Price competition between a macrocell and a small-cell service provider with limited resources and optimal bandwidth user subscription: a game-theoretical model

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Abstract The ever-increasing demand for higher data rates in wireless commutations provides a rationale for small cells deployment. While the physical and technological aspects of small-cell networks have been extensively studied in recent years, the economic analysis has received much less attention. We focus on the economic rationale for a Small-cell Service Provider (SSP) operating a market where an incumbent Macrocell Service Provider (MSP) exists, and competition develops. We analyze such scenario for the case of fixed users by means of Game Theory, specifically through a two-stage game: in the first stage each service provider posts its price according to a Stackelberg game where the MSP is the leader and the SSP is the follower; and, in the second stage, each user chooses both which provider to subscribe to and the optimal amount of bandwidth. A subgame perfect Nash equilibrium is used as a solution concept, and it is derived analytically. We show that the SSP has an incentive to operate in the market and its profit gets higher as SSP's resources increase. Furthermore, users benefit from SSP's operation, which may provide a rationale for a regulatory authority to grant the SSP access to the market, despite the fact that MSP's profit is harmed. Finally, we identify two modes of operation of the system, which depend on the SSP coverage: one where SSP's deployment is limited and the MSP strategy is not affected by SSP competition and takes only the users outside the SSP coverage; and another, where the SSP covers a large area and the MSP competes against the SSP taking a fraction of the users inside the small cells.

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1 Introduction

Currently, there are a lot of wireless devices in the world which demand a greater bandwidth every day, because the data service has become a basic need: from 2013 to 2018 it is expected that the average traffic per smartphone increases from 529 MB to 2672 MB per month [12]. This trends require that wireless service provision is in constant innovation to meet users' needs. At the same time, providing broadband wireless connectivity anytime and anywhere poses a great challenge in indoor scenarios due to signal attenuation. This is specially true for conventional cellular service providers whose radio network infrastructure is primarily based on base stations servicing macrocells. These factors have generated an attractive business opportunity for new service providers (SPs) that want to enter the wireless market.

A great deal of research effort has been put in new techniques and technologies to increase the capacity of the wireless network by making a more efficient use of spectrum, such as spectrum sharing techniques by implementing cognitive radio, and new and innovative technologies that increase network capacity, especially in indoor scenarios, by using multiple radio access technologies [8]. An example of this is the concept of heterogeneous networks (HetNets), which improves the quality of service received by users, especially in indoor scenarios [14], [25], [5]. HetNets are implemented by deploying a network of short-range base stations (corresponding to small cells, e.g., picocells or femtocells) as shown in Fig. 1. Due to the smaller area of coverage, the same licensed frequency band can be efficiently reused several times within elements of a second level of a HetNet; thus improving spectral efficiency per unit area and therefore the capacity of the network [1]. Recently, HetNets/small cells have been proposed as an enabler for 5G networks [7] and [11]. HetNets usually lack a business model that makes them feasible once they are deployed. Our paper, like [20] and [19], proposes a business model that makes the incentives of SPs deploying this technology explicit.

More specifically, this paper investigates a business model in which a new Small-cell Service Provider (SSP) enters a market and competes against an existing Macrocell Service Provider (MSP). The competition between the MSP and the SSP is an asymmetric one where each of them has a competitive advantage over the other. The access network of the SSP is based on small cells and is deployed in such a way that the SSP is prepared to offer a better quality of service at strategic indoor locations. On the other hand, the access network of the MSP, based on macrocells, can cover all the service area and reach some customers that cannot be serviced by shorter-range small cells. Additionally, we show that this model is economically viable. Moreover, our analysis of the model provides an insight into the behavior and profits of users and SPs, and how these are affected when a new SP featuring technological innovation enters the market.

The paper is structured as follows. We describe the network model in Section 2. In Section 3, we perform the analysis of the game model. The numerical results for a range of scenarios in Section 4. Finally, conclusions are drawn in Section 5.

1.1 Related work

There are papers dedicated to the study of the adoption of HetNets by the wireless communications market, but one of the challenges for the deployment of this technology is the need of a viable business model [17]. Therefore, this paper focuses on the study of a business model that allows the integration of small-cell technology in the communications service market.

Recent papers study the business model that allows the incorporation of HetNets. In [5], the authors apply Game Theory to study the economic incentives of an MSP to implement a femtocell service. They prove that the SP's profits increase because it can serve more users at a higher price. It is shown that the service is viable for the existing SP because it increases its network resources, it gets more profits and the users can choose which service the prefer to subscribe. In our model, unlike [5], a new service provider enters the market and competes with the incumbent MSP and the users choose not only which service provider to subscribe to, but also the amount of data rate.

The authors of [25] proposed a business model that encourages SSPs to lease bandwidth to the MSP to increase the network capacity. This model performs a dynamic control of resources that are being leased by SSPs to the MSP, and also takes into account the evolutionary behavior of users over time. This models departs from the model analyzed in this paper, since the SSP's strategy is the price here, whereas it is the amount of bandwidth leased in the above work.

In [10], the entry of a new SP (which uses femtocells) that competes against an incumbent MSP is studied. It is showed that all agents in the system improve their situation with the implementation of femtocell technology. In this paper we model the competition between the two service providers using a leader-follower model, instead of a simultaneous-move model in the above work. Additionally, in this paper the users choose not only which service provider to subscribe to, but also the amount of data rate.

In [24] a pricing-based resource allocation scheme for multiple providers is studied. In [13], the optimal strategies for resource allocation based on prices in a femtocell network with shared spectrum are discussed. In [23], the optimal decisions of SPs and users are analyzed. And the economic incentives for an MSP to provide a service based on small cells is investigate in [6]. All the above works suffer from the important limitation that the users' subscription game is not analyzed. In this paper the users' subscription game is explicitly modeled as part of the game.

2 Model description

We consider a scenario in which there are two operators that provide fixed wireless service. One of them, which we refer to as Macrocell Service Provider (MSP), is a conventional one and owns a set of BS, each servicing a macrocell, that provides full coverage on the service area. The second operator, which we refer to as Small-cell Service Provider (SSP), deploys a radio access network (RAN) consisting only of small cells. The coverage areas of the small cells are disjoint, included in the service area of the MSP and covering only a fraction of the latter.

While the MSP holds a license to exploit a spectrum band, the SSP does not hold such a license, but a generic authorization for providing wireless communications services. Spectrum trading is currently allowed by most National Regulatory Authorities (e.g., the European Union included this possibility in the 2009 Telecommunications Regulatory Reform). Thus, an SSP is allowed to use spectrum that a primary licensee makes available through a secondary trading agreement.

In the sequel, to simplify notation, we will consider without any loss of generality that the RAN of the MSP is

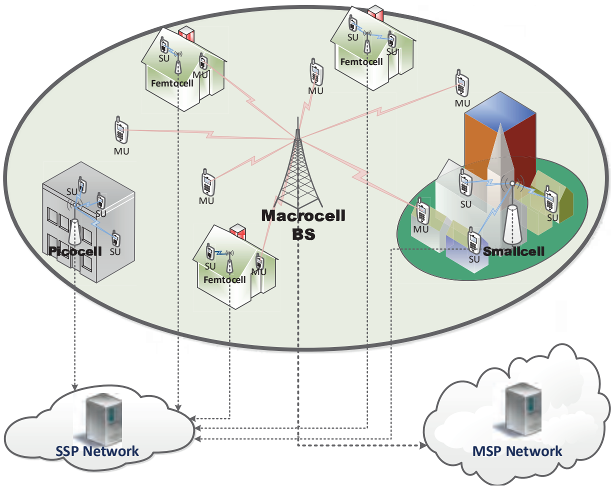

Fig. 1 Scenario.

composed of a single macrocell (see Fig. 1). Note that all our analysis and results could be generalized to any number of macrocells simply by multiplying, where appropriate, by the number of macrocells. We denote by B_m the bandwidth available to the MSP. The SSP deploys a total of K small cells. We refer to the i-th small cell as s_i and to the available bandwidth at this small cell as B_{s_i}. In addition to the K small cells, $s_1,\dots,s_K$, the part of the macrocell that does not overlap with the coverage area of any small cell is referred to as s_0; clearly $B_{s_0}=0$.

The users inside the small cells can be served by both the MSP and the SSP, which compete in prices for these users. Each SP posts a price per nominal–data–rate unit, which is p_m for the MSP and p_s for the SSP. As detailed in the following subsections, users subscribe to either MSP's or SSP's service and pay for a nominal data rate, which is equal to the bandwidth allocated by the SP. We will then say that the users *subscribe bandwidth from the SPs.* The actual data rate obtained by the user will depend on the spectral efficiency of the radio channel. Seminal papers such as [5] follow this same approach.

2.1 Transmission and channel model

For simplicity, we assume that operators only provide one type of service [18]. For the MSP, we consider a channel model similar to that in [25], [5] and [22]. The channel between the BS and the users is affected by pathloss, shadow fading and fast fading. It is assumed that M-QAM with adaptive modulation and no power control is used. The rate in a subchannel of bandwidth b (in Hz) is in a set of L discrete rates and can be expressed as [9]

$$w_l = b\log_2(1+\Gamma_l/G), \quad \text{if SNR} \in [\Gamma_l,\Gamma_{l+1}),\ l=1,\dots,L, \tag{1}$$

where G models the deviation from the Shannon capacity of the achievable transmission rate with M-QAM, and Γ_l is the minimum SNR to achieve a rate of w_l bps.

Users experience different channel conditions due to the distance between the BS and the user, interferences, obstacles and other factors. We assume that no interference between users exists. With these conditions the spectral efficiency that a user subscribing to the MSP, whose SNR is in the interval $[\Gamma_l,\Gamma_{l+1})$, obtains is given by [6]

$$\theta_m = w_l/b = \log_2(1+\Gamma_l/G). \tag{2}$$

Hence, from our previous discussion, we can consider that there is a set of L discrete values for the spectral efficiency, θ_m, which we assume to be in the interval $[0,1]$.

Since the coverage area of each small cell is relatively small, we assume that users subscribing to the MSP which are in the same small cell get the same spectral efficiency; and denote by θ_{m_i} the spectral efficiency for MSP users in s_i. For the same reason, we assume that the spectral efficiency of all users subscribing to the SSP is equal to 1. [1] We call b_m the bandwidth subscribed by a MSP user and b_s the bandwidth subscribed by a SSP user. Then, a user subscribing to the SSP obtains a rate of b_s and user subscribing to the MSP obtains $\theta_m b_m$ [5].

2.2 Users

We assume that there are N users uniformly distributed throughout the coverage area of the MSP. Thus, the number of users in s_i is $N_i = N(A_{s_i}/A_m)$, where A_m is the area covered by the MSP and A_{s_i} is the area of s_i. Users that are inside s_i $(i=1,\dots,K)$ can subscribe to either the MSP or the SSP; or not to subscribe. Users make their subscription decision according to the expected utility and independently from one another.

We call x_{m_i} and x_{s_i} the fraction of users who subscribe, respectively, to the MSP and to the SSP at a small cell s_i. If N_{m_i} is the number of users subscribing to MSP in s_i, then $x_{m_i}=N_{m_i}/N_i$ and if N_{s_i} is the number of users subscribing to SSP in s_i, then $x_{s_i}=N_{s_i}/N_i$. Obviously, $N_{s_0}=0$ and $x_{s_0}=0$. The fraction of users who fail to subscribe to the service in s_i is $x_{o_i}=1-x_{m_i}-x_{s_i}$. In the whole service area, the fraction of users subscribing to MSP is

$$x_m = \sum_{i=0}^{K}\frac{N_{m_i}}{N} = \sum_{i=0}^{K}\frac{N_i}{N}x_{m_i}; \tag{3}$$

[1] Actually, the spectral efficiency is not upper limited by a value of 1. Nevertheless, by using a spectral efficiency equal to 1 for the SSP and a spectral efficiency in the interval $[0,1]$ for the MSP, we are just normalizing the maximum spectral efficiency attainable by the MSP to the one achieved by the SSP.

the fraction of users subscribing to the SSP is

$$x_s = \sum_{i=1}^{K} \frac{N_{s_i}}{N} = \sum_{i=1}^{K} \frac{N_i}{N} x_{s_i}; \tag{4}$$

and the fraction of users that do not subscribe is $x_o = 1 - x_m - x_s$.

We assume that there is no mobility, so that the number of users in each small cell remains constant in time.

2.2.1 *Users utility*

For users utility, we propose a function that integrates all the factors involved in the choice between operators: the rate perceived, the amount of bandwidth subscribed and the payment charged.

If a user subscribes to the MSP, his utility is

$$u_m(\theta_m, b_m, p_m) = \theta_m b_m e^{-p_m b_m}, \tag{5}$$

whereas if the user subscribes to the SSPhis utility is

$$u_s(b_s, p_s) = b_s e^{-p_s b_s}. \tag{6}$$

Note that in both cases, the factor $\theta_m \cdot b$ or $1 \cdot b$ reflects the fact that the higher rate the user is allocated, the greater the utility is. Furthermore, the payment for the (maximum) achievable rate b affects the utility through a negative exponential function (i.e., $e^{-b \cdot p}$). This is a similar effect to the one achieved by a quasilinear utility and a budget constraint, where the payment for the rate is linear (i.e., $-b \cdot p$). Although the latter is a more common model in network economics, we argue that our proposal reflects more realistically how the spectrum scarcity faced by a service provider is translated to the user. Finally, the proposed utility expression is more amenable for the analytical treatment detailed below.

Lastly, the utility perceived by the users who do not subscribe the service is $u_o^n = 0$, which is consistent with a user subscribing zero bandwidth to either the MSP or the SSP.

2.2.2 *Optimal bandwidth subscribed by users.*

Each user will subscribe the amount of bandwidth that maximizes its utility, given the price and the spectral efficiency. If a user subscribes to the MSP, the optimal amount of bandwidth is given by

$$b_m^* = \arg\max_{b_m^n \geq 0} \quad u_m(\theta_m, b_m, p_m), \tag{7}$$

and if he subscribes to the SSP,

$$b_s^* = \arg\max_{b_s^n \geq 0} \quad u_s(b_s, p_s). \tag{8}$$

By maximizing (5) and (6), we obtain

$$b_m^* = 1/p_m \tag{9}$$

and

$$b_s^* = 1/p_s. \tag{10}$$

It follows that the amounts of bandwidth that maximize the utility functions depend only on the prices, which are the same for all users. For that reason, and given that users are rational, all users will be willing to subscribe the same amounts of bandwidth from the respective providers [16].

The maximum utility values are:

$$u_{m_i}^* = \frac{\theta_{m_i}}{p_m} e^{-1}, \tag{11}$$

where $u_{m_i}^*$ denotes the maximum utility of the users that are in s_i and subscribe to the MSP, and depends only on the small cell that the user is in;

$$u_s^* = \frac{1}{p_s} e^{-1}, \tag{12}$$

which is the same for all users, and

$$u_o^* = 0. \tag{13}$$

Observe that $u_{m_i}^*, u_s^* > 0 = u_o^*$. Consequently, all users would prefer to subscribe the service (actually, with the SP offering the highest utility) over not to subscribe it. However, as we describe next, each SP can accommodate a maximum number of subscribers owing to its limited resources. Hence, if the SP offering the highest utility sells out all its capacity, some users may have to subscribe to the other SP or even fail to subscribe to any of them.

The user welfare (UW) is defined as the aggregate utility of all users and is given as

$$\mathrm{UW} = \sum_{i=0}^{K} (N_i x_{m_i}^* u_{m_i}^* + N_i x_{s_i} u_s^*). \tag{14}$$

2.3 Service providers

At small cell s_i, the bandwidth demanded to the MSP is $Q_{m_i}(p_m, x_{m_i}) = N_i x_{m_i} b_m^*$ and the bandwidth demanded to the SSP is $Q_{s_i}(p_s, x_{s_i}) = N_i x_{s_i} b_s^*$. These demands are limited by the available bandwidth of the corresponding operator, i.e.,

$$\sum_{i=0}^{K} N_i x_{m_i} b_m^* \leq B_m \tag{15}$$

and

$$N_i x_{s_i} b_s^* \leq B_{s_i} \quad i = 1, \ldots, K. \tag{16}$$

To simplify the analysis of the game in Section 3, the bandwidth of the MSP is divided into $K+1$ portions,

B_{m_i} $(i = 0, \ldots, K)$, where B_{m_i} is the amount of bandwidth that the MSP can use to serve its subscribers in s_i,

$$N_i x_{m_i} b^*_m \le B_{m_i}, \quad i = 0, \ldots, K. \tag{17}$$

In other words, a bandwidth constraint at each small cell is artificially imposed on the MSP. These artificial constraints simplify the analysis of the game, which is carried out for a general allocation

$$B_{m_0} + \cdots + B_{m_K} = B_m, \quad B_{m_i} \ge 0. \tag{18}$$

Then, to obtain the results in Section 4 the bandwidth allocation of the MSP is appropriately redistributed to maximize its profit. This way, the final bandwidth allocation to each a small cell s_i, B_{m_i}, will be the same as the one that the MSP will use to serve users in s_i if only the global constraint (15) was considered; and consequently, the artificial constraints in (17) have no impact on the results.

From the above we can write

$$x_{m_i} \le \min\left(\frac{p_m}{N_i/B_{m_i}}, 1\right), \tag{19}$$

$$x_{s_i} \le \min\left(\frac{p_s}{N_i/B_{s_i}}, 1\right). \tag{20}$$

The total amount of bandwidth demanded to the MSP is

$$Q_m(p_m, x_m) = \sum_{i=0}^{K} Q_{m_i}(p_m, x_{m_i}) = b^*_m N x_m \tag{21}$$

and the total amount of bandwidth demanded to the SSP is

$$Q_s(p_s, x_s) = \sum_{i=1}^{K} Q_{s_i}(p_s, x_{s_i}) = b^*_s N x_s \tag{22}$$

2.3.1 Service providers profits

The decisions of the providers are made based on the profits obtained, which are defined as revenues minus costs. Revenues are given by the payment received from the subscribers; costs are assumed to be of an operational nature. Without any loss of generality, we assume that they are zero. Then, the MSP utility function is

$$\pi_m(p_m, x_m) = p_m Q_m(p_m, x_m), \tag{23}$$

and the SSP utility function is

$$\pi_s(p_s, x_s) = p_s Q_s(p_s, x_s). \tag{24}$$

It should be stressed that our work is focused on analyzing the competition between operators at a short time frame, which is the time frame where only the price can be used as a strategy. At this time frame, only operational expenses are relevant. Capital expenditures incurred by the SSP, as those involved in deploying the required small cell infrastructure (see e.g. the analysis conducted in [15] and [3]), are relevant at a longer time frame. At this time frame, the relevant strategies would be not only the deploying strategy by the SSP, but also the very same entry decision by the SSP, as well as potential deterring strategies by the MSP (e.g., upgrading the technology, which may improve the spectral efficiency). This is deferred for further study.

2.4 Monopolistic scenario

In this section we determine the optimal price chosen by the MSP in a monopolistic scenario, that is, when the MSP is the only provider in the service area. In this case, users only have the option to subscribe to the MSP, their utility is given by (11) and the bandwidth subscribed is given by (9).

If possible, all users will be willing to subscribe because $u^*_m > 0 = u^*_o$. However, the fraction of users that can subscribe depends on the available bandwidth. Since $N x_m b^*_m \le B_m$ and $0 < x_m < 1$, by recalling (9) we can write

$$x^*_m(p_m) = \min\left(1, \frac{p_m B_m}{N}\right)$$

Thus, the MSP profit is given as $\pi_m(p_m) = N \min\left(1, \frac{p_m B_m}{N}\right)$ and the MSP will choose the price, p_m, so as to maximize this profit. It is easily shown that with any price $p_m \ge p^*_m$, where

$$p^*_m = \frac{N}{B_m}, \tag{25}$$

the MSP obtains the maximum profit, $\pi^*_m = N$, and a 100% user subscription fraction ($x_m = 1$). Throughout this paper, when this situation arises we assume that the SP will choose the minimum price among those that maximize its profit. While this decision does not affect the profits of the SP, the UW is maximized.

3 Analysis

We analyze the competition between the MSP and the SSP based on the models developed in the previous section. We assume that at a first stage they compete according to a leader-follower model, which can be modeled as a Stackelberg's game. The MSP is the leader because it is the incumbent company owning a spectrum license and the SSP is the follower. At a second stage, each user will subscribe to the service providing the highest utility.

The strategic interaction between the SPs and the users is shown in Fig. 2. The game is solved by backward induction [2], which means that at Stage I players proceed strategically anticipating the solution of Stage II.

Table 1 Population ratio when $p_s > p_m/\theta_{m_i}$, the users prefer to subscribe to the MSP

High supply ($x^*_{m_i}+x^*_{s_i}=1$)		Low supply ($x^*_{m_i}+x^*_{s_i}<1$)
$p_m \ge \frac{N_i}{B_{m_i}}$	$p_{m_i}(p_s) < p_m < \frac{N_i}{B_{m_i}}$	$p_m \le p_{m_i}(p_s)$
$x^*_{m_i}=1$ $x^*_{s_i}=0$ $x^*_{o_i}=0$	$x^*_{m_i}=\frac{p_m B_{m_i}}{N_i}$ $x^*_{s_i}=1-\frac{p_m B_{m_i}}{N_i}$ $x^*_{o_i}=0$	$x^*_{m_i}=\frac{p_m B_{m_i}}{N_i}$ $x^*_{s_i}=\frac{p_s B_{s_i}}{N_i}$ $x^*_{o_i}=1-x^*_{m_i}-x^*_{s_i}$

Table 2 Population ratio when $p_s < p_m/\theta_{m_i}$, the users prefer to subscribe to the SSP

High supply ($x^*_{m_i}+x^*_{s_i}=1$)		Low supply ($x^*_{m_i}+x^*_{s_i}<1$)
$p_s \ge \frac{N_i}{B_{s_i}}$	$p_{s_i}(p_m) < p_s < \frac{N_i}{B_{s_i}}$	$p_s \le p_{s_i}(p_m)$
$x^*_{m_i}=0$ $x^*_{s_i}=1$ $x^*_{o_i}=0$	$x^*_{m_i}=1-\frac{p_s B_{s_i}}{N_i}$ $x^*_{s_i}=\frac{p_s B_{s_i}}{N_i}$ $x^*_{o_i}=0$	$x^*_{m_i}=\frac{p_m B_{m_i}}{N_i}$ $x^*_{s_i}=\frac{p_s B_{s_i}}{N_i}$ $x^*_{o_i}=1-x^*_{m_i}-x^*_{s_i}$

Table 3 Population ratio when $p_s = p_m/\theta_{m_i}$

	$p_s > \frac{N_i}{2B_{s_i}}$	$p_{s_i}(p_m) \le p_s \le \frac{N_i}{2B_{s_i}}$	$p_s < p_{s_i}(p_m)$
$p_m > \frac{N_i}{2B_{m_i}}$	$x^*_{m_i}=1/2$ $x^*_{s_i}=1/2$ $x^*_{o_i}=0$	$x^*_{m_i}=1-\frac{p_s B_{s_i}}{N_i}$ $x^*_{s_i}=\frac{p_s B_{s_i}}{N_i}$ $x^*_{o_i}=0$	$x^*_{m_i}=\frac{p_m B_{m_i}}{N_i}$ $x^*_{s_i}=\frac{p_s B_{s_i}}{N_i}$ $x^*_{o_i}=1-x^*_{m_i}-x^*_{s_i}$
$p_{m_i}(p_s) \le p_m \le \frac{N_i}{2B_{m_i}}$	$x^*_{m_i}=\frac{p_m B_{m_i}}{N_i}$ $x^*_{s_i}=1-\frac{p_m B_{m_i}}{N_i}$ $x^*_{o_i}=0$	$x^*_{m_i}=1/2$ $x^*_{s_i}=1/2$ $x^*_{o_i}=0$	N/A
$p_m < p_{m_i}(p_s)$	$x^*_{m_i}=\frac{p_m B_{m_i}}{N_i}$ $x^*_{s_i}=\frac{p_s B_{s_i}}{N_i}$ $x^*_{o_i}=1-x^*_{m_i}-x^*_{s_i}$	N/A	$x^*_{m_i}=\frac{p_m B_{m_i}}{N_i}$ $x^*_{s_i}=\frac{p_s B_{s_i}}{N_i}$ $x^*_{o_i}=1-x^*_{m_i}-x^*_{s_i}$

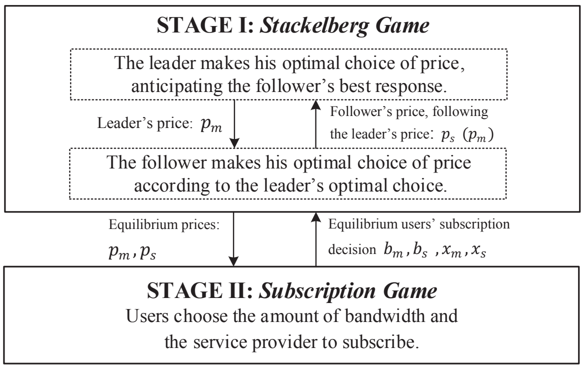

Fig. 2 Game structure

3.1 Stage II: subscription game

At Stage II, the users know the prices announced by the MSP and the SSP at Stage I. Given this, they decide which SP to subscribe to and how much bandwidth to subscribe. User population strategies are $\mathscr{S} = \{m,\ s,\ o\}$ which correspond to: subscribing to the MSP, subscribing to the SSP and not subscribing to the service, respectively.

We adopt the Wardrop equilibrium [21] as the solution concept for this stage. In this equilibrium, no user can increase their utility function either by varying the amount of bandwidth to subscribe or by changing SP.

It can be easily inferred that the decisions made by the population at small cell s_i do not depend on decisions made by the population at the other small cells, because each small cell has independent resources. In addition, if users only have MSP coverage, they will subscribe to it, so that the following analysis applies only to the areas covered by both SPs.

Clearly, all users in small cell s_i subscribe to one of the two SPs if

$$N_i \le B_{m_i}/b^*_m + B_{s_i}/b^*_s = B_{m_i} p_m + B_{s_i} p_s.$$

Now, for a given p_s, the minimum price p_m, that will allow all users in s_i to subscribe the service is given by

$$p_{m_i}(p_s) = \frac{N_i - p_s B_{s_i}}{B_{m_i}}. \tag{26}$$

Likewise, for a given p_m, the minimum price p_s, that will allow all users in s_i to subscribe the service is given by

$$p_{s_i}(p_m) = \frac{N_i - p_m B_{m_i}}{B_{s_i}}. \tag{27}$$

Let $(x^*_{m_i}, x^*_{s_i}, x^*_{o_i})$ represent the fraction of users in s_i following each strategy in equilibrium. The following three cases are possible:

1. If $p_s > p_m/\theta_{m_i}$, then $u_{m_i} > u_{s_i}$, which means that all users tend to subscribe to the MSP. We have the following alternatives
   (a) If $p_m \geq N_i/B_{m_i}$, the MSP has enough bandwidth to serve all users, so that all users can subscribe to it: $x^*_{m_i} = 1, x^*_{s_i} = 0$ and $x^*_{o_i} = 0$.
   (b) If $p_m < N_i/B_{m_i}$, the MSP does not have enough bandwidth to serve all users. We have the following possibilities:
      i. If $p_m > p_{m_i}(p_s)$, then the SSP has enough bandwidth to serve the users not served by the MSP, obtaining $x^*_{m_i} = p_m B_{m_i}/N_i$, $x^*_{s_i} = 1 - p_m B_{m_i}/N_i$ and $x^*_{o_i} = 0$.
      ii. If $p_m \leq p_{m_i}(p_s)$, then the SPs does not have enough bandwidth to serve all users, obtaining $x^*_{m_i} = p_m B_{m_i}/N_i$, $x^*_{s_i} = p_s B_{s_i}/N_i$ and $x^*_{o_i} = 1 - x^*_{m_i} - x^*_{s_i}$.

   All these possibilities are summarized in Table 1.
2. If $p_s < p_m/\theta_{m_i}$, then $u_{m_i} < u_{s_i}$, which means that all users tend to subscribe to the SSP. Applying similar considerations as in $u_{m_i} > u_{s_i}$, we obtain the results in Table 2.
3. If $p_s = p_m/\theta_{m_i}$, then $u_{m_i} = u_{s_i}$, which means that users are indifferent between subscribing to the MSP or to the SSP. Therefore, users choose randomly (with equal probability) between the MSP and the SSP [4]. We have the following possibilities:
   (a) If $p_m > \frac{N_i}{2B_{m_i}}$, the MSP has enough bandwidth to serve more than $N_i/2$ users. We have the following possibilities:
      i. If $p_s > \frac{N_i}{2B_{s_i}}$, the SSP has enough bandwidth to serve more than $N_i/2$ users, so that all users can subscribe to it: $x^*_{m_i} = 1/2$, $x^*_{s_i} = 1/2$ and $x^*_{o_i} = 0$.
      ii. If $p_{s_i}(p_m) \leq p_s \leq \frac{N_i}{2B_{s_i}}$, the SSP does not have bandwidth to serve more than $N_i/2$ users. Still, all users subscribe to the service: $x^*_{m_i} = 1 - p_s B_{s_i}/N_i$, $x^*_{s_i} = p_s B_{s_i}/N_i$ and $x^*_{o_i} = 0$.
      iii. If $p_s < p_{s_i}(p_m)$, the SSP does not have enough bandwidth to serve all users, obtaining $x^*_{m_i} = p_m B_{m_i}/N_i, x^*_{s_i} = p_s B_{s_i}/N_i$ and $x^*_{o_i} = 1 - x^*_{m_i} - x^*_{s_i}$.
   (b) If $p_{m_i}(p_s) \leq p_m \leq \frac{N_i}{2B_{m_i}}$, the MSP does not have bandwidth to serve more than $N_i/2$ users. Still, all users subscribe to the service. We have the following possibilities:
      i. If $p_s > \frac{N_i}{2B_{s_i}}$, the SSP has enough bandwidth to serve more than $N_i/2$ users: $x^*_{m_i} = p_m B_{m_i}/N_i$, $x^*_{s_i} = 1 - p_m B_{m_i}/N_i$ and $x^*_{o_i} = 0$.
      ii. If $p_{s_i}(p_m) \leq p_s \leq \frac{N_i}{2B_{s_i}}$, all users subscribe to the service, so the only possible population ratio is $x^*_{m_i} = 1/2$, $x^*_{s_i} = 1/2$ and $x^*_{o_i} = 0$.
      iii. The case $p_s < p_{s_i}(p_m)$ does not exist, because if $p_m \geq p_{m_i}(p_s)$ then $p_s \geq p_{s_i}(p_m)$.
   (c) If $p_m < p_{m_i}(p_s)$, the SPs does not have bandwidth to serve all users. We have the following possibilities:
      i. If $p_s > \frac{N_i}{2B_{s_i}}$, the population ratio is: $x^*_{m_i} = p_m B_{m_i}/N_i, x^*_{s_i} = p_s B_{s_i}/N_i$ and $x^*_{o_i} = 1 - x^*_{m_i} - x^*_{s_i}$.
      ii. The case $p_{s_i}(p_m) \leq p_s \leq \frac{N_i}{2B_{s_i}}$ does not exist, because if $p_m < p_{m_i}(p_s)$ then $p_s < p_{s_i}(p_m)$.
      iii. If $p_s < p_{s_i}(p_m)$, the population ratio is: $x^*_{m_i} = p_m B_{m_i}/N_i, x^*_{s_i} = p_s B_{s_i}/N_i$ and $x^*_{o_i} = 1 - x^*_{m_i} - x^*_{s_i}$.

   All these possibilities are summarized in Table 3.

3.2 Stage I: Stackelberg game

At Stage I, the SPs assume that users will behave as derived above, and each SP chooses its price according to a Stackelberg game (Fig. 2). The SSP sets its price in response to MSP's price, in order to maximize its profits given MSP's choice, while the objective of the leader is to maximize its profits anticipating SSP's choice

$$\begin{aligned} p^*_m &= \arg\max_{p_m \geq 0} \quad \pi_m(p_m, x_m(p_m, p^*_s)), \\ \text{s. t.} \quad & p^*_s = \arg\max_{p_s \geq 0} \quad \pi_s(p_s, x_s(p_m, p_s)). \end{aligned}$$

For the sake of clarity, we first describe the analysis with just two small cells that cover the entire service area. Then, at the end of the analysis, the solution is extended to a general number, K, of small cells. Without loss of generality we assume that $\theta_{m_1} > \theta_{m_2}$. We also assume that $N_1/B_{s_1} \leq N_2/B_{s_2}$; although the analysis is not included here, it can be shown that the case $N_1/B_{s_1} > N_2/B_{s_2}$ would lead to the same equilibrium.

We divide the strategy space into the following regions (Fig. 3):

– Regions of user preferences:
  – **Region A** is defined by $p_s \geq p_m/\theta_{m_1}$ and it corresponds to the region where, in both small cells, all users prefer to subscribe to the MSP. Consequently, this region is also called MSP preference region. The population ratios in this region are as shown in Table 1.

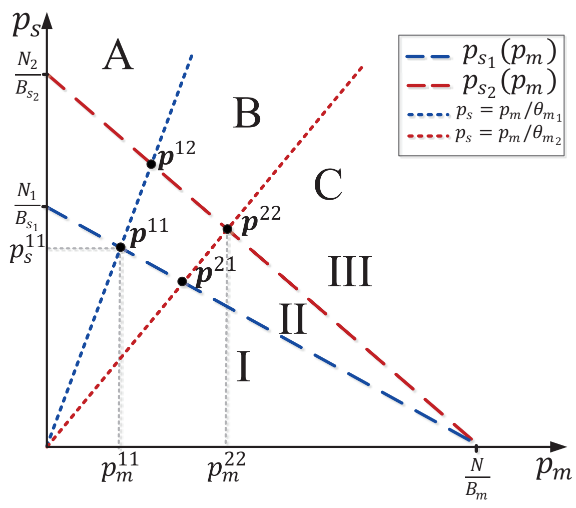

Fig. 3 Different regions.

- **Region B** is defined by $p_m/\theta_{m_2} < p_s \leq p_m/\theta_{m_1}$ and it corresponds to the region where users in s_2 will prefer to subscribe to the MSP, and users in s_1 will prefer to subscribe to the SSP. Consequently, this region is also called user's indecision region. The population ratios in this region are as shown in Table 1 for s_2, and in Table 2 for s_1.
- **Region C** is defined by $p_s < p_m/\theta_{m_2}$ and it corresponds to the region where, in both small cells, all users prefer to subscribe to the SSP. Consequently, this region is also called SSP preference region. The population ratios in this region are as shown in Table 2.

- Regions of SPs supply
  - **Region I** is defined by $p_s \leq p_{s_1}(p_m)$ and it corresponds to the case in which the SPs do not have enough bandwidth to serve all users in s_1 or s_2. Consequently, this region is also called low supply region.
  - **Region II** is defined by $p_{s_1}(p_m) \leq p_s \leq p_{s_2}(p_m)$ and it corresponds to the case in which the SPs do not have enough bandwidth to serve all users in s_2, but they can serve all users in s_1.
  - **Region III** is defined by $p_s \geq p_{s_2}(p_m)$ and it corresponds to the case in which the SPs have enough bandwidth to serve all users in both s_1 and s_2. Consequently, this region is also called high supply region.

Let $p^{ij} = (p_m^{ij}, p_s^{ij})$, with $i, j \in \{1,2\}$, denote the points defined by the intersection of the borders between the different regions as shown in Fig. 3.

The intersection of the regions defined by users preferences (A, B, C) and those defined by SPs supply (I, II, III) defines nine new regions that we denote as AI, AII, AIII, ..., CIII.

3.2.1 SSP's best response

Here we analyze the SSP's Best Response

$$\mathrm{BR}(p_m) = \arg\max_{p_s} \pi_s.$$

From (22) and (24) the SSP utility function is

$$\pi_s = N_1 x_{s_1}^* + N_2 x_{s_2}^*, \tag{28}$$

where $x_{s_1}^*$ and $x_{s_2}^*$ are as shown in Tables 1 and 2.

Since $0 \leq x_{s_i} \leq 1$, it is clear that $0 \leq N_i x_{s_i}^* \leq N_i$. However, in the following analysis, when $N_i x_{s_i}^*$ is substituted in (28) by the corresponding expression, its lower and upper limits are considered to be implicit for simplicity of notation; otherwise our notation would be cluttered with the use of $\max(0,\cdot)$ and $\min(N_i,\cdot)$, or with the introduction of more regions in the strategy space.

First we study the best response in each of the nine regions (AI, AII, ..., CIII). For a given region R we denote by $\mathrm{BR}_R(p_m)$ the best response of the SSP in the region R, i.e., such that the strategy $\left(p_m, \mathrm{BR}_R(p_m)\right) \in R$. Similarly, we define $\pi_s^R(p_m) \equiv \pi_s\left(p_m, \mathrm{BR}_R(p_m)\right)$. Then, $\mathrm{BR}(x)$ will be easily obtained by comparing the values $\pi_s^{\mathrm{R}}(x)$ for all the regions crossed by the line $p_m = x$.

$$\mathrm{BR}_{\mathrm{AI}}(p_m) = \arg\max_{p_s} \; p_s(B_{s_1} + B_{s_2}) = p_{s_1}(p_m) \tag{29}$$

$$\pi_s^{\mathrm{AI}}(p_m) = (N_1 - p_m B_{m_1}) \frac{B_{s_1} + B_{s_2}}{B_{s_1}} \tag{30}$$

$$\mathrm{BR}_{\mathrm{AII}}(p_m) = \arg\max_{p_s} \; N_1 - p_m B_{m_1} + p_s B_{s_2} = p_{s_2}(p_m) \tag{31}$$

$$\pi_s^{\mathrm{AII}}(p_m) = N_1 + N_2 - p_m(B_{m_1} + B_{m_2}) \tag{32}$$

$$\mathrm{BR}_{\mathrm{AIII}}(p_m) = \arg\max_{p_s} \; N_1 + N_2 - p_m(B_{m_1} + B_{m_2}) = p_{s_2}(p_m) \tag{33}$$

$$\pi_s^{\mathrm{AIII}}(p_m) = N_1 + N_2 - p_m(B_{m_1} + B_{m_2}) \tag{34}$$

$$\mathrm{BR}_{\mathrm{BI}}(p_m) = \arg\max_{p_s} \; p_s(B_{s_1} + B_{s_2}) = \begin{cases} p_m/\theta_{m_1} & \text{if } p_m < p_m^{11} \\ p_{s_1}(p_m) & \text{if } p_m \geq p_m^{11} \end{cases} \tag{35}$$

$$\pi_s^{\mathrm{BI}}(p_m) = \begin{cases} p_m/\theta_{m_1}(B_{s_1} + B_{s_2}) & \text{if } p_m < p_m^{11} \\ p_{s_1}(p_m)(B_{s_1} + B_{s_2}) & \text{if } p_m \geq p_m^{11} \end{cases} \tag{36}$$

$$\mathrm{BR}_{\mathrm{BII}}(p_m) = \arg\max_{p_s} \; p_s(B_{s_1}+B_{s_2}) = \begin{cases} p_m/\theta_{m_1}-\varepsilon & \text{if } p_m < p_m^{12} \\ p_{s_2}(p_m) & \text{if } p_m \geq p_m^{12} \end{cases} \quad (37)$$

where ε is an arbitrarily small positive number.

$$\pi_s^{\mathrm{BII}}(p_m) = \begin{cases} p_m/\theta_{m_1}(B_{s_1}+B_{s_2})-\varepsilon & \text{if } p_m < p_m^{12} \\ p_{s_2}(p_m)(B_{s_1}+B_{s_2}) & \text{if } p_m \geq p_m^{12} \end{cases} \quad (38)$$

$$\mathrm{BR}_{\mathrm{BIII}}(p_m) = \arg\max_{p_s} \; p_sB_{s_1}+N_2-p_mB_{m_2} = \begin{cases} p_m/\theta_{m_1}-\varepsilon & \text{if } p_m < \frac{N_1\theta_{m_1}}{B_{s_1}} \\ N_1/B_{s_1} & \text{if } p_m \geq \frac{N_1\theta_{m_1}}{B_{s_1}} \end{cases} \quad (39)$$

$$\pi_s^{\mathrm{BIII}}(p_m) = \begin{cases} p_m(\frac{B_{s_1}}{\theta_{m_1}}-B_{m_2})+N_2-\varepsilon & \text{if } p_m < \frac{N_1\theta_{m_1}}{B_{s_1}}, \\ N_1+N_2-p_mB_{m_2} & \text{if } p_m \geq \frac{N_1\theta_{m_1}}{B_{s_1}} \end{cases} \quad (40)$$

$$\mathrm{BR}_{\mathrm{CI}}(p_m) = \arg\max_{p_s} \; p_s(B_{s_1}+B_{s_2}) = \begin{cases} p_m/\theta_{m_2} & \text{if } p_m < p_m^{21} \\ p_{s_1}(p_m) & \text{if } p_m \geq p_m^{21} \end{cases} \quad (41)$$

$$\pi_s^{\mathrm{CI}}(p_m) = \begin{cases} p_m/\theta_{m_2}(B_{s_1}+B_{s_2}) & \text{if } p_m < p_m^{21}, \\ p_{s_1}(p_m)(B_{s_1}+B_{s_2}) & \text{if } p_m \geq p_m^{21}, \end{cases} \quad (42)$$

$$\mathrm{BR}_{\mathrm{CII}}(p_m) = \arg\max_{p_s} \; p_s(B_{s_1}+B_{s_2}) = \begin{cases} p_m/\theta_{m_2} & \text{if } p_m < p_m^{22} \\ p_{s_2}(p_m) & \text{if } p_m \geq p_m^{22} \end{cases} \quad (43)$$

$$\pi_s^{\mathrm{CII}}(p_m) = \begin{cases} p_m/\theta_{m_2}(B_{s_1}+B_{s_2}) & \text{if } p_m < p_m^{22} \\ p_{s_2}(p_m)(B_{s_1}+B_{s_2}) & \text{if } p_m \geq p_m^{22} \end{cases} \quad (44)$$

$$\mathrm{BR}_{\mathrm{CIII}}(p_m) = \arg\max_{p_s} \; p_s(B_{s_1}+B_{s_2}) = \begin{cases} p_m/\theta_{m_2}-\varepsilon & \text{if } p_m < \frac{N_2\theta_{m_2}}{B_{s_2}} \\ N_2/B_{s_2} & \text{if } p_m \geq \frac{N_2\theta_{m_2}}{B_{s_2}} \end{cases} \quad (45)$$

$$\pi_s^{\mathrm{CIII}}(p_m) \begin{cases} p_m/\theta_{m_2}(B_{s_1}+B_{s_2})-\varepsilon & \text{if } p_m < \frac{N_2\theta_{m_2}}{B_{s_2}} \\ N_1+N_2 & \text{if } p_m \geq \frac{N_2\theta_{m_2}}{B_{s_2}} \end{cases} \quad (46)$$

Comparing (30), (32), (34), (36), (38), (40), (42), (44) and (46), we obtain the $\mathrm{BR}(p_m)$

$$\mathrm{BR}(p_m) = \begin{cases} p_{s_2} & \text{if } p_m \leq p_m^{AB} \\ \frac{p_m}{\theta_{m_1}}-\varepsilon & \text{if } p_m^{AB} < p_m \leq \frac{N_1\theta_{m_1}}{B_{s_1}} \\ \frac{N_1}{B_{s_1}} & \text{if } \frac{N_1\theta_{m_1}}{B_{s_1}} < p_m \leq p_m^{BC} \\ \frac{p_m}{\theta_{m_2}}-\varepsilon & \text{if } p_m^{BC} < p_m < \frac{N_2\theta_{m_2}}{B_{s_2}} \\ \frac{N_2}{B_{s_2}} & \text{if } p_m \geq \frac{N_2\theta_{m_2}}{B_{s_2}} \end{cases} \quad (47)$$

The threshold prices p_m^{AB} and p_m^{BC} are given as

$$p_m^{AB} = \frac{\theta_{m_1}(N_1+N_2)}{\theta_{m_1}(B_{m_1}+B_{m_2})+B_{s_1}+B_{s_2}}$$

$$p_m^{BC} = \begin{cases} \frac{N_2\theta_{m_2}}{B_{s_2}+\theta_{m_2}B_{m_2}+B_{s_1}(1-\frac{\theta_{m_2}}{\theta_{m_1}})} & \text{if } \mathrm{BR}(p_m) = \frac{p_m}{\theta_{m_1}}-\varepsilon, \\ \frac{(N_1+N_2)\theta_{m_2}}{B_{s_2}+\theta_{m_2}B_{m_2}+B_{s_1}} & \text{if } \mathrm{BR}(p_m) = \frac{N_1}{Bs_1}. \end{cases}$$

3.2.2 MSP's optimal decision

Given SSP's $\mathrm{BR}(p_m)$ derived in (47), the expressions for π_m are the following ones:

$$\pi_m = \begin{cases} \pi_m^{\mathrm{i}} & \text{if } p_m \leq p_m^{AB} \\ \pi_m^{\mathrm{ii}} & \text{if } p_m^{AB} < p_m \leq \frac{N_1\theta_{m_1}}{B_{s_1}} \\ \pi_m^{\mathrm{iii}} & \text{if } \frac{N_1\theta_{m_1}}{B_{s_1}} < p_m \leq p_m^{BC} \\ \pi_m^{\mathrm{iv}} & \text{if } p_m^{BC} < p_m < \frac{N_2\theta_{m_2}}{B_{s_2}} \\ \pi_m^{\mathrm{v}} & \text{if } p_m \geq \frac{N_2\theta_{m_2}}{B_{s_2}} \end{cases} \quad (48)$$

where $\pi_m^{\mathrm{i}}, \pi_m^{\mathrm{ii}}, \pi_m^{\mathrm{iii}}, \pi_m^{\mathrm{iv}}, \pi_m^{\mathrm{v}}$ are

$$\pi_m^{\mathrm{i}} = p_m(B_{m_1}+B_{m_2})$$
$$\pi_m^{\mathrm{ii}} = p_m(B_{m_2}-\frac{B_{s_1}}{\theta_{m_1}})+N_1-\varepsilon$$
$$\pi_m^{\mathrm{iii}} = p_mB_{m_2}$$
$$\pi_m^{\mathrm{iv}} = N_1+N_2-\frac{p_m}{\theta_{m_2}}(B_{s_1}+B_{s_2})-\varepsilon$$
$$\pi_m^{\mathrm{v}} = 0$$

We analyze the MSP's best price for each possibility

$$p_m^{\mathrm{i}*} = \arg\max_{p_s} \; \pi_s^{\mathrm{i}} = p_m^{AB} \quad (49)$$

$$p_m^{\mathrm{ii}*} = \arg\max_{p_s} \; \pi_s^{\mathrm{ii}} = \frac{N_1\theta_{m_1}}{B_{s_1}} \quad (50)$$

$$p_m^{\mathrm{iii}*} = \arg\max_{p_s} \; \pi_s^{\mathrm{iii}} = p_m^{BC} \quad (51)$$

$$p_m^{\mathrm{iv}*} = \arg\max_{p_s} \; \pi_s^{\mathrm{iv}} = p_m^{BC} \quad (52)$$

$$p_m^{\mathrm{v}*} = \arg\max_{p_s} \; \pi_s^{\mathrm{v}} = \frac{N_2\theta_{m_2}}{B_{s_2}} \quad (53)$$

There is a Stackelberg equilibrium when the price announced by the MSP generates the highest MSP's profits, given SSP's $\mathrm{BR}(p_m)$. Since $\pi_m^{\mathrm{iii}}(p_m^{\mathrm{iii}*}) \geq \pi_m^{\mathrm{ii}}(p_m^{\mathrm{ii}*}) \geq \pi_m^{\mathrm{iv}}(p_m^{\mathrm{iv}*}) \geq \pi_m^{\mathrm{v}}(p_m^{\mathrm{v}*})$, then there are two alternatives:

– If $\pi_m^{\mathrm{i}}(p_m^{\mathrm{i}*}) > \pi_m^{\mathrm{iii}}(p_m^{\mathrm{iii}*})$ the Stackelberg equilibrium is

$$\begin{aligned} p_m^* &= p_m^{\mathrm{i}*} = p_m^{AB}, \\ p_s^* &= \frac{N_2 - p_m^* B_{m_2}}{B_{s_2}} \end{aligned} \tag{54}$$

– If $\pi_m^{\mathrm{iii}}(p_m^{\mathrm{iii}*}) \geq \pi_m^{\mathrm{i}}(p_m^{\mathrm{i}*})$ the Stackelberg equilibrium is

$$\begin{aligned} p_m^* &= p_m^{\mathrm{iii}*} = p_m^{BC}, \\ p_s^* &= p_m^*/\theta_{m_2} - \varepsilon \end{aligned} \tag{55}$$

Since this equilibrium anticipates the equilibrium in Stage II as derived in the previous section, the Stackelberg equilibrium is a Subgame Perfect Nash Equilibrium for the whole game.

For the general case with K small cells, again, without loss of generality, we assume that $\theta_{m_1} > \theta_{m_2} \ldots > \theta_{m_K}$ and that $N_1/B_{s_1} \leq N_2/B_{s_2} \ldots \leq N_K/B_{s_K}$. There are three possible cases:

– $\pi_m^{\mathrm{i}}(p_m^{\mathrm{i}*}) > \max(\pi_m^{\mathrm{iii}}(p_m^{\mathrm{iii}*}), N_{m_0})$
The Stackelberg equilibrium is

$$\begin{aligned} p_m^* &= \frac{\theta_{m_1} N}{\theta_{m_1} B_m + B_s}, \\ p_s^* &= \frac{N_K - p_m^* B_{m_K}}{B_{s_K}} \end{aligned} \tag{56}$$

– $\pi_m^{\mathrm{iii}}(p_m^{\mathrm{iii}*}) > \max(\pi_m^{\mathrm{i}}(p_m^{\mathrm{i}*}), N_{m_0})$.
Let $p_m^{BC_{ij}}$ be the MSP price where SSP changes its strategy from $p_s = p_m/\theta_{m_i} - \varepsilon$ to $p_s = p_m/\theta_{m_j} - \varepsilon$, where $\theta_{m_j} < \theta_{m_i}$. $p_m^{BC_{ij}}$ is given as

$$p_m^{BC_{ij}} = \begin{cases} \dfrac{N_j \theta_{m_j}}{B_{s_j} + \theta_{m_j} B_{m_j} + B_{s_i}\left(1 - \frac{\theta_{m_j}}{\theta_{m_i}}\right)} & \text{if } \mathrm{BR}(p_m) = \frac{p_m}{\theta_{m_i}} - \varepsilon, \\ \dfrac{(N_i + N_j)\theta_{m_j}}{B_{s_j} + \theta_{m_j} B_{m_j} + B_{s_i}} & \text{if } \mathrm{BR}(p_m) = \frac{N_i}{B s_i}. \end{cases}$$

The Stackelberg equilibrium is

$$\begin{aligned} p_m^* &= p_m^{BC_{i_0, i_0+1}}, \\ p_s^* &= p_m^*/\theta_{m_{i_0+1}} - \varepsilon \end{aligned} \tag{57}$$

where

$$i_0 = \arg\max_k \; \pi_m^{\mathrm{iii}}(p_m^{BC_{k,k+1}}) \tag{58}$$

– $N_{m_0} \geq \max(\pi_m^{\mathrm{i}}(p_m^{\mathrm{i}*}), \pi_m^{\mathrm{iii}}(p_m^{\mathrm{iii}*}))$
The Stackelberg equilibrium is

$$\begin{aligned} p_m^* &= \frac{N}{B_m}, \\ p_s^* &= \max_i \left(\frac{N_i}{B_{s_i}}\right) = \frac{N_K}{B_{s_K}} \end{aligned} \tag{59}$$

Table 4 Parameter setting

Parameter	Value
N	1000 users
A_m	10000 m^2
B_m	150 MHz
K	5 small cells

4 Results

In this section some results are presented in order to illustrate the capabilities of our model and analysis, and to provide an insight into the system behavior. The effect of SSP coverage ratio, MSP spectral efficiency and SSP available bandwidth on the system key indicators are analyzed. The indicators calculated are: population ratios, prices and users welfare. Note that, from (9), (21) and (23), MSP profit is proportional to the number of subscribers, $\pi_m^* = N x_m^*$, and from (10), (22) and (24), SSP profit is $\pi_s^* = N x_s^*$). The system parameters values are those shown in Table 4.

4.1 Effect of SSP coverage ratio

Figures 4 and 5 show the effect of the SSP coverage ratio (A_s/A_m) on equilibrium population ratios (x_m^*, x_s^* and x_o^*), equilibrium prices (p_m^* and p_s^*) and users welfare (UW). These results have been obtained for a scenario with five small cells with identical area ($A_i = A_s/5$) and a number of users proportional to the area ($N_i = NA_i/A$). The bandwidth available to the SSP is $B_s = 130$ and equally distributed among all the small cells, $B_{s_i} = B_s/5$ MHz. The MSP spectral efficiency is $\theta_{m_i} = \theta_m = 0.8$ bits/s/Hz at each small cells, and the MSP spectral efficiency at c_0 (the area not covered by any small cell) is also $\theta_{m_0} = \theta_m = 0.8$ bits/s/Hz.

As can be seen, across the whole range of As/Am, all users subscribe with one of the two SPs ($x_0^* = 0$) and in all cases they obtain the same UW. It can be shown that in this scenario UW= $(\theta_m B_m + B_s)e^{-1} = 91.97$.

These figures clearly show that the range of A_s/A_m can be divided into two regions. When A_s/A_m is below a certain threshold, the system displays a completely different behavior from the one when A_s/A_m is above that threshold. For the current values of the configuration parameters, the threshold that separates the two regions is $A_s/A_m = B_s/(\theta B_m + B_s) = 13/25 = 0.52$.

When the coverage ratio is low (under the mentioned threshold), the SSP sets a price sufficiently lower than the price of the MSP so that all users into the small cells prefer to subscribe to the SSP. The SSP sets the minimum price that allows to serve all the users in the small cells. As the

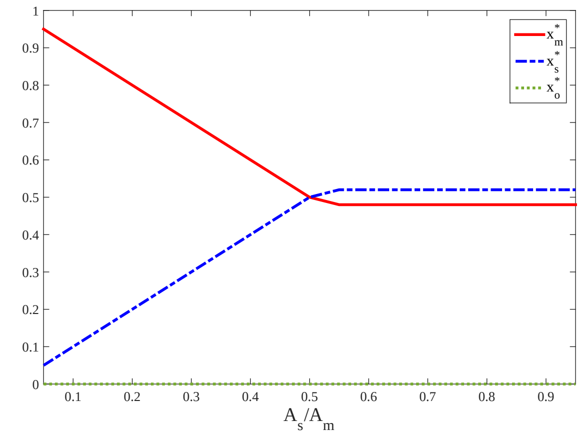

Fig. 4 Population ratios as a function of A_s/A_m.

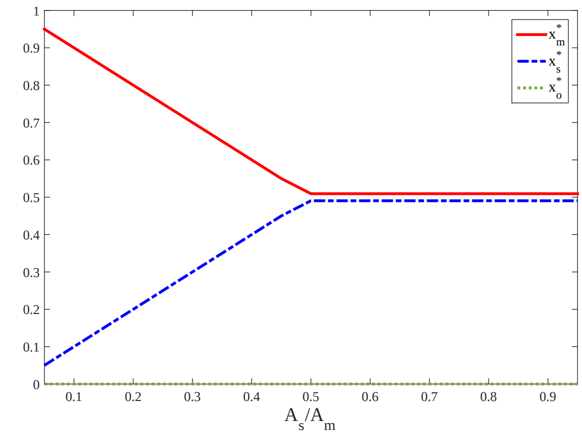

Fig. 6 Population ratios as a function of A_s/A_m with heterogeneous θ_i's.

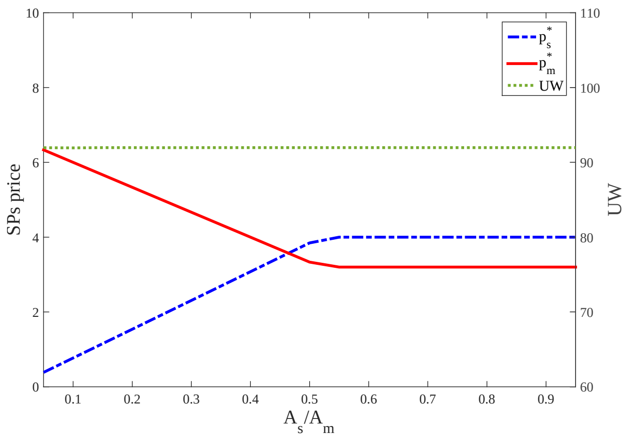

Fig. 5 Prices and users welfare as a function of A_s/A_m.

coverage ratio increases, and consequently the number of users to serve, the SSP raises the price linearly. In this range of low coverage ratio, the MSP gets no subscribers in the small cells, and it fixes its price as the minimum one that allows to serve all the users in s_0. As the coverage ratio increases, the number of users in s_0 decreases, and the MSP lowers the price accordingly.

This can be interpreted as follows. When the SSP coverage is low, the MSP strategy is not to compete, that is, it behaves as if the users in the small cells did not exist. In this situation, all users in the small cells subscribe to the SSP and all users in s_0 subscribe to the MSP, and the population ratios of both SPs vary linearly with the coverage ratio, with a gradual transfer of users from the MSP to the SSP as the coverage ratio increases. As a result of this transfer of users, the MSP profit is gradually transferred to the SSP, while the UW is held constant.

When the coverage ratio exceeds the threshold mentioned above, SPs behaviors changes abruptly. This happens when the price of the SSP equals the price of the MSP divided by θ_m, which means that users inside the small cells are indifferent between subscribing to the MSP or to the SSP. From this point, as the coverage ratio increases both prices remain constant, so that a fraction of the users in the small cells will subscribe to the MSP and the rest will subscribe to the SSP.

Both the MSP and SSP retain their respective population ratios at the point where A_s/A_m crossed the threshold, and these are maintained irrespective of the value of A_s/A_m. Consequently, both SPs profits remain constant.This can be interpreted as that, when the SSP coverage is high, MSP strategy is to compete against the SSP to get a fraction of the users into the small cells while keeping all the users in s_0.

The above results correspond to a homogeneous scenario in which the MSP gets the same spectral efficiency at all the small cells. We now examine the effect of a heterogeneous distribution of the MSP spectral efficiency. Figures 6 and 7 show the results obtained by repeating the above calculations, but this time being the spectral efficiencies obtained by the MSP at the small cells $\{\theta_{m_i}\} = \{0.9,\ 0.8,\ 0.7,\ 0.6,\ 0.5\}$ bits/s/Hz, and the spectral efficiency at s_0, $\theta_{m_0} = 0.8$ bits/s/Hz.

In this scenario, the values of the indicators in equilibrium show a behavior similar to that of the homogeneous scenario but with some slight differences. Now, the coverage ratio threshold that separates the two regions happens when the price of the SSP equals the price of the MSP divided by $\theta_1 = 0.9$, which corresponds to the small cell with the highest spectral efficiency (s_i). This means that, when the prices reach this threshold, the MSP starts to compete against the SSP in s_1. From this point, as the coverage ratio increases both prices remain constant, and the MSP is gradually gaining users to the SSP in the small cells, and losing users in s_0 (because the area and, consequently the population, in s_0 decrease), keeping the

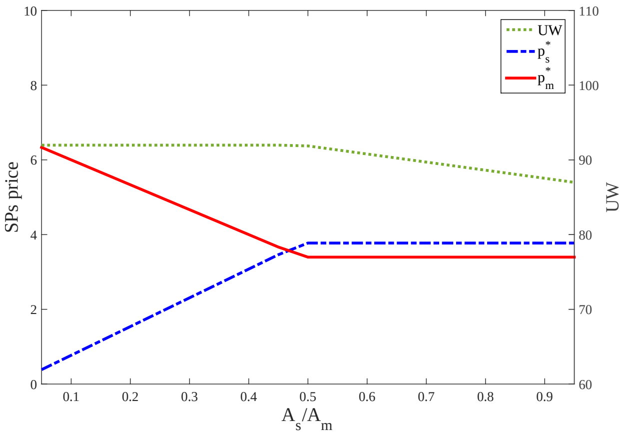

Fig. 7 Prices and users welfare as a function of A_s/A_m with heterogeneous θ_i's.

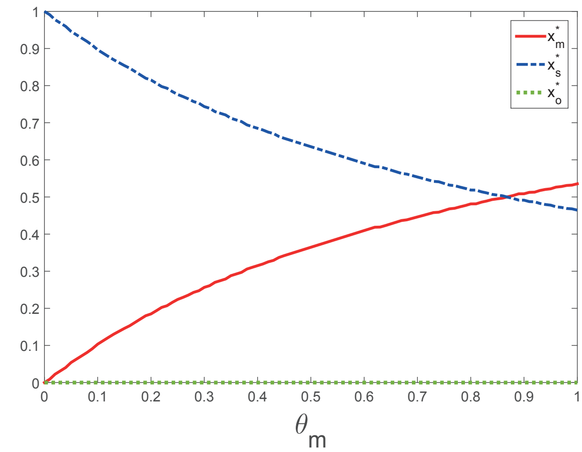

Fig. 8 Population ratios as a function of θ_m.

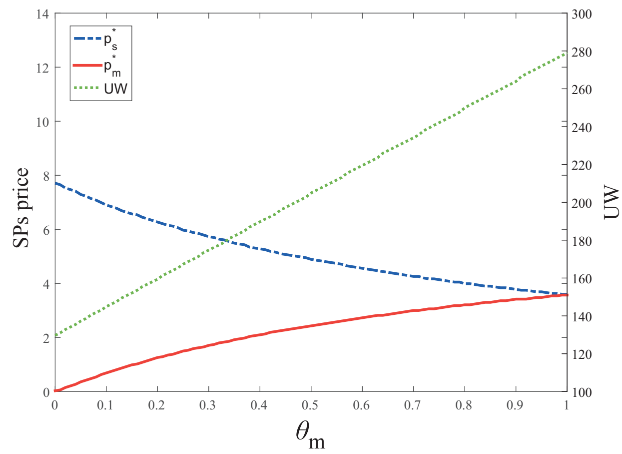

Fig. 9 Prices and users welfare as a function of θ_m.

overall population ratio constant. As the price no longer changes, the MSP starts to obtain users in the small cells with less efficiency ($\{c_2 \ldots c_5\}$) when in these the SSP does not have, for the given prices, sufficient bandwidth to satisfy the demand.

With respect to the UW, we now see that in the zone of high coverage it does not stay constant, but decreases slightly. This is because as more users subscribed to the MSP that were in s_0 go from being inside s_0 to being inside a small cell, they continue to be subscribed to the MSP, but their average utility decreases, because the spectral efficiency of the MSP in the small cells is, on average, lower than in s_0, ($\frac{1}{5}\sum_{i=1}^{5}\theta_{m_i} = 0.7 < \theta_{m_0} = 0.8$).

4.2 Effect of spectral efficiency

Figures 8 and 9 show the effect of the spectral efficiency obtained by the MSP at the small cells (θ_m) on the equilibrium population ratios (x_m^*, x_s^* and x_o^*), equilibrium prices (p_m^* and p_s^*) and equilibrium users welfare (UW). It is assumed that the MSP spectral efficiency is $\theta_{m_i} = \theta_m$ bits/s/Hz at all the small cells, and at s_0, $\theta_{m_0} = \theta_m$ bits/s/Hz. These results have been obtained for the case where the coverage ratio is $A_s/A_m = 1$, the number of users at each cell is $N_i = 200$, and the bandwidth available to the SSP is $B_s = 130$ and equally distributed among all the small cells, $B_{s_i} = B_s/5$ MHz.

It can be seen that, the higher θ_m, the harder is for the SSP to compete. When θ_m is low, SSP's price is high and MSP's price must be low, because the MSP has to compensate for its low service quality. As θ_m increases, the MSP increases its price, and the SSP is forced to compete and lower its price. Accordingly, users and profit are transferred from SSP to MSP as θ_m increases. Moreover, the competition favors the users, and UW increases as θ_m increases.

4.3 Effect of SSP's available bandwidth

Figures 10 and 11 show the effect of the SSP's available bandwidth at the small cells (B_s) on equilibrium population ratios (x_m^*, x_s^* and x_o^*), equilibrium prices (p_m^* and p_s^*) and equilibrium users welfare (UW). It is assumed that SSP's available bandwidth is equally distributed among all the small cells, $B_{s_i} = B_s/5$. These results have been obtained for the case where the coverage ratio is $A_s/A_m = 1$, the number of users at each cell is $N_i = 200$, and the MSP spectral efficiency is $\theta_{m_i} = \theta_m = 0.8$ bits/s/Hz at all the small cells and at s_0, $\theta_{m_0} = \theta_m = 0.8$ bits/s/Hz.

In this case the results obtained for population ratios mirrors that of section 4.2; the higher B_s, the more opportunities the SSP has to compete. The figures show that, as B_s increases, the SSP cuts down the price and users are

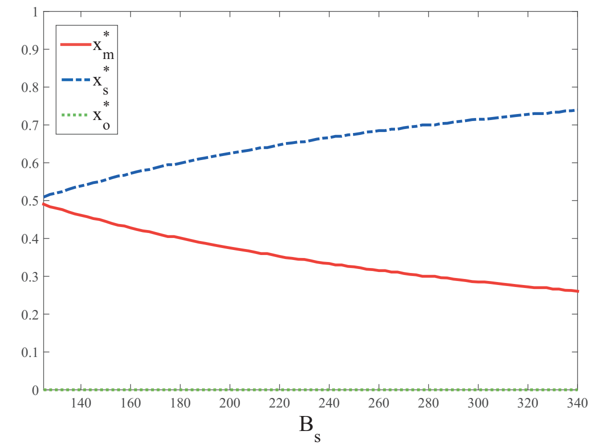

Fig. 10 Population ratios as a function of B_s.

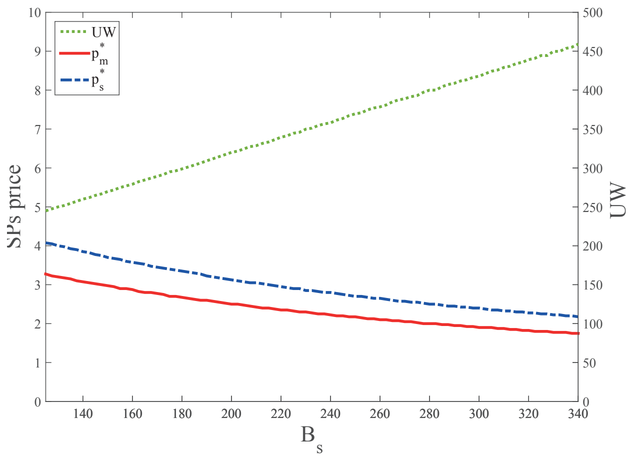

Fig. 11 Prices and users welfare as a function of B_s.

gradually transferred from the MSP to the SSP. Accordingly, profit is transferred from MSP to SSP as B_s increases. As before, competition favors the users, and UW increases as B_s increases.

5 Conclusions

We have proposed a business model for a small-cell service provider consisting on, first, deploying a network of small cells, and second, competing in prices against the macro cell service provider.

Analyzing the previous results it can be concluded that:

– The SSP has an incentive to operate in the market. Actually, the greater the SSP's resources are, and the lower the MSP's spectral efficiency is, the higher the SSP's profit is.
– Users benefit from SSP's operation and the greater the bandwidth available to the SSP and the SSP's espectral efficiency in relation to MSP spectral efficiency, the greater the user welfare is.
– The MSP will obtain lower profits when the SSP operates. A regulatory authority may grant access to the SSP, however, based on the user welfare improvement.
– If the SSP has a low coverage compared to the MSP coverage, competition does not change the behavior of the MSP, which behaves the same way as it would in a monopoly in the area not covered by the SSP, setting a price that makes all the users in this area to subscribe, while SSP takes all the subscribers inside the small cells. As SSP coverage increases, more profit and subscribers are transferred from the MSP to the SSP, while the users welfare remains constant.
– If the SSP has a high coverage ratio, the MSP strategy is to compete against the SSP to get a fraction of the users into the small cells while keeping all the users in the area not covered by the SSP. As the SSP coverage increases, prices and population ratios remain constant, while the user welfare does not increase.

As a future work, we will extend the static model analyzed here to a dynamic one, where the available bandwidth for the SSP varies over time. This variation will translate in time-varying subscribing populations. The pricing strategies for the MSP and the SSP will also be time varying and will aim to maximize discounted profits over time. A differential game will be the appropriate model in this case.

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