UNIVERSITAT POLITÈCNICA DE VALÈNCIA

Design of isolated foundation pads considering normal stresses

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1. Summary

This document sets out the conditions that isolated foundation pads have to fulfil considering only normal stresses in the contact surface between the foundation pad and the soil.

A worked example is included in order to show how to proceed. It is proposed a practical exercise to consolidate the process.

2. Introduction

When designing isolated foundation pads, it must be taken into account that the design tensile strength of the soil is zero.

Therefore, the foundation pad must transmit only compressions to the surface between the bottom cross section of the foundation pad and the soil.

Considering the four cases in Figure 1:

Case a) corresponds with a simple compression situation, where the internal forces transmitted by the structure to the foundation pad is a centred compression axial force.

Case b) corresponds with a combined compression axial force plus bending moment situation, equivalent to an eccentric axial force, being the eccentricity small enough to generate only compression stresses on the soil.

Case c) is again a combined compression axial force plus bending moment situation, equivalent to an eccentric axial force, being the eccentricity of the axial force a particular value that guarantees that the minimum normal stress σ_{min} is zero.

Finally, case d) corresponds to a combined compression axial force plus bending moment situation, equivalent to an eccentric axial force, where the eccentricity of the axial force is big enough to produce tensile stresses on the contact surface between the concrete pad and the soil. As has been said before, this is a not admissible situation, as the design tensile strength of the soil is zero.

As the foundation pad must transmit only compressions to the soil, the design of isolated foundation pads is directly related with the concept of central core, as will be shown in the following epigraph.

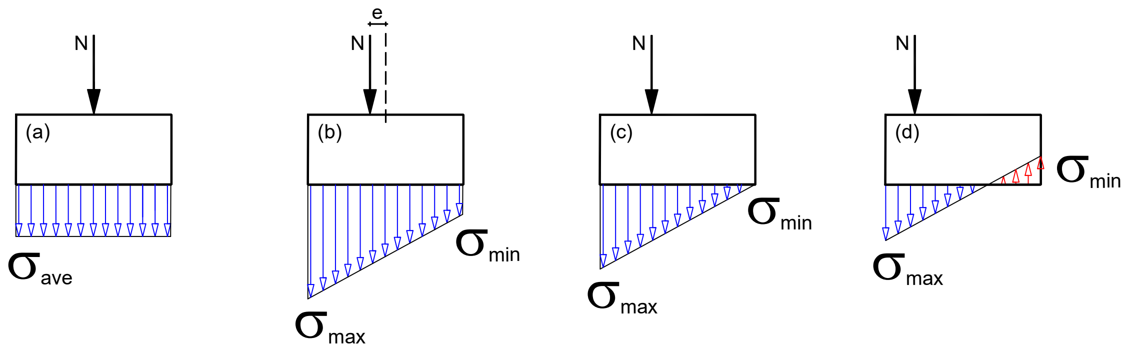

Figure 1

3. Aims

At the end of this document, the student will be able to check an isolated foundation pad considering the proposed dimensions and the type of soil.

Students will use the central core concept and will be able to propose a new design of the foundation pad, in case the initial one fails.

4. Design conditions and assumptions

4.1 Conditions for designing foundation pads

Considering that the design tensile strength of the soil is zero, the foundation pad must transmit only compressions on the cross section in contact with the soil, being the concept of central core directly related with the design of foundation pads.

The conditions to take into account are:

1. The minimum normal stress should be positive

$$\sigma_{\min,d} \leq 0 \quad \rightarrow \quad \gamma_F \sigma_{\min,k} \leq 0 \qquad \textit{Equation 1}$$

If the eccentric axial force is inside the central core, this condition will be fulfilled.

2. The maximum normal stress must be smaller than 1.25 times the allowable bearing strength of the soil. The allowable values are shown in table 1

$$\sigma_{\max,d} \leq 1.25\, \sigma_{all} \quad \rightarrow \quad \gamma_F \sigma_{\max,k} \leq 1.25 \frac{\sigma_{soil,k}}{\gamma_M} \qquad \textit{Equation 2}$$

3. The average normal stress, which is the value of the normal stress at the centroid of the cross section of the foundation pad in contact with the soil should be smaller or equal to the allowable bearing strength of the soil:

$$\sigma_{ave,d} \leq \sigma_{all} \quad \rightarrow \quad \gamma_F \sigma_{ave} \leq \frac{\sigma_{soil,k}}{\gamma_M} \qquad \textit{Equation 3}$$

4.2 Normal stress distribution in simple compression

This situation corresponds to case a in Figure 1

The normal stress distribution is obtained with Equation 4

$$\sigma_{\max,k} = \sigma_{\min,k} = \sigma_{ave,k} = -\frac{N}{A} \qquad \textit{Equation 4}$$

where N is the compression axial force, including the weight[1] of the footing pad and A is the area of the footing pad.

[1] The weight of the foundation pad is a compression axial force at the centroid of the footing pad. Its value is obtained considering the dimensions of the footing pad and the density of concrete equal to 25 kN/m^3

4.3 in combined biaxial bending and axial force

This situation corresponds to cases b, c and d in Figure 1

Considering the bending moment is equivalent to an eccentric axial force Figure 2

$$M_z = N \times e_y$$

$$M_y = N \times e_z$$

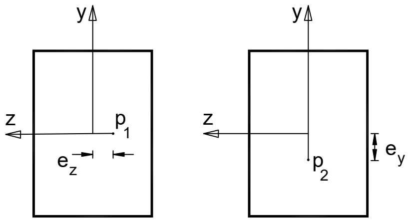

Figure 2

The normal stress distribution is obtained with Equation 5

$$\sigma_{\max,k} = -\left|\frac{N}{A}\right| - \left|\frac{N \times e}{W_{el}}\right|; \qquad \sigma_{\min,k} = -\left|\frac{N}{A}\right| + \left|\frac{N \times e}{W_{el}}\right|$$

Equation 5

Where N is the compression axial force, including the weight of the footing pad, A is the area of the footing pad, and W_{el} is the elastic modulus[2] about the same axis than the considered bending moment ($W_{el,z}$ if the bending moment is a M_z, and $W_{el,y}$ if the bending moment is a M_y)

4.4 in combined biaxial bending and axial force

Combined biaxial bending and axial force situation is occurs when the axial force is applied considering both eccentricities, e_z and e_y, as shown in Figure 3.

In that situation, the normal stress distribution is obtained

$$\sigma_{\max,k} = -\left|\frac{N}{A}\right| - \left|\frac{N \times e_y}{W_{el,z}}\right| - \left|\frac{N \times e_z}{W_{el,y}}\right|$$

$$\sigma_{\min,k} = -\left|\frac{N}{A}\right| + \left|\frac{N \times e_y}{W_{el,z}}\right| + \left|\frac{N \times e_z}{W_{el,y}}\right|$$

Equation 6.

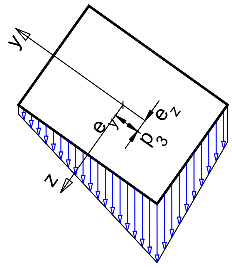

Figure 3

$$\sigma_{\max,k} = -\left|\frac{N}{A}\right| - \left|\frac{N \times e_y}{W_{el,z}}\right| - \left|\frac{N \times e_z}{W_{el,y}}\right|$$

$$\sigma_{\min,k} = -\left|\frac{N}{A}\right| + \left|\frac{N \times e_y}{W_{el,z}}\right| + \left|\frac{N \times e_z}{W_{el,y}}\right|$$

Equation 6

[2] For a rectangular cross section b ×h, $W_{el,z} = \frac{bh^2}{6}; \qquad W_{el,y} = \frac{hb^2}{6}$

Where N is the compression axial force, including the weight of the footing pad, A is the area of the footing pad, and $W_{el,z}$ is the elastic modulus about z axis, and $W_{el,y}$ is the elastic modulus about y axis.

4.5 Central core concept

The Central Core is the geometric place of all the points of application of the eccentric compression axial force, N, that guarantee that none fibre is in tension.

Therefore, if the eccentric axial force N is inside the central core, all the fibres will be in compression. When the axial force N is applied on the central core contour, the stress on the cross-section contour will be zero (case c in Figure 1)

The central core of a rectangular cross-section is a rhombus with b/3 and h/3 diagonals, as shown in Figure 4.

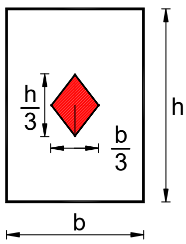

Figure 4

According to that, if the eccentricity[3] of the compression axial force is smaller than b/6 or h/6, all the cross section will be in compression. If the eccentricity of the compression axial force is equal to b/6 or h/6, all the cross section will be in compression being the stress at some points of the contour equal to zero.

Finally, if the eccentricity of the compression axial force is bigger than b/3 or h/3, part of the cross section will be in tension, failing to fulfil the first condition in epigraph 4.1.

4.6 Partial factors to be considered

According to the Spanish Building Code (CTE), for the design of foundation pads considering normal stresses transmitted to the soil, the partial factors to be taken into account are: $\gamma_F = 1$ and $\gamma_M = 1$

4.7 Bearing strength of different soils

Table 1 shows the allowable bearing strength of different soils according to the Spanish Building Code.

Types of soils	MPA = N/mm^2
Rock	10 to 4
Gravel	0.2 to 0.6
Clay, dry hard	0.3 to 0.6
Sand, coarse	> 0.3
Sand with clay. Dry fine	< 0.2
Sand, dry fine	< 0.1
Clay, soft	< 0.075

[3] The eccentricity is measured from the centroid of the cross section

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table 1. Allowable bearing strength of different soils (CTE DB SE C)

5. Worked example

Given the column and the footing pad in the figure,

Considering the square isolated footing dimensions are: 2 m x 2 m x 0.5 m (Breadth x Length x Depth), it is requested:

1. To obtain the central core of the footing pad.

Considering the concentrated load values are:

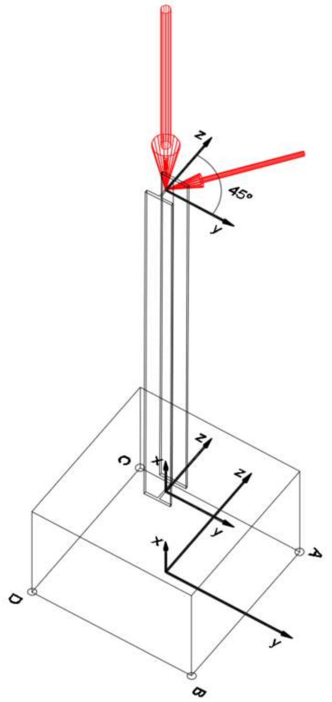
Vertical load: 100 kN and Horizontal load: 10 kN

Taking into account that the length of the column is 3 metres, and the concrete density is 25 kN/m2, it is requested:

2. To analyse if the eccentric axial force is inside or outside of the central core.

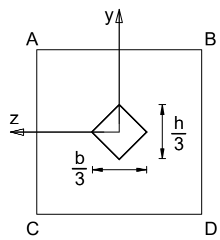
Assuming that the tensile strength of the soil is 2 MPa it is requested:

3. To check the design of the footing pad



1. Central core of the footing pad will be a diamond, in which the length of both diagonals will be $d_z = d_y = \frac{2000}{3} = 666.6\ mm$

   as can be seen in the figure

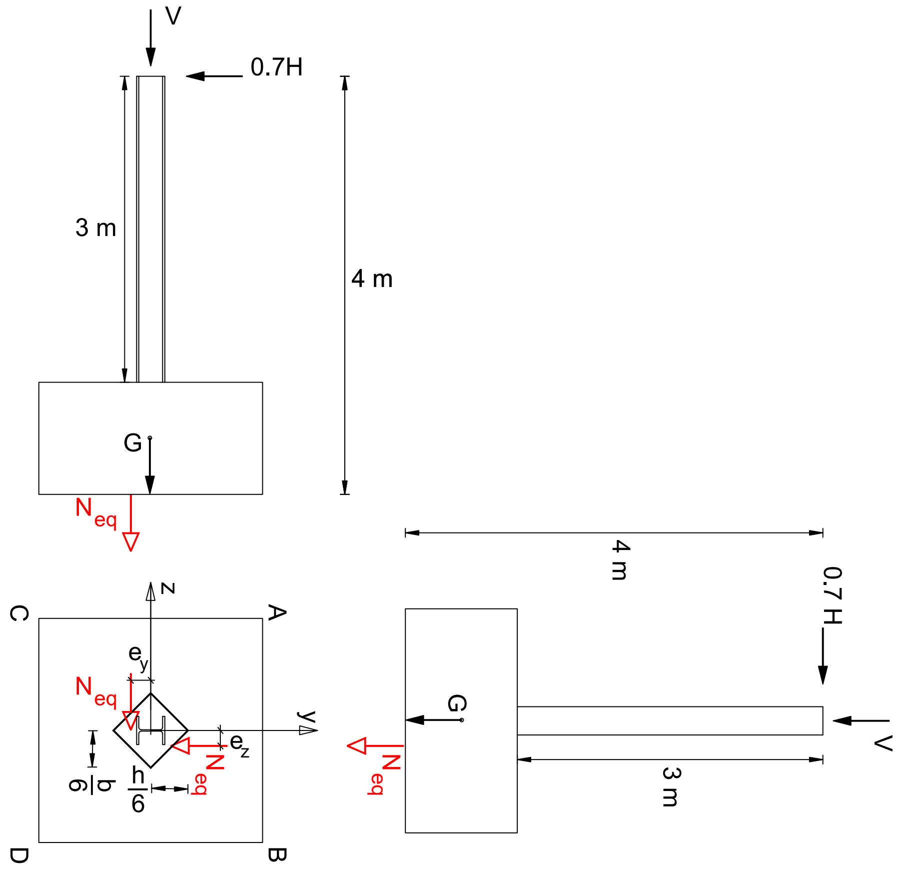


2. Internal forces:

Vertical load: N =100 kN (compression at the centroid of the column)

Weight of the footing pad: $2x2x0.5x25 = 50\ kN$ (at the centroid of the footing pad)

Bending moment: considering the axes in the figure, the column, and the footing pad is in biaxial bending, being the bending moment about z axis negative and the bending moment about y axis positive.



At the cross section in contact with the soil: $M_z = -10\frac{\sqrt{2}}{2}\times(3+0.5) = -24.74\ kNm$

And $M_y = 10\frac{\sqrt{2}}{2}\times(3+0.5) = 24.74\ kNm$

Being both bending moments equivalent to an axial force with two eccentricities equal to: $e_y = e_z = \dfrac{24.74\times10^6\,Nmm}{150\times10^3} = 164.93\ mm$

As both eccentricities are smaller than h/6 and b/6, the axial force is inside the central core and all the fibres are in compression.

3. The conditions to be considered are:

$\sigma_{\min,d} \le 0 \quad \rightarrow \quad \gamma_F\sigma_{\min,k} \le 0$ which is going to be fulfilled as the eccentric axial force is inside the central core

$$\sigma_{\max,d} \le 1.25\ \sigma_{adm} \quad \rightarrow \quad \gamma_F\sigma_{\max,k} \le 1.25\ \frac{\sigma_{soil,k}}{\gamma_M}$$

$$\sigma_{ave,d} \le \sigma_{adm} \quad \rightarrow \quad \gamma_F\sigma_{ave} \le \frac{\sigma_{soil,k}}{\gamma_M}$$

Where $\sigma_{\max,k} = -\left|\frac{N}{A}\right| - \left|\frac{N \times e_y}{W_{el,z}}\right| - \left|\frac{N \times e_z}{W_{el,y}}\right|$ $\quad\quad \sigma_{\min,k} = -\left|\frac{N}{A}\right| + \left|\frac{N \times e_y}{W_{el,z}}\right| + \left|\frac{N \times e_z}{W_{el,y}}\right|$

and $\sigma_{ave,k} = -\left|\frac{N}{A}\right|$ is the normal stress at the centroid of the foundation pad

considering that partial factors, when designing foundations are: $\gamma_F = \gamma_M = 1$

The normal stress admissible value for the soil will be: $\sigma_{adm} = \frac{1}{1} MPa \equiv 1 \frac{N}{mm^2} \equiv 100 \frac{kN}{m^2}$

Being the mechanical properties of the footing pad:

$$A = 2 \times 2 = 4\ m^2$$

$$W_{el,z} = \frac{bh^2}{6} = \frac{2 \times 2^2}{6} = 1.33\ m^3; \quad\quad W_{el,y} = \frac{hb^2}{6} = \frac{2 \times 2^2}{6} = 1.33\ m^3$$

Substituting and operating:

$$\sigma_{\min} = -\left|\frac{(100+50)}{4}\right| + \left|\frac{-24.74}{1.33}\right| + \left|\frac{24.74}{1.33}\right| = -75 + 18.6 + 18.6 = -37.6 \frac{kN}{m^2} = \sigma_{x_{at\ A}}$$

being $\sigma_{\min,d} = 1(-37.6) \frac{kN}{mm^2} \leq 0$

$$\sigma_{\max} = -\left|\frac{(-100-50)}{4}\right| - \left|\frac{-24.74}{1.33}\right| - \left|\frac{24.74}{1.33}\right| = -75 - 18.6 - 18.6 = -112.2 \frac{kN}{m^2} = \sigma_{x_{at\ D}}$$

where $\sigma_{\max,d} = 1(-112.2) \frac{kN}{mm^2} \leq 1.25\, \sigma_{adm} = 1.25 \times \frac{-100}{1} = -125 \frac{kN}{mm^2}$

and $\sigma_{ave,d} = 1(-75) \frac{kN}{mm^2} \leq \sigma_{adm} = \frac{-100}{1} \frac{kN}{mm^2}$

In conclusion, the foundation pad fulfils the three conditions.

6. Proposed exercise

It is requested to check the same foundation pad considering the horizontal load is equal to 100 kN.

At the end of this document is explained the solution of this exercise.

7. Conclusion

This document explains the conditions that the isolated foundation pads must fulfil considering normal stress distribution and soil strength.

The concept of central core has been introduced in order to explain when the foundation pad transmits compression stresses and when try to transmit tensile stresses.

A worked example has been developed to consolidate the method.

A variation of the worked example is presented to encourage the student to practice about this topic

8. Bibliography

Building Spanish code "Código Técnico de la Edificación" Ministerio de Fomento 2006. It can be downloaded from: https://www.codigotecnico.org/

Solution to the proposed exercise

Considering the horizontal load value is 100 kN, the bending moments will be equal to:

$$M_y = M_z = 100\frac{\sqrt{2}}{2} \times 3 = 212\ kNm$$

Being the eccentricities: $e_y = e_z = \dfrac{212 \times 10^6\, Nmm}{150 \times 10^3} = 1413\ mm$, the axial compression force is outside the central core. Therefore, first condition is not fulfilled, and the proposed design fails.