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Additional Information

A stochastic dynamic model to evaluate the influence of economy and well-being on unemployment control

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Abstract

This paper presents a stochastic dynamic mathematical model to study the evolution of the unemployment rate and other relevant related variables in a country. This model is composed by three basic interrelated subsystems: demographic, economic and well-being ones. A key aspect of this model is that it considers three UN well-being variables simultaneously: Human Development Index, Gender Empowerment Index and Gender Differentiation Index. These variables involve key concepts for human development, as Health, Education, Economy and Female Labor.

With this model, the most outstanding variables found in the literature in relation with unemployment control can be used to design strategies and scenarios to reduce the unemployment rate in the future. The model has been fitted for the case of Spain in the 2002-2014 period, the largest one with information about all the variables involved in the model. Finally, several tentative scenarios and strategies have been tested to reduce the unemployment rate in Spain in the horizon of year 2025, and the corresponding forecast evolution of the Gender Differentiation Index, National income per capita, Public debt and the ratio between Public debt and Gross domestic product are shown.

Keywords: Unemployment rate; United Nations well-being variables; sex/age-structured population dynamics; stochastic model; forecasting.

1. Introduction

Unemployment is one of the most important problems today in the world, and particularly in many southern European countries. For instance, Greece had a 24.14% unemployment rate in March 2016 and Spain had a 21% unemployment rate also in March 2016. But there are countries with more than 40% of jobless rate, like Bosnia and Herzegovina, Congo or Haiti [1].

This is not only an economic problem, but also a social and a physical and mental health problem. In addition, politicians and professionals do not succeed in reducing the current unemployment rate and in the way to create jobs without reducing the well-being of a country. In order to progress in the way to provide a tool to tackle this problem, a mathematical model that relates the unemployment rate with demographic, well-being and economic variables is presented in this paper. Some of the input variables can be controllable and others do not. With this information, governments can design feasible strategies to be simulated under different

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scenarios. The results of the simulations lead to assign suitable values to the controlled input variables in order to decrease the unemployment rate respecting well-being.

The reviewed literature about the problem here treated reveals several papers that present some kind of relation between demographic or well-being variables and the unemployment rate. For example, Pompili et al. [2] and Milner et al. [3] relate the unemployment with mental health and suicide. Worach-Kardas and Kostrzewski [4] explain that unemployment entails many negative health consequences and that it is related with well-being. Cherlin et al. [5] show the relation between unemployment and fertility in a specific time, the Great Recession. Adsera and Menéndez [6] study the relation between fertility and the business cycle in Latin America, observing that the decline of fertility is due to unemployment. Hopcroft and Whitmeyer [7] find that the correlation between fertility and female employment status is negative. Jackson and Victor [8], using a System Dynamics model, find that inequality does not necessarily increase as growth slows down and discuss the implications of this finding for questions of employment, government fiscal policy and the politics of de-growth.

On the other hand, the use of the United Nations (*UN*) well-being variables seems a universal way to measure the evolution of well-being in the context of a mathematical model. These variables allow studying well-being by countries with general models whose equations depend only on the particular historical data. For instance, taking into account a *UN* well-being variable, the Gender Differentiation Index (*GDI*) [9], Sanz et al. [10] present a general stochastic human population-dynamics model to select government inversions for the 2006-2015 period. Another example is the work by Caselles et al. [11], where well-being is measured by the Human Development Hybrid Index (*HDI-Hybrid*) [9]. In this case the model is used as a tool to study how to increase the life expectancy per gender in the horizon of the 2025 year by stating different strategies and scenarios.

Recent studies that build models trying to connect unemployment with other socioeconomic variables consider a low number of variables but do not simulate and test different strategies to control unemployment. For instance, Peiró et al. [12] study the relation between output and unemployment concluding that they are countercyclical and their relation varies over time, gender and age; Canova et al. [13] study the relations among 10 variables (growth rates of oil prices, world trade, US gross domestic product (*GDP*), US nominal interest rate, and growth rates of output, industrial production, employment, consumption, investment and prices) and build a dynamic stochastic model with the objective to determine the possible relation between business cycles and institutional changes in Europe; De Nicco [14] builds a model to connect *GDP*, unemployment and Employment-At-Will Exceptions in USA concluding that they are positive to reduce unemployment; Volna [15] builds a theoretical differential equations based dynamical model with 11 variables (aggregate income, long-term real interest rate, production, capital, labor, technical progress, money supply, money stock determined by central bank, money demand, maturity premium, and expected inflation rate) and studies the possible presence of chaos, concluding that when chaos appears the evolution of the macroeconomic variables are unpredictable.

In the present paper the model proposed by Caselles et al. [11] is generalized (widened) to study unemployment control in a country through the demographic, economic and well-being variables. The here proposed generalization deals with an extension of the economic subsystem to introduce the unemployment rate and related variables into the model, and the consideration of

three *UN* well-being variables simultaneously: the Human Development Index (*HDI*), the Gender Empowerment Index (*GEM*) and the Gender Differentiation Index (*GDI*) [9].

Basically, the *HDI* is a summary measure of the average achievement made in some key dimensions of human development: a long and healthy life, being knowledgeable and enjoying a decent standard of living. The *GDI* measures the gender gap in human development achievements in three basic dimensions of human development (health, education and command) over economic resources. Finally, the *GEM* is a measure of inequalities between men's and women's opportunities in a given country. Thus, the model becomes a richer tool than the classical approaches that use only economic variables to face the reduction of the unemployment rate.

This new model has been fitted for the case of Spain in the 2002-2014 period, and several tentative scenarios and strategies have been tested in order to reduce this rate in the horizon of year 2025. Note that, as this model is stochastic, the estimations of the values of the considered variables in each period have been obtained with their respective confidence intervals specified by a maximum and a minimum value, which contains the corresponding real value with a given probability. This way to present results provides a measure of its reliability, unlike deterministic models [16]. Nevertheless, an obvious assumption is that when unthinkable (at the study time) events occur (at forecast times) the model maybe will need to be refitted or even redesigned.

The methodology used to construct the model is the General Modeling Methodology (*GMM*), proposed by Caselles [17,18], which is implemented by using *SIGEM*: the Complex-Systems Based Programmes' Generator. The *GMM* is a hypothetic-deductive methodology to construct complex models which contains Data Mining Methods in some of its phases.

The rest of this paper is organized as follows. Section 2 presents and explains the model in its different parts: demographic, economic and well-being subsystems. Section 3 shows the model validation for the case of Spain in the 2002-2014 period. Section 4 develops tentative strategies and scenarios. Finally, Section 5 presents some conclusions and suggestions for future research.

2. The mathematical model

The mathematical model is divided into three subsystems: demographic, well-being and economic. The links between these three subsystem are given in Fig. 1.

(Please Insert Figure 1 about here)

Note that the well-being variables (*HDI*, *GDI* and *GEM*) need other variables related with health or education to be computed. These variables arise from the disaggregation of the well-being subsystem [9].

2.1. The Demographic Subsystem

The basis of our demographic subsystem is the one provided by Caselles et al. [11], where a sex and age-structured von Foerster-McKendrick model is presented.

Fig. 2 shows a part of the Forrester Diagram [19] of the model, which provides the demographic subsystem disaggregated and linked with the well-being variables, and Table 1 shows the demographic variables used in this diagram. Note that the sub-indices $i=1$ and $i=2$ represents the variables related with females and males, respectively.

(Please insert Figure 2 about here)

(Please Insert Table 1 about here)

The equations are the following:

$$\frac{\partial POPL_i(t,x)}{\partial t} + c \frac{\partial POPL_i(t,x)}{\partial x} = (RINM_i(x) - REMI_i(x) - RDEA_i(WEL1(t), x)) \cdot POPL_i(t, x) \quad (1)$$

$$POPL_i(t, 0) = \int_0^\infty RFER_i(WEL2(t), x) \cdot POPL_2(t, x) dx + (RINM_i(0) - REMI_i(0) - RDEA_i(WEL1(t), 0)) \cdot POPL_i(t, 0) \quad (2)$$

$$POPL_i(t_0, x) = U_i(x) \quad (3)$$

In Eqs. 1, 2 and 3, t and x are the independent variables, time and age respectively, which the $POPL_i$ depend on. The constant c of Eq. (1) is the conversion constant between time and age, whose value is the unit, due to both variables are measured in years. Typically in a von Foerster-McKendrick model, the boundary conditions are given by the time functions of the $POPL_i$ at zero age, given by the births; and births are calculated through $RFER_i$. Meanwhile, the initial conditions are given, for both age-density populations, by age functions $U_i(x)$ at the considered initial time t_0 . In addition, the immigration flows are computed as the product $RINM_i(x) \cdot POPL_i(t, x)$, where the $RINM_i(x)$ depend only on age. The emigration flows are computed as the product $REMI_i(x) \cdot POPL_i(t, x)$, where the $REMI_i(x)$ also depend only on age. Both migration rates are computed through fitting functions of age at the initial time. This simplified hypothesis is supported by the good model validation results. In addition, the demographic rates ($RDEA_i$ and $RFER_i$) are computed as functions of time and of the well-being variable $WEL1$ and $WEL2$. This is a key hypothesis used by Caselles et al. [11] here brought: get the demographic rates depending on the well-being given by the *HDI-Hybrid*. This hypothesis has been used as well by Sanz et al. [10] with the *GDI* measuring well-being.

However, the hypothesis here considered to measure well-being is working simultaneously with three *UN* well-being variables, *HDI*, *GEM* and *GDI*. Concretely, in the case of the $RDEA_i$, the well-being variable is computed as $WEL1(t) = HDI(t) \cdot GDI(t) \cdot GEM(t)$, and in the case of the $RFER_i$, the well-being variable is computed as $WEL2(t) = HDI(t) \cdot GDI(t)$. These two different considerations to compute well-being are obtained by a process of trial and error to validate the model. This validation has been obtained through software tool *Regint* [18]. The validation is provided by high determination coefficient and random residuals. In addition, the fitting functions to the historical data produce random residuals that support both equations. The generic fitted functions corresponding to these variables with all their parameters are:

$$RDEA_i(wel1(t), x) = \begin{cases} \alpha_{i0} + \alpha_{i1} e^{-\frac{x}{\beta_{i1}}} + \sum_{j=2}^4 \alpha_{ij} e^{-\frac{(\frac{x}{wel1(t)} - \mu_{ij})^2}{2\beta_{ij}}} & 0 \leq x \leq 45 \\ \alpha_{i5} + \alpha_{i6} e^{-\frac{x}{\beta_{i6}}} + \sum_{j=7}^8 \alpha_{ij} e^{-\frac{(\frac{x}{wel1(t)} - \mu_{ij})^2}{2\beta_{ij}}} & 46 \leq x \leq 100 \end{cases} \quad (4)$$

$$RFER_i(wel2(t), x) = \alpha_{i0} + \alpha_{i1} e^{-\frac{(\frac{x}{wel2(t)} - \mu_{i1})^2}{2\beta_{i1}}} + \alpha_{i2} e^{-\frac{(\frac{x}{wel2(t)} - \mu_{i2})^2}{2\beta_{i2}}} \quad (5)$$

The details about the parameter values of the fitted functions by sex are shown in Appendix A. The dependence of the demographic rates on time (Eqs. 4 and 5) through the *WEL1* and *WEL2* variables and on age seems to be a general pattern. It will be confirmed once this model can be validated for other countries. In fact, this pattern is also observed in Caselles et al. [11] for the dependence of the demographic rates on the *HDI-Hybrid* variable and on age. Concretely, for the fertility rates, the pattern consists of a sum of Gaussian functions (Eq. 5), and for the death rates the pattern is similar but structured as a piecewise function (Eq. 4).

2.2. The Economic Subsystem

Fig. 3 shows the corresponding Forrester Diagram to this subsystem, where its variables are emphasized in relation to the other subsystems. Note that the unemployment rate is calculated through Demographic subsystem variables and, the demographic variables are calculated with well-being variables. Table 2 defines the variables shown in Fig. 3.

(Please Insert Figure 3 about here)

(Please Insert Table 2 about here)

Note that the variables added to the economic subsystem are the main variables considered to obtain the unemployment rate. Also note that business cycles affecting input variables and some endogenous ones (those whose structure are not evident) have been generally modeled by cosine functions. However, as Canova et al. [13] detect, the characteristics of real fluctuations may change over time (maybe as a consequence of global shocks). This behavior is observed in some of the model variables, what leads us to model them as a combination of logistics instead of cosine functions.

GDPR is calculated as:

$$GDPR(t) = FCEX(t) + GRCF(gdpd) + EBSE(xvnj, xmov, xvco) - IBSE(xnvj, xmov) \quad (6)$$

XNVJ and *XMOV* are input variables but *NAIC* is calculated through *RDUP*, which is an input variable.

$$RUNM(t) = \frac{XUNP(t)}{EAPO(t)} \cdot 100 \quad (7)$$

$$XUNP(t) = EAPO(t) - PWOR(t) \quad (8)$$

$$EAPO(t) = FEWO(t) + MAWO(t) \quad (9)$$

$$FEWO(t) = RFFE(t) \cdot PWFE(t) \quad (10)$$

$$MAWO(t) = RFMA(t) \cdot PWMA(t) \quad (11)$$

$$PWOR(t) = JOBS(t) \cdot CUCP(t) \quad (12)$$

$$JOBS(t) = \frac{CTOI(t)}{XVNI(t) \cdot GDPD(t)} \quad (13)$$

$$CTOT(t) = CTOI(t) + GRCF(gdpd) - COCF(t) \quad (14)$$

$$COCF(t) = CCPC(t) \cdot POPL(t) \quad (15)$$

So far we have introduced the variables used for the direct calculation of the unemployment rate. Now we add three major variables for the economy of any country that complete the economic subsystem and affect the unemployment rate, and that we name here again due to their importance: National income per capita (*NAIC*), Public debt (*PUDE*) and National patrimony (*XPAT*). To calculate these three variables we need a list of input and auxiliary variables that we show below.

- For *NAIC*:

$$NAIC(t) = NAIN(t)/POPL(t) \quad (16)$$

$$NAIN(t) = GRNI(t) + COCF(t) \quad (17)$$

$$GRNI(t) = XWSA(t) \cdot EAPO(t) + XVIM(t) - NPIN(t) + GRSU(t) \quad (18)$$

GRSU is an input variable, *XWSA* and *XVIM* are calculated in Eqs. (19) and (20) respectively.

$$XWSA(t) = HOWO(t) \cdot NUEM(t) \cdot PWOH(t) \cdot e^{-7} \quad (19)$$

$$XVIM(t) = IMTA(t) \cdot IBSE(xvni, xmov) \quad (20)$$

HOWO, *PWOH* and *IMTA* are input variables, and *NUEM* is calculated in Eq. (21).

$$NUEM(t) = PWOH(t) \cdot EMOP(t) \quad (21)$$

Hence wages and salaries without quotes are calculated as:

$$XSAO(t) = XWSA(t) - XSCC(t) \quad (22)$$

Where

$$XSCC(t) = HOWO(t) \cdot NUEM(t) \cdot PWOH(t) \cdot e^{-7} \cdot 0.227 \quad (23)$$

- For *PUDE*:

$$PUDE(t) = PUDI(t) - CNFI(t) - AMDP(t) \quad (24)$$

Where *PUDI* and *AMDP* are input variables.

$$CNFI(t) = RECC(t) - CAJO(t) \quad (25)$$

$$RECC(t) = XVIM(t) + TAIN(t) + XVAT(t) + TAXO(t) \quad (26)$$

$$CAJO(t) = GPUB(t) + REST(t) \quad (27)$$

$$TAIN(t) = XRPF(t) \cdot GRNI(t) \quad (28)$$

$$GPUB(t) = GRCF(gdpd) - GPRI(gdpd) \quad (29)$$

Hence the degree of nationalization calculated through *GPRI* and *GPUB*.

$$XAC1(t) = \left(\frac{GPUB(t)}{GRCF(gdpd)} \right) \cdot 100 \quad (30)$$

$$XAC2(t) = \left(\frac{GPRI(t)}{GRCF(gdpd)} \right) \cdot 100 \quad (31)$$

And the Total tax:

$$TAXT(t) = XVIM(t) + XVAT(t) + TAIN(t) \quad (32)$$

- For *XPAT*:

$$XPAT(t) = XAHO(t) + XCAT(t) \quad (33)$$

$$XAHO(t) = NAIN(t) - DOCO(t) \quad (34)$$

Where *DOCO* is an input variable.

The details of the model equations (with its stochastic formulation) are shown in Appendix A.

3. Model validation

The historical data used in this article to fit the model have been obtained from the Spanish National Statistics Institute database [20] in the 2002-2014 period (the largest one with information about all the variables considered inside the model). The model fitting and verification is performed for both the deterministic and the stochastic formulations of the model.

The deterministic verification has been carried out by writing the model as a set of finite difference equations, and its solutions have been calculated with the Euler approach. This approach is chosen because it provides results that do not accumulate too many calculation errors, as Djidjeli et al. [21] demonstrate. These authors make a comparison with the Runge-Kutta method and deduce that the latter method often produced false results. Moreover, Letellier et al. [22] follow Euler's method and show by means of adequate time discretization that the solutions of the discretized model are equivalent to those of the continuous model.

The software tool used for the model verification is *SIGEM*. The verification is performed for the 2002-2014 period, and it is considered successful for three reasons:

- The visual evaluation of the graphic overlapping of the historical data and the calculated data is satisfactory (see Fig. 4).
- The determination coefficients, R^2 , are very high.
- The randomness of the residuals is verified by the maximum relative error, which do not exceed the 5%.

(Please insert Figure 4 about here)

The procedure to verify that the stochastic formulation of the model is acceptable is the following: (a) Observing that all results have a normal distribution (for this purpose, *SIGEM* automatically programs a χ^2 test); (b) Creating (for instance) 99% confidence intervals for every result and checking that 11 out of 12 points are within the interval, and that the only point that is outside the interval, is very close to it (see Fig. 5).

(Please insert Figure 5 about here)

The stochastic model is used to simulate the future because it allows determining the reliability of the results (each result is obtained with its confidence interval or with its respective mean value and standard deviation).

To strengthen the validation, results on some economic variables of the model are compared with the analogous ones presented by the own Spanish Government through [20]. The chosen variables to perform such comparison are the rate of growing of GDP, the unemployment rate, and the ratio (public debt)/GDP (Figs. 6 and 7).

$$\text{rate of growing of GDP}(t) = 100 - \frac{GDPR(t)}{GDPR(t+1)} \cdot 100 \quad (35)$$

$$(\text{public debt})/GDP(t) = \frac{PUDE(t)}{GDPR(t) \cdot POPL(t)} \cdot 100 \quad (36)$$

(Please insert Figures 6 to 7 about here)

Tables 3 to 5 present the trend previsions for these variables, performed by world and Spanish institutions jointly with the corresponding ones performed with our model. In these tables, BE=Banco de España (Central Bank of Spain), CEOE=Confederación Española de Organizaciones Empresariales (Spanish companies confederation), EC=European Commission, IMF=International Monetary Fund, and MEP=Spanish economy ministry.

(Please insert Tables 3 to 5 about here)

4. Forecasts

Fig. 8 and Table 6 show the maximum and the minimum simulated values of unemployment rate with a 99% confidence interval as well as their time tendencies.

(Please insert Figure 8 about here)

(Please insert Table 6 about here)

As a tentative exercise to try to reduce the unemployment rate trend, we classify the input variables into control variables and scenario variables (non-controllable ones). We design four strategies with those variables that seem controllable and three scenarios (Expansive (1), Recessive (2) and Trend (3)) with the remaining ones. See Tables 7 and 8, where \uparrow means increase (2% above tendency), \downarrow means decrease (2% below tendency), and \approx means keep tendency.

Strategy 1: to reduce public expenditures and taxes (2% below its extrapolated tendency), and to increment Research and Development (RD), investment and quality of work (2% above its extrapolated tendency).

Strategy 2: to reduce public expenditures, RD, investment and quality of work (2% below its extrapolated tendency), and to increment taxes (2% above its extrapolated tendency).

Strategy 3: to reduce public expenditures, investment and quality of work (2% below its extrapolated tendency), and to increment taxes and RD (2% above its extrapolated tendency).

Strategy 4: to keep the trend in all control variables.

(Please insert Tables 7 and 8 about here)

At a starting point, the concrete objective has been to decrease the unemployment rate in the horizon of the 2025 year. The corresponding objective-variable is named as $RUNM_{ij}^k$. This variable represents the unemployment rate in each year k of the simulated period, with $i \in \{1,2,3,4\}$ describing strategies and $j \in \{1,2,3\}$ describing scenarios. The variable to optimize is given by Eq. (37).

$$RUNM_i^k = \sum_j RUNM_{ij}^k * p_j \quad (37)$$

Where p_j is the probability that experts assign to scenario j . In this case, we tentatively assign $p_1=p_2=p_3=1/3$.

The corresponding calculations are performed with the simulator generated by *SIGEM* and with *MATHEMATICA* 10.4 [23]. The optimal strategy to reach our goal is chosen by observing the evolution of $RUNM_i^k$ ($i=1$ to 4) ($k=2015$ to 2025) (see Table 9). The conclusion is that Strategy 2 makes minimum the unemployment rate in the horizon of the 2025 year. This conclusion is related with the probabilities assigned to the three scenarios (see Eq. (37)). The greatest decrease in unemployment rate corresponds to Scenario 1 (see Table 10).

(Please insert Tables 9 and 10 about here)

As an instance of other consequences of Strategy 2 and Scenario 1, Figs. 9 to 12 show the corresponding forecast evolution of the well-being variable (*XIDG*), National income per capita (*NAIC*), Public debt (*PUDE*) and the ratio (public debt)/GDP. Therefore, it is determined that *XIDG* and *NAIC* grow, but *PUDE* and Ratio (public debt)/GDP decrease, in the 2015-2025 period. Note that the values are presented in stochastic formulation.

(Please insert Figures 9 to 12 about here)

5. Conclusion

An abstract complex demographic-economic model has been presented and used to attempt to control the unemployment rate evolution in a country as well as its main related variables. The model includes three well-being variables defined by the United Nations (*HDI*, *GDI* and *GEM*) that consider key concepts for human development, as Health, Education, Economy and Female

Labor, as well as detailed demographic and economic sub-models. Some tentative strategies and scenarios have been designed to attempt to find the best feasible strategy (within the control variables considered in the model) to reduce unemployment in a country and to determine the collateral consequences (over the other variables considered in the model: economical, demographical and wellbeing ones) of these strategy.

The study has been performed with two model formulations: deterministic (for simplicity) and stochastic (to obtain confidence intervals for forecasts). Both formulations of the model have been fitted and verified with the corresponding criteria and real data from Spain in the 2002-2014 period (the only period with enough adequate data). The rate of growing of GDP, the unemployment rate, and the ratio (public debt)/GDP have been chosen to strengthen the model validation, by comparing the obtained values with those given by other institutions as OECD, European Commission, IMF and Spanish Government among others.

The most relevant input variables in relation with unemployment have been selected from literature and have been used in a tentative simulation exercise to attempt to reduce the unemployment rate in Spain in the horizon of the 2025 year. Three scenarios (built with non-controllable input variables) and four strategies (built with controllable input variables) have been considered in this exercise. The conclusion of the simulation exercise has been that the lowest unemployment rate is provided by Strategy 2 (reducing public expenditures, research and development, investment and quality of work, and increasing taxes) under Scenario 1 (expansive scenario).

For future research we expect to obtain the information required to apply this model to other countries, as well as to increase the level of description detail of the model and to perform other kinds of experiments. Let us remark that the simulations presented in this paper are only one of the possible experiments achievable with the model.

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APPENDIX A

List of functions involved in the model and corresponding to the application case

(This is a part of an input file for *SIGEM*).

(Please insert Table A.1 about here)

- DEMOGRAPHIC SUBSYSTEM

```
RFER1 h=0.8240352596128939+47.591690689802576*Exp(-0.008963540065121161*((i1-18)/(xhdi*xgdi*xgem))^2)+7.5294385339624395*Exp(-0.03802752727011444*((i1-18)/(xhdi*xgdi*xgem))^2)
A=Exp(-0.008963540065121161*((i1-18)/(xhdi*xgdi*xgem))^2)-0.300330:B=Exp(-0.03802752727011444*((i1-18)/(xhdi*xgdi*xgem))^2)-0.145785
s=1.262584*sqr(1+1/37+0.238666336561*A*A+0.356233070691*B*B+2*0.078850706692*A*B)
RFER2 h=0.26227914737865154+50.89337873225642*Exp(-0.008417296444939507*((i1-18)/(xhdi*xgdi*xgem))^2)+8.163684806681216*Exp(-0.03532695012060796*((i1-7)/(xhdi*xgdi*xgem))^2)
A=Exp(-0.008417296444939507*((i1-18)/(xhdi*xgdi*xgem))^2)-0.309912:B=Exp(-0.03532695012060796*((i1-7)/(xhdi*xgdi*xgem))^2)-0.151036
s=1.112388*sqr(1+1/37+0.246446049586*A*A+0.360493443158*B*B+2*0.098498672186*A*B)
RDEA2 if i1<46 then
h=3.442781465438052e-001+1.010569531656582*Exp(-0.004058935532574176*((i1-51)/(xhdi*xgdi))^2)+(-8.518539115880452e-002)*Exp(-0.0010681230415243226*((i1-22)/(xhdi*xgdi))^2)+(-1.758461572788987e-001)*Exp(-0.010929963115862426*((i1-9)/(xhdi*xgdi))^2)+3.526298355508211*Exp(-2.2003124412745194*(i1-1))
A=Exp(-0.004058935532574176*((i1-51)/(xhdi*xgdi))^2)-0.143305:B=Exp(-0.0010681230415243226*((i1-22)/(xhdi*xgdi))^2)-0.792239:C=Exp(-0.010929963115862426*((i1-9)/(xhdi*xgdi))^2)-0.294473:D=Exp(-2.2003124412745194*(i1-1))-0.023952
s=0.034837*sqr(1+1/46+2.121867293689*A*A+3.122321863006*B*B+0.415664035134*C*C+1.521704042799*D*D+2*(2.205012537556)*A*B+2*(0.724434154456)*A*C+2*(0.946458617069)*A*D+2*(0.773834300298)*C*B+2*(1.236769940397)*B*D+2*(0.314441109020)*C*D)
else
h=4.023001258772503+(2.563879960148310e+002)*Exp(0.08707606339844953*((i1-96)/(xhdi*xgdi)))+(-1.360251354357019e+001)*Exp(-0.002410901549920305*((i1-71)/(xhdi*xgdi))^2)
A=Exp(0.08707606339844953*((i1-96)/(xhdi*xgdi)))-0.215052:B=Exp(-0.002410901549920305*((i1-71)/(xhdi*xgdi))^2)-0.563091
s=5.120049*sqr(1+1/54+0.319569970349*A*A+0.274473660210*B*B+2*(0.146935822453)*A*B)
endif
RDEA1 if i1<46 then
h=1.630059960279515+1.8313843087925823*Exp(-0.007314643477456073*((i1-51)/(xhdi*xgdi))^2)+0.3403974191295738*Exp(-0.047170721779348175*((i1-21)/(xhdi*xgdi))^2)-1.4837894021370026*Exp(-0.0012600706804197506*((i1-9)/(xhdi*xgdi))^2)+5.21538682806462*Exp(-2.8274163515709*(i1-1))
A=Exp(-0.007314643477456073*((i1-51)/(xhdi*xgdi))^2)-0.082214:B=Exp(-0.047170721779348175*((i1-21)/(xhdi*xgdi))^2)-0.151832:C=Exp(-0.0012600706804197506*((i1-9)/(xhdi*xgdi))^2)-0.632246:D=Exp(-2.8274163515709*(i1-1))-0.023008
s=0.040841*sqr(1+1/45+1.650066994209*A*A+0.286034492034*B*B+0.463779933775*C*C+1.053611833693*D*D+2*(0.093872502707)*A*B+2*(0.614920181449)*A*C+2*(-0.010513495673)*A*D+2*(-0.025016423518)*C*B+2*(0.061070900538)*B*D+2*(-0.078173233376)*C*D)
else
h=-41.02968411304129-0.29734721220768406*Exp(-0.11742647060363397*((i1-67)/(xhdi*xgdi))^2)+45.57570691741214*Exp(0.0007755934844817768*((i1-52)/(xhdi*xgdi))^2)-11.372001295821937*Exp(0.2766368369954939*(i1-91)/(xhdi*xgdi))
A=Exp(0.2766368369954939*(i1-91)/(xhdi*xgdi))-1.264318:B=Exp(-0.11742647060363397*((i1-67)/(xhdi*xgdi))^2)-0.078735:C=Exp(0.0007755934844817768*((i1-52)/(xhdi*xgdi))^2)-3.042968
```

```

s=3.421862*sqr(1+1/55+0.006096889503*A*A+0.391063513912*B*B+0.009381775830*C*C+
2*(-0.006178935102)*A*B+2*(-0.006607350595)*A*C+2*(0.012896035940)*C*B)
endif

```

• INPUT VARIABLES

```

FCEX h= (6.4688568941910681e11+(-1.930955180139670e11)*Sin(0.261799*(temi+1)))
A = Sin(0.261799*(temi+1))+ 0.292144
s = 10982483169.780403* Sqr(1 + 1 / 13 + 0.177290142327* A * A)
CUCP h = 1.6691 / (1 + 1.57344 * Exp(-0.827016 * (-2006 + (TEMI + 1)))) + 0.999558 / (1
+ 0.00421719 * Exp(1.33067 * (-2002 + (temi + 1))))
A = 1 / (1 + 1.57344 * Exp(-0.827016 * (-2006 + (temi + 1)))) - 0.540228: B = 1 /
(1 + 0.00421719 * Exp(1.33067 * (-2002 + (temi + 1)))) - 0.419184
s = 0.0183 * Sqr(1 + 1 / 11 + 0.217328110242 * A * A + 0.267900864318 * B * B + 2
* (-0.044009159217) * A * B)
XVCO h= (3.108909333658162e+002)+(-1.540082923866549e+002)*Cos(0.357658*(temi+1))
A = Cos(0.357658*(temi+1)) + 0.146646
s = 54.911525 * Sqr(1 + 1 / 13 + 0.134247493860* A * A)
CCPC h= (-33332.3 + 33368.5/( 1 + 0.00116093*Exp(-0.0178892*(-2008 + (temi+1)))) +
5.23492/( 1 + 0.194053*Exp(0.960632*(-2002 + (temi+1)))))*100
XMOV h=(1.046844186664150)+(3.544767413316030e-1)*Sin(0.241661*(temi+1))
A = Sin(0.241661*(temi+1))- 0.863301
s = 0.093438* Sqr(1 + 1 / 14 + 3.214132058206* A * A)
RFFE h= (4.913833333333335e1+(-5.157416116316625)*Cos(0.285599*(temi+1)))/100
A = Cos(0.285599*(temi+1))- 0.612031
s = 1.674584* Sqr(1 + 1 / 12 + 0.617661420402* A * A)
RFMA h= (6.660600645448993e1+(2.563268289155783)*Sin(0.285599*(temi+1)))/100
A = Sin(0.285599*(temi+1))- 0.588031
s = 1.041293* Sqr(1 + 1 / 12 + 0.575825100577* A * A)
EMOP h=((8.648557383176074e-1)+(2.439194827818366e-2)*Cos(0.224399*(temi+1)))
A = Cos(0.224399*(temi+1))- 0.441416
s = 0.004494* Sqr(1 + 1 / 15 + 0.510296026398* A * A)
PWOH h=((1.357438571736864e1)+(2.170629992990047)*Cos(0.224399*(temi+1)))
A = Cos(0.224399*(temi+1))- 0.441416
s = 0.563319* Sqr(1 + 1 / 15+ 0.510296026398* A * A)
HOWO h=((1.659234769580475e3)+(-5.143168529094768e1)*Sin(0.157080*(temi+1)))
A = Sin(0.157080*(temi+1))+ 0.934030
s = 30.409427* Sqr(1 + 1 /21+ 8.998005331815* A * A)
NPIN h=(0.07+ 0.32/(1 + 0.391042*Exp(0.500687*(-2008 + (temi+1)))) - 0.264132/(1 +
0.00122894*Exp(1.30777*(-2001 + (temi+1)))))*10000000000
A = 1/(1 + 0.391042*Exp(0.500687*(-2008 + (temi+1))))- 0.762645:B=1/(1 +
0.00122894*Exp(1.30777*(-2001 + (temi+1))))- 0.509568
s =0.034579 * Sqr(1 + 1 /13+5.986162882426* A * A+1.898318667406*B*B+2*(-
2.992353584981)*A*B)
GRSU h=((3.522564937208650e11)+(-1.047821126705812e11)*Sin(0.261799*(temi+1)))
A = Sin(0.261799*(temi+1))- 0.304057
s = 9999907012.577366 * Sqr(1 + 1 /13+ 0.179332736306* A * A)
IMTA h=(0.4/(1 + 15.789*Exp(0.484268*(-2008 + (temi+1)))) + 0.283242/(1 +
0.409622*Exp(-1.10303*(-2003 + (temi+1))))))
A = 1/(1 + 15.789*Exp(0.484268*(-2008 + (temi+1)))) -0.250558:B=1/(1 +
0.409622*Exp(-1.10303*(-2003 + (temi+1))))- 0.789677
s =0.016492* Sqr(1 + 1 /13+0.735203856908* A * A+0.127281098509*B*B+2*(-
0.124008315377)*A*B)
AMDP h=(-12.9053 + 50/( 1 + 0.480767*Exp(-0.355124*(-2009 + (temi+1)))) + 10/( 1 +
0.0116166*Exp(1.39817*(-1998 + (temi+1)))))*10000000000
A = 1/( 1 + 0.480767*Exp(-0.355124*(-2009 + (temi+1))))- 0.388670:B=1/( 1 +
0.0116166*Exp(1.39817*(-1998 + (temi+1))))- 0.246023
s =9.679804* Sqr(1 + 1 /15+1.853639253063* A *
A+1.060501381976*B*B+2*(1.034765802719)*A*B)
REST h=(-38.5278 + 150/( 1 + 0.08573*Exp(-0.363478*(-2009 + (temi+1)))) + 80/( 1 +
0.019483*Exp(1.044*(-1998 + (temi+1)))))*10000000000
A = 1/( 1 + 0.08573*Exp(-0.363478*(-2009 + (temi+1))))- 0.678374:B=1/( 1 +
0.019483*Exp(1.044*(-1998 + (temi+1))))- 0.267672
s =5.720352 * Sqr(1 + 1 /16+8.922332659580* A *
A+4.890881037109*B*B+2*(6.289841367491)*A*B)

```

XVAT $h=(15/(1 + 3.94074*Exp(0.397063*(-2009 + (temi+1)))) + 24/(1 + 2.27342*Exp(-0.153196*(-2001 + (temi+1)))))*10000000000$
 $A = 1/(1 + 3.94074*Exp(0.397063*(-2009 + (temi+1)))) - 0.502659$:B=1/(1 + 2.27342*Exp(-0.153196*(-2001 + (temi+1)))) - 0.470634
 $s = 44.167900 * Sqr(1 + 1 / 16 + 0.281492374458 * A * A + 0.405378870214 * B * B + 2 * (-0.208083277352) * A * B)$

TAXO $h=(160/(1 + 23.6536*Exp(-1.21142*(-2009 + (temi+1)))) + 80/(1 + 0.0394985*Exp(0.347823*(-2003 + (temi+1)))))*1000000000$
 $A = 1/(1 + 23.6536*Exp(-1.21142*(-2009 + (temi+1)))) - 0.178568$:B=1/(1 + 0.0394985*Exp(0.347823*(-2003 + (temi+1)))) - 0.766934
 $s = 11.380975 * Sqr(1 + 1 / 11 + 0.945386203375 * A * A + 0.168615714197 * B * B + 2 * (-0.142183837635) * A * B)$

XRPF $h=0.31/(1 + 0.975933*Exp(-0.483402*(-2010 + (temi+1)))) + 0.370385/(1 + 0.0129316*Exp(0.791497*(-2003 + (temi+1))))$
 $A = 1/(1 + 0.975933*Exp(-0.483402*(-2010 + (temi+1)))) - 0.348595$:B=1/(1 + 0.0129316*Exp(0.791497*(-2003 + (temi+1)))) - 0.543724
 $s = 0.012480 * Sqr(1 + 1 / 16 + 0.520693424463 * A * A + 0.233837643824 * B * B + 2 * (-0.110294599797) * A * B)$

DOCO $h=((8.520968735788187e11)+(-1.618950457014195e11))*Cos(0.448799*(temi+1))$
 $A = Cos(0.448799*(temi+1)) - 0.512591$
 $s = 9391237383.118979 * Sqr(1 + 1 / 8 + 0.619256829494 * A * A)$

XCAT $h=(6.914637829932951e9)+(-2.185251956776419e9)*Cos(0.650758*(temi+1))$
 $A = Cos(0.650758*(temi+1)) - 0.080889$
 $s = 432355283.504294 * Sqr(1 + 1 / 8 + 0.225325862212 * A * A)$

GRFE $h=(-4.743242389328714) + (1.048943855340064e+002)/(1 + 0.0557828*Exp(-0.649628*(-2002 + (temi+1))))$
 $A=1/(1 + 0.0557828*Exp(-0.649628*(-2002 + (temi+1)))) - 0.990615$
 $s=0.484934*sqr(1+1/12+351.359171375263*A*A)$

GRMA $h=(8.265876903617055e-001)+ (9.972155099977370e+001)/(1 + 0.0531139*Exp(-0.502242*(-2004 + (temi+1))))$
 $A=1/(1 + 0.0531139*Exp(-0.502242*(-2004 + (temi+1)))) - 0.971940$
 $s=1.387181*sqr(1+1/12+58.151838244150*A*A)$

RLIF $h=(9.747918420506343e+001)+ 2.103885023291785/(1 + 3664.17*Exp(-0.968704*(-2006 + (temi+1))))$
 $A=1/(1 + 3664.17*Exp(-0.968704*(-2006 + (temi+1)))) - 0.036871$
 $s=0.052105*sqr(1+1/12+29.727874587001*A*A)$

RLIM $h=100/(1 + 1.26157*Exp(-0.283777*(-2008 + (temi+1)))) + 99/(1 + 0.145705*Exp(0.286953*(-2002 + (temi+1))))$
 $A=1/(1 + 1.26157*Exp(-0.283777*(-2008 + (temi+1)))) - 0.424370$:B=1/(1 + 0.145705*Exp(0.286953*(-2002 + (temi+1)))) - 0.570093
 $s=0.036156*sqr(1+1/12+0.692562579011*A*A+0.419844263795*B*B+2*(-0.367548192044) * A * B)$

EPIF $h=(-5.700511938878761e+001) + (5.817465303809334e+001)/(1 + Exp(-5.66054 + 0.655496*(-2004 + (temi+1)))) + (8.941916034859274e+001)/(1 + Exp(2.89649-0.113565*(-1982 + (temi+1))))$
 $A=1/(1 + Exp(-5.66054 + 0.655496*(-2004 + (temi+1)))) - 0.951757$:B=1/(1 + Exp(2.89649-0.113565*(-1982 + (temi+1)))) - 0.286031
 $s=3.731899*sqr(1+1/32+3.701710981029*A*A+1.626437580248*B*B+2*(1.652187581833) * A * B)$

PAEF $h=(3.154663635790649e+001)+ (2.993608789244036e+001)/(1 + 35.1019*Exp(-0.427072*(-2007 + (temi+1))))$
 $A=(1/(1 + 35.1019*Exp(-0.427072*(-2007 + (temi+1)))) - 0.021311$
 $s=0.517329*sqr(1+1/13+102.150772109747*A*A)$

PPPF $h=175541+ 98654/(1 - 1.56214*Exp((1.43312e-6)*(-1998 + (temi+1))))$
 $A=1/(1 - 1.56214*Exp((1.43312e-6)*(-1998 + (temi+1)))) + 1.778882$
 $s=0.402582*sqr(1+1/13+109478353.13997672*A*A)$

SNAF $h=(1.748840130607686e4+(2.511346006913204 e3)*Sin(0.485202*(temi+1)))$
 $A = Sin(0.485202*(temi+1))+0.090263$
 $s = 0.093438 * Sqr(1 + 1 / 10 + 0.173966951127 * A * A)$

SNAM $h=(2.326657771263647e4+(2.371198353433891e3)*Sin(0.485202*(temi+1)))$
 $A = Sin(0.485202*(temi+1))+0.090263$
 $s = 213.069314 * Sqr(1 + 1 / 10 + 0.173966951127 * A * A)$

- ECONOMIC SUBSYSTEM

```

PWFE pwfe=0:for il=14 to 65: pwfe = pwfe +pomi(il):next
PWMA pwma=0:for il=14 to 65: pwma = pwma +pohi(il):next
GRCF h=(2.994963627378788+(1.083845989945927)*Sin(15.662469*gdpd))*10000000000
A = Sin(15.662469*gdpd)+ 0.483959
s =0.323260* Sqr(1 + 1 / 12 + 0.801160325394* A * A)
EBSE
h=4.490610525068581e10+(5.344307025854781e8)*(xuco)+(9.044549458741611e10)*Sen(0.000000*
xuco*xvnj*xmov)
A = xuco- 320.989091: Sin(0.000000*xuco*xvnj*xmov)- 0.000055
s= 30279993955.138332* Sqr(1 + 1 / 11 + 0.000011255850* A * A+ 488105619*B*B+2*(
-51.335910428495)*A*B)
IBSE h=1.367926443345826e11+(9.237247334095247e10)*(xmov^2)-
(6.224140573039864e10)*Cos(0.000879*xvnj/xmov)
A= xmov^2-0.752813:B= Cos(0.000879*xvnj/xmov)-0.445071
s=0.009448*sqr(1+1/9+1.317903065844*A*A+1.872214134015*B*B+2*(-
1.104412969274)*A*B)
GDPD h=(9.668114200945517e-1)+(5.826478809101661e-2)*Cos(1.087971*rpud)
A = Cos(1.087971*rpud)+ 0.426912
s = 0.014636* Sqr(1 + 1 / 12 + 0.295419592334* A * A)
GPRI h=((2.852115770150065)+(1.207670814078404)*Cos(15.952514*(gdpd)))*100000000000
A = Cos(15.952514*(gdpd))+ 0.656618
s = 0.249934* Sqr(1 + 1 / 12 + 1.591182159329* A * A)
RUNM runm=(xunp/eapo)*100
FEWO fewo=rffe*pwfe*1000
MAWO mawo=rffma*pwma*1000
EAPO eapo=fewo+mawo
XUNP xunp=eapo-pwor
PWOR pwor=jobs*cucp
JOBS jobs=ctoi/(xvnj*gdpd)
CTOT ctot=ctoi+grcf-cocf
COCF cocf=ccpc*(popl1+popl2)
GDPR gdpr=(fcex+grcf+ebse-ibse)/(popl1+popl2)
XWSA xwsa=howo*nuem*pwoh*1e-7
XSCC xsc=howo*nuem*pwoh*1e-7*0.227
XSAO xsao=xwsa-xsc
NUEM nuem=pwor*emop
XCPI xcpi=100-(gdpr(t-1)/gdpr(t))*100
GRNI grni=xwsa*eapo+xvim-npin+grsu
XVIM xvim=imta*xibs
NAIN nain=grni+cocf
NAIC naic=nain/(popl1+popl2)
PUDE pude=pudi-cnfi-amdp
DEGD degd=100*pude/(gdpr*(popl1+popl2))
CAJO cajo=gpub+rest
CNFI cnfi=recc-cajo
RECC recc=xvim+tain+xvat+taxo
GPUB gpub=grcf-gpri
TAIN tain=xrpf*grni
TAXT taxt=xvim+xvat+tain
XAC1 xac1=(gpub/grcf)*100
XAC2 xac2=(gpri/grcf)*100
XAHO xaho=nain-doco
XPAT xpat=xaho+xcat

```


(Please insert these tables and figures inside text as indicated)

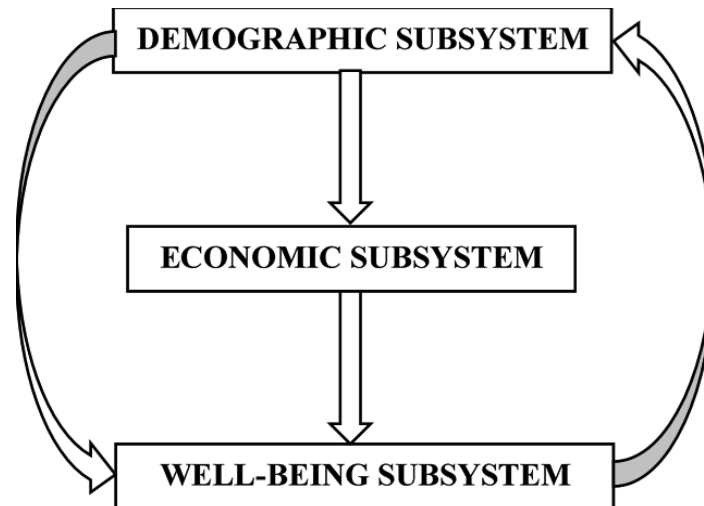


Fig. 1. Relationships between well-being, demographic and economic subsystems.

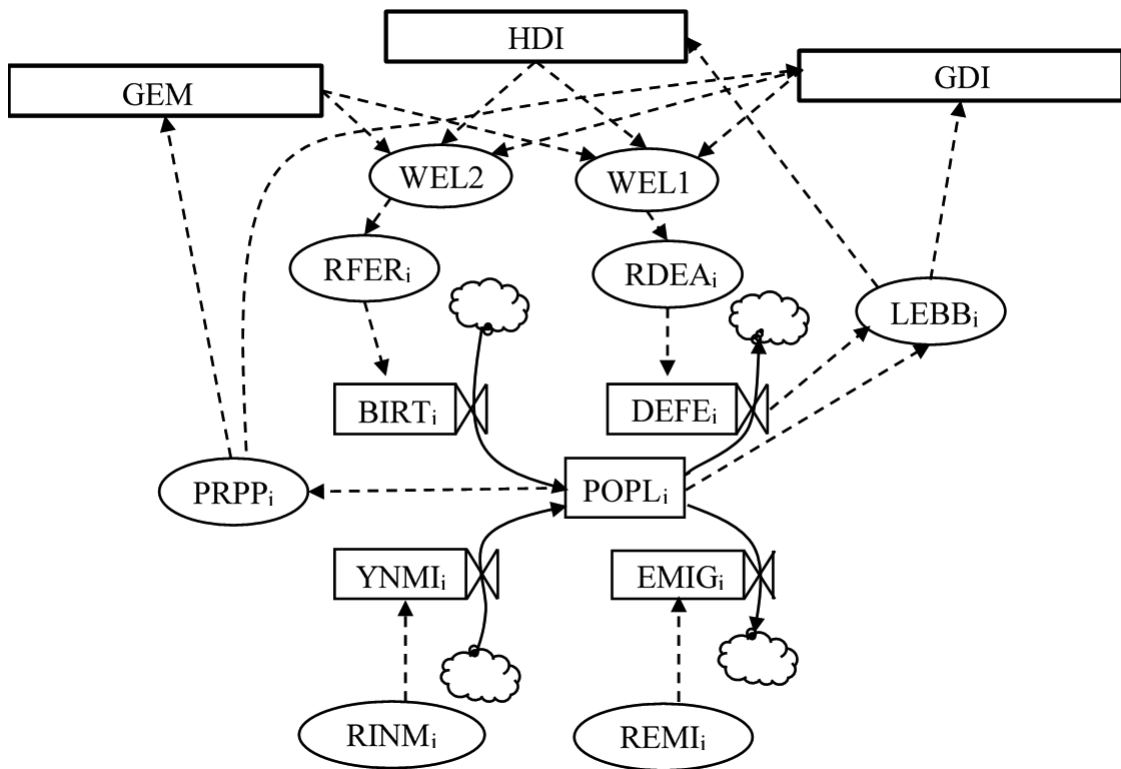


Fig. 2. Forrester Diagram of the demographic subsystem and the link with the well-being variables.

Table 1

Variables used in the demographic subsystem.

Variable	Definition
BIRT	Birth by year
DEFE	Death by year
EMIG	Emigrates by year
LEBB	Life Expectancy at Birth
POPL	Total Population
PRPP	Population proportion (Population by sex/Total Population)
RDEA	Death Rate
REMI	Emigration Rate
RFER	Fertility Rate
RINM	Immigration Rate
YNMI	Immigrates by year
WEL1	Well-being variables with <i>HDI</i> , <i>GDI</i> and <i>GEM</i>
WEL2	Well-being variables with <i>HDI</i> and <i>GDI</i>

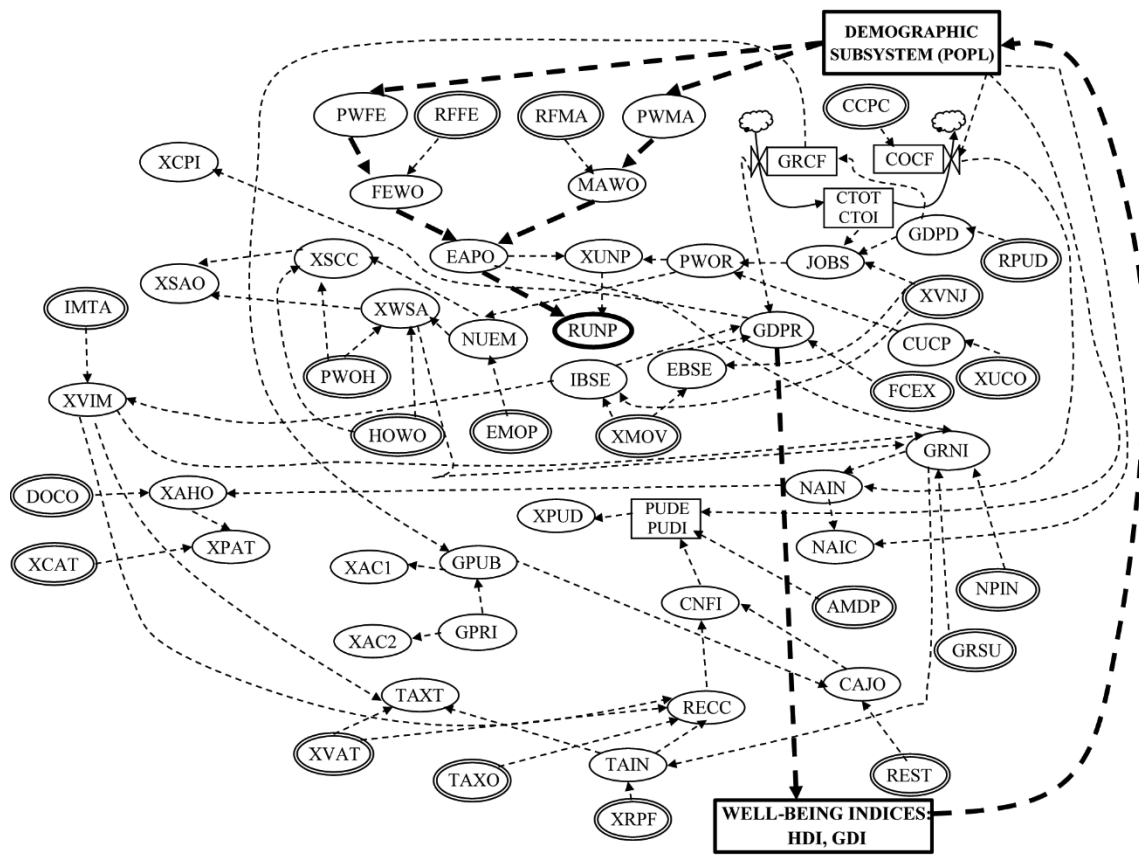


Fig. 3. Forrester Diagram of the economic subsystem.

Table 2
Variables used in the economic subsystem.

Variable	Definition	Variable	Definition
<i>AMDP</i>	Amortization	<i>PWMA</i>	Male population at working age
<i>CAJO</i>	Capital jobs	<i>PWOH</i>	Average price per worked hour
<i>CCPC</i>	Consumption of fixed capital per capita	<i>PWOR</i>	Number of people working
<i>CNFI</i>	Capacity or need of financing	<i>RECC</i>	Capital resources
<i>COCF</i>	Consumption of fixed capital	<i>REST</i>	Current transfers and acquisition of land and assets
<i>CTOI</i>	Initial consumption of fixed capital	<i>RFFE</i>	Female labor force rate
<i>CTOT</i>	Total fixed capital at the end of the current year	<i>RFMA</i>	Male labor force rate
<i>CUCP</i>	Coefficient of utilization of productive capacity	<i>NPIN</i>	Net property incomes
<i>DEGD</i>	The ratio (public debt)/GDP	<i>RPUD</i>	Interest rate on public debt
<i>DOCO</i>	Spending on domestic consumption	<i>RUNM</i>	Unemployment rate
<i>EAPO</i>	Economically active population	<i>EMOP</i>	Percentage of employees on the occupied population
<i>EBSE</i>	Goods and services exports	<i>TAIN</i>	Taxes related to income
<i>FCEX</i>	Final consumption expenditure	<i>TAXO</i>	Other taxes
<i>FEWO</i>	Number of female workers	<i>TAXT</i>	Total taxes
<i>GDPD</i>	Gross domestic product deflator	<i>XAC1</i>	PVIN/XOIN
<i>GDPR</i>	Gross domestic product	<i>XAC2</i>	PVIN/XOIN
<i>GPRI</i>	Gross formation of private fixed capital	<i>XAHO</i>	National savings
<i>GPUB</i>	Gross formation of public fixed capital	<i>XCAT</i>	Capital transfer
<i>GRCF</i>	Gross capital formation	<i>XCPI</i>	Gross Domestic Product increase rate
<i>GRNI</i>	Gross national income	<i>XMOV</i>	Monetary value (\$ for 1€)
<i>GRSU</i>	Gross surplus	<i>XPAT</i>	National patrimony
<i>HOWO</i>	Number of hours worked per year and per person	<i>XPUD</i>	Rate of public debt
<i>IBSE</i>	Goods and services imports	<i>XRPF</i>	Income tax rate
<i>IMTA</i>	Import tax rate	<i>XSAO</i>	Wages and salaries without quotes
<i>JOBS</i>	Number of jobs	<i>XSCC</i>	Social security quotation
<i>MAWO</i>	Number of male workers	<i>XUCO</i>	Unemployment compensation
<i>NAIC</i>	National income per capita	<i>XUNP</i>	Number of the unemployed people
<i>NUEM</i>	Number of employees	<i>XVAT</i>	Taxes related to VAT
<i>PUDE</i>	Public debt	<i>XVIM</i>	Taxes related to imports
<i>PUDI</i>	Initial Public debt	<i>XVNJ</i>	Average value of a new job
<i>NAIN</i>	National income	<i>XWSA</i>	Gross wages and salaries
<i>PWFE</i>	Female population at working age		

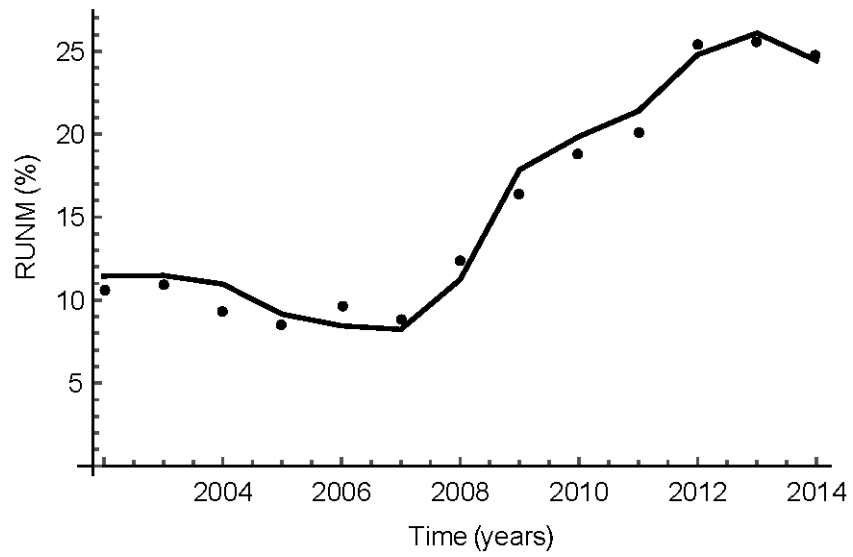


Fig. 4. Deterministic validation. Variable *RUNM (%)*, real data (dots), tendency values (line), for 2002-2014 period in Spain. $R^2=0.963313$.

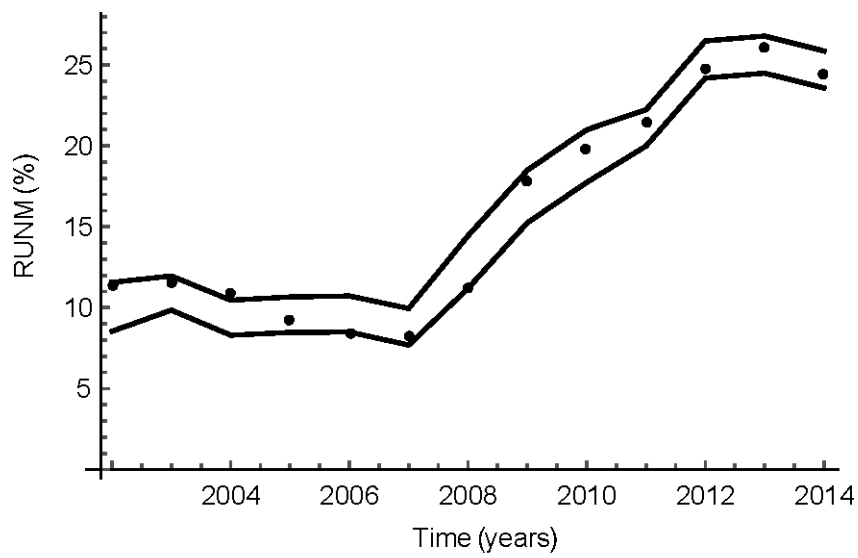


Fig. 5. Stochastic validation. Variable *RUNM (%)*, real data (dots), maximum and minimum values 99% confidence (lines), for 2002-2014 period in Spain.

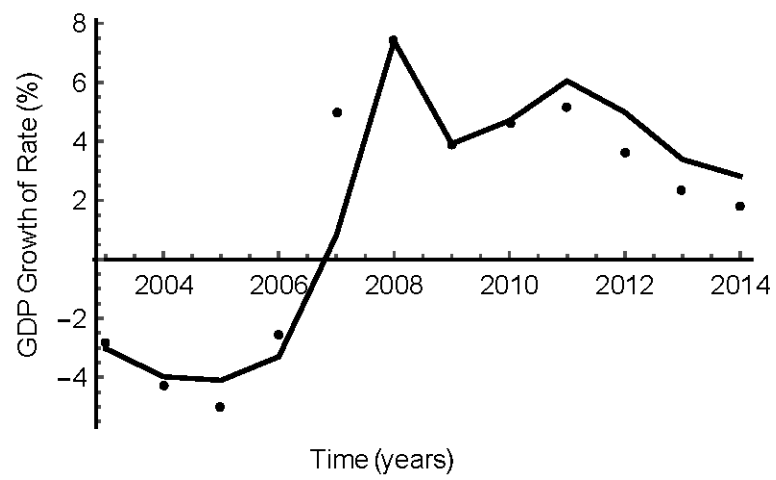


Fig. 6. Variable Rate of growth of GDP (%), real data (dots), tendency values (lines), for 2002-2014 period in Spain.

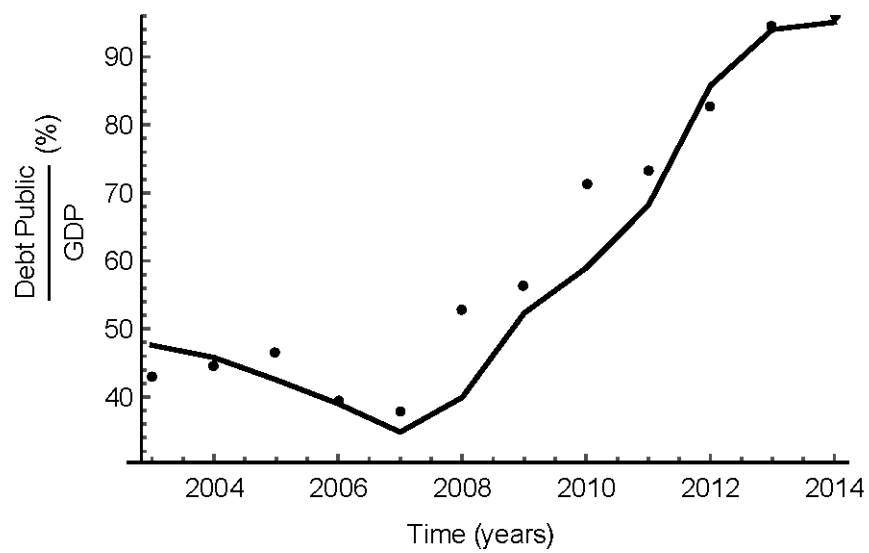


Fig. 7. Variable (public debt) / GDP (%), real data (dots), tendency values (lines), for 2002-2014 period in Spain.

Table 3
 Forecasts of *GDP growing rate (%)*.

Year	Model			BE	CEOE	EC	IMF	MEP	OECD
	Min	Mean	Max						
2015	3.75	3.82	4.18	3.1	2.8	2.8	3.1	2.9	3.2
2016	2.18	2.45	2.73	2.7	2.6	2.6	2.5	2.9	2.7
2017	1.35	1.70	2.05	--	--	--	--	3.0	2.5
2018	0.32	0.51	0.99	--	--	--	--	3.0	--

Table 4Forecasts of *Ratio (public debt)/GDP (%)*.

Year	Model			BE	CEOE	EC	IMF	MEP	OECD
	Min	Mean	Max						
2015	120.35	121.62	122.52	--	--	100.4	--	98.9	118.9
2016	111.56	112.93	113.84	--	--	100.4	--	98.5	118.7
2017	102.98	104.77	105.76	--	--	--	--	96.5	117.7
2018	97.65	98.78	100.02	--	--	--	--	93.2	--

Table 5
Forecasts of *Unemployment rate (%)*.

Year	Model			BE	CEOE	EC	IMF	MEP	OECD
	Min	Mean	Max						
2015	20.98	22.25	23.52	--	--	22.4	--	22.1	22.1
2016	19.08	20.34	22.60	--	--	20.5	--	19.8	19.8
2017	14.28	15.55	17.84	--	--	--	--	17.7	18.2
2018	11.63	12.93	14.23	--	--	--	--	15.6	--

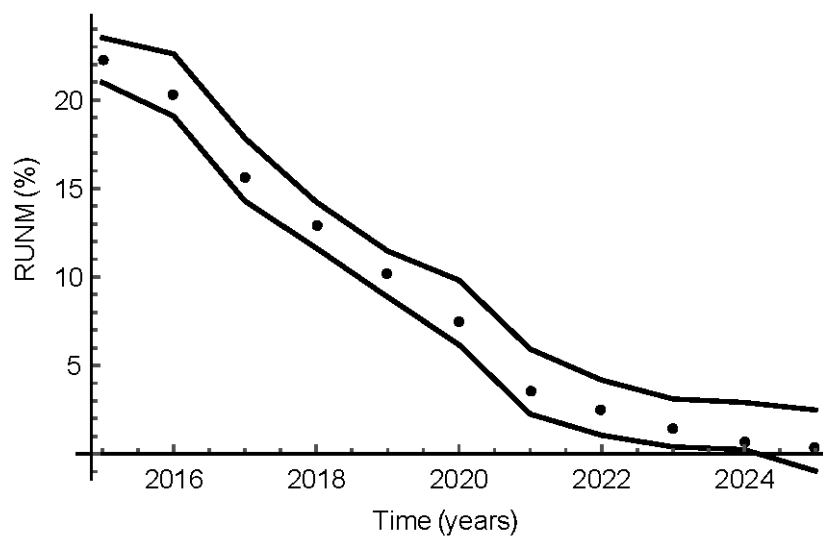


Fig. 8. Unemployment rate forecasts. Variable *RUNM* (%), tendency values (dots), maximum and minimum values 99% confidence (lines), for 2015-2025 period in Spain.

Table 6

Unemployment rate forecasts. Variable *RUNM (%)*: tendency values and maximum and minimum values (99% confidence) for 2015-2025 period in Spain.

Year	Minimum	Tendency	Maximum
2015	20.99	22.25	23.52
2016	19.08	20.34	22.60
2017	14.28	15.56	17.84
2018	11.63	12.93	14.23
2019	8.86	10.16	11.46
2020	6.18	7.49	9.81
2021	2.25	3.59	5.92
2022	1.05	2.56	4.17
2023	0.40	1.45	3.11
2024	0.25	0.72	2.91
2025	-0.97	0.32	2.48

Table 7

Strategies. ↑ increase, ↓ decrease and ≈ keep the tendency.

Variable	Strategy 1	Strategy 2	Strategy 3	Strategy 4
<i>XVNJ</i>	↑	↓	↑	≈
<i>XIDE</i>	↑	↓	↓	≈
<i>XMOV</i>	↓	↓	↑	≈
<i>PWOH</i>	↑	↓	↓	≈
<i>GPRI</i>	↑	↓	↓	≈
<i>AMDP</i>	↑	↓	↓	≈
<i>HOWO</i>	↓	↑	↑	≈
<i>GPUB</i>	↓	↑	↑	≈
<i>XVAT</i>	↓	↑	↑	≈
<i>XRPF</i>	↓	↑	↑	≈
<i>DOCO</i>	≈	≈	≈	≈
<i>XCAT</i>	≈	≈	≈	≈
<i>REST</i>	≈	≈	≈	≈

Table 8

Scenarios. ↑ increase, ↓ decrease and ≈ keep the tendency.

Variable	Scenario 1	Scenario 2	Scenario 3
<i>GRFE</i>	↑	≈	≈
<i>RLIF</i>	↑	≈	≈
<i>GRMA</i>	↑	≈	≈
<i>RLIM</i>	↑	≈	≈
<i>RPUD</i>	↓	↑	≈
<i>EPIF</i>	↑	≈	≈
<i>PAEF</i>	↑	≈	≈
<i>PPPF</i>	↑	≈	≈
<i>FCEX</i>	↓	↑	≈
<i>RFFE</i>	↑	≈	≈
<i>RFMA</i>	↑	≈	≈
<i>EMOP</i>	↑	≈	≈
<i>CUCP</i>	↑	≈	≈
<i>NPIN</i>	↑	↓	≈
<i>GRSU</i>	↑	↓	≈
<i>CCPC</i>	↑	↓	≈

Table 9Strategies. Variables $RUNM_i^k$ for Spain in the period 2015-2025.

Year	Strategy 1	Strategy 2	Strategy 3	Strategy 4
2015	26.74	23.66	26.74	25.23
2016	23.36	20.15	23.36	21.79
2017	21.49	18.21	21.50	19.88
2018	16.84	13.37	16.84	15.13
2019	14.28	10.70	14.28	12.53
2020	11.59	7.91	11.59	9.79
2021	8.99	5.20	9.00	7.14
2022	5.17	3.25	5.19	3.26
2023	3.16	1.97	3.18	2.86
2024	2.53	0.95	2.53	1.69
2025	1.60	0.32	1.61	0.91

Table 10Scenarios. Variable $RUNM_{2j}^k$ in Spain in the 2015-2025 period.

Year	Scenario 1	Scenario 2	Scenario 3
2015	22.68	25.16	23.15
2016	19.17	21.66	19.61
2017	17.28	19.68	17.66
2018	12.49	14.83	12.78
2019	9.93	12.08	10.10
2020	7.27	9.17	7.28
2021	4.72	6.34	4.55
2022	2.90	4.28	2.56
2023	1.18	2.05	1.70
2024	0.76	1.07	0.83
2025	0.28	0.52	0.45

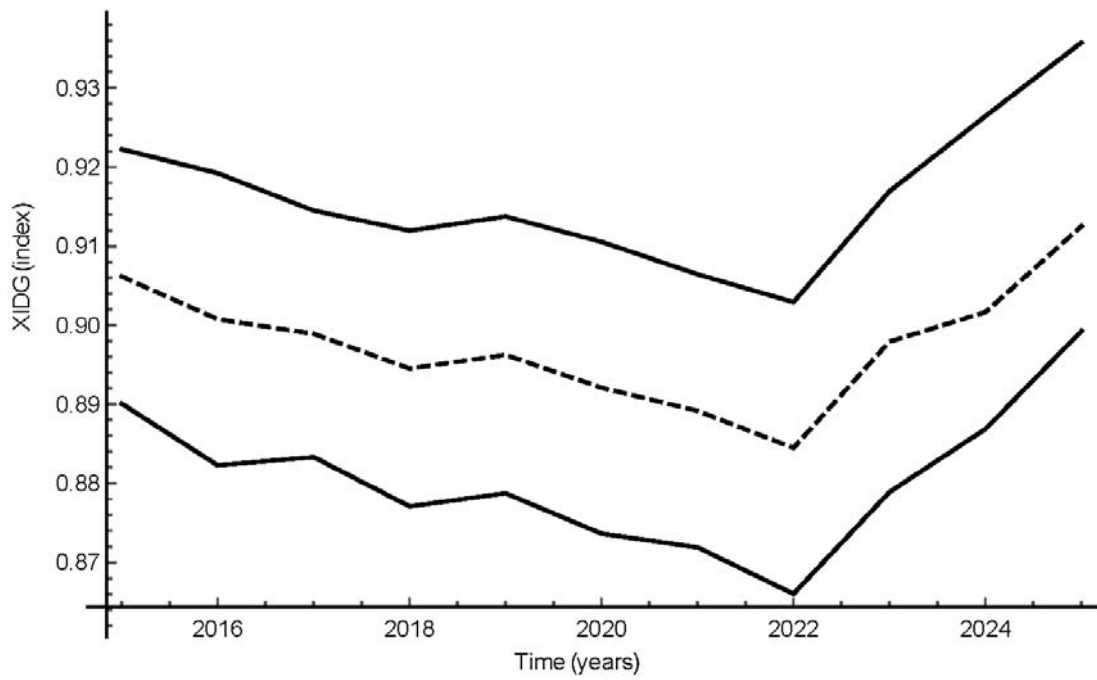


Fig. 9. Gender Development Index forecasts. Variable *XIDG*, tendency values (dots), maximum and minimum values 99% confidence (lines), for 2015-2025 period in Spain.

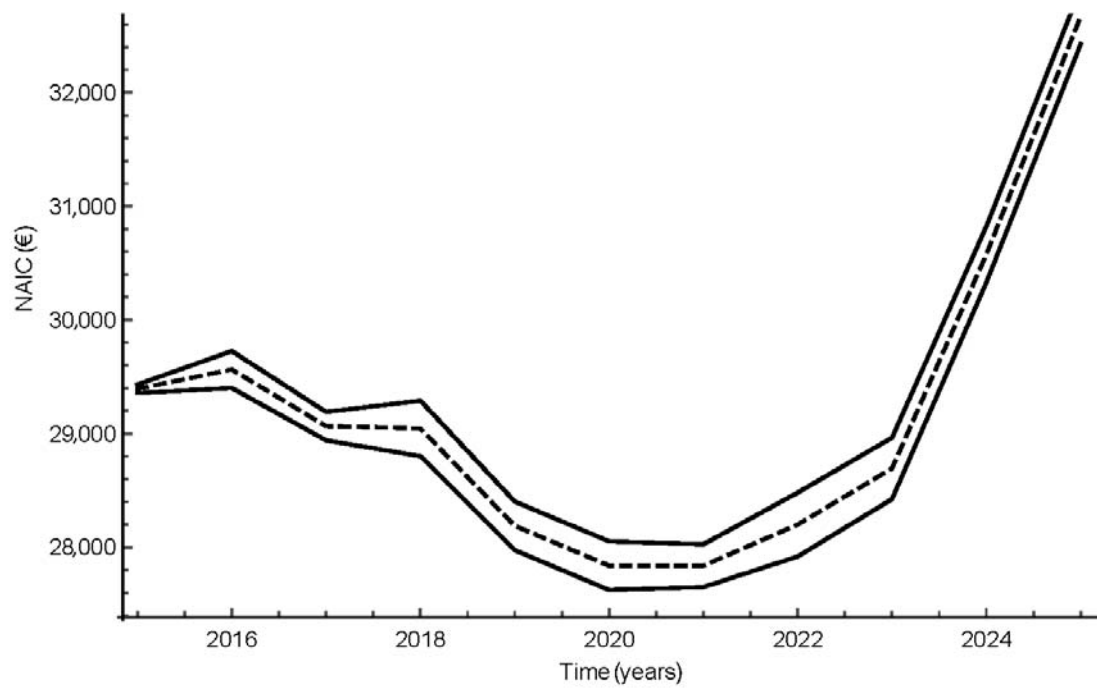


Fig. 10. National income per capita forecasts. Variable *NAIC* (€), tendency values (dots), maximum and minimum values 99% confidence (lines), for 2015-2025 period in Spain.

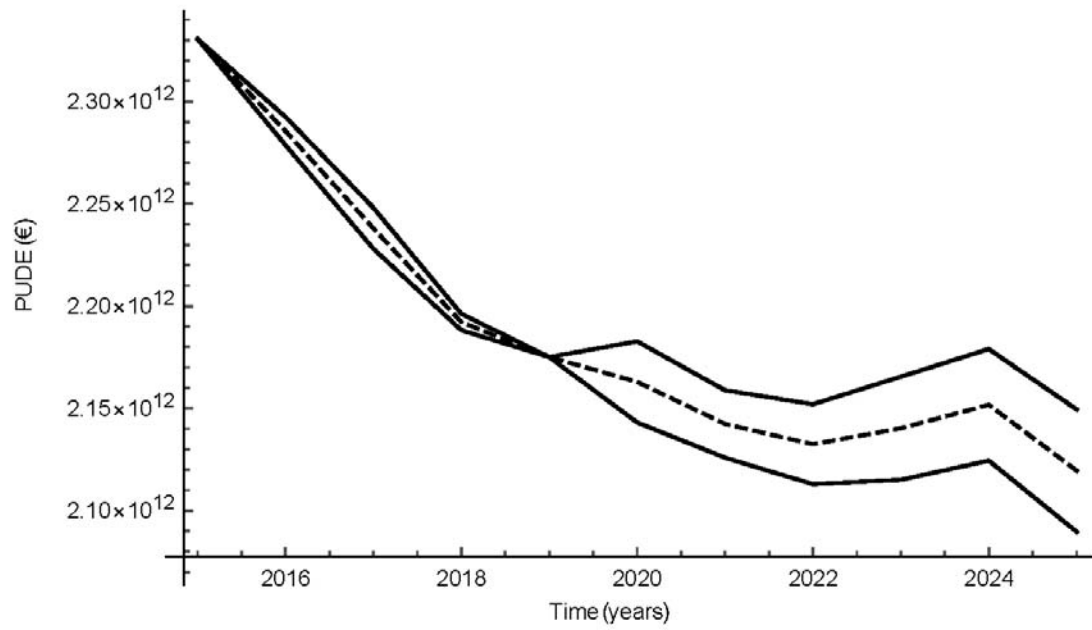


Fig. 11. Public debt forecasts. Variable *PUDE* (€), tendency values (dots), maximum and minimum values 99% confidence (lines), for 2015-2025 period in Spain.

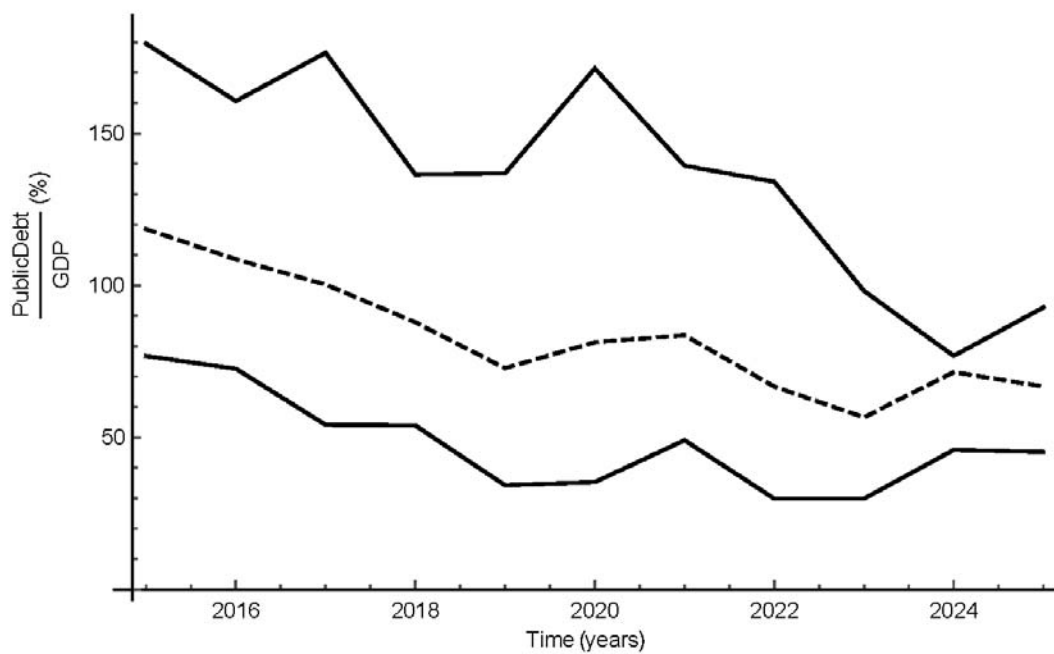


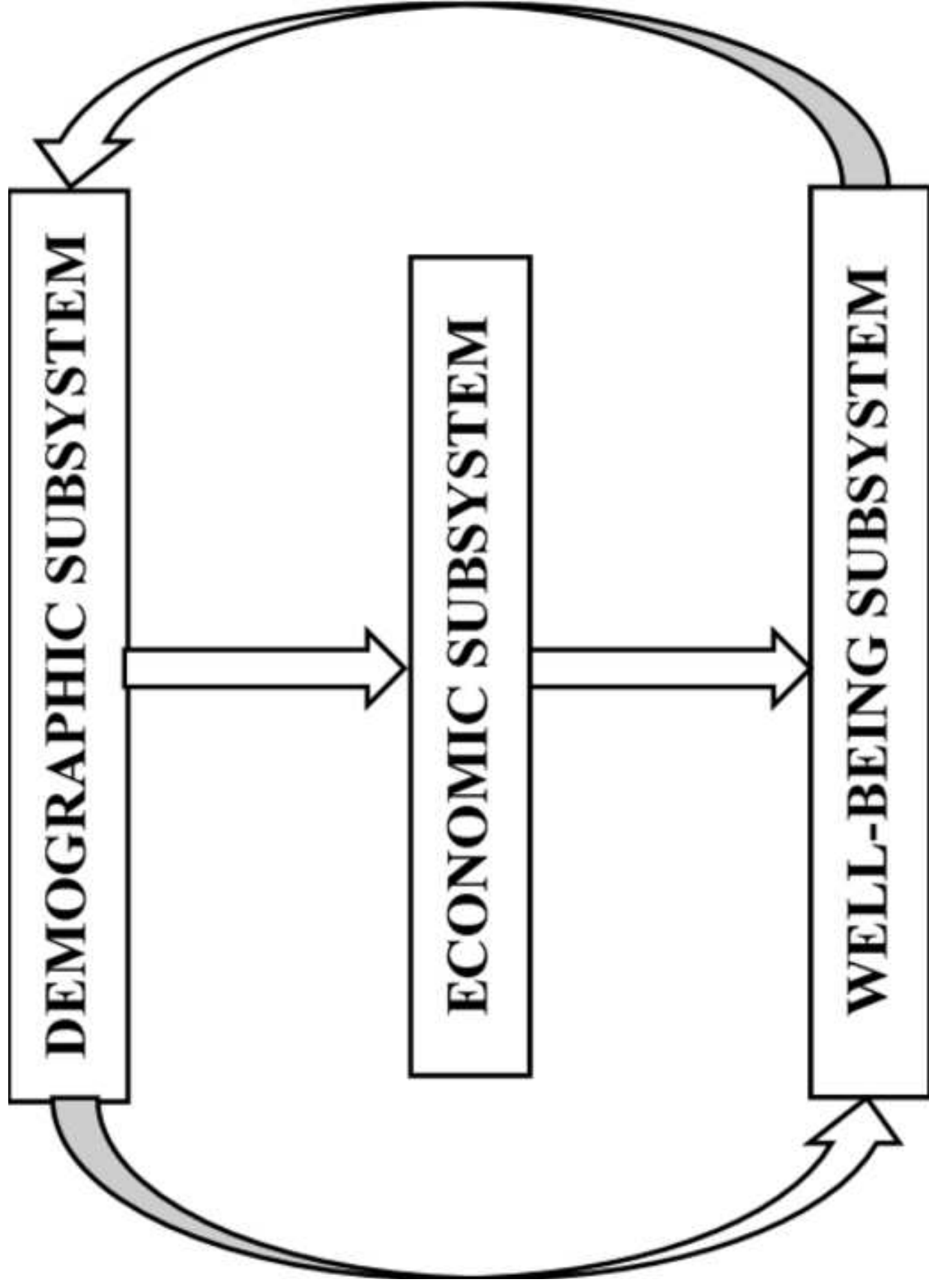
Fig. 12. Public debt/GDP forecasts. Tendency values (dots), maximum and minimum values 99% confidence (lines), for 2015-2025 period in Spain.

Table A.1

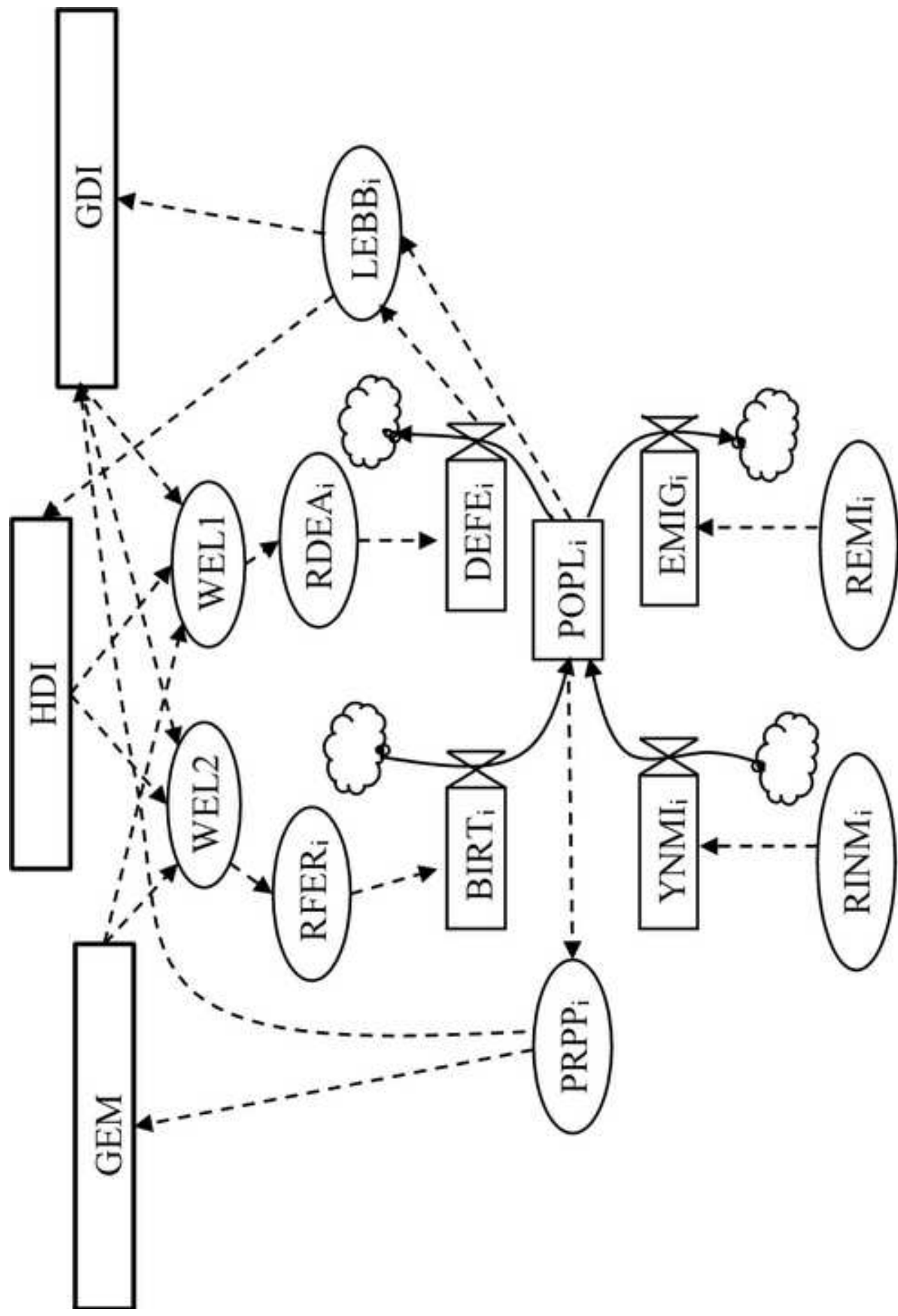
Variables to complete the model.

Variable	Definition	Variable	Definition
<i>EPIF</i>	Female Percentage Parliamentary Representation	<i>RLIM</i>	Male literacy rate
<i>GRFE</i>	Percentage of Female Gross Registered to primary, secondary and tertiary level	<i>PPPF</i>	Female percentage shares of professional and technical positions
<i>GRMA</i>	Percentage of Male Gross Registered to primary, secondary and tertiary level	<i>RLIM</i>	Male literacy rate
<i>IBSE</i>	Wellbeing and services imports	<i>SNAF</i>	Female non-agricultural wage
<i>PAEF</i>	Female percentage shares of positions as legislators senior officials and managers	<i>SNAM</i>	Male non-agricultural wage
<i>RLIF</i>	Female literacy rate		

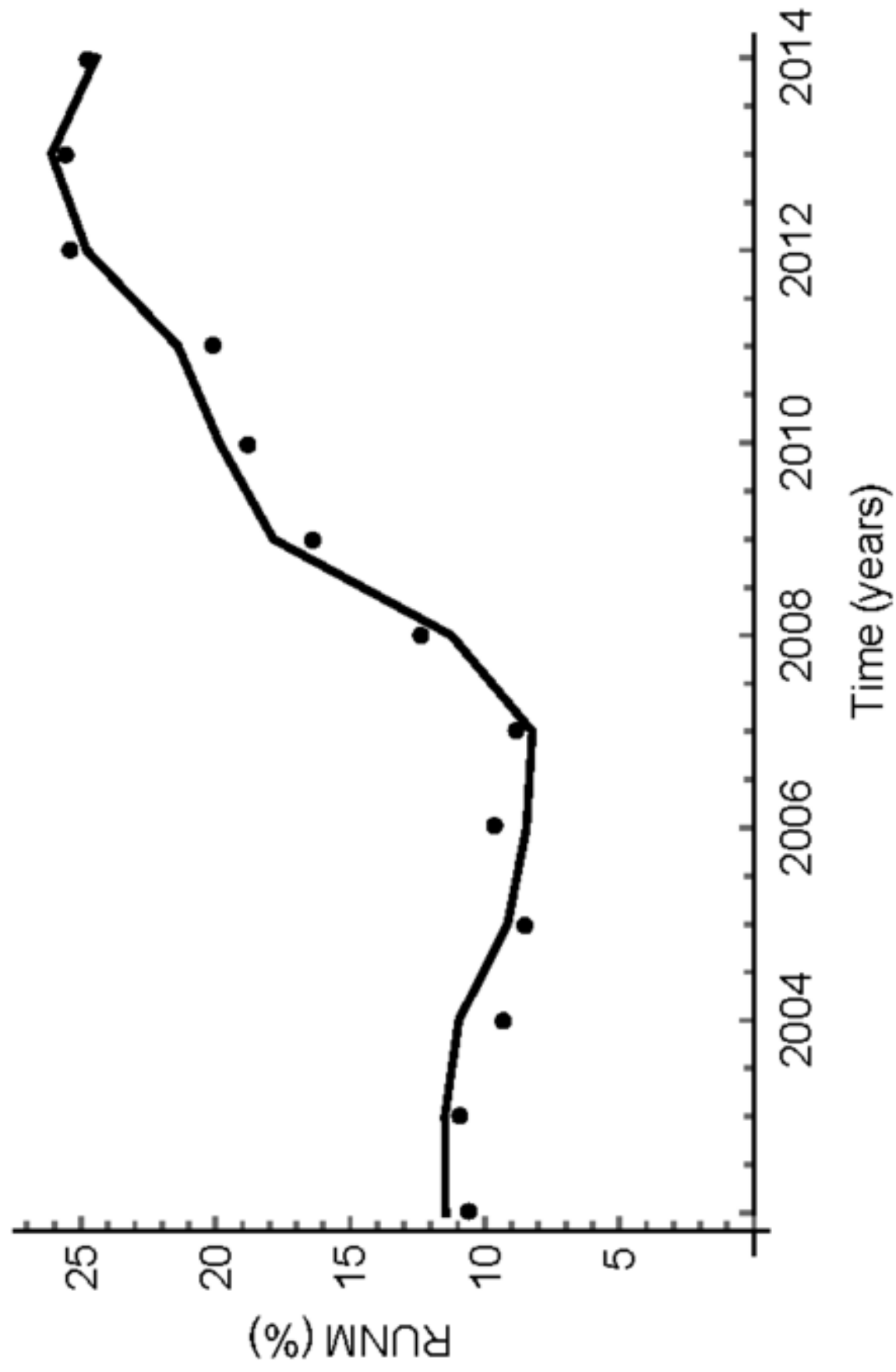
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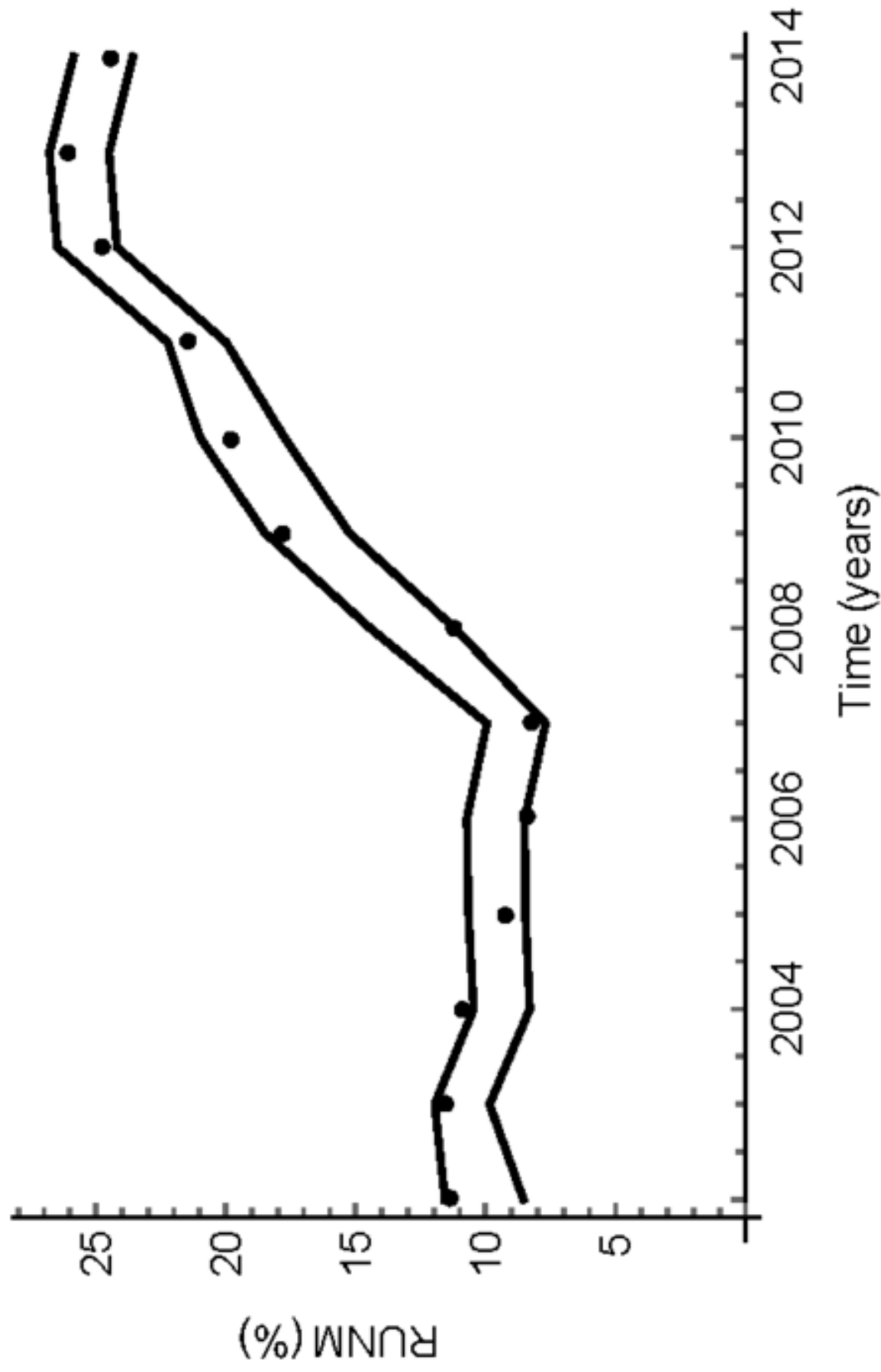
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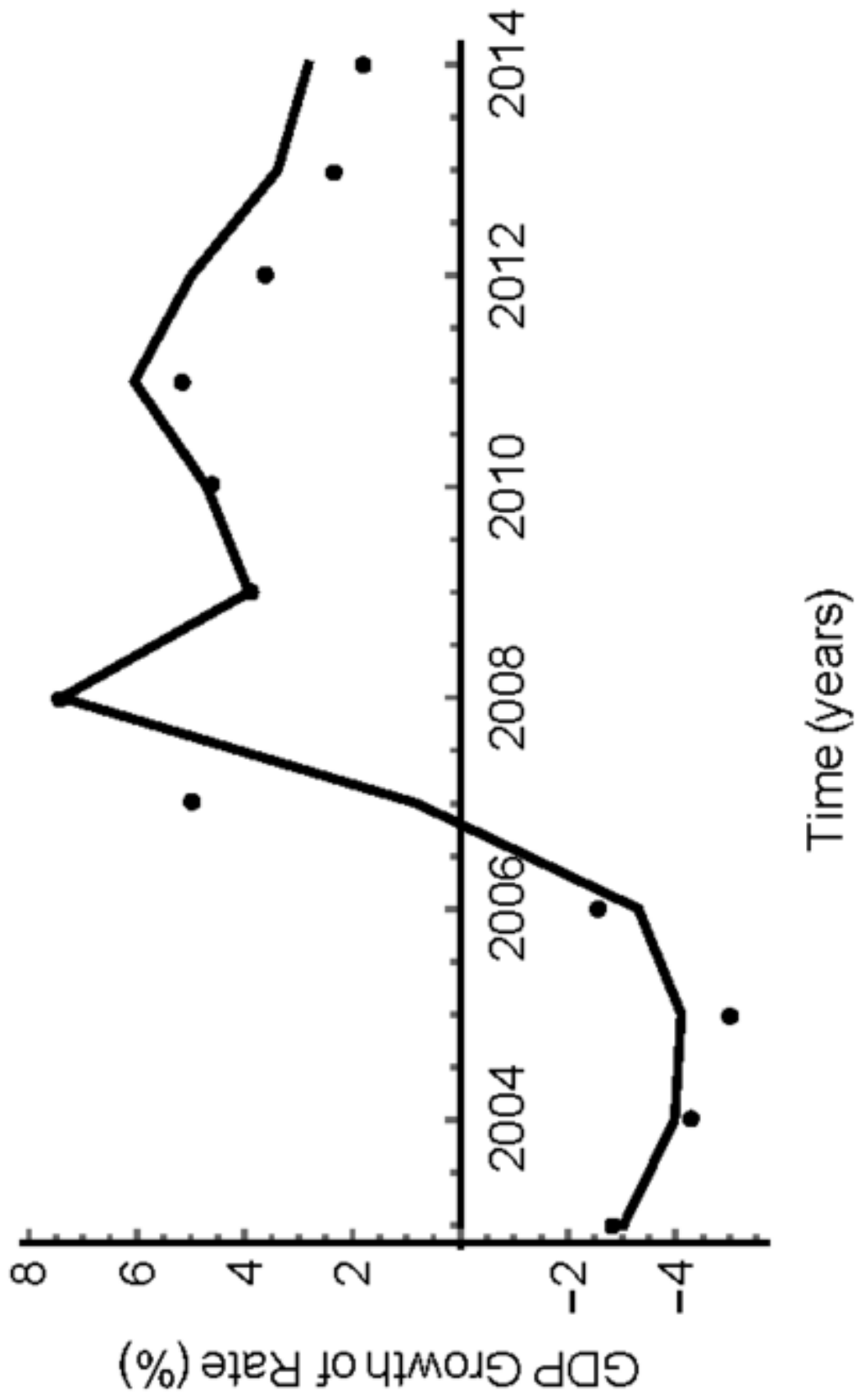
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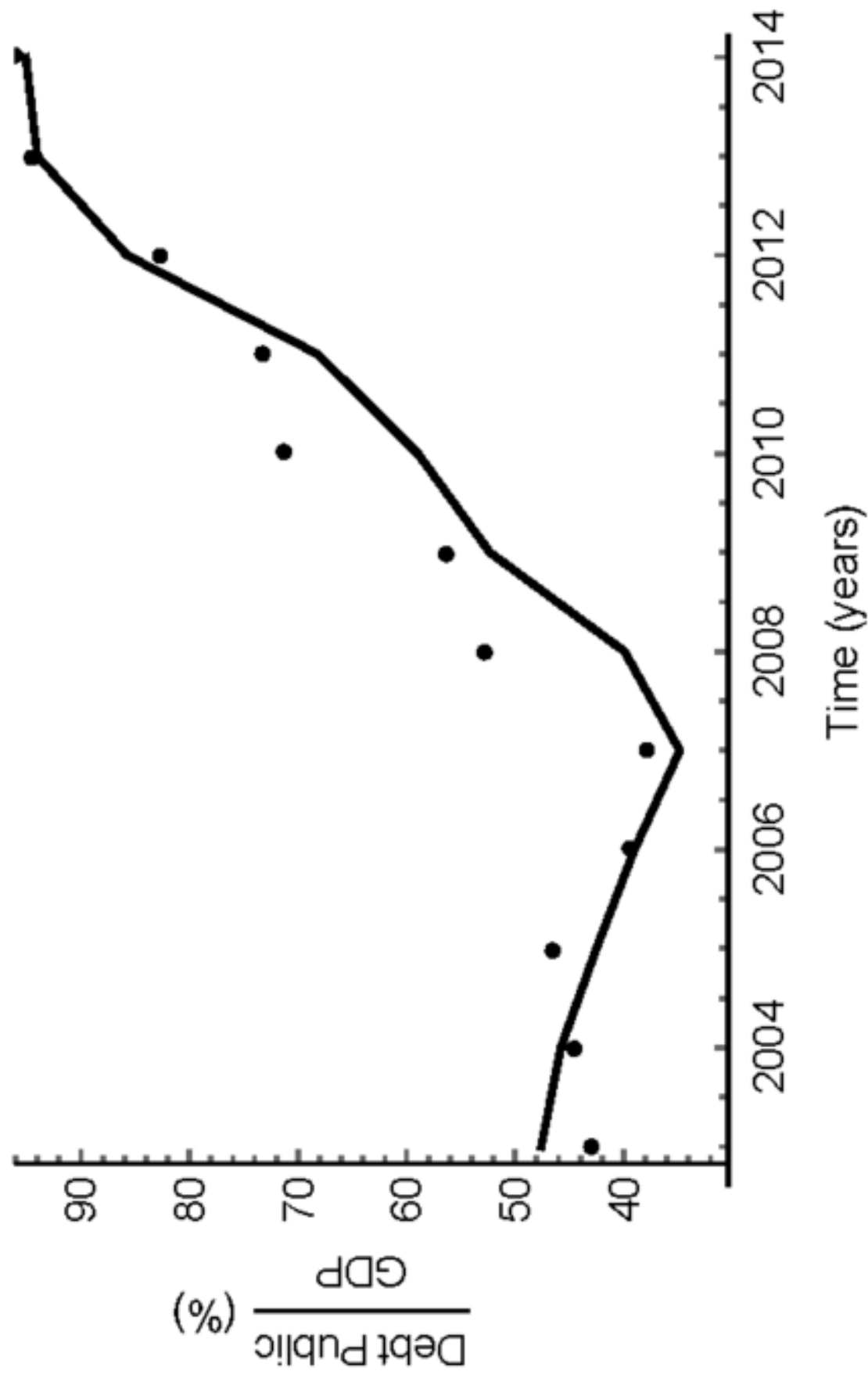
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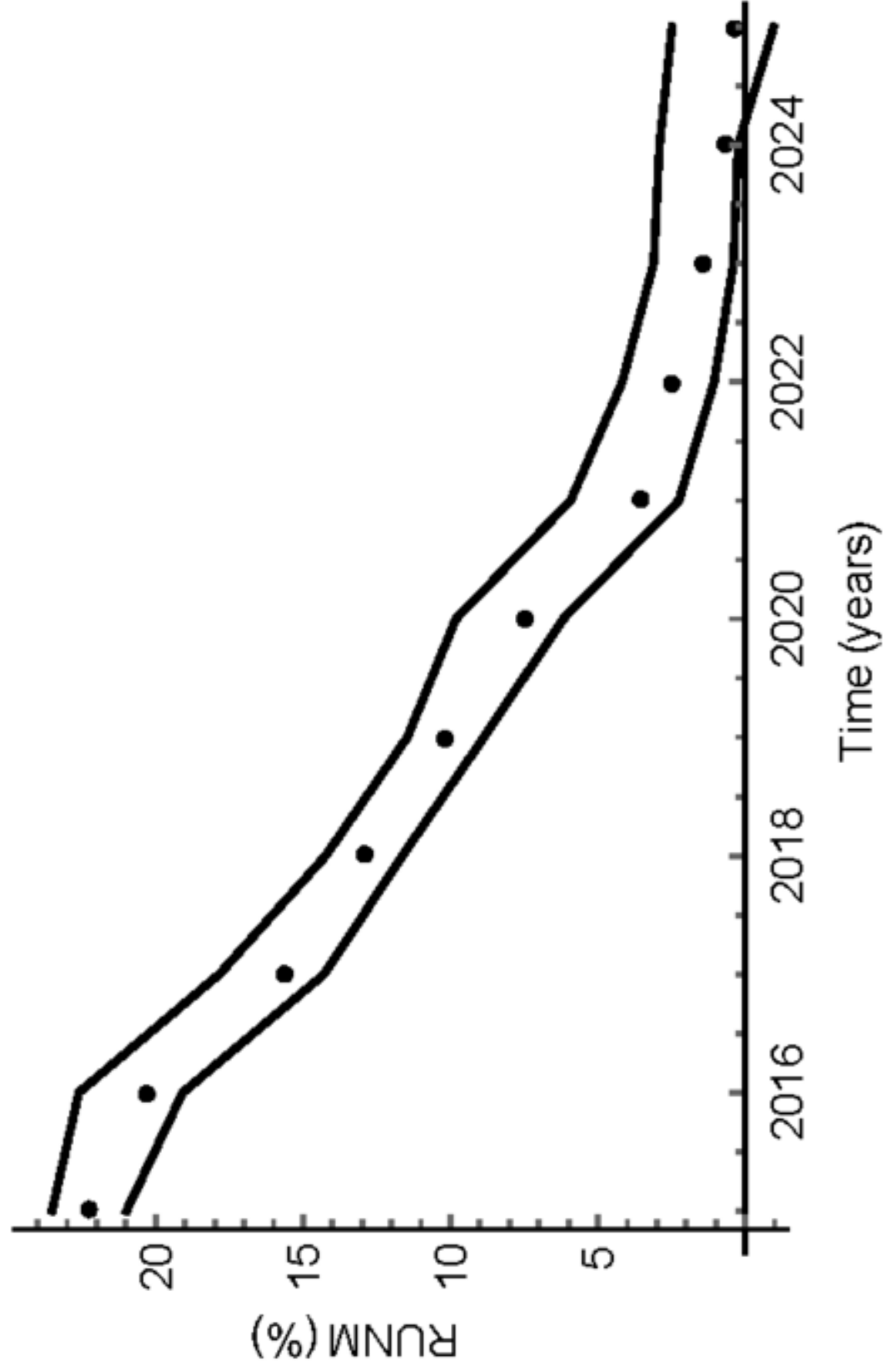
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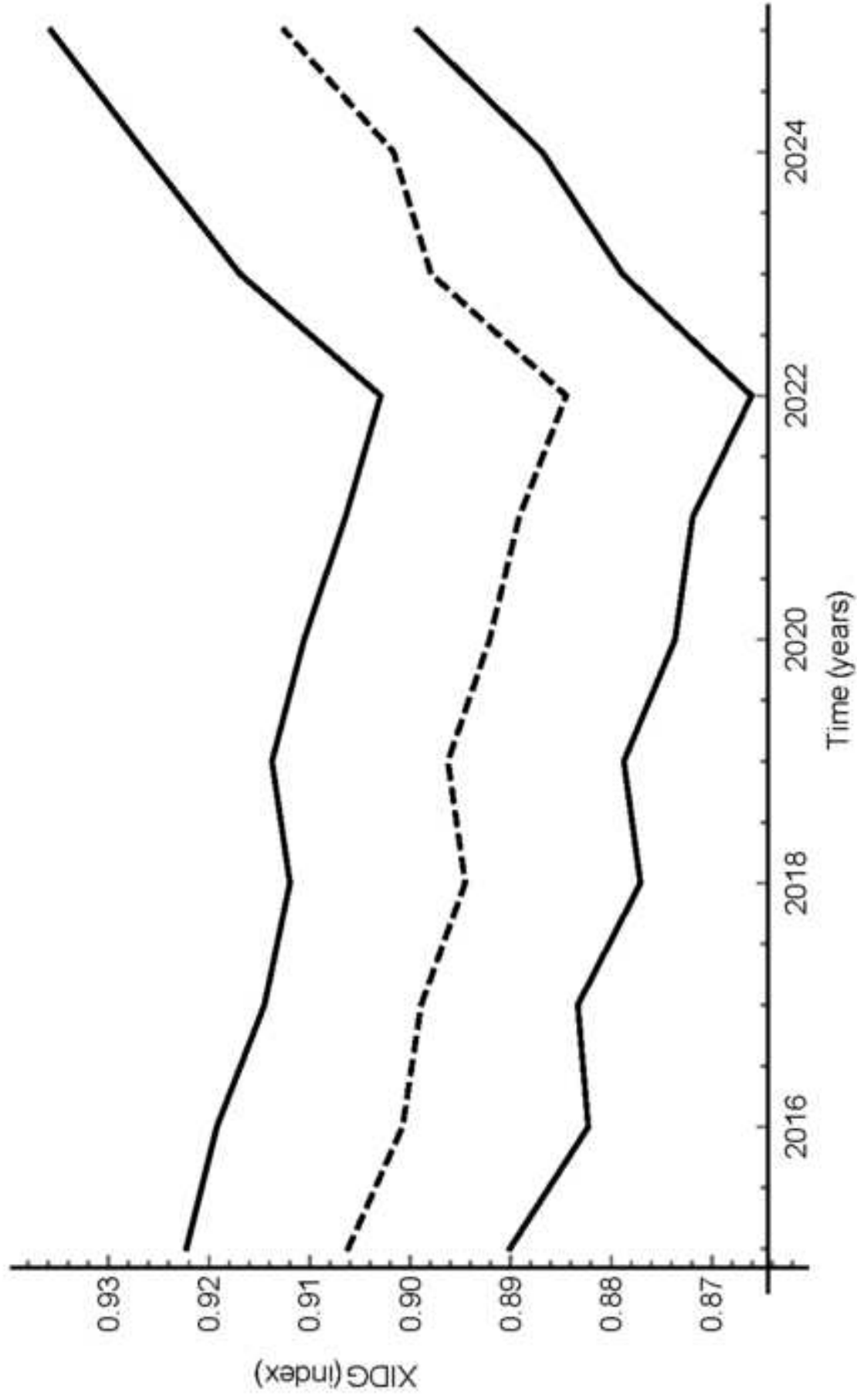
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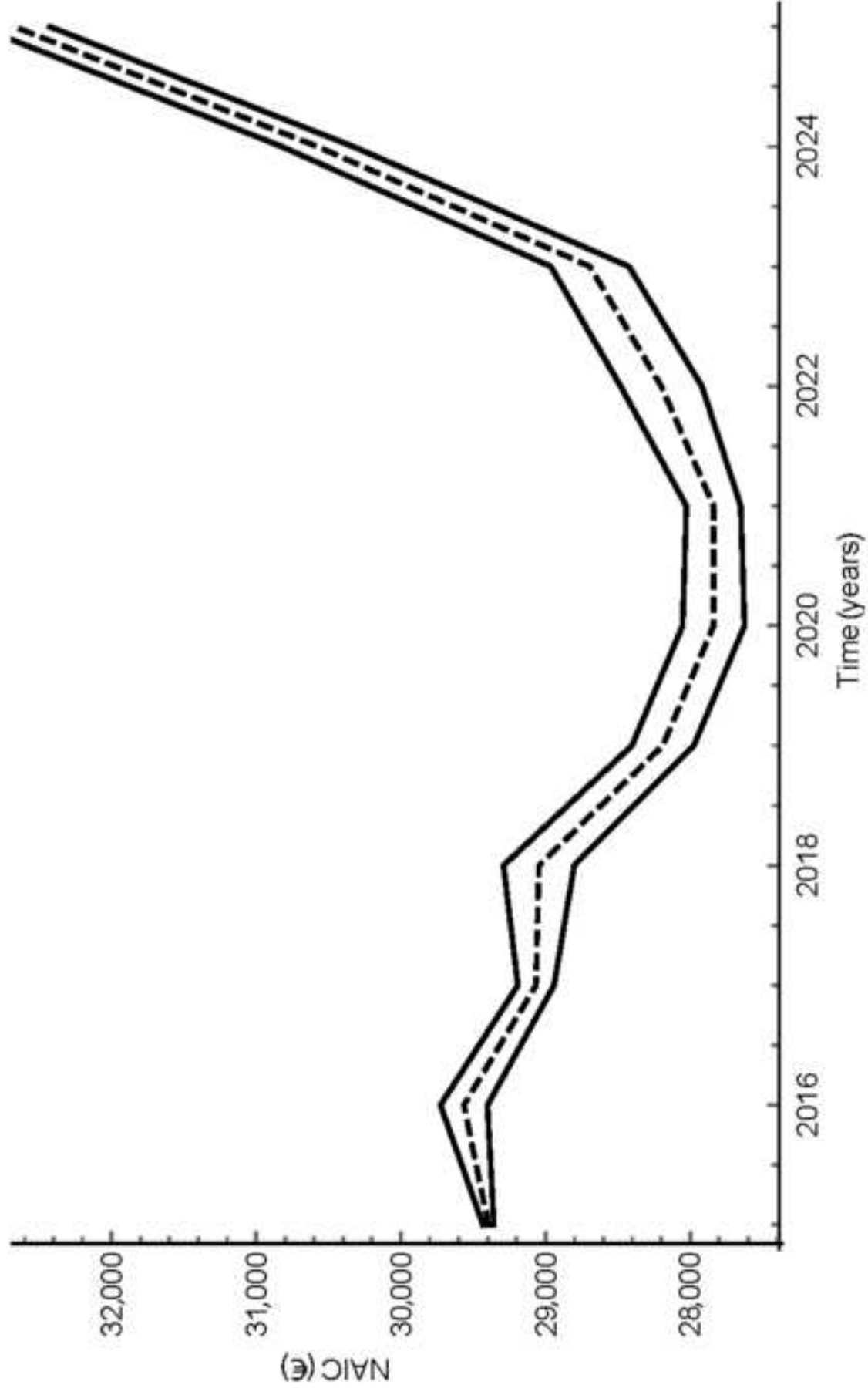
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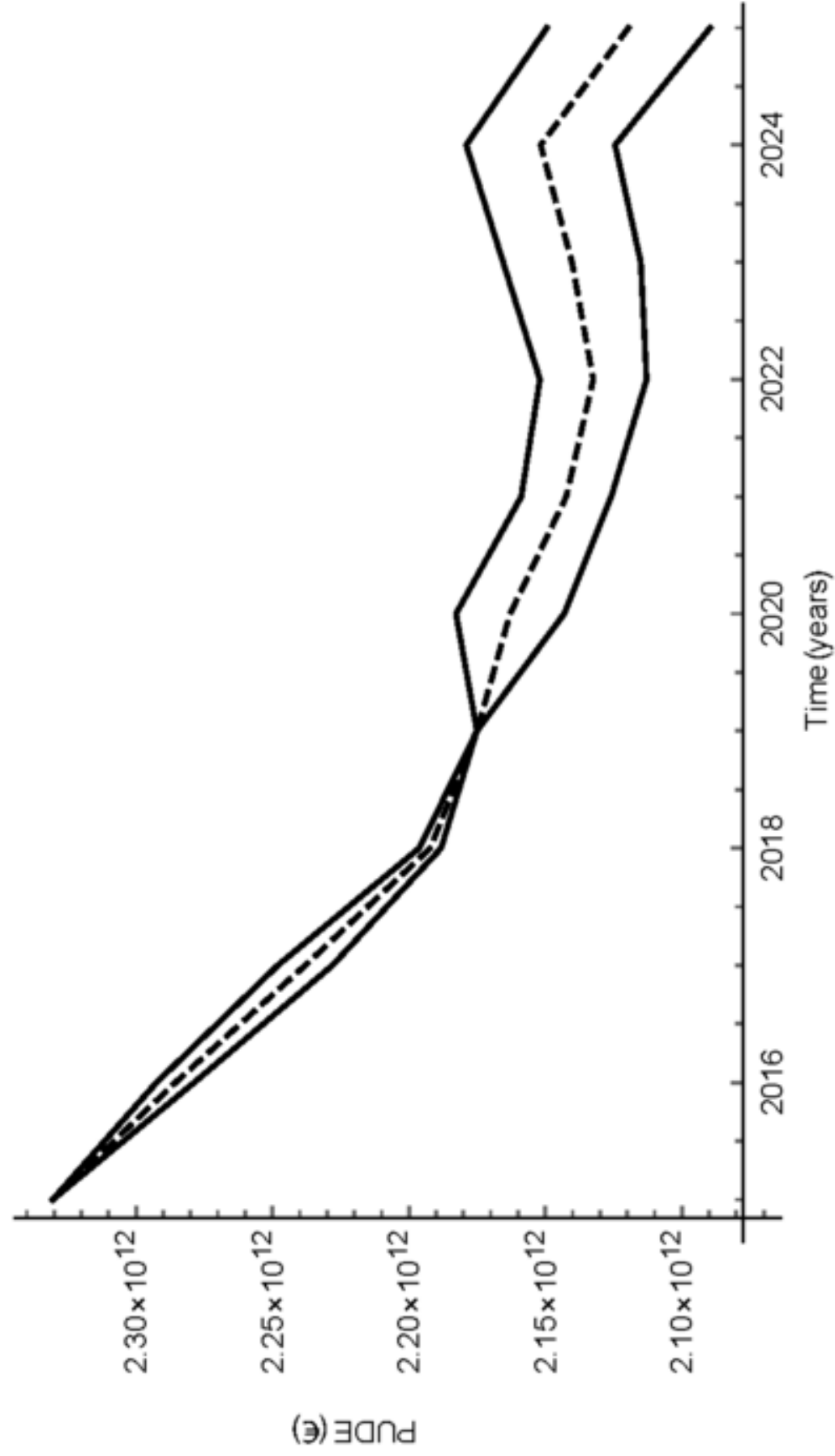
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