OFDM ICI Self-Cancellation Scheme Based on Five Weights

Oscar Real, Vicenç Almenar

Abstract—**OFDM systems suffer from inter-carrier interference when they are used in mobile environment due to the appearance of Doppler frequency produced by the movement of the terminals. Recently, different schemes have been proposed in the scientific literature in order to cancel this type of interference. This paper deals with the improvement of these methods, it will be shown that our approach is able to achieve better results. Moreover, we propose to perform the signal processing only on the transmitter side allowing a reduction of the receiver complexity without any performance degradation.**

***Index Terms*—OFDM, ICI cancellation, mobile channels.**

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a data transmission technique used in multiple digital communication standards. OFDM can manage efficiently the intersymbol interference (ISI) and its associated channel deep fading in time dispersive channels by means of a cyclic prefix with adequate length [1][2]. However, if an OFDM system is used in scenarios with high mobility, the Doppler frequency will shift constantly producing channel variations that cause a loss of orthogonallity between subcarriers, taking place the appearance of the effect known as intercarrier interference (ICI) [3].

The ICI increases with the time variation of the channel and introduces a bit error floor that is independent of the signal-to-noise ratio (SNR) level. Several methods have been proposed to reduce this type of interference among subcarriers [4][5][6], one of them is Zhao-Haggman's self-cancellation scheme [7] which shows a solution with low complexity at the expense of a reduction in bandwidth efficiency.

This solution consists of transmitting each symbol in two consecutive subcarriers: odd subcarriers are the negative copies of even subcarriers. On the receiver side, both subcarriers are employed to make a decision on the transmitted symbol. Later, Seyedi and Saulnier extended in [8] this method showing first that the paired modulation proposed by Zhao-Haggman in transmission is equivalent to convolve, in frequency domain, the sequence of symbols alternated with zeros that modulates the subcarriers with a filter of two coefficients [1 -1]. Besides, the operations done by the receiver in order to joint use two consecutive subcarriers for symbol detection can also be seen as a convolution between the received sequence in frequency domain and a filter of two coefficients. After this analysis and after taking into account that the filtering in frequency domain can be substituted by windowing in time domain, Seyedi and Saulnier proposed two methods to design new windows in order to reduce the ICI effect at the receiver.

In this paper we extend the ideas from [8] to propose a new window that improves the performance of the ICI self-cancellation scheme given in [8]. Whereas in that reference the window design makes use of three weights in frequency domain, our proposal makes use of five weights. Moreover, we have observed that using a window only on the transmitter side gives as good results as using two windows: one for transmission and another for reception. In this way, we reduce the computational complexity of the receiver compared with the solutions given in [7] and [8]. The proposed method can be considered as a general ICI self-cancellation where both Zhao-Haggman's and Seyedi-Saulnier's algorithms are particular cases of our ICI self-cancellation scheme based on five weights.

This paper is divided in eight sections, in the first one an introduction where the problem is presented, second section describes the OFDM system, section III gives a model of the ICI effect, in section IV an analysis of the BER as a function of the ICI effect is done, section V presents the method in order to design the window for ICI cancellation, section VI describes a design based on SIR, and finally sections VII and VIII show a comparison of the ICI cancellation methods and a conclusion of the paper, respectively.

II. SYSTEM DESCRIPTION

In Fig. 1, it is shown a block diagram of the proposed system. At the transmitter, an inverse FFT (IFFT) of size N is used to modulate $N/2$ symbols: they are located in subcarriers with even indexes meanwhile the rest of subcarriers are set to zero, this insertion of zeros is done by the up-sampling block. Once the N samples of the signal have been generated in time domain, they are windowed by $w[n]$. As it has been commented above, an equivalent process can be done before the IFFT if we convolve the block of N symbols with alternated zeros through a filter with coefficients c_m. The relationship between this filter and the window can be written as:

$$w[n] = \sum_{m=-M}^{M} c_m \, e^{j2\pi\delta_m n/N} \quad n = 0,1,\ldots,N-1, \tag{1}$$

where δ_m are the normalized digital frequencies and c_m are the amplitude coefficients. Note that, in a discrete filter, δ_m can only take integer values. This is the case of Zhao-Haggman's

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scheme where only two coefficients are employed: $c_0 = 1$ and $c_{-1} = -1$, and they are located in $\delta_0 = 0$ and $\delta_{-1} = -1$. However, when the window is used in time domain, the position of the coefficients is not restricted to integer values, allowing a more flexible design. This approach is used by the Seyedi-Saulnier's scheme where tree weights are employed, the central one c_0 is set to 1 and $c_{-1} = c_1^*$. This design gives a window which is split in two equivalent windows: $w[n] = w_{TX}[n] \cdot w_{RX}[m]$, where one is used in the transmitter and the other in the receiver. In [7] the receiver makes use of two consecutive subcarriers to take a decision on the received symbol, this process can be modelled as a convolution in frequency domain with a filter of two coefficients. If we convolve the filter in transmission with the filter in reception, we will obtain a filter with three weights; this is similar to the designed window in [8] with three coefficients and its subsequent split in two windows.

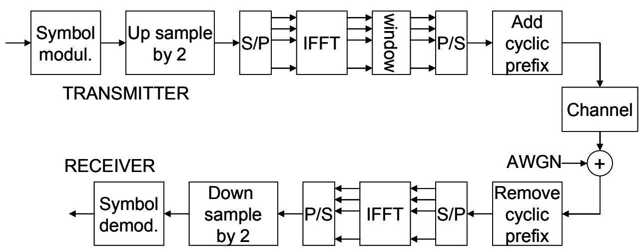

Fig. 1. The system block diagram of the proposed algorithm

In this paper, we propose the design of a window using five weights in frequency domain instead. Moreover, we have checked that windowing only at the transmitter, we can reach the same performance as splitting it between transmitter and receiver. Then, at the receiver side, once the OFDM symbol has been equalized in frequency domain, the symbols placed on even subcarriers are taken to estimate the transmitted symbols. In this way, the processing at the receiver is simplified.

III. OFDM Model with ICI

In an OFDM communication system, assuming that the channel Doppler frequency offset normalized by the subcarrier separation is ε and the only distortion is an additive white Gaussian noise (AWGN), the received signal on subcarrier m can be expressed as [7]:

$$Y_m = S_m G_m + \sum_{\substack{k=0 \\ k \neq m}}^{N-1} S_k G_{m-k} + N_m , \tag{2}$$

where S_m is the transmitted symbol for the m subcarrier, N_m is the added noise and G_{m-k} is the ICI coefficient that represents the interference of subcarrier k on subcarrier m:

$$G_{m-k} = \frac{\sin(\pi(m-k-\varepsilon))}{\sin(\pi(m-k-\varepsilon)/M)} e^{-j\frac{\pi}{N}(m-k-\varepsilon)(N-1)} . \tag{3}$$

The first term on the right-hand side of (2) is the desired signal. When there is no Doppler frequency ($\varepsilon = 0$), then there is no ICI and therefore $G_m = 1$ and $G_{m-k} = 0$.

IV. BER Analysis

In this section, the bit error rate (BER) of an OFDM system with ICI is calculated using the method proposed in [9] to evaluate the error probabilities for BPSK signalling in systems with ISI. In our case, the interference among symbols in consecutive subcarriers is evaluated in a similar way as the inter-symbol interference between consecutive temporal symbols done in [9]. Moreover, we assume that subcarriers are modulated using a QPSK mapping.

In a QPSK modulation two bits are mapped to a data symbol with the form $a + jb$, where a is the real part that carries one bit and can take integer values of 1 or –1, while b is the imaginary part and carries the other bit and can also take integer values of 1 or –1. The receiver decides if a particular bit is +1 or –1 when $a > 0$ ($b > 0$) or $a < 0$ ($b < 0$), respectively. Therefore, by symmetry, the average probability of error P_e in the real part of the QPSK symbol located on subcarrier m is given by

$$P_e = \Pr\left(\mathrm{Re}(Y_m) > 0 \,\middle|\, a_m = -1\right). \tag{4}$$

For $a_m = -1$ the real part of the symbol in subcarrier m is

$$\mathrm{Re}(Y_m) = -\mathrm{Re}(G_m) + \mathrm{Re}(z_m) + \mathrm{Re}(N_m), \tag{5}$$

where

$$z_m = \sum_{\substack{k=0 \\ k \neq m}}^{N-1} S_k G_{m-k} \tag{6}$$

is the interference coming from the symbols of the rest of subcarriers. Using the same reasoning as in [9] and taking into account that we deal with QPSK signalling in additive Gaussian noise of zero mean and variance σ_n^2, we can obtain the probability of error from the real part of the transmitted data symbols as (eq. (30) in [9]):

$$P_e = \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{l=1 \\ l\ odd}}^{\infty} \frac{\sin\left(\dfrac{l\,\omega_0 \,\mathrm{Re}(G_m)}{\sigma_n}\right)}{l} e^{-\frac{l^2 \omega_0^2}{2\sigma_n^2}} \Phi_z\left(\frac{-l\,\omega_0}{\sigma_n}\right), \tag{7}$$

where ω_0 is the angular frequency of the Fourier series approximation and the last term is:

$$\Phi_z(\omega_0) = \prod_{\substack{k=0 \\ k \neq m \\ k\ even}}^{N-1} \cos(\omega_0 \,\mathrm{Re}(G_k)) \cdot \cos(\omega_0 \,\mathrm{Im}(G_k)) \cdot \tag{8}$$

This equation changes from an equivalent equation found in [9] due to the employment of QPSK signalling and the use of only even subcarriers for data transmission. The probability of error for the real part of the transmitted data symbols is equal to the probability of error for the imaginary part in a QPSK mapping, furthermore, without loss of generality, (7) corresponds to the BER of the system.

V. Window Design for ICI Cancelation

The windowing applied after the IFFT introduces a known and controlled intercarrier interference, this ICI and the transmission in even subcarriers cause at the receiver side that the signal on subcarrier m becomes:

$$Y_m = S_m\, G'_m + \sum_{\substack{k=0 \\ k\neq m \\ k\ even}}^{N-1} S_k\, G'_{m-k} + N_m, \tag{9}$$

where G'_m is the modified ICI coefficient due to the windowing:

$$G'_m = \sum_{k=-2}^{2} c_k\, G_{m-\delta_k}\,. \tag{10}$$

Therefore, the BER obtained at the receiver side using (7) would now depend on G'_m instead of G_m:

$$BER = \frac{1}{2} - \frac{2}{\pi}\sum_{\substack{l=1 \\ l\ odd}}^{\infty} \frac{\sin\left(\dfrac{l\,\omega_0\,\mathrm{Re}(G'_m)}{\sigma_n}\right)}{l}\, e^{-\frac{l^2\,\omega_0^2}{2\sigma_n^2}}\, \Phi_z\left(\frac{-l\,\omega_0}{\sigma_n}\right), \tag{11}$$

with

$$\Phi_z(\omega_0) = \prod_{\substack{k=0 \\ k\neq m \\ k\ even}}^{N-1} \cos\left(\omega_0\,\mathrm{Re}(G'_{m-k})\right)\cdot\cos\left(\omega_0\,\mathrm{Im}(G'_{m-k})\right) \tag{12}$$

Equations (11), (12) and (10) can be used to design the transmission window, thus, to obtain the values of the parameters c_0, c_1, c_{-1}, c_2, c_{-2}, δ_1, δ_{-1}, δ_2 and δ_{-2}, which improve the BER for a maximum normalized Doppler frequency ε and a given signal to noise ratio. This means that if we design the window appropriately, we will minimize the BER through the self-cancellation of the ICI introduced by the windowing.

In the design, we will assume that $c_0 = 1$, thus, we do not modify even subcarriers where data symbols are contained, and next symmetry: $c_1 = c_{-1}^*$, $c_2 = c_{-2}^*$, $\delta_1 = -\delta_{-1}$ and $\delta_2 = -\delta_{-2}$, because the Doppler shift can raise or reduce the carrier frequency. In case of $c_2 = c_{-2}^* = 0$, our system implements Seyedi-Sauliner's algorithm. Furthermore, if $c_1 = -0.5$ and $\delta_1 = 1$, then the transmission window will be equivalent to the Zhao-Haggman's algorithm.

VI. SIR-Based Window Design Method for ICI Cancelation

Another way to obtain parameters c_0, c_1, c_{-1}, c_2, c_{-2}, δ_1, δ_{-1}, δ_2 and δ_{-2}, is to maximize the signal to interference ration (SIR) of the system for a given ε. In this section, we obtain the SIR for the proposed system over an AWGN channel.

Given (2), the SIR can be described for an OFDM signal on subcarrier m as

$$SIR = \frac{|S_m|^2\,|G'_m|^2}{\sum_{\substack{k=0 \\ k\neq m}}^{N-1} |S_k|^2\,|G'_{m-k}|^2}, \tag{13}$$

If we assumed that data symbols are statistically independent and have zero mean and equal power, such as in QPSK modulation, (13) provides the average SIR and is equal to

$$SIR = \frac{|G'_m|^2}{\sum_{\substack{k=0 \\ k\neq m}}^{N-1} |G'_{m-k}|^2}, \tag{14}$$

In this design, we look for the parameters that result in a SIR equal to or larger than a minimum desired SIR, SIR_{min}, for a maximum normalized Doppler frequency. Assuming $c_0 = 1$, $c_1 = c_{-1}^*$, $c_2 = c_{-2}^*$, $\delta_1 = -\delta_{-1}$ and $\delta_2 = -\delta_{-2}$, it is possible to calculate the values of these parameters using equations (3), (10) and (14).

VII. Results

In this section, the performance of the exposed methods is compared in terms of the obtained BER. Table I shows the parameters used by each method when QPSK modulation is employed.

Table I
Design parameters for $N = 64$ and QPSK modulation

	c_1	δ_1	c_2	δ_2
Zhao-Hagmann algorithm [7]	–0.5	1	0	0
Seyedi-Saulnier Algorithm based on BER design [8]	–0.50–0.63j	0.72	0	0
Proposed Algorithm based on BER design	0.25–0.25j	0.5	–0.35–0.4j	0.83
Seyedi-Saulnier Algorithm based on SIR design [8]	–0.39–0.33j	0.77	0	0
Proposed Algorithm based on SIR design	0.4+0.4j	0.2	-0.5-0.3j	0.9

Given the number of subcarriers $N = 64$, a level of signal to noise ratio of $E_b/N_0 = 20$ dB and a maximum normalized Doppler frequency of $\varepsilon_{max} = 0.3$, it is possible to find the optimized parameters in our proposed method based on BER design through equations (11), (12) and (10) that minimize the BER. This value of signal to noise ratio has been chosen in order to achieve that the BER is mainly determined by the ICI effect; furthermore, the Doppler frequency has been set to 0.3 because higher values are unusual. Also, window parameters have been calculated for the SIR-Based design through equations (3), (10) and (14) for $\varepsilon_{max} = 0.3$ and SIR_{min}=25 dB, which will be compared later with the obtained results using the BER-Based design.

Fig. 2 shows the performance of six transmission schemes in AWGN channels and with an $\varepsilon = 0.3$: using even subcarriers without any windowing, with Zhao-Hagmann windowing, with Seyedi-Saulnier windowing and with five-weights windowing. It is clear from these results that our BER-Based design outperforms the rest of the methods, obtaining a gain of 1 dB compared with Seyedi-Saulnier method based also on BER design.

Although the method for ICI reduction has been developed for AWGN channel, we will show that it also performs well over frequency-selective Rayleigh fading channels. We have used the same channel model as the one used in [8], this is a four-tap channel where the fading coefficient on every tap is a Rayleigh distributed random variable, with a power spectral density given by Jakes' model [10]. The power-delay profile of this channel is described in TABLE 2.

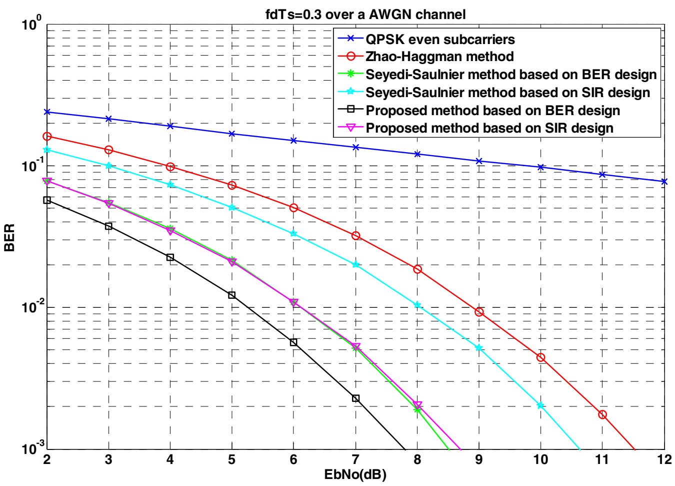

Fig 2. BER performance in an AWGN channel

TABLE 2
DELAY PROFILE OF THE RAYLEIGH FADING CHANNEL

Tap	Excess Delay	Relative Average Power
1	0	0 dB
2	$T/64$	–6 dB
3	$T/32$	–12 dB
4	$3T/64$	–18 dB

Where T is the time duration of the OFDM symbol. It is assumed that the guard interval of the cyclic prefix is longer than the maximum delay of the channel to prevent ISI. Considering a coherent system at the receiver with a perfect one tap equalization of each subcarrier, the BER results are represented in Fig. 3 and Fig. 4.

Fig. 3 shows the performance of the six methods in a Rayleigh frequency-selective channel. In this case the normalized Doppler frequency is again set to 0.3. We can observe that the gain of our method over the Seyedi-Saulnier is again of 1 dB better. Moreover, in Fig. 4 these methods are compared for a range of normalized Doppler frequencies at an E_b/N_0 of 20 dB, it can be seen that our method always obtains lower BER than the rest of methods.

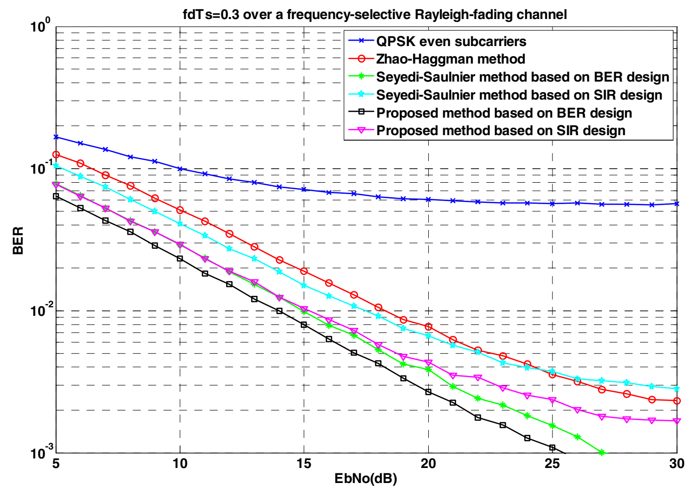

Fig. 3. BER in frequency selective channel

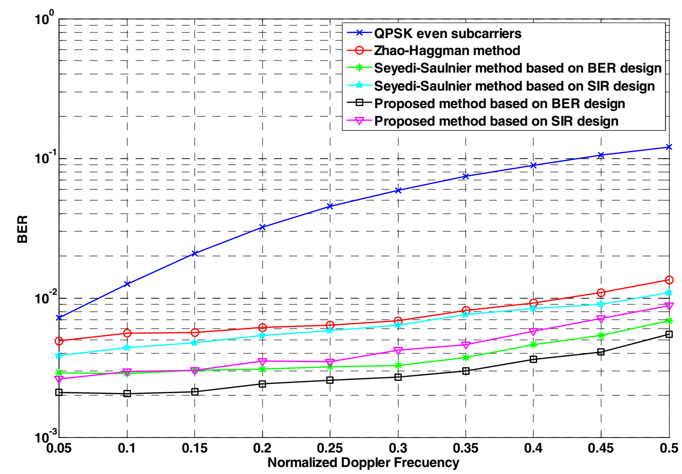

Fig. 4. BER for a given $E_b/N_0 = 20$ dB and different normalized Doppler frequencies

Observing Fig. 3 and Fig. 4 we can conclude that the Five-Weigths' method based on BER design achieves a better BER than the other methods for the same E_b/N_0 including $\varepsilon > 0.3$ (Fig. 4), in spite of the fact that the parameters of the Seyedi-Saulnier's method based on BER design were estimated to support up to a normalized Doppler frequency of 0.54 whereas in our method they have been obtained for $\varepsilon_{max} = 0.3$.

Finally, Fig. 5 compares the performance of the Zhao-Haggman method in two configurations: using only a transmission window as it is proposed in this paper and splitting it in two windows, one for the transmitter and another for the receiver as it was proposed originally in [7] and also in [8]. This result shows that both configurations are similar; this is an interesting observation since this allows the receiver to ignore the ICI cancellation method used by the transmitter, simplifying the design of the receiver.

VIII. CONCLUSION

We have just exposed a self-cancellation scheme based on a window design in frequency domain. Our proposal uses five weights, whereas previous solutions only use three coefficients.

We have shown that both Zhao-Hagmann's and Seyedi-Saulnier's methods are particular cases of our approach. In order to reduce the receiver complexity, we have proposed the use of the windowing only in transmission, instead of both in transmission and in reception. Simulation results show that our method is not only able to reduce the ICI in AWGN channels, but also in frequency-selective Rayleigh channels.

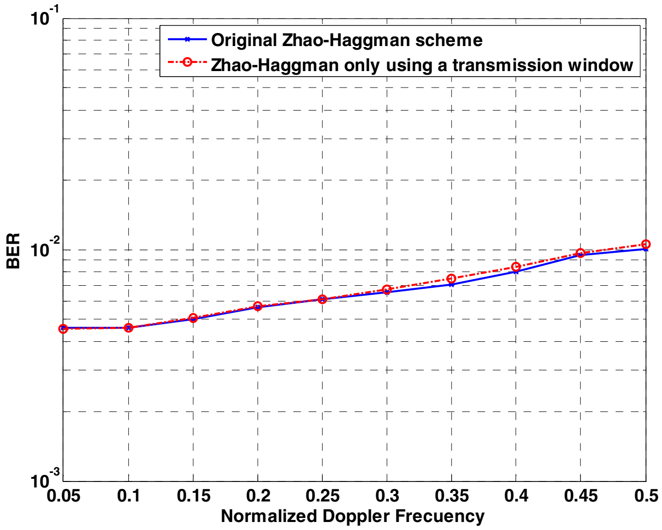

Fig. 5. Comparison of windowing at transmission vs. both transmission and reception

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