Central core of a rectangular cross section

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1 Summary

This document explains the concept of central core for a rectangular cross section and shows how to calculate it.

Practical application about how to obtain the central core is presented.

A couple of proposed exercises are included in order to involve the student in thinking about central core concept.

2 Introduction

Considering Neutral Axis definition as the straight line in a beam or other structural member subjected to a bending action where the normal stress is zero, it is known that this Neutral axis can be inside or outside the cross section, as shown in Figure 1 (inside) Figure 2 (in the limit) and Figure 3 (outside)

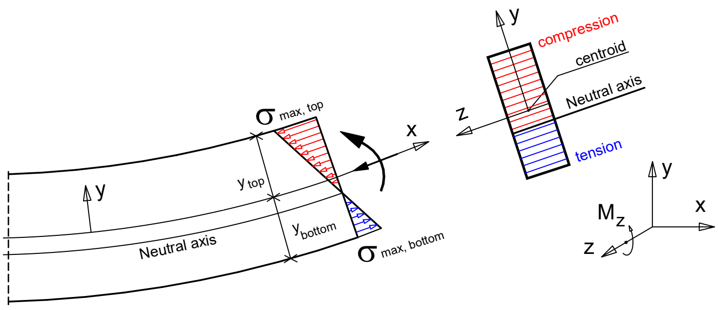


Figure 1

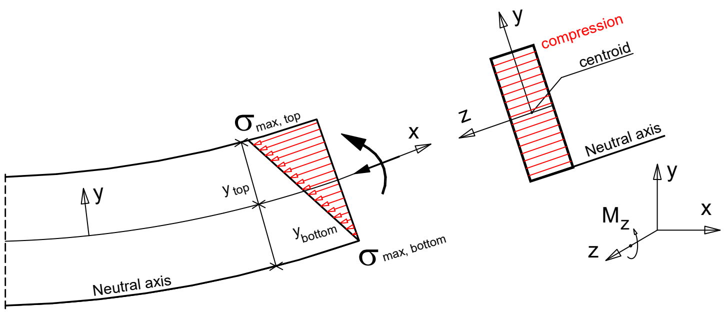


Figure 2

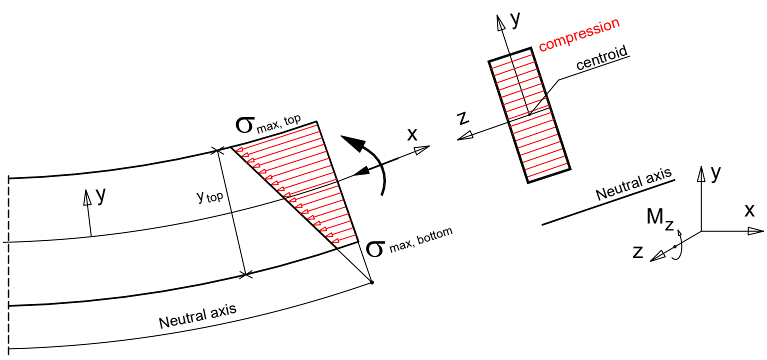


Figure 3

Being the physical meaning of the Neutral Axis as the set of points where the fibres are neither stretched nor compressed, it is assumed that, when Neutral axis is inside the cross section, fibres on one side of the Neutral axis are in tension and those on the opposite side are in compression.

On the other hand, when Neutral Axis is outside the cross section, whole fibres would be in tension or in compression,

Considering that some construction materials, have a weak design tensile strength (concrete, masonry, or the soil) it is recommended for those elements to be mostly in compression.

For that reason, it is necessary to obtain the domain for a given cross section, where applying an eccentric axial force, none fibre is in tension. This domain is called central core or kern of the cross section.

3 Aims

At the end of this document, the student will be able to:

- Explain the concept of central core

- Obtain the central core of a rectangular cross section

4 Central core

4.1 Definition and properties

Central core is the domain, inside a cross section where none fibre is in tension when applying an eccentric axial force.

Geometric properties of the central core or cross section kern are:

1. The central core will always represent an area around the cross-section centroid.
2. If the cross section presents symmetry conditions, the central core will present same symmetry.
3. Each Neutral Axis, tangent to the cross-section contour has its corresponding vertex of the central core, being situated in different parts of the cross section with respect of the centroid of the cross section.

Mechanical properties, according with its definition, are:

1. If the point of application of the eccentric axial force is inside the central core, the neutral axis is outside the cross section.
2. If the point of application of the eccentric axial force is on the central core contour, the neutral axis is tangent to the cross section contour.
3. If the point of application of the eccentric axial force is outside the central core, the neutral axis will intersect the cross section.

As the Neutral Axis is the limit between fibres in tension and fibres in compression, the central core will be obtained considering that the cross-section contour is the Neutral Axis that corresponds to an eccentric axial compression force.

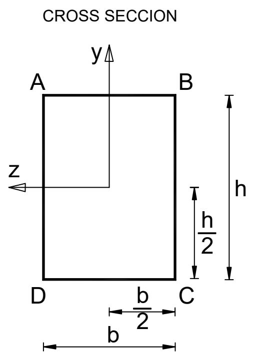
Therefore, to obtain the central core of a cross section, it is assumed that every straight line of the cross-section contour is the neutral axis that corresponds to one of the vertex of the central core (e_y, e_z).

The process consists on obtaining, on turn, each of the vertex of the central core.

In the following epigraph it is shown how to obtain the central core of a rectangular cross-section.

4.2 Central core for a rectangular cross section

As the rectangle is a double symmetrical cross-section, the central core will be double symmetrical too. Therefore, it will be enough to obtain two coordinates of the central core.



Being the rectangular cross section in the figure, the first step may be to consider the neutral axis is the line tangent to the cross section in AD, being the equation of that straight line equal to: $z = \frac{b}{2}$

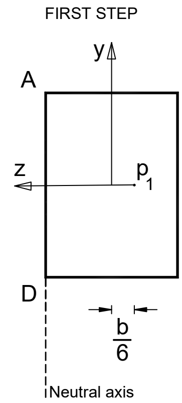
And the equation of the neutral axis:

$$\frac{N}{A} + \frac{N \times e_z}{I_y} z = 0$$ s

ubstituting the value of z coordinate:

$$\frac{N}{A} + \frac{N \times e_z}{I_y} \frac{b}{2} = 0$$

$$N\left(\frac{1}{A} + \frac{e_z\, b}{2\, I_y}\right) = 0 \quad \rightarrow \quad \left(\frac{1}{A} + \frac{e_z\, b}{2\, I_y}\right) = 0$$



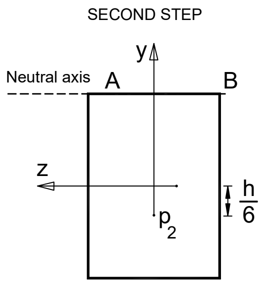
Taking into account the formulae[1] for the area and the second moment of area of the rectangle, the z coordinate of first vertex of the central core is:

$$\frac{1}{b\,h} + e_z \frac{b}{2} \frac{1}{\frac{h\,b^3}{12}} = 0$$

$$\frac{1}{b\,h} + e_z \frac{1}{\frac{h\,b^2}{6}} = 0 \quad \rightarrow \quad e_z = -\frac{b}{6}$$

[1] Area A = b × h and second moment of area $I_y = hb^3/12$

The second step will be to consider the neutral axis is the line tangent to the cross section in AB, being the equation of that straight line equal to: $y = \dfrac{h}{2}$

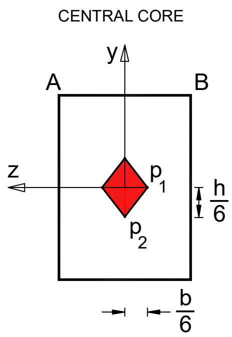


Being the equation[2] of the neutral axis:

$$\frac{N}{A} - \frac{N \times (-e_y)}{I_z} y = 0$$

$$\frac{1}{b\,h} + e_y \frac{h}{2} \frac{1}{\frac{b\,h^3}{12}} = 0$$

$$\frac{1}{b\,h} + e_y \frac{1}{\frac{b\,h^2}{6}} = 0 \quad \rightarrow \quad e_y = -\frac{h}{6}$$



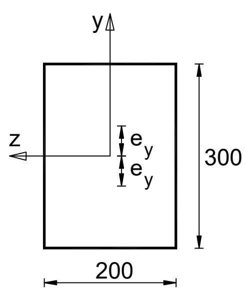
Finally, considering symmetry, third and fourth vertexes of the central core will be (b/6,0) and (0,h/6) as can be seen in the image.

In conclusion, the central core of a rectangular cross-section is a rhombus with b/3 and h/3 diagonals.

If the compression axial force is applied inside this rhombus or in the perimeter, all the points in the cross-section will be in compression. On the other hand, if it is applied outside the central core, part of the cross section will be in tension and part in compression.

4.3 Practical application

Given a rectangular b × h cross section, which dimensions are 200 × 300 mm, it is requested to obtain the maximum eccentricity e_y that a 100 kN compression axial force can admit in order to guarantee that the whole cross section is in compression.



The right answer is $e_y \le \pm \dfrac{h}{6} = 50\ mm$

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[2] Note: e_y should be negative to generate a positive M_z

4.4 Proposed exercises

1. Considering previous cross section in epigraph 4.3, it is requested to obtain the maximum eccentricity e_z that a 100 kN compression axial force can admit in order to guarantee that the whole cross section is in compression.

2. Considering previous cross section, it is requested to obtain the maximum eccentricity e_z that a 1000 kN compression axial force can admit in order to guarantee that the whole cross section is in compression.

The answer to both questions can be found at the end of this document.

5 Conclusion

This document explains the concept of central core and its importance in building materials.

It shows the method, step by step, to obtain the central core of a rectangular cross section, giving the mathematical expression,

A practical example has been developed and two proposed exercises included

6 Bibliography

Gere, James M: "Mechanics of materials" Thomson Learning, Inc. 2004

7 Proposed exercise solution

The maximum eccentricity e_z that a 100 kN compression axial force can admit in order to guarantee that the whole cross section is in compression is:

$$e_z \leq \pm \frac{b}{6} = 33.3\ mm$$

The maximum eccentricity e_z that a 1000 kN compression axial force can admit in order to guarantee that the whole cross section is in compression is the same that the maximum eccentricity e_z that a 100 kN compression axial force can admit in order to guarantee that the whole cross section is in compression, because central core is a geometrical property of the cross section and does not depend on the magnitude of the compression axial force.