STABILITY OF TILTED BUILDINGS

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SUMMARY

This paper shows a procedure to follow when a tall building tilts in order to obtain resistance soil parameters and predict the evolution of movement in a near future. Two soil behavior hypotheses are used, although other can be used, the paper showing the general way to act. An example is given where some of the predictions are fairly good.

RESUMEN

En este artículo se muestra un procedimiento para obtener algunos parámetros resistentes del suelo y predecir el futuro inmediato de un edificio alto que se inclina. Se hacen dos hipótesis sobre el comportamiento del suelo, aunque otras más sofisticadas también pueden hacerse, mostrando el artículo la manera general de proceder. Se da un ejemplo en el que alguna de las predicciones es bastante buena.

KEYWORDS: Settlement, tilt, soil elasticity, soil plasticity, dynamics.

INTRODUCTION

This paper analyzes the inmediate future of a tilted building in base of a series of settlement measurements at different times.

For each instant the rigid body movements are supposed to have been calculated as indicated in the other paper by the same authors, presented in this Congress.

Rational Mechanics is used, the weight and moment due to eccentricity being the acting forces.

Two soil behavior hypotheses are used so that the one that better fits the data seems to be more probable to explain what has happened so far.

Integration of the dynamic equations yields the building most probable behavior in the immediate future.

In general, only the vertical motion and one tilt equations are necessary. This is because one tilt direction predominates and because horizontal movements are much more difficult to obtain than vertical.

APPLIED FORCES

The vertical forces acting on the building are its weight and the soil reaction. The acting moments are the one due to the weight eccentricity and the eccentric soil reaction.

The vertical dynamic equation is then

$$m_B.\ \frac{d^2u}{dt^2} = W_B - F_S$$

and the moment equation is

$$I_B.\ \frac{d2\alpha}{dt_2} = M_b - M_s$$

where

$\mathbf{m}_B$ = mass of building
$\mathbf{u}$ = vertical displacement of the measured points center of gravity
$\mathbf{W}_B$ = weight of building
$\mathbf{F}_S$ = vertical soil reaction
$\mathbf{I}_B$ = moment of inertia of building
α = angle rotated by the building
$\mathbf{M}_B$ = moment due to the weight eccentricity
$\mathbf{M}_s$ = moment due to the soil reaction

The moment due to the building can be written as

$$\mathbf{M}_B = \mathbf{W}_B\ .\ \mathbf{H}\ .\ \alpha \qquad (\alpha \text{ is an approximation to } \sin(\alpha))$$

where

H = heighth of the building center of gravity.

The formulation of soil reaction, $\mathbf{F}_S$, and soil moment, MS, depend on the soil behavior hypothesis.

PSEUDOELASTIC SOIL BEHAVIOR

If the foundation is composed of several portions (isolated footings or piles) the assumption that, for each portion, the displacement is proportional to the stress can be used.

The proportionality constant is the coefficient of subgrade reaction, **K**. For each portion, the following equation holds

$$\sigma_i = \mathbf{K}\ .\ \mathbf{u}_i$$

and the total soil reaction is

$$\mathbf{F}_S = \Sigma\,\sigma_i\ \mathbf{A_i} = \mathbf{K}\,\Sigma\,\mathbf{A_i}\ .\ \mathbf{u}i$$

In a rigid body, the displacement can be expressed as

$$\mathbf{u}_i = \mathbf{u} + \alpha\ .\ \mathbf{x}_i$$

and, therefore, the total soil reaction is

$$\mathbf{F}S = K\ .\ A_C\ .\ \mathbf{u}$$

since the α term vanishes because it is the static moment of the foundation area about the foundation center of gravity.

In the foregoing expressions the following meanings are used

$\mathbf{A}_i$ = area of a foundation portion
σ_i = stress of the foundation against the soil
$\mathbf{u}_i$ = displacement of the foundation portion
$\mathbf{u}$ = displacement of the foundation center of gravity
$\mathbf{X}_i$ = distance of foundation portion to the total foundation center
$\mathbf{A}_C$ = foundation area

The soil moment is

$$\mathbf{M}_S = \Sigma\,\sigma_i\ .\ \mathbf{A}_i\ .\ \mathbf{X}_i$$

Following the same steps as before, the soil moment reaction is now

$$\mathbf{M}_S = \mathbf{K} . \mathbf{I}_C . \alpha$$

where

$\mathbf{I}_C$ = inertia moment of the foundation area.

EQUATIONS OF THE DISPLACEMENT INCREMENTS

When the foregoing forces and moments are introduced in the movement equations, they become

$$\frac{\mathbf{d}^2 \Delta \mathbf{u}}{\mathbf{dt}^2} = \mathbf{B}_O + 2 . \mathbf{C}_O . \Delta_{\mathbf{u}}$$

$$\frac{\mathbf{d}^2 \Delta œ}{\mathbf{dt}^2} = \mathbf{B}_1 + \mathbf{2} . \mathbf{C}_1 . \Delta\alpha$$

where Δ accounts for the increment of the variable to its right and $\mathbf{B}_O$, $\mathbf{B}_I$, $\mathbf{C}_O$ and $\mathbf{C}_1$ are unknown constants which include initial values and the unknown coefficient of subgrade reaction.

By multiplying these equations by **du** or **dœ**, the energy equations are obtained, after integration

$$\frac{\mathbf{1}}{\mathbf{2}} . \mathbf{V}O^2 = \mathbf{A}_O + \mathbf{B}_O . \Delta\mathbf{u}_O + \mathbf{C}_O \; \Delta\mathbf{u}_O^2$$

$$\frac{\mathbf{1}}{\mathbf{2}} . \mathbf{W}^2 = \mathbf{A}_1 + \mathbf{B}_1 . \Delta\alpha + \mathbf{C}_1 . \Delta\alpha^{\mathbf{2}}$$

FINDING THE UNKNOWN COEFFICIENTS

The measurements taken allow to compute the vertical velocity and the tilt velocity. Values are attributed to middle point of the interval. The data can be adjusted to a minimum square parabola.

As it can be seen, three measurements are necessary, apart from the initial setting of measuring points, to define the parabolas.

In this manner the coefficients **A**, **B** and **C** are obtained. If the **C** value is negative, the solution is stable and an equilibrium position exists, given by the no acceleration position (**B/2C**)..

If the **C** value is positive, the solution is unstable and the displacements grow indefinitely.

The equations can be easily solved for the displacements and analytical prediction of building behavior can be obtained.

If the equations are used just as shown, no long term predictions can be made since mathematical simplifications have been made and because the soil and other conditions change.

VISCOPLASTIC SOIL BEHAVIOR

As a viscoplastic material, the soil reaction can be expreesed as being proportional to the velocity.

In this case, the soil stress at a foundation portion may be written

$$\sigma_i = \mathbf{K} . \mathbf{V}_i$$

where **K** is a constant and $\mathbf{V}_i$ is the velocity of a foundation portion.

Following the same steps as before, the equations of the Mechanics of a rigid body are

$$\frac{\mathbf{d}^2 \Delta \mathbf{u}}{\mathbf{dt}^2} = \mathbf{A}_O - \mathbf{B}_O . \frac{\mathbf{d}\Delta\mathbf{u}}{\mathbf{dt}}$$

$$\frac{\mathbf{d}^2 \Delta\alpha}{\mathbf{dt}^2} = \mathbf{A}_1 . \Delta\alpha - \mathbf{B}_1 \frac{\mathbf{d}\Delta\alpha}{\mathbf{dt}}$$

where it can be seen that the tilt equation always results in unstability, since all coefficients shown are positive. After a certain time has elapsed, the acceleration vanishes and the movement becomes uniform.

Unknown coefficients are found in a similar way as before.

AN EXAMPLE

Because its recent development, it is not possible to give direct reference to this case.

When the movement problem was recognized, a period of 4 months was expected to be necessary until repairs could begin.

A question to answer was if at that time the building tilt would be small enough for the building to be still usable.

Twelve points have been measured during three months. The data have been treated as indicated in Rechea and Basset (1).

Fig. 1 show the data measurements in settlements and tilt.

Fig. 2 shows the predictions for the settlement and tilt based on then, pseudoelastic assumption.

Fig. 3 gives the predictions of settlement and tilt based on the viscous assumption

It can be seen that the viscoplastic solution gives a better approximation.

BIBLIOGRAFIA

(1) Rechea, M.; Basset, L.; **"Settlement monitoring. A mathematical treatment"**. I Congreso sobre rehabilitación del Patrimonio Arquitectónico y Edificación. La Laguna, 1992. (Este volumen).

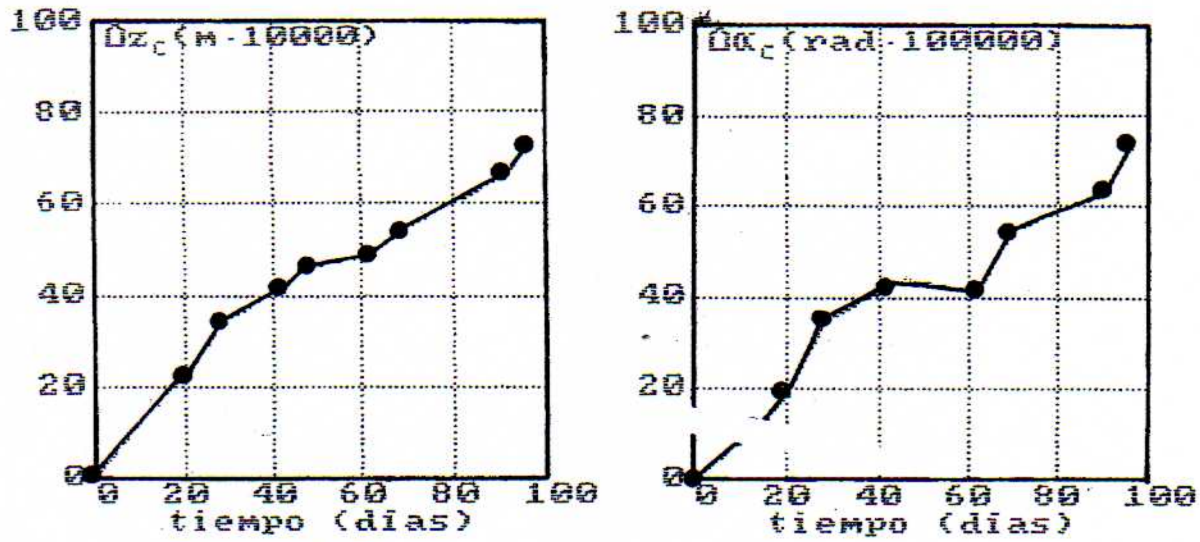

Figura 1. Datos

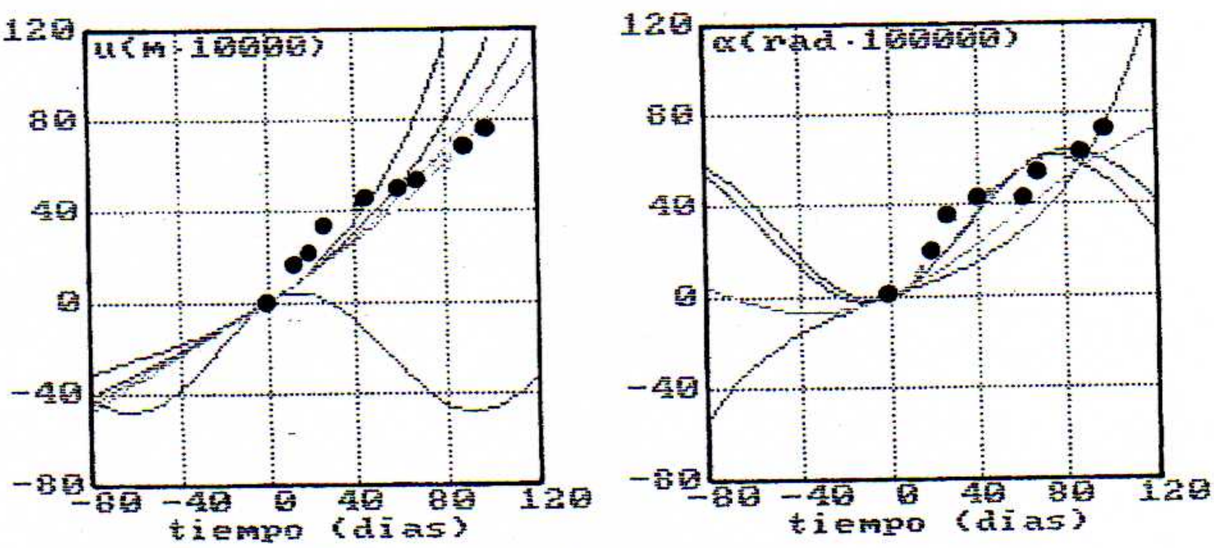

Figura 2. Pseudoelasticidad

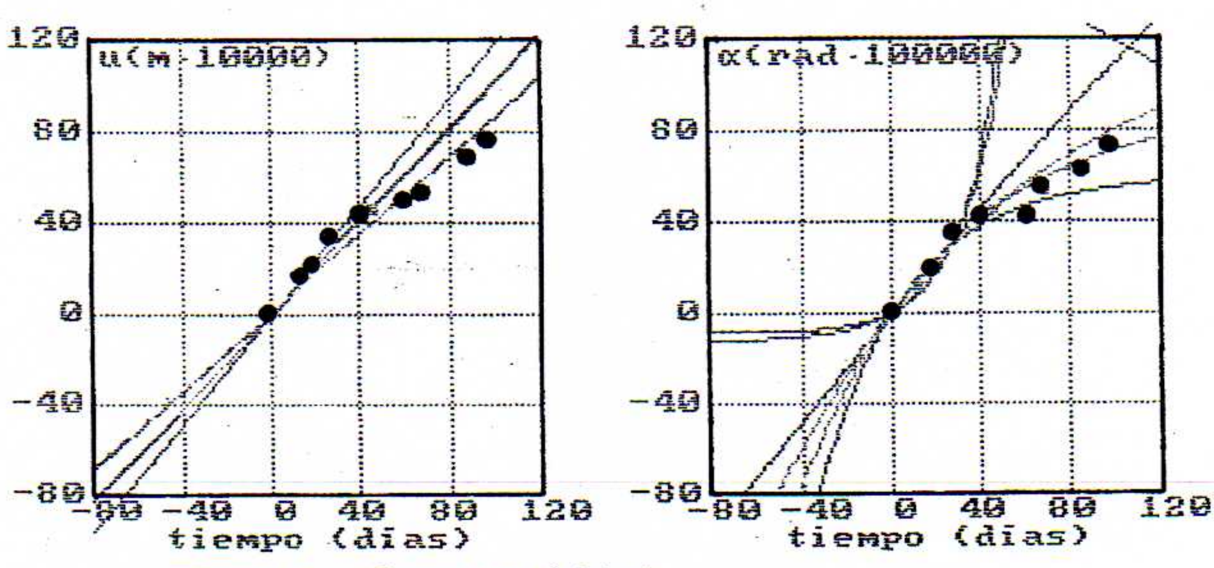

Figura 3. Viscoplasticidad