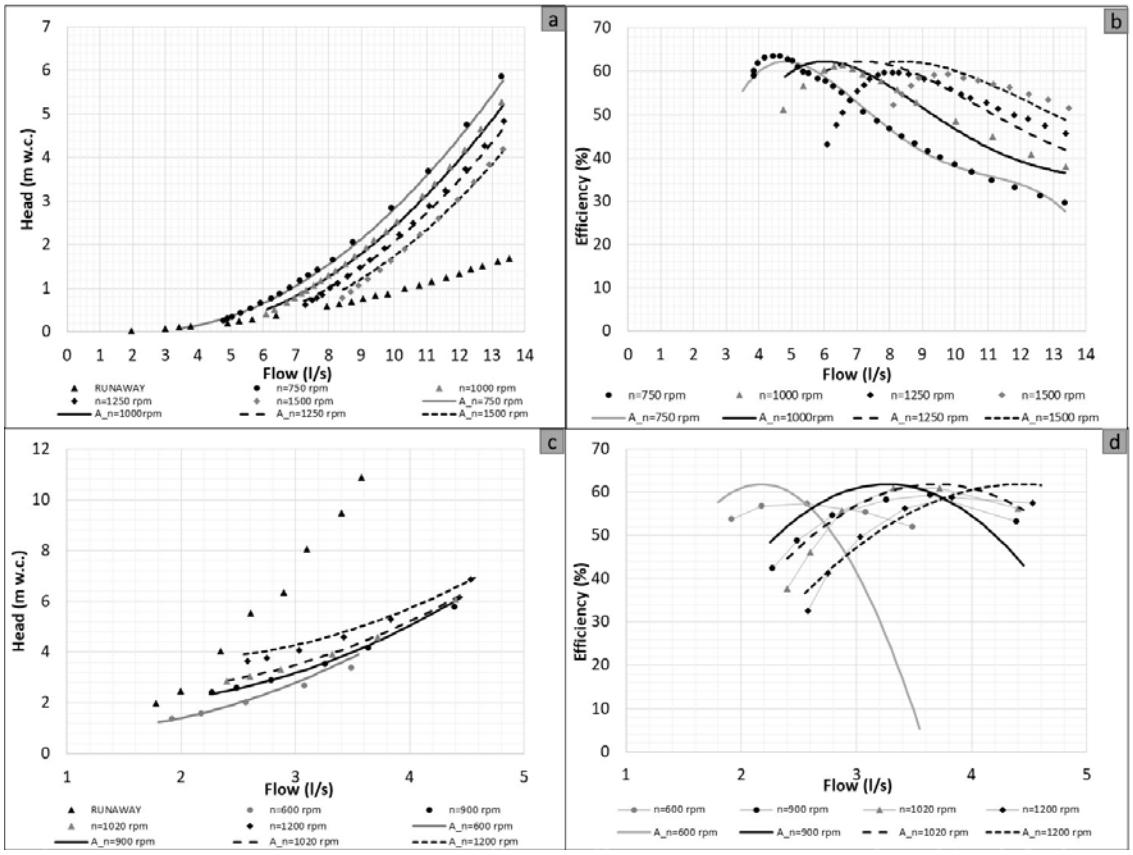
3.3 Analytical versus experimental curves

The need to know the head and efficiency values of each machine can lead to develop new curves with different rotational speeds through affinity laws or Suter parameters, when the

296 water manager uses *VOS*. The obtained results in analyses of axial and radial machines are
297 shown in Figure 3. The head values of the affinity laws can be observed, coinciding on a great
298 range of flow conditions. Therefore, the use of these laws to develop new equation of recovered
299 head curve can be used. Otherwise, if water managers want to determine the efficiency, the
300 affinity laws can fail. Figures 4b and 4d show the predicted efficiencies given by the affinity
301 laws, which are different, especially when the maximum values are considered (*i.e.*, maximum
302 efficiency and flow value).

303 As shown in Figures 3a and 3b, in the axial machine when the rotational speed is 750 rpm, the
304 maximum efficiency is 63.60% for a flow of 4.66 l/s, while the maximum efficiency by affinity
305 laws is 62.19%. For the rotational speed of 1000 rpm, the maximum efficiency is 61.56% when
306 the flow is 6.56 l/s, being the maximum efficiency by the affinity laws 62.27% when the flow
307 is 6.03 l/s. This error raises with rotational speeds of 1250 and 1500 rpm. For the rotational
308 speed of 1250 rpm, the maximum measured efficiency is 59.98% and the maximum estimated
309 efficiency is 62.27 %, being the flow values of 8.28 and 7.21 l/s, respectively. Similar tendency
310 can be observed in Figure 3b when the rotational speed is 1500 rpm.

311 Figures 3c and 3d show results between measured efficiencies and estimated values by affinity
312 laws for different rotational speeds of 600, 900, 1020, and 1200 rpm in the radial machine, not
313 being coincident the *BEP* for each rotational speed (*e.g.*, when the rotational speed is 900 rpm,
314 the maximum measured efficiency is 59.23% and the maximum estimated efficiency is 61.82%,
315 being the flows 3.65 l/s and 3.25 l/s, respectively).



316

317 **Fig. 3** Experimental vs affinity laws in the axial (a and b) and radial machine (c and d)

318 Figure 4a shows the absolute error between experimental data and estimated values by affinity
319 laws used to predict head and efficiency for flows (Q) and rotational speeds (N) in the axial
320 machine. The majority of head errors can attain 10% in both cases (axial and radial machines).
321 If efficiency predictions errors are analyzed, the best fits are located in low value zone of
322 efficiency curve, being these errors below to 1%. Figure 4a also shows the maximum errors are
323 near the maximum efficiency point. In the case of radial machine (Figure 4b), this trend does
324 not occur as for axial machine. These efficiency errors are important to be considered, because
325 when the mean errors are averaged taken into account for all flow range, the obtained mean
326 error is lower than the absolute relative error near the *BEP* (*e.g.*, when the radial machine is run
327 to 900 rpm, the absolute relative error is 0.06 and the mean square error is 0.02). Therefore, if
328 the mean error is determined for each speed, this can mislead to erroneous analyses. In the axial
329 machine, the mean square errors are 0.05, 0.04, and 0.06 for rotational speeds of 1000, 1250,

and 1500 rpm, respectively. These error values are not representative to analyze the maximum efficiency in the machine when the recovery system is operating by *VOS*. The mean square errors in radial machine are 0.09, 0.02, and 0.01 for speed of 600, 900, and 1200 rpm, respectively.

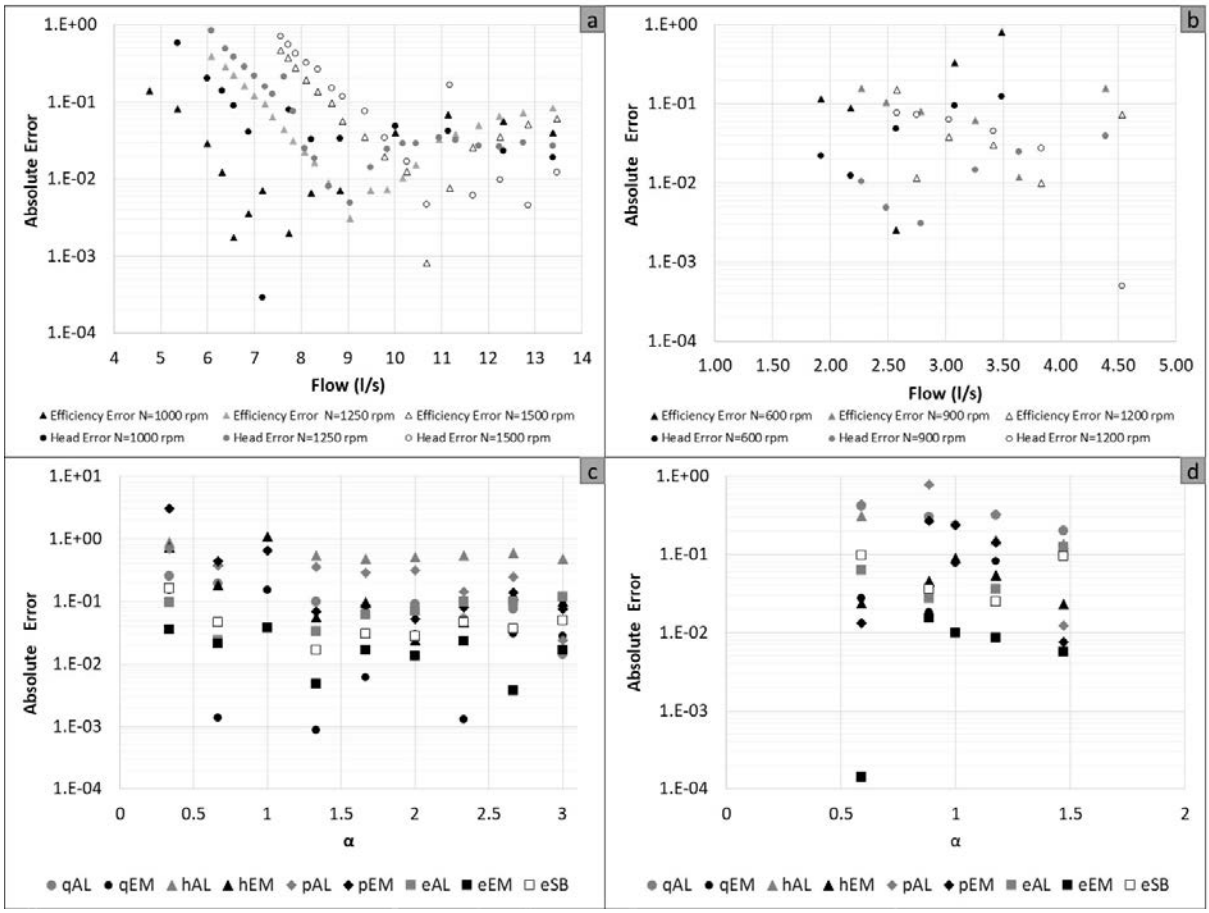


Fig. 4 Absolute error as a function of the flow and rotational speed in axial (a) and radial (b) machines considering the affinity laws. Absolute error in axial (c) and radial (d) machines when the parameters were determined by affinity laws (AL), empirical method (EM), and Sarbu-Borza method (SB)

3.4 Empirical method towards new affinity laws

If the shown errors in Figures 4a and 4b are taken into account, it is necessary to search for new solutions. These solutions have to allow modelers to determine functions with a less error, to

342 be used in the energy studies based on Suter Parameter (*SP*). To fit the affinity laws, the
343 described methodology in section 2.3 enables to obtain such functions (f_1, f_2, and f_3). These
344 functions lead for the determination of the *BEH* and the *BEL* in both machines, maximizing the
345 efficiency in all ranges of flow by rotational speed variations.

346 Figures 5 and 6 show the proposed fitted functions for the axial and radial machines,
347 respectively. To do a better analysis, other maximum values of efficiency were measured in the
348 machine for rotational speeds of 250, 1750, 2000, and 2250 rpm. In the case of the radial
349 machine, the maximum efficiency for the rotational speed of 1500 rpm has also been measured.
350 In both machines, the obtained functions present good regression indexes that are higher to 0.94
351 to parameters of *q*, *h*, *p*, and efficiency ratio, except the value of efficiency ratio in axial
352 machine. For this machine, the value is 0.719.

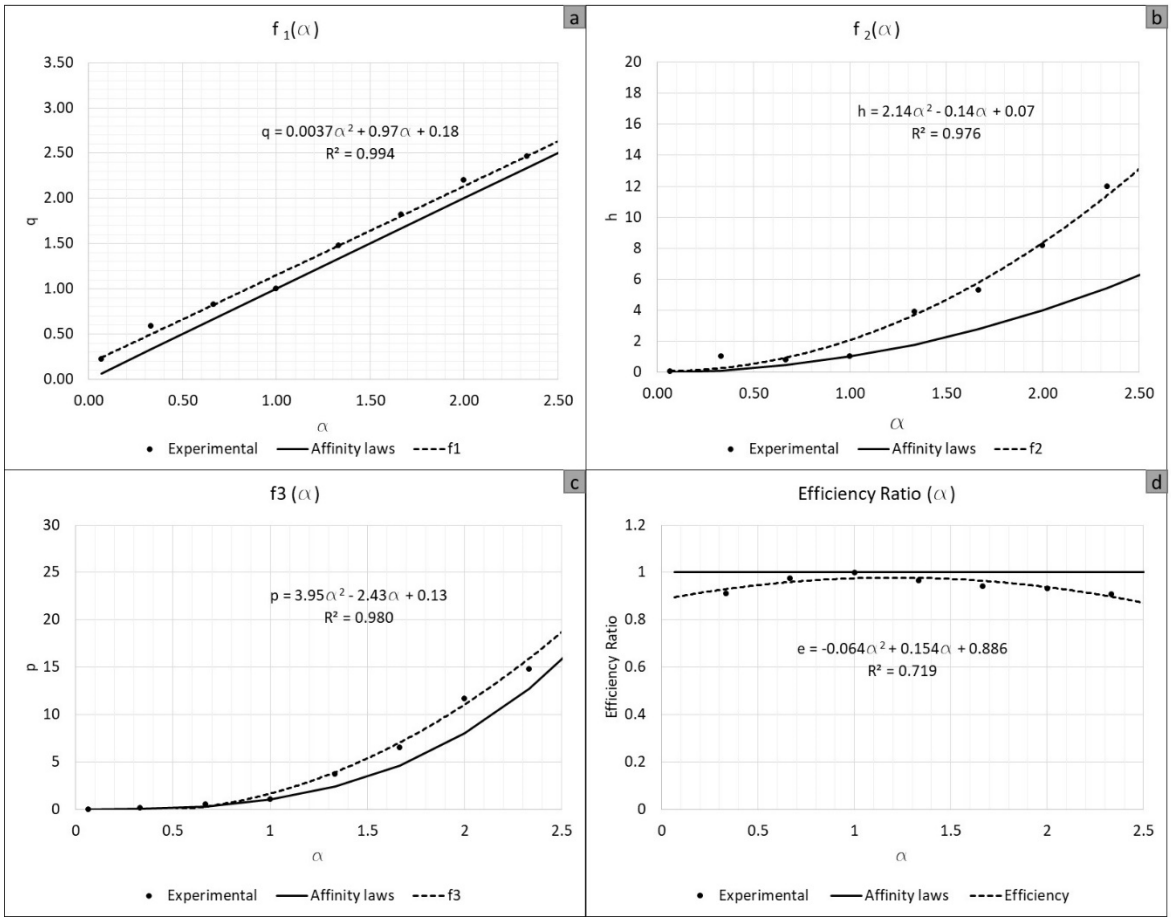


Fig. 5 Proposed functions for axial machine.

Once the functions for each parameter were adjusted by least squares, the errors can be calculated. The error results are shown in Figures 4c and 4d for both machines. The errors of *q*, *h*, *p*, and efficiency ratio (*e*) are shown when the parameters are determined by affinity laws (*AL*), the proposed empirical method (*EM*), and Sarbu-Borza's Method (*SB*) (since this method is only used to determine efficiency values according to equation (13)). In all cases, the less error was obtained by *EM*. The obtained errors with *SB* were between the *AL* and *EM*. For the *EM*, the mean square errors were 0.02, 0.02, 0.004 for *q*, *h*, and efficiency ratio in the radial machine. When these values were determined for the axial machine, the errors were 0.02, 0.03, and 0.008, respectively. The mean square error by *SB* was 0.02 for both machines.

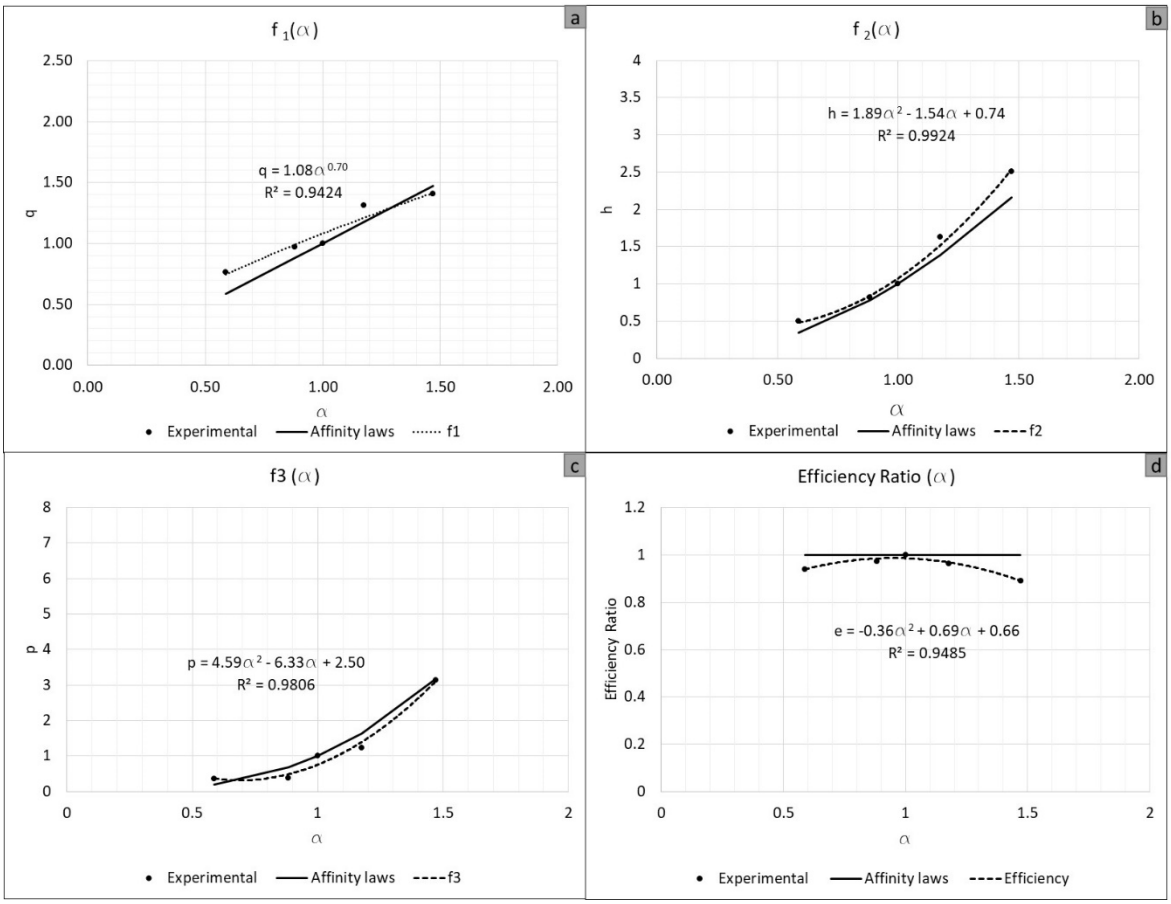


Fig. 6 Proposed functions for radial machine.

Once the errors were analyzed, the functions (f_1, f_2, and f_3) were known depending on the rotational speed. These functions enabled to draw the *BEL* and *BEH* of each machine. Figure 7 shows these lines for the axial (Figure 7a) and the radial (Figure 7b) machines as function of flow, according to *VOS* described in Figure 1b. In the particular case of the radial machine, the *BEL* indicates that recovery system is working upper to 55% in the all range of flows. *BEL* and *BEH* are significant information in the optimization methodologies for water managers to develop the *VOS* in each hydraulic machine, thus maximizing the system efficiency.

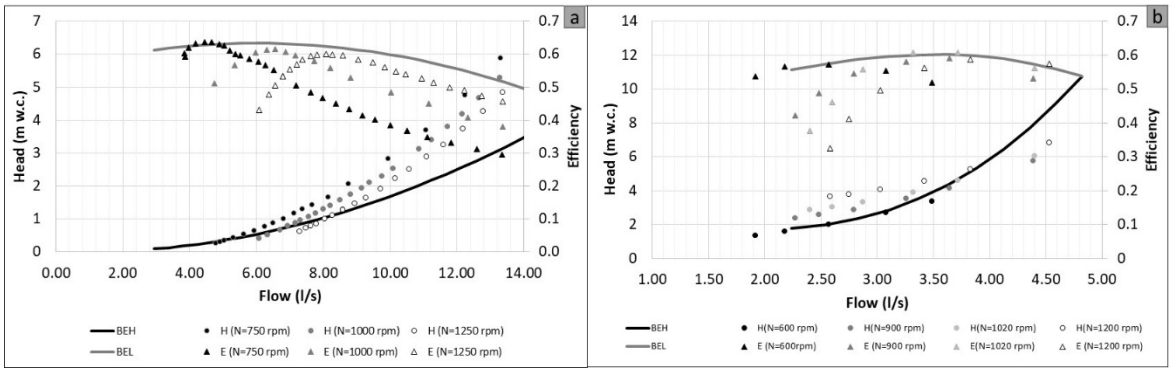


Fig. 7 *BEH* and *BEL* for axial (a) and radial (b) hydraulic machines

Finally, different functions as a function of the rotational speed can be drawn (*i.e.*, f_1, f_2, f_3 and e) , joining the depicted procedure to modify the affinity laws with the analysis developed for semi-axial machines by Fecarotta et al. (2016). This analysis is shown in Figure 8 which correlates functions (*i.e.*, f_1, f_2, f_3, and e) with the variables: velocity ratio and specific rotational speed.

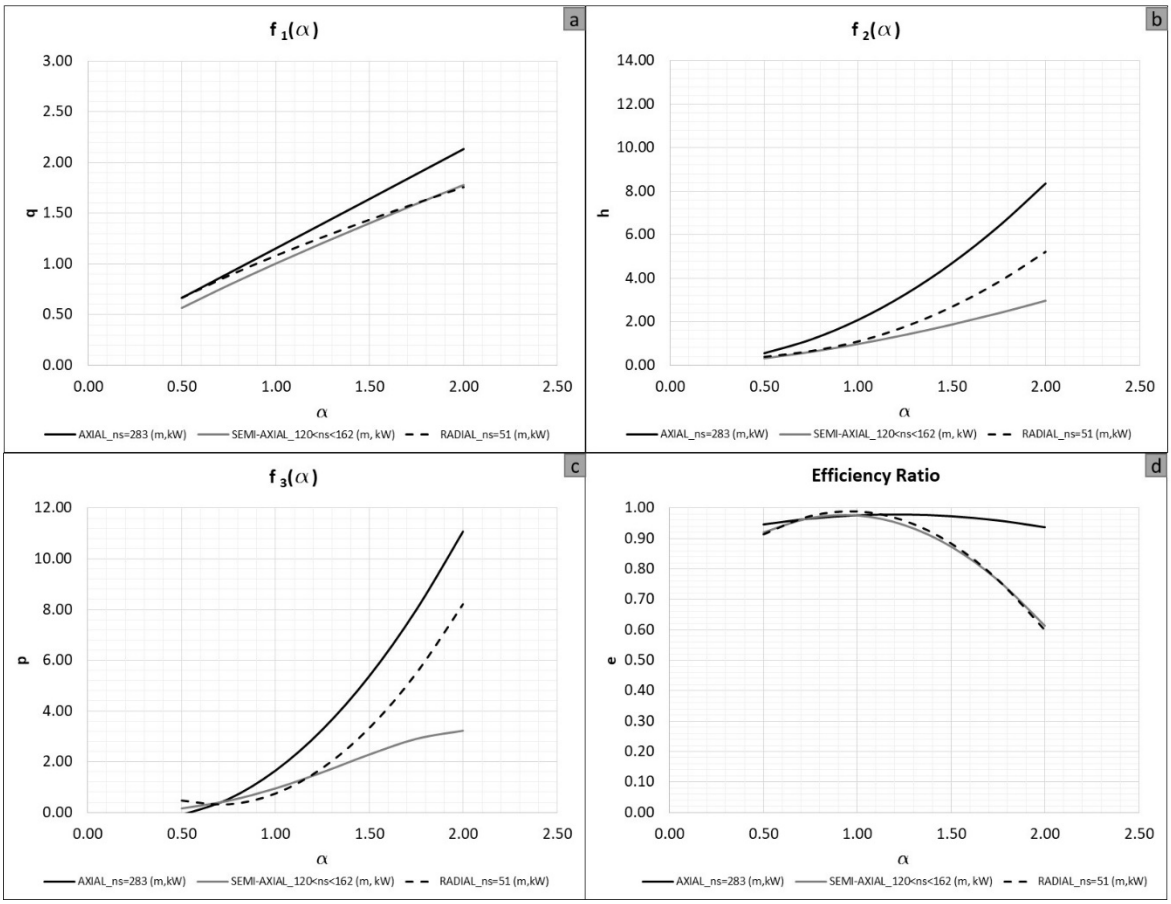


Fig. 8 Fit function to different hydraulic machines.

These functions (f_1, f_2, f_3, and e) presented relative and mean errors smaller than affinity laws and Sarbu-Borza's method when they were used to predict the real behavior of hydraulic turbines. When the radial machine was analyzed, the empirical mean square errors were 0.02, 0.02, and 0.004 for q, h, and e parameters and the mean square error of affinity laws were 0.11, 0.04, and 0.03, respectively. Therefore, the application of this empirical method reduced the mean error values in 81.81 %, 50%, and 86.67%, respectively. For the axial machine, the empirical mean errors were 0.02, 0.03, and 0.008, for q, h, and e parameters, respectively. Against, these errors were 0.06, 0.18, and 0.03 when the efficiency was determined by affinity laws, reducing them to 66.67%, 83.33%, and 73.33%, respectively.

4 CONCLUSIONS

This research presents experimental tests for two different hydraulic turbines (axial and radial) that have a specific rotational speed of 283 and 51 (m, kW) respectively. For these machines, the characteristic curves (flow-head, flow-mechanical power and flow-efficiency) were obtained for different rotational speeds. The variable operating strategies (*VOS*) in recovery systems of water distribution networks involve the need to predict the efficiency and head curve of each machine for different rotational speeds. These curves will enable to develop viability studies based on energy and economic analyses. These obtained curves are for a specific reaction turbine and not general for all turbines.

The prediction of the efficiency with different rotational speeds by the classic affinity laws can imply significant erroneous values. These errors induce the need to develop a modification in the affinity laws to take into account losses in the impeller of each machine. In this research, as novelty, new modified functions are proposed to know the flow, head, power and efficiency parameters that depend on the rotational speed ratio for radial and axial machines.

These functions (f_1, f_2, f_3, and e) were validated for the tested hydraulic machines, presenting relative and mean errors smaller than affinity laws and the Sarbu-Borza's method. When the radial machine was analyzed, the mean errors were reduced 81.81 %, 50%, and 86.67% for q, h, and e parameters when the errors were compared with the affinity laws. In contrast, the mean errors were reduced 66.67%, 83.33%, and 73.33%, respectively for the axial machine.

The knowledge of these functions enables to develop the *BEL* and *BEH* lines as function of the flow of each machine. These lines help managers to choose the best operating rules in order to achieve the best efficiency point for each flow. These rules maximize the real recovered energy under optimization strategies. Regarding the analysis of the rotational speeds, when the flows are reduced, keeping a constant head the rotational speed increases in radial machine. This

phenomenon can cause that *PAT* reaches the runaway conditions when the machine is installed in a line or point of a network in which the flow variation is significant.

Finally, the need to correlate the rotational speed and the specific speed with the different characteristic parameters (*q, h, p,* and *e*) is of utmost importance. The development of similar studies for different specific rotational speeds will enable to define the *BEL* and *BEH* as well as the selection of the best hydraulic machine for each system characteristics.

Acknowledgments

This research is supported by Program to support the academic career of the faculty of the Universitat Politècnica de Valencia 2015/2016 in the project "Methodology for Analysis of Improvement of Energy Efficiency in Irrigation Pressurized Network".

Compliance with Ethical Standards

Conflict of Interest The authors declare that they have no conflict of interest.

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