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A proposal toward a possibilistic multi-robot task allocation

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Abstract

One of the main problems to solve in a multi-robot systems is to select the best robot to execute each task (task allocation). Several ways to address this problem have been proposed in the literature. This paper focuses on one of them, the so-called response threshold methods. In a recent previous work, it was proved that the possibilistic Markov chains outperform the classical probabilistic approaches when they are used to implement response threshold methods. The aim of this paper is to summarize the advances given by or research group toward a new possibilistic swarm multi-robot task allocation framework.

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1. Introduction

A multi-agent (multi-robot) system is defined as a group of two or more robots with a common mission. These systems provide several advantages compared to the systems with only one robot, like for example, they can perform tasks that one robot would be impossible to execute or could take a very long time. Furthermore, such systems are more robust, scalable and flexible than those with only one robot. A great number of complex problems must be addressed in order to take all these advantages. This paper focuses on one of them, referenced as multi-robot task allocation (MRTA for short), which consists of selecting the best robot or robots to execute each of the tasks that must be performed. MRTA problem is still an open issue in real environments where the robots have a limited number of computational resources. A lot of work have been done in order to solve the MRTA problem. The solution developed to solve it can be grouped in two main strategies: swarm methods and auction methods. Concretely, we will only focus on the swarm methods. The auction-like approaches are out of the scope of this paper.

Swarm intelligence methods provide very simple solutions for the MRTA problem. One of the most widely used swarm methods are the so-called Response Threshold algorithms, where the behavior of the systems is modeled as a Markov chain and the robots in each time step select the next task to execute according to a transition probability function. Among other factors, this probability depends on a stimulus (for example the distance between the robot and the task). This classical probabilistic approach presents a lot of disadvantages: the transition function must meet the constraints of a probabilistic distribution, the system only convergences to a stationary asymptotically, and so on. In order to overcome these problems, we proposed a new theoretical framework based on fuzzy (possibilistic) Markov chains in [6]. As was proved, the possibilistic Markov chains outperform the classical probabilistic when a Max-Min algebra is considered for matrix composition. For example, fuzzy Markov chains convergence to a stable state in a finite number of steps 10 times faster than its probability counter part. More recent works extents this first paper in order to analyze the behavior of the system when other algebras are considered for matrix composition [4]. Moreover, in [5] we studied the impact of the possibility transition function on the system's performance. Thus, the propose of this paper is to summarize the aforementioned recent advances given toward a new possibilistic swarm multi-robot task allocation framework and propose some new future research lines in this field.

2. Probabilistic Response Threshold Task Allocation

This section introduces the main concepts on classical RTM approaches, where the decision process is modeled as a probabilistic Markov chain.

The definition of the MRTA problem depends on the characteristics of the problem. In our case, we assume that only one robot can be assigned to each task at the same time. This kind of problem is defined as follows: Let $\mathbb{N}$ denote the set of positive integer numbers and let $n, m \in \mathbb{N}$. Denote by R the set of robots with $R = \{r_1, ..., r_n\}$ and by T the set of tasks to carry out with $T = \{t_1, ..., t_m\}$. A task allocation is a function $TA : T \to R$ such that $TA(t_i) \bigcap TA(t_j) = \emptyset$ provided that $i \neq j$.

The classical response threshold method (see [1]) defines for each robot r_i and for each task t_j, a stimulus $s_{r_i,t_j} \in \mathbb{R}$ that represents how suitable t_j is for r_i, where $\mathbb{R}$ stands for the set of real numbers. The task selection is usually modeled by a probabilistic response function that depends on s_{r_i,t_j} and a given threshold value θ_{r_i} ($\theta_{r_i} \in \mathbb{R}$). Thus, a robot r_i will select a task t_j to execute with a probability $P(r_i, t_j)$ according to a probabilistic Markov decision chain. There are different kind of probabilities response functions that defines a transition, but one of the most widely used (see [2]) is given by

$$P(r_i, t_j) = \frac{s^n_{r_i,t_j}}{s^n_{r_i,t_j} + \theta^n_{r_i}}, \tag{1}$$

where $n \in \mathbb{N}$, where $\mathbb{N}$ stands for the set of natural numbers. The preceding response function has tested in our previous work [6]. Another transition function that presents similar characteristics to the given in (1), which was tested in [5], is given by:

$$P(r_i, t_j) = \mathrm{e}^{-\frac{\theta^n_{r_i}}{s^n_{r_i,t_j}}} \tag{2}$$

It could be checked that both transition functions are are indistinguishably operators whenever s_{r_i,t_j} only depends on the distance between the robot following this expression: $s_{r_i,t_j} = \frac{1}{d(r_i,t_j)}$.

In general none of these transition functions meet the equality $\sum_{j=1}^{m} P(r_k, t_j) = 1$ and, therefore the transition between states is not a probability distribution. In order to solve this problem a normalization processes must be introduced. In most cases, this implies a modification of the behavior of the system. Moreover, the transition P_{r_k} is regular. According [7], under this condition the evolution of the system to a stable state is, in general, only guaranteed asymptotically. From the above-said probabilistic Markov chains problems, we can conclude that the probability theoretical foundation may be inappropriate. As will be seen, the possibilitic (or Fuzzy) Markov chains are able to solve the problems of their probabilistic counterparts.

3. Possibilistic and Fuzzy MRTA

This section summarizes the contributions proposed toward the aforementioned new possibilistic task allocation framework. This work has been developed by the members of the research groups MOTIBO (Models for Information Processing. Fuzzy Information) and SRV (Systems, robotics and Vision) at the University of the Balearic Islands.

3.1. **Possibility Theory and Markov Chains.** A possibility Markov (memoryless) process can be defined as follows [4]: let $S = \{s_1, \ldots, s_m\}$ $(m \in \mathbb{N})$ denote a finite set of states. If the system is in the state s_i at time τ $(\tau \in \mathbb{N})$, then the system will move to the state s_j with possibility p_{ij} at time $\tau + 1$. Let $x(\tau) = (x_1(\tau), ..., x_m(\tau))$ be a fuzzy state set, where $x_i(\tau)$ is defined as the possibility that the state s_i will occur at time τ for all $i = 1, \ldots, m$. Thus, the evolution of the Markov chain admits a matrix formulated as follows:

$$x(\tau) = x(\tau - 1) \circ P = x(0) \circ P^{\tau}, \tag{3}$$

where $P = \{p_{ij}\}_{i,j=1}^{m}$ and $\circ$ denotes the matrix composition. A possibility distribution $x(\tau)$ of the system states at time τ is said to be stationary, or stable, whenever $x(\tau) = x(\tau) \circ P = x(0) \circ P^{\tau}$. In [6] we used a Max-Min algebra to compose the matrices and then in [4] the aforementioned composition was extended to a more general algebras $([0,1], S_M, T)$, where S_M denotes the maximum t-conorm and T any t-norm on $[0,1]$. In this work the following t-norms are analyzed: Lukasiewicz T_L, Product T_P (see [8]). Therefore, evolution of the possibilistic Markov chain in time is given by

$$x_i(\tau) = {S_M}_{j=1}^{m} \left(T(p_{ji}, x_j(\tau - 1))\right).$$

In [3], J. Duan gave the conditions that guarantee that a possibilistic Markov chain converges to a stationary state in a finite number of steps in at most $m-1$ steps. It is not hard to check that the possibilistic response threshold method, that will be introduced in Section 3.2, meets these conditions when a $([0,1], S_M, T)$ algebra is used to compose the matrices. This is one of the main advantages of possibilistic Markov chains compared to its probabilistic counterparts which, according to [7], the only convergence, in general, asymptotically.

3.2. **Possibilistic Response Threshold.** In this section we will see how to use possibilistic (fuzzy) Markov chains for implementing a RTM method. The possibility response function that will be explained here was tested and introduced in [4] and [5].

The task that the robot must carried out is defined as follows: a set of randomly placed robots in an environment must gather, or gets closer, to a set of tasks randomly placed too. It will be assumed that the stimulus only depends on the distance between the robot and the task. Consider the position space endowed with a distance (metric) d and denote by $d(r_i, t_j)$ the distance between the current position of r_i. It is also assumed that when a robot is assigned to a task, then the distance between the task and the robot is 0. Following the RTM notation, define the stimulus of the robot r_k to carry out task t_j as follows:

$$s_{r_k,t_j} = \begin{cases} \frac{U_{t_j}}{d(r_k,t_j)} & \text{if } d(r_k,t_j) \neq 0 \\ \infty & \text{if } d(r_k,t_j) = 0 \end{cases}. \tag{4}$$

When the stimulus s_{r_k,t_j} is used in Equation (1), we get following possibility response function:

$$p_{r_k,ij} = \frac{U_{t_j}^n}{U_{t_j}^n + d(r_k,t_j)^n \theta_{r_k}^n}. \tag{5}$$

In the same way, the same stimulus is used in (2), then the following exponential possibility response function is obtained:

$$p_{r_k,ij} = \mathsf{e}^{-\frac{\theta_{r_k}^n d(r_k,t_j)^n}{U_{t_j}^n}}. \tag{6}$$

As was see in [5], both function (5) and (6) meets the conditions (column diagonally dominant and power dominant) that guarantee the finite convergence (see [3]) in at most $m-1$ steps.

4. Experimental Results

In order to experimentally compute the number of steps to converge to stationary state we executed a set of experiments with possibilistic Markov chains with several t-norms. We consider that a possibilistic Markov chain converges in k steps wherever $P^k = P^{k+1}$, where P is the possibility transition matrix. In order to compare the results obtained with possibilistic Markov chains to its probabilistic counter part, the transition matrix P_{r_k} must be converted into a probabilistic matrix. To make this conversion each element of P_{r_k} is divided by the sum of all the elements in its row. A more detailed descriptions of all the experiments and results presented in this paper can be found in our previous works [4, 5, 6].

All the experiments have been executed using MATLAB with 500 different environments and with different number of randomly placed tasks: 50 and 100 ($m = 50, 100$). For the sake of simplicity, we have assumed that all tasks have the same utility, i.e., $U_{t_j} = 1$ for all $j = 1, \ldots, m$. In order to analyze the impact of the threshold on the system's performance, the θ_{r_k} will depend on the maximum distance between tasks as follows: $\theta_{r_k} = \frac{nTH}{d_{max}}$, where $d_{max} = 800$ is the maximum distance between two objects and nTH $(nTH = 1, 4, 8, 12, 16)$ is a parameter of the system.

Table 1 shows the average number of steps to converge with an algebra $([0,1], S_M, T_M)$. In all cases the possibility transition function 4 has been used to compose the matrices. As can be observed, the number of steps needed by the fuzzy Markov chains to converge is about 10 times lower than the time needed by its probabilistic counterpart (whenever they converge in a finite number of steps). The results do not change whichever possibility function is used, 5 or 6, and therefore, we it can be concluded that the number of steps required to converge with a Max-Min algebra does not depends on the transition function applied to compose the matrices.

Tasks	Possibilistic	Probabilistic	% Prob. Conv.
50	15.8	150.4	49.2%
100	23.4	256.8	51%

TABLE 1. Number of iterations needed to converge with the algebra $([0,1], S_M, T_M)$. Last column shows the percentage of probabilistic experiments that do not converge.

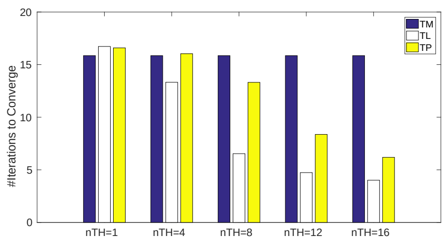

FIGURE 1. Number of iteration required to converge with 50 tasks.

Figure 1 shows the number of steps (iterations) needed to converge to a stationary state with different vales of nTH $(nTH = 1, 4, 8, 12, 16)$, 50 tasks $(m = 50)$, the power value $n = 2$ and when the t-norms T_M, T_L, T_P are used. In all cases, the possibility transition function 5 is used for the matrices composition. As can be seen, the iterations number when the t-norms T_L and T_P are used for matrix composition depends on the nTH parameter values. In general, the number of iterations decreases as the nTH increases. In contrast, the t-norm T_M always provides a system convergence in a same number of steps (15.85).

5. Conclusion and Future works

This paper has summarized the work developed by our research group towards a new multi-robot task allocation possibilistic framework based on response threshold algorithms. The classical RT algorithms, based on probabilistic Markov chains, in general only converges to a stable state asymptotically. In contrast the fuzzy Markov chain converges in at most $m-1$ of steps, where m is the number of tasks. In addition, the results of the experiments carried out to validate our approach also show that the possiblistic Markov chains converges 10 time faster than its possibilistic counter part. Furthermore, several transition possibility function and algebras for the composition the matrices has been considered. On the one hand, the number of steps needed to converge to stationary state with T_M does not depend on the possibility transition function used in the Markov chain. On the other hand, the results obtained for T_L and T_P are affected by the threshold value (θ_{r_k}). A lot of new challenges, problems and improvements must be addressed as future work. For the time begin, we focus on provide a deeper analysis about how the position of the tasks impacts on the convergence time. Moreover we are planning to study the behavior of the system when the distance between task ($d(t_i, t_j)$) is asymmetric.

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