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# New Predictor and 2DOF Control Scheme for Industrial Processes with Long Time Delay

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**Abstract**—To address the difficulty of controlling industrial processes with long time delay, a novel design of dead-time compensator (DTC) is introduced, which can be used to predict the undelayed output response of any process (no matter stable or unstable) such that the control design may be focused on the delay-free part of the process for performance optimization. Based on the undelayed output estimation, a two-degree-of-freedom control scheme is analytically developed for optimizing the set-point tracking and disturbance rejection, respectively. By proposing the desired transfer functions, the corresponding controllers are analytically derived based on commonly used low-order process models. A notable merit is that there is a single adjustable parameter in the proposed DTC as well as in each controller, which can be monotonically tuned to meet a good trade-off between the prediction (or control) performance and its robustness. Illustrative examples from the literature and a practical application to a temperature control system of a jacketed reactor are used to demonstrate the effectiveness of the proposed predictor-based control scheme.

**Index Terms**—Long time delay, time compensator, delay-free output prediction, two-degree-of-freedom control, discrete-time control design, disturbance rejection

## I. INTRODUCTION

Long time delays are involved in a lot of industrial processes, due to mass transportation, energy exchange, remote signal processing, and valve stickiness etc [1]. A typical scenario is the heating-up process of a jacketed reactor for mixing chemical materials. It may be viewed as an integrating process with long input delay, because the solution temperature in the reactor does not change for quite a while after the electric heater of the thermostatic circulator is turned on, and then the temperature rises up continuously without reaching a steady value. The existing control methodologies for delay-free plants may fail or result in degraded performance when applied to time-delay systems, if the delay is not considered in the control design. Other control methods aiming at counteracting the delay effect are inefficient or even inapplicable in the presence of long time delay. The classical PID controller could be used for a time delay system only when the delay is small, but may result in poor control performance when the ratio of the time delay over the dominant time constant of the process is larger than one [2].

The well-known Smith predictor (SP) [3] has been effectively used to control stable processes, but it cannot be

used for integrating and unstable processes due to the problem of internal instability [4]. Modified SPs and different dead-time compensators (DTCs) were explored (e.g., [5-12]) to estimate the undelayed output of an integrating or unstable process with time delay. The main advantage of using an SP or DTC lies in the possibility to apply any of the developed control methods for the ‘undelayed’ plant, and alleviating control limitations due to the time delay. However, a notable drawback of the SP and its modifications, including some of the aforementioned, is the sensitivity to external disturbances and delay uncertainties [10, 13]. To eliminate the error in the presence of static disturbance, an external loop with certain filtering capacity was proposed, e.g. the modified SP [5, 6], the generalized predictor (GP) [10], or the disturbance estimator [14], bringing additional complexity for controller design and robust stability analysis.

Besides, for the control of integrating and unstable processes, severe water bed effect between the set-point tracking and load disturbance rejection in terms of using the classical unity feedback control structure or the standard internal model control (IMC) structure [4] inevitably occurs. To overcome this deficiency, two-degrees-of-freedom (2DOF) control schemes in combination with a DTC for output prediction have been developed (e.g. [15-22]), based on the use of practical low-order process models with time delay. Among these references, the IMC theory for optimizing the set-point tracking was adopted in [15, 17, 22, 23], and in contrast, a few set-point filtering strategies were developed in [16, 18, 19, 21]. For optimizing the disturbance rejection, a few desired transfer functions of disturbance response were proposed in [15, 18, 19, 22], taking into account some practical types of disturbances.

In practice, process uncertainties or model mismatch may provoke dynamic errors in estimating the undelayed output. These errors must be counteracted by the control structure, that is, the control design should be robust against output prediction errors. Moreover, when there exist internal or external disturbances, the control design also needs to maintain the internal stability. In this sense, the DTC should be as simple as possible, while avoiding any specific loop purely for prediction compensation that may complicate the control structure and robust stability analysis.

To address the above problems, a new DTC design is proposed in this paper for general application to different types of industrial processes with time delay, as inspired by the idea introduced in [10]. The proposed DTC, consisting of two stable filters to estimate the undelayed output, does not require any

additional filtering loop as explored in the references [5, 6, 10, 14] to eliminate the steady-state error estimation under static or asymptotically stable disturbance. Accordingly, a new predictor-based 2DOF control scheme to optimize the set-point tracking and disturbance rejection performance is proposed. The corresponding controllers are analytically derived by proposing the desired transfer functions. To allow for a trade-off between the prediction performance and its robustness, a single adjustable parameter is introduced in the proposed DTC, while there is also a single adjustable parameter in each controller of the proposed 2DOF control scheme, which can be monotonically increased or decreased to obtain the desired set-point tracking performance together with a good trade-off between the disturbance rejection performance and the system robust stability.

The paper is organized as follows. In Section II, the ideal situation of having access to the ‘undelayed’ plant output for closed-loop control is described. Then a novel DTC to estimate the undelayed output of a single-input-single-output (SISO) process is proposed in Section III. Based on the designed DTC, a 2DOF control scheme is analytically developed in Section IV. Illustrative examples are shown in Section V, followed by the application to a temperature control system of a jacketed reactor in Section VI. Some conclusions are drawn in the last section.

## II. IDEAL DELAY-FREE CONTROL LOOP

Since computer-aided control systems have been widely applied for industrial process operations, the control system will be designed in a discrete-time (DT) framework. The external representation of an input-output delayed SISO process is generally described in discrete-time domain by

$$P(z) = G(z)z^{-d} = \frac{N(z)}{D(z)}z^{-d} \quad (1)$$

where  $N(z)$  is an  $m$ -order polynomial whose roots can be out of the unit circle in the  $z$ -plane if the plant is non-minimum phase (NMP),  $D(z)$  is an  $n$ -order polynomial whose roots may be out of the unit-circle if the plant is unstable, and  $d$  is the time delay. The size of  $d$  depends on the sampling period. In fact, to consider a long time delay, what matters is the ratio between the time delay and the dominant time constant of the process. Notice that, if the actual behavior of the plant is described by  $P_p(z) = G_p(z)z^{-d_p}$ , for perfect model matching, it is  $G(z) = G_p(z)$  and  $d = d_p$ .

Let us consider an ideal delay-free control loop where  $r(z)$  is the set-point command,  $u(z)$  is the control input,  $w(z)$  is a disturbance entering the plant from the input side, and  $n(z)$  is measurement noise. Denote by  $C_f(z)$  the closed-loop feedback controller and by  $C_s(z)$  the set-point filter. The relevant closed-loop transfer functions are

$$\begin{aligned} y(z) &= P_{yr}(z)r(z) + P_{yw}(z)w(z) + P_{yn}(z)n(z) \\ &= C_s(z) \frac{C_f(z)G_p(z)}{1 + C_f(z)G_p(z)} z^{-d_p} r(z) \end{aligned} \quad (2)$$

$$+ \frac{G_p(z)}{1 + C_f(z)G_p(z)} z^{-d_p} w(z) - \frac{C_f(z)G_p(z)}{1 + C_f(z)G_p(z)} z^{-d_p} n(z) \quad (3)$$

**Definition 1 (Ideal delay-free control loop).** The above

scenario where the undelayed output denoted by  $\bar{y}(z)$  is accessible, leading to the closed-loop transfer functions (2)-(3), is referred to as the ideal delay-free control loop.

However, the undelayed output is not accessible in practice.

## III. PROPOSED DTC

Since the undelayed output  $\hat{y}(z)$  should be predicted based on the available information, that is,  $u(z)$ ,  $y(z)$ , and the plant model, the proposed DTC is shown in Fig. 1.

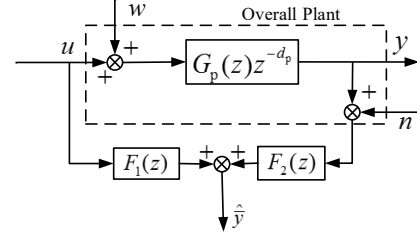


Fig. 1 Proposed DTC structure

The estimated undelayed output is computed by

$$\hat{y}(z) = F_1(z)u(z) + F_2(z)[y(z) + n(z)] \quad (4)$$

Obviously, the predicted output  $\hat{y}(z)$ , will not perfectly match the actual one  $\bar{y}(z)$ . The prediction error is defined as

$$\bar{e}(z) = \bar{y}(z) - \hat{y}(z) \quad (5)$$

From Fig. 1, this error can be rewritten as

$$\begin{aligned} \bar{e}(z) &= \{[1 - z^{-d_p} F_2(z)]G_p(z) - F_1(z)\}u(z) \\ &+ [1 - z^{-d_p} F_2(z)]G_p(z)w(z) - F_2(z)n(z) \end{aligned} \quad (6)$$

The estimation of the process rational part is given by

$$\hat{G}(z) = F_1(z) + F_2(z)G(z)z^{-d} \quad (7)$$

Note that the proposed DTC (for predicting the undelayed output) is different from a disturbance estimator (DOB) that aims at disturbance estimation for feedforward control design [24], although both use the process input and output information for prediction or estimation. Besides, the developed DOB methods as surveyed in [24] require a model-based DTC or the inverse of the process model to estimate load disturbance for a time-delay system, therefore inapplicable to unstable processes with time delay due to the internal stability issue.

### A. Simplified Generalized Predictor (SGP)

Decompose the plant model in (1) as studied in [10] into

$$P(z) = \tilde{G}(z)\Gamma(z)z^{-d} \quad (8)$$

where  $\Gamma(z)$  is a proper stable transfer function including  $N(z)$  with  $m$  zeros of the plant and an  $m$ -multiple pole, i.e.,

$$\Gamma(z) := \frac{N(z)}{(z - \lambda)^m} H(z, \lambda) \quad (9)$$

where  $\lambda$  is a basic design parameter satisfying  $|\lambda| < 1$ , and  $H(z, \lambda)$  is a filter introduced herein to be designed.

To match with (1) and (8),  $\tilde{G}(z)$  is written in the form of

$$\tilde{G}(z) = \frac{(z - \lambda)^m}{D(z)} H^{-1}(z, \lambda) = c(zI - A)^{-1} b \quad (10)$$

where  $(A, b, c)$  is a minimum-order state space model of  $\tilde{G}(z)$  and  $D(z)$ , the model denominator, is composed of all the plant poles.

For practical implementation, it is proposed to take

$$H(z) = \frac{(z-1)}{(z-\lambda)} \cdot \frac{(1-\lambda)^{n_h} z^{n_h}}{(z-\lambda)^{n_h}}, \quad (11)$$

which has null static gain, and  $[(1-\lambda)^{n_h} z^{n_h}] / (z-\lambda)^{n_h}$  is an all-pass filter for reducing sensitivity to measurement noise, with  $n_h$  a user specified order in practice.

Similar to (10), define a new transfer function including  $A^d$ ,

$$\tilde{G}^*(z) = c(zI - A)^{-1} A^d b = \frac{\tilde{N}^*(z)}{\tilde{D}(z)} \quad (12)$$

**Lemma 1** For any process of stable, integrating, or unstable type, minimum-phase (MP) or NMP, the prediction computed by (4) using the stable filters,

$$F_1(z) = c \sum_{i=1}^d A^{i-1} b z^{-i} \Gamma(z), \quad (13)$$

$$F_2(z) = \frac{\tilde{N}^*(z)}{(z-\lambda)^{m+1+n_h}}, \quad (14)$$

provides an undelayed output estimation without steady-state error under static or asymptotically stable disturbances, and correspondingly,

$$F_1(z) + z^{-d} F_2(z) G(z) = G(z) \quad (15)$$

**Proof.** The proof follows a similar derivation as developed in [10] and thus it is omitted for brevity.  $\square$

**Remark 1.** As  $D(z)$ , which is composed of all the (possibly unstable) plant poles is excluded from  $F_1(z)$  and  $F_2(z)$ , the proposed SGP can be used for any time delay process, with no need to design specific predictors for unstable processes as studied in the literature (e.g. [5, 6, 8, 10]). Moreover, the proposed SGP can also be used for NMP processes without any specific treatment of the plant NMP zeros, as  $N(z)$  composed of all the (possibly NMP) plant zeros is included in  $F_1(z)$   $\diamond$

**Remark 2.** The decomposition in (8)-(10) is different from that given in [10] by introducing an additional filter  $H(z)$  which is chosen to eliminate the steady-state error under static or asymptotically stable disturbances and reduce the noise sensitivity. Therefore, any additional filtering loop for static disturbance rejection as studied in [10] is not needed.  $\diamond$

## B. SGP Tuning

With the above DTC design, the undelayed output prediction error shown in (6), under perfect model matching and taking into account (15), is reduced to

$$\bar{e}(z) = F_1(z)w(z) - F_2(z)n(z) \quad (16)$$

As  $\lim_{z \rightarrow 1} F_1(z) = 0$ , the first term at the right-hand side of (13) vanishes in the steady state. Furthermore, the structure of  $F_1(z)$  indicates that a smaller value of  $\lambda$  will lead to faster disturbance rejection. The second term at the right-hand side indicates that the noise is related to the magnitude of  $F_2(z)$ .

For practical application, two indices are defined herein for evaluating the prediction performance and the noise rejection. The prediction performance is assessed by the integral-absolute-error (IAE) of the output prediction. As a measure of the prediction error arising from the measurement noise, the magnitude of  $F_2(z)$  is defined as

$$M_{F_2} = \|F_2(z, \lambda)\|_{\infty} \quad (17)$$

Hence, by numerically comparing these two indices, both of them should be as small as possible for optimality in terms of a tuning range of  $\lambda \in (0, 1)$ . A suitable choice of  $\lambda$  can be determined to meet a good trade-off between the prediction performance and the noise rejection. As a practical guideline it is suggested to choose  $\lambda$  corresponding to the minimum of a weighted cost function defined by using the normalized values in  $(0, 1)$  of the above IAE and  $M_{F_2}$ .

For illustration, consider an NMP process with time delay,

$$P(s) = \frac{s-1}{(s+2)(s+3)} e^{-3s}$$

With a sampling period  $T_s = 0.1s$ , its DT transfer function is

$$P(z) = \frac{0.07367(z-1.106)}{(z-0.8187)(z-0.7508)} z^{-30}$$

The proposed SGP is applied to reconstruct the undelayed output. Assume there is a unity step change in the input  $u$  at the initial time, followed by a random measurement noise with a standard deviation of  $\sigma = 0.001$  appearing three seconds later owing to the time delay, and a unity step type disturbance is added at  $t = 10s$ . The prediction error is depicted in Fig. 2 for three different choices of  $\lambda$  in the SGP.

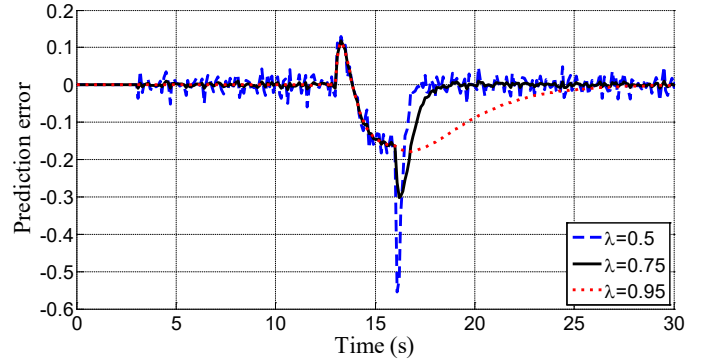


Fig. 2 Plot of the undelayed output prediction errors by tuning  $\lambda$

It is seen that no steady-state error is guaranteed by taking different choices of  $\lambda$ . Note that for the input change, only the error due to the noise appears. By comparison, taking a lower value of  $\lambda$  leads to a faster filtering but provokes an undesired noise amplification, and vice versa.

The normalized values in  $(0, 1)$  of both indices as well as a joint one computed by  $J = (\text{IAE} + M_{F_2}) / 2$ , are plotted in Fig. 3. According to this balance, the tuning parameter should be adopted in the neighborhood of  $\lambda = 0.75$  to meet a good trade-off for this example.

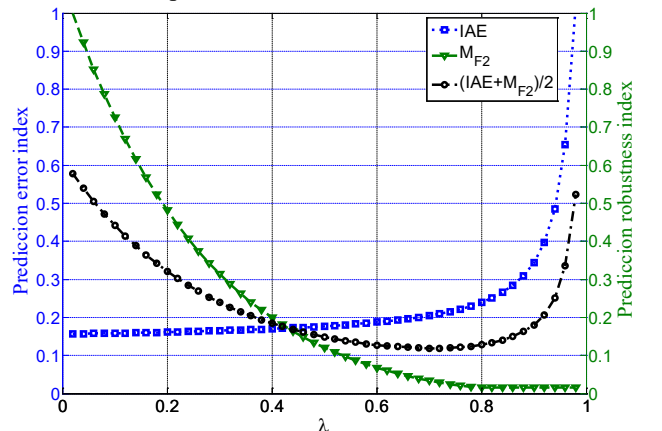


Fig. 3 Indices of prediction error and noise rejection by tuning  $\lambda$

#### IV. SGP-BASED 2DOF CONTROL SCHEME

Based on the designed SGP, a predictive 2DOF control scheme is proposed as shown in Fig. 4.

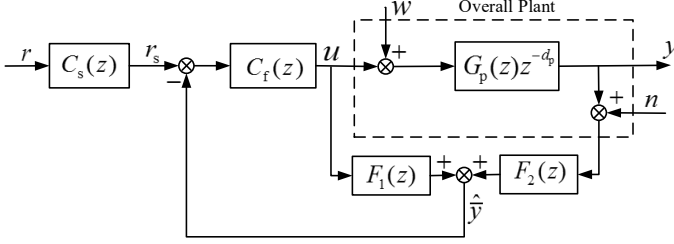


Fig. 4 Proposed predictor-based 2DOF control scheme

where  $\hat{y}(z)$  is the predicted undelayed output,  $C_s(z)$  is a controller for the set-point tracking and  $C_f(z)$  is the closed-loop controller for disturbance rejection.

The system transfer functions can be derived from Fig. 4,

$$y(z) = C_s(z) \frac{C_f(z)G_p(z)}{1 + C_f(z)\hat{G}(z)} z^{-d_p} r(z) \quad (18)$$

$$+ \frac{G_p(z)}{1 + C_f(z)\hat{G}(z)} [1 + C_f(z)F_1(z)] z^{-d_p} w(z) \quad (19)$$

$$- \frac{C_f(z)G_p(z)}{1 + C_f(z)\hat{G}(z)} F_2(z) z^{-d_p} n(z) \quad (20)$$

By inspecting (2)-(3), it is evident that with the proposed  $F_1(z)$  and  $F_2(z)$  to obtain  $\hat{G}(z) = G_p(z)$ ,  $C_f(z)$  can be designed similarly to that of a delay-free closed-loop system, thus facilitating the improvement of control performance.

As many industrial processes can be effectively approximated by reduced order models [1, 2, 4], the following typical discrete-time models for describing stable, integrating and unstable processes with time delay are studied here,

$$P_1(z) = \frac{k_p}{z - z_p} z^{-d} \quad (21)$$

$$P_2(z) = \frac{k_p(z - z_0)}{(z - 1)(z - z_p)} z^{-d} \quad (22)$$

$$P_3(z) = \frac{k_p(z - z_0)}{(z - z_u)(z - z_p)} z^{-d} \quad (23)$$

where  $|z_p| < 1$ ,  $|z_u| > 1$ , and  $|z_0| < 1$ .

##### A. Controller Design for Disturbance Rejection

Based on the above undelayed output prediction, the transfer functions between  $\hat{y}$ ,  $u$ , and  $w$  are derived as

$$\frac{u(z)}{w(z)} = T_d(z) = \frac{C_f(z)G(z)}{1 + C_f(z)G(z)} \quad (24)$$

$$\frac{\hat{y}(z)}{w(z)} = G(z)(1 - T_d) \quad (25)$$

Note that a stable pole of  $G(z)$  close to the unity circle or larger than the closed-loop transfer function pole will result in a slow behavior of the controlled plant. Thus, inspired by the optimal closed-loop transfer function for disturbance rejection developed in the recent work [22], but also considering the slow stable pole  $z_p$  if it exists, the desired  $T_d$  is proposed to satisfy the following asymptotic constraints in order to cancel the influence from undesired plant poles that are close to, on or outside the unit circle in the  $z$ -plane,

$$\lim_{z \rightarrow \eta} (1 - T_d) = 0, \quad \eta = z_u \text{ or } z_p \text{ (if necessary)}. \quad (26)$$

Note that for an integral process (corresponding to  $\eta = 1$ ), the constraint in (26) should be modified as

$$\lim_{z \rightarrow 1} \frac{d}{dz} (1 - T_d) = 0 \quad (27)$$

The desired closed-loop transfer function from  $w$  to  $u$  is therefore proposed as

$$T_d(z) = \frac{(1 - \lambda_r)^{n_d}}{(z - \lambda_r)^{n_d}} \sum_{i=0}^l \beta_i z^i, \quad \sum_{i=0}^l \beta_i = 1 \quad (28)$$

where  $\lambda_r$  is a tuning parameter for the closed-loop control performance,  $\beta_i$  ( $i = 1, 2, \dots, l$ ) are coefficients to be determined by the asymptotic constraints in (26) and (27),  $l$  is the number of these constraints used for stable, integrating, and unstable processes, respectively, and  $n_d$  is a practically specified order to maintain the properness of  $T_d(z)$  for implementation, e.g.,  $n_d = l + 1$ .

Accordingly, the closed-loop controller can be inversely derived from (24) as

$$C_f(z) = \frac{T_d(z)}{1 - T_d(z)} \cdot \frac{1}{G(z)} \quad (29)$$

Note that if the zero  $z_0$  of  $G(z)$  has a negative real part in  $z$ -plane, i.e.,  $z_0 < 0$ , the designed  $C_f(z)$  in (29) will present the corresponding pole that may provoke inter-sample rippling in the output response or control signal. To circumvent the problem, the controller is modified as

$$\hat{C}_f(z) = C_f(z) \frac{z - z_0}{z(1 - z_0)}, \quad \text{if } z_0 < 0 \quad (30)$$

Given the models in (21)-(23), two cases of the stable pole ( $z_p$ ) location in the  $z$ -plane are categorized for controller design: case (i)  $z_p$  is far away from the unit circle or smaller than the desired closed-loop pole in (28) by tuning  $C_f(z)$ , (i.e.,  $|z_p| < \lambda_r$ ); case (ii)  $z_p$  is very close to the unit circle or  $|z_p| > \lambda_r$  in tuning  $C_f(z)$ . The controller design is exemplified for a stable process described by (21) as below.

For case (i), it follows from (28) with  $l = 0$  that

$$T_d(z) = \frac{(1 - \lambda_r)^{n_d}}{(z - \lambda_r)^{n_d}} \quad (31)$$

Substituting (31) into (29) it yields

$$C_f(z) = \frac{(1 - \lambda_r)^{n_d} (z - z_p)}{k_p (z - 1) \sum_{i=0}^{n_d-1} (z - \lambda_r)^i (1 - \lambda_r)^{n_d-i-1}} \quad (32)$$

For case (ii), the asymptotic constraint in (26) should be considered, corresponding to  $l = 1$  in using (28). It follows that

$$T_d(z) = \frac{(1 - \lambda_r)^{n_d} (\beta_0 + \beta_1 z)}{(z - \lambda_r)^{n_d}} \quad (33)$$

Substituting (33) into (28) for  $\eta = z_p$ , it yields

$$\begin{cases} \beta_1 = \frac{(z_p - \lambda_r)^{n_d} - (1 - \lambda_r)^{n_d}}{(z_p - 1)(1 - \lambda_r)^{n_d}} \\ \beta_0 = 1 - \beta_1 \end{cases} \quad (34)$$

Substituting (21), (33) and (34) into (29), the closed-loop controller is obtained as

$$C_r(z) = \frac{(1 - \lambda_r)^{n_d} (\beta_1 z + \beta_0)}{k_p (z - 1) \sum_{i=0}^{n_d-1} [(1 - \lambda_r)^{n_d-i-1} \sum_{j=0}^{i-1} (z - \lambda_r)^j (z_p - \lambda_r)^{i-j-1}]} \quad (35)$$

Similarly, the closed-loop controllers can be derived for integrating and unstable processes described by (22) and (23), respectively, which are omitted for brevity.

### B. Set-point Tracking Controller

Firstly, the desired  $T_d(z)$  is factorized into an all-pass part,  $T_{dA}(z)$ , and an MP part,  $T_{dM}(z)$ , i.e.,

$$T_d(z) = T_{dA}(z) T_{dM}(z) \quad (36)$$

Then the controller to achieve the  $H_2$  optimal control performance for set-point tracking according to the IMC theory [4] is proposed in the form of

$$C_s(z) = \frac{1}{z^{n_g} T_{dM}(z)} \cdot \frac{(1 - \lambda_s)^{n_f} z^{n_f}}{(z - \lambda_s)^{n_f}} \quad (37)$$

where  $\lambda_s$  is a tuning parameter,  $n_g$  is a specified integer to keep  $z^{n_g} T_{dM}(z)$  bi-proper, and  $n_f$  is a user specified filter order.

In practice,  $\lambda_s$  in  $C_s(z)$  can be monotonically tuned in a range of  $(0, 1)$ . When it is tuned to a smaller value, the set-point tracking may be expedited but at the cost of larger control effort, while the tracking robustness may be degraded in the presence of process uncertainties, and vice versa.

### C. Robust Stability Analysis

Considering the process uncertainties described in a multiplicative form,  $\Delta(z) = [G_p(z)z^{-d_p} - G(z)z^{-d}] / G(z)z^{-d}$ , it can be derived from the proposed scheme in Fig. 4 that the transfer function from the output to the input of  $\Delta(z)$  is

$$M = F_2 \frac{C_r G z^{-d}}{1 + C_r G} = F_2 T_d z^{-d} \quad (38)$$

According to the  $M - \Delta$  form of the small gain theorem for robust stability analysis (see e.g., [4]), the closed-loop structure in Fig. 4 holds robust stability if and only if

$$\|F_2 T_d \Delta\|_\infty < 1 \quad (39)$$

With a fixed  $\lambda$  in the proposed predictor, substituting (14) and (28) into (39) yields the robust stability constraint for tuning  $\lambda_r$  in the closed-loop controller  $C_r(z)$ ,

$$\left\| \frac{(1 - \lambda_r)^{n_d} \sum_{i=0}^l \beta_i z^i \sum_{j=0}^{m+1+n_h} \alpha_j z^j}{(z - \lambda_r)^{n_d} (z - \lambda)^{m+1+n_h}} \right\|_\infty < \frac{1}{\|\Delta(z)\|_\infty} \quad (40)$$

which may be reformulated as

$$\left\| \frac{\sum_{i=0}^l \sum_{j=0}^{m+1+n_h} \beta_i \alpha_j z^{i+j}}{\sum_{i=0}^{n_d} \sum_{j=0}^{m+1+n_h} \gamma_i \delta_j z^{i+j}} \right\|_\infty < \frac{1}{\|\Delta(z)\|_\infty (1 - \lambda_r)^{n_d}} \quad (41)$$

where  $\alpha_j$  ( $j = 0, 1, \dots, m+1+n_h$ ) is the expansion coefficients of  $\tilde{N}^*(z)$  in (14),  $\gamma_i = C_{n_d}^i (-\lambda)^{n_d-i}$  and  $\delta_j = C_{m+1+n_h}^j (-\lambda_r)^{m+1+n_h-j}$ .

Consider the following process uncertainty description,

$$\Delta = \left(1 + \frac{\Delta k_p}{k_p}\right) z^{-\Delta d} - 1 \quad (42)$$

By defining  $z = e^{j\theta}$  ( $0 < \theta < 2\pi$ ) and substituting (42) into (41), the robust stability constraint is derived as

$$\frac{(1 - \lambda_r)^{n_d} \sqrt{x_1^2 + x_2^2}}{\sqrt{x_3^2 + x_4^2}} < \frac{1}{\sqrt{\left[\left(1 + \frac{\Delta k}{k}\right) \cos \Delta d \theta - 1\right]^2 + \left[\left(1 + \frac{\Delta k}{k}\right) \sin \Delta d \theta\right]^2}} \quad (43)$$

where  $x_1 = \sum_{i=0}^l \sum_{j=0}^{m+1+n_h} \beta_i \alpha_j \cos[(i+j)\theta]$ ,  $x_2 = \sum_{i=0}^l \sum_{j=0}^{m+1+n_h} \beta_i \alpha_j \sin[(i+j)\theta]$ ,

$x_3 = \sum_{i=0}^{n_d} \sum_{j=0}^{m+1+n_h} \gamma_i \delta_j \cos[(i+j)\theta]$ ,  $x_4 = \sum_{i=0}^{n_d} \sum_{j=0}^{m+1+n_h} \gamma_i \delta_j \sin[(i+j)\theta]$ .

In practice, the single adjustable parameter  $\lambda_r$  in  $C_r(z)$  may be initially taken around 0.9~0.95, which can be monotonically increased or decreased in a range of  $(0, 1)$  to meet a good trade-off between the closed-loop control performance and its robust stability. Given an upper bound of  $\Delta(z)$  as shown in (42), the robust stability constraints in (43) can be used to check if  $\lambda_r$  has been properly tuned.

## V. ILLUSTRATIVE EXAMPLES

To assess the control performance, the commonly used performance index of IAE of the process output and the total-variation (TV) of the control input are adopted.

**Example 1.** Consider the stable process with long input delay studied in [11],

$$P(s) = \frac{e^{-27.5s}}{52.5s + 1}$$

With a sampling period of  $T_s = 0.5s$ , a discrete-time model of the process is obtained as

$$G(z) = \frac{0.009479}{z - 0.9905} z^{-55}$$

In the proposed SGP,  $F_1(z)$  and  $F_2(z)$  are configured by the design formulae in (13) and (14) with  $m = 0$  and  $n_h = 0$ . By taking  $\lambda = 0.8$  in terms of the prediction performance and robustness indices given in section III.B, we have

$$F_1(z) = c \sum_{i=1}^{55} A^{i-1} b z^{-i} \Gamma(z), \quad F_2(z) = \frac{(z - 0.9749)(z + 0.9408)}{(z - 0.8)^2}$$

where  $A = \begin{bmatrix} 1.9905 & -0.9905 \\ 1 & 0 \end{bmatrix}$ ,  $b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ ,  $c = [0.3905 \quad -0.3505]$ ,

and  $\Gamma = \frac{0.009479(z-1)}{(z-0.8)^2}$ .

The 2DOF controllers are designed by using the formulae in (35) and (37) with  $n_d = 2$  and  $n_f = 2$ , in consideration of the slow process pole,  $z_p = 0.9905$ , obtaining

is also shown in Fig. 5 for comparison.

Disturbance rejection	Example 1				Example 2				
	Proposed	Ref.[11]	Ref.[17]	Ref.[18]	Proposed	Ref.[10]	Ref.[20]	Ref.[22]	
IAE	Nominal	17.77	31.26	33.24	19.44	3.41	4.47	5.69	4.42
	Perturbed	18.01	31.33	33.33	20.33	3.47	4.95	5.47	4.63
TV	Nominal	1.12	1.00	1.00	1.62	0.78	0.89	1.07	0.85
	Perturbed	1.30	1.23	1.12	1.72	1.33	0.93	1.46	1.43

$$C_f(z) = \frac{(1 - \lambda_f)^2(\beta_1 z + \beta_0)}{0.009479(z - 1)}$$

$$C_s(z) = \frac{(z - \lambda_s)^2(1 - \lambda_s)^2 z}{(1 - \lambda_f)^2(\beta_0 + \beta_1 z)(z - \lambda_s)^2}$$

where  $\beta_1 = (z_p - \lambda_f)^2 - (1 - \lambda_f)^2 / (z_p - 1)(1 - \lambda_f)^2$  and  $\beta_0 = 1 - \beta_1$ .

For illustration, a unity step change of the set-point is added at  $t = 0s$  and a step type load disturbance with a negative unity magnitude is added to the process input at  $t = 200s$ . By taking

TABLE I INDICES FOR DISTURBANCE REJECTION

(a)

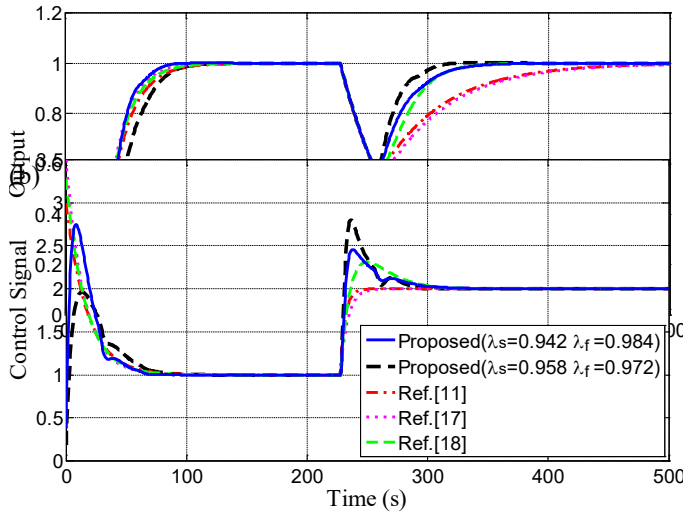


Fig. 5 Control results for Example 1

$\lambda_s = 0.942$  and  $\lambda_f = 0.984$  in the above controllers to obtain a similar rising speed of the set-point response and a similar disturbance response peak as those in [11, 17, 18] for comparison, the control results are shown in Fig. 5, and the IAE and TV indices for disturbance rejection listed in Table I.

It is seen that improved control performance is obtained by the proposed method. Note that faster (or slower) set-point tracking performance and smaller (or larger) disturbance response peak can be easily obtained by monotonically decreasing (or increasing)  $\lambda_s$  and  $\lambda_f$  in the proposed 2DOF controllers, respectively, but in exchange for inferior (or better) robustness against process uncertainties. An illustrative control result by increasing  $\lambda_s$  to 0.958 while decreasing  $\lambda_f$  to 0.972

Assume that the output measurement is blurred by a Gaussian white noise with zero mean and a variance of 0.002 causing the noise-to-signal ratio about 5.6%, Fig. 6 shows the control results by using the above methods, demonstrating good robustness of the proposed control scheme.

Then assume that the process time delay and time constant are 10% larger than the model. The corresponding IAE and TV indices are listed in Table I, indicating good robust stability by using the proposed method.

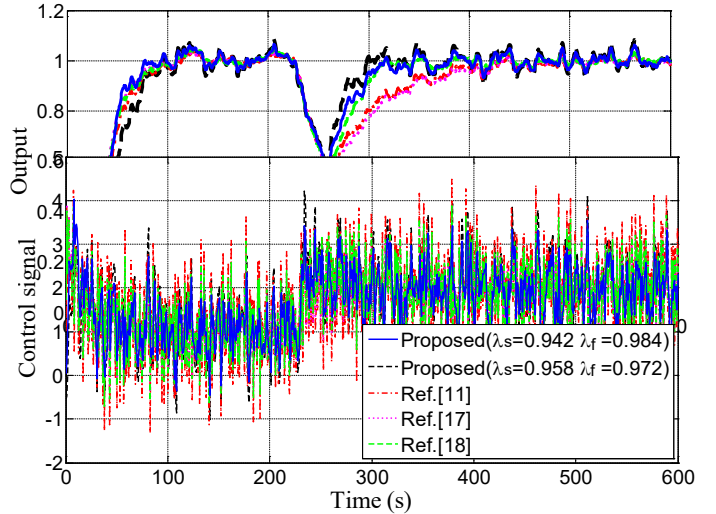


Fig. 6 Control results under measurement noise for Example 1

**Example 2.** Consider the unstable process with time delay studied in [10, 20, 22],

$$P(s) = \frac{2e^{-5s}}{(10s - 1)(2s + 1)}$$

With a sampling period of  $T_s = 0.1s$ , a discrete-time model of the process was obtained as

$$G(z) = \frac{0.00049342(z + 0.9868)}{(z - 1.0101)(z - 0.9512)} z^{-50}$$

In the proposed predictor,  $F_1(z)$  and  $F_2(z)$  are configured based on the design formulae in (13) and (14) with  $m = 1$  and  $n_h = 0$ . By choosing  $\lambda = 0.98$  in terms of a trade-off between

the prediction performance and robustness indices, we have

$$F_1(z) = c \sum_{i=1}^{50} A^{i-1} b z^{-i} \Gamma(z), \quad F_2(z) = \frac{1.7261(z-0.9952)(z-0.9521)}{(z-0.98)^2}$$

where  $c = [1 \quad -1.9600 \quad 0.9604]$ ,  $b = [1 \quad 0 \quad 0]^T$ ,

$$A = \begin{bmatrix} 2.9613 & -2.9221 & 0.9608 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \Gamma = \frac{0.0004934(z+0.9868)(z-1)}{(z-0.98)^2}$$

The 2DOF controllers are configured by using the formulae in (26), (28), (29) and (37) with  $n_d = 3$  and  $n_f = 2$ , obtaining

$$C_f(z) = \frac{(1-\lambda_r)^3(\beta_2 z^2 + \beta_1 z + \beta_0)}{0.00098033z(z-1)}$$

$$C_s(z) = \frac{(1-\lambda_s)^2 z(z-\lambda_r)^3}{(z-\lambda_s)^2 (1-\lambda_r)^3 (\beta_2 z^2 + \beta_1 z + \beta_0)}$$

where  $\beta_0 = -1949.36 + 351.42 \frac{(0.9512-\lambda_r)^3}{(1-\lambda_r)^3} + 1598.95 \frac{(1.0101-\lambda_r)^3}{(1-\lambda_r)^3}$ ,

$$\beta_1 = 4078.27 + 699.33 \frac{(0.9512-\lambda_r)^3}{(1-\lambda_r)^3} - 327.99 \frac{(1.0101-\lambda_r)^3}{(1-\lambda_r)^3}, \quad \beta_2 = 1 - \beta_1 - \beta_0.$$

For illustration, a unity step change is added to the system input at  $t = 0s$  and an inverse step type load disturbance is added to the process input at  $t = 80s$ . By taking  $\lambda_s = 0.98$  and  $\lambda_r = 0.95$  to obtain a similar rising speed of the set-point response as those in [10, 20, 22], the control results are shown in Fig. 7.

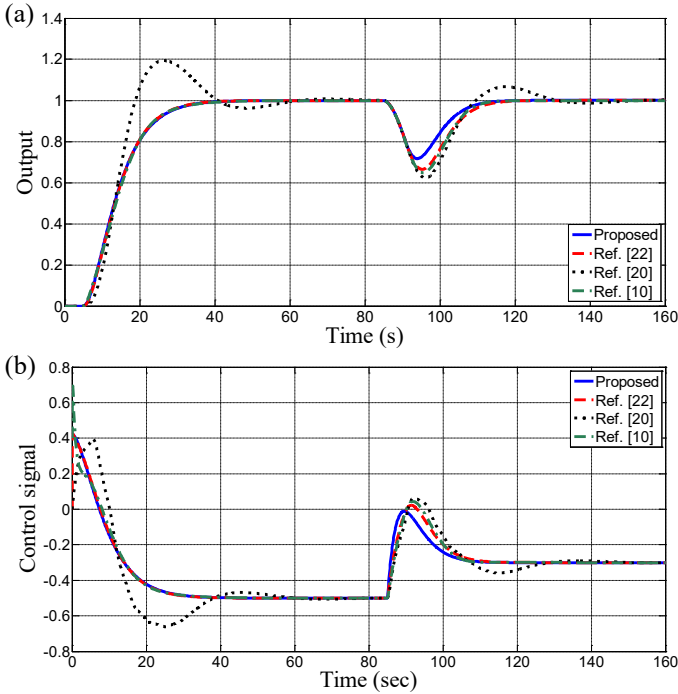


Fig. 7 Control results for Example 2

It is again seen that evidently improved control performance is obtained by the proposed method. Then a perturbation test assumed in [22] is performed, i.e., the process time delay and proportional gain are actually 5% larger while the stable pole is 5% smaller than the model. The corresponding IAE and TV indices are listed in Table I, once again demonstrating good robust stability by the proposed method.

## VI. EXPERIMENTAL RESULTS

Consider the temperature control system for a laboratory 4-liter jacketed reactor, as shown in Fig. 8, which is a typical integrating process with long input delay. The temperature control system consists of a thermostatic circulator filled with 9-liter ethylene glycol and distilled water in proportion of 2:3, an electric heater with a capacity of 2000(W) regulated via a solid state relay with pulse-width modulation, a thermocouple of Pt100, a programmable logic controller (made by Siemens), and a 64-bit data acquisition card (AT-MIO-64X, made by NI company) used for analog-to-digital conversions.

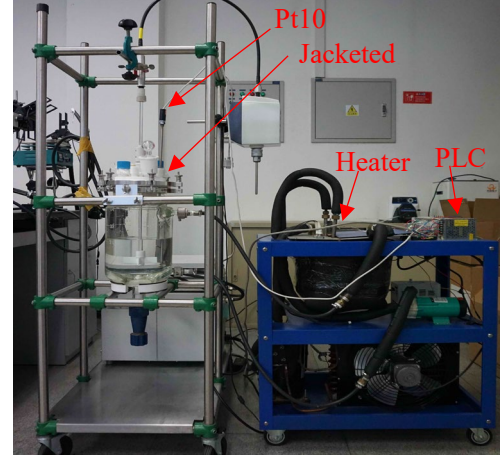


Fig. 8 Temperature control system for a jacketed reactor

The jacketed reactor is used for pharmaceutical crystallization such as the L-glutamic acid (LGA). For operating the crystallization process, it is required to heat up the aqueous solution in the reactor to the dissolving temperature of 45°C. The control task is therefore specified as quickly heating up the solution in the reactor from the room temperature (25°C) to 45°C, and then maintaining the operating temperature of 45°C against load disturbance (e.g. feeding raw LGA solute and solvent for continuous operation of the crystallization process).

An open-loop step test is performed by fully turning on the electric heater. The temperature response is shown in Fig. 9.

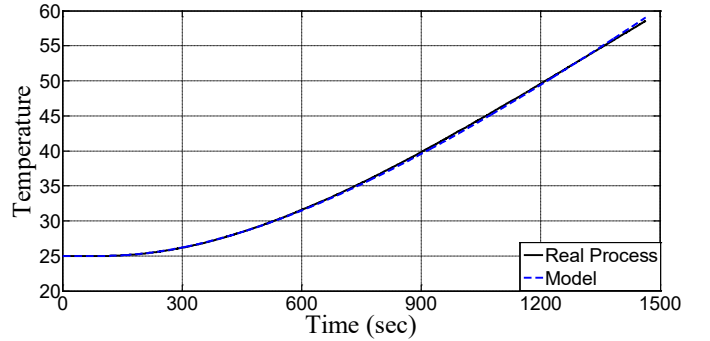


Fig. 9 Temperature response under a step test for identification

By using a step response identification method [25], a transfer function model of the temperature response is obtained,



$$G(s) = \frac{0.0004529}{s(760.40s + 1)} e^{-100.25s}$$

Due to the slow dynamics of the temperature response with long input delay, the sampling period is taken as  $T_s = 3s$  for implementation. A discrete-time model is therefore obtained as

$$G(z) = \frac{2.6765 \times 10^{-6} (z + 0.9989)}{(z - 1)(z - 0.9961)} z^{-34}$$

Using the proposed control scheme, the output predictor is designed with  $\lambda = 0.98$  in terms of the aforementioned indices. Taking into account the measurement noise in the Pt100 thermocouple, the design formulae in (11), (13) and (14) are used with  $m = 1$  and  $n_h = 2$ , obtaining

$$H(z) = \frac{(1 - \lambda)^2 z^2 (z - 1)}{(z - \lambda)^3} \quad F_1(z) = c \sum_{i=1}^{34} A^{i-1} b z^{-i} \Gamma(z),$$

$$F_2(z) = \frac{0.0028916 (z^2 - 1.987z + 0.9872)}{(z - 0.98)^4},$$

where  $c = 10^4 \times [0.25, -0.98, 1.4406, -0.9412, 0.2306]$ ,

$$\Gamma = \frac{1.071 \times 10^{-9} z^2 (z - 1)(z + 0.9987)}{(z - 0.98)^4},$$

$$A = \begin{bmatrix} 2.9961 & -2.9921 & 0.9961 & 0 & 0 \\ 1.0000 & 0 & 0 & 0 & 0 \\ 0 & 1.0000 & 0 & 0 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 \\ 0 & 0 & 0 & 1.0000 & 0 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.$$

The 2DOF controllers are designed by using the formulae in (26)-(28) and (37) with  $n_d = 4$  and  $n_r = 3$ , obtaining

$$C_r(z) = \frac{(1 - \lambda_r)^4 (\beta_0 + \beta_1 z + \beta_2 z^2)}{4.9557 \times 10^{-6} z (z - 1)(z - 0.9899)}$$

$$C_s(z) = \frac{z(z - \lambda_s)^4 (1 - \lambda_s)^3}{(1 - \lambda_r)^4 (\beta_0 + \beta_1 z + \beta_2 z^2)(z - \lambda_s)^3}$$

where  $\beta_1 = 4 / (1 - \lambda_r) - \beta_2$ ,  $\beta_0 = 1 - \beta_1 - \beta_2$ , and

$$\beta_2 = \frac{(0.9961 - \lambda_r)^4}{(0.9961 - 1)^2 (1 - \lambda_r)^3} - \frac{4}{(1 - 0.9961)(1 - \lambda_r)} - \frac{1}{(0.9961 - 1)^2}.$$

To comply with the input constraint of  $0 \leq u \leq 100$  for regulating the heating power ( $u = 100$  corresponds to the full power), the adjustable parameters are taken as  $\lambda_r = 0.9765$  and  $\lambda_s = 0.99$  for implementation.

A control test is performed to heat up the aqueous solution from the room temperature. After the solution temperature has risen up to  $45^\circ\text{C}$ , a load disturbance arising from feeding the raw material is added by pouring 200(ml) solvent of distilled water with the room temperature ( $25^\circ\text{C}$ ) into the reactor.

For comparison, the control method given in [18] is also used for implementation. In order to obtain a similar rising speed of the heating-up response with the proposed method, the controllers are taken as  $C = (760.4s + 1) / [0.4529t_0(7.604s + 1)]$ ,  $F_r = [(4t_0 + 100.25)s + 1] / (t_0s + 1)^3$ , and  $F_d = 1 / (t_r s + 1)^2$  with the tuning parameters chosen as  $t_0 = 380$  and  $t_r = 260$ , by applying the design formulae given therein. In addition, the recently developed PID design with a set-point filter for suppressing the overshoot [21] is performed by taking the maximum sensitivity function,  $M_s = 1.8$ , for controller tuning according to the guidelines given therein, which had already demonstrated its

superiority over previous PID tuning methods for time-delay processes, based on only the delayed output feedback.

The control results are shown in Fig. 10. It is seen that fast and smooth heating-up response without overshoot is obtained by the proposed method, and more than 20 minutes are saved for recovering the solution temperature to the operation zone of  $(45 \pm 0.1)^\circ\text{C}$  against the load disturbance provoked by feeding the solvent, compared to using the control method given in [18]. Note that the filtered PID control method [21] has spent a much longer time (about 50 minutes) to recover the solution temperature, without using a DTC for delay-free control design.

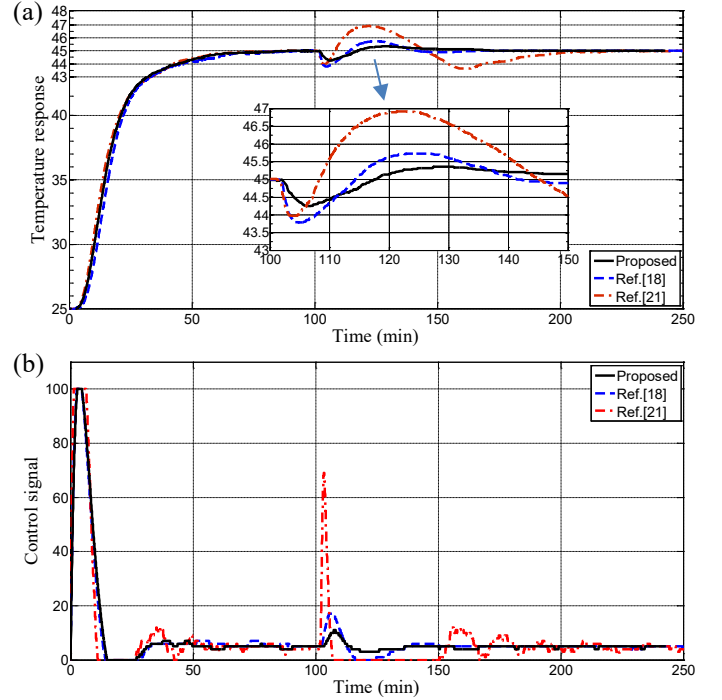


Fig. 10 Temperature responses by using different control methods

Note that if the controller parameters of [18] and [21] are tuned to yield further aggressive control action for expediting the load disturbance response, the solution temperature will not recover to the operating temperature, i.e., a steady-state temperature deviation will be turned out, because additional control signal of negative values (corresponding to cooling action) are thus required, which in fact cannot be implemented due to the control limit of  $0 \leq u \leq 100$ . Such control invalidity indicated by  $u = 0$  at certain moments is shown in Fig. 10(b).

## VII. CONCLUSIONS

For industrial processes with long time delay, a generalized DTC has been proposed in this paper. The proposed predictor has a simple structure, filtering the information available from the plant by means of two stable filters, no matter the plant poles/zeros position in the  $z$ -plane. In other words, the proposed predictor can be generally applied for any process of stable, integrating, or unstable type, no matter the plant is MP or NMP. No additional filtering loop as explored in the literature (e.g., [5, 6, 10, 14]) is required to eliminate the steady-state prediction error under static or asymptotically stable disturbance. Based

on the undelayed output prediction, a new 2DOF control scheme is analytically developed for optimizing the set-point tracking and disturbance rejection, respectively. Analytical controller design formulae are derived by proposing the desired transfer functions, while asymptotic tracking constraints are introduced to cancel the influence from the plant poles close to, on, or outside the unity circle in the  $z$ -plane. There is a single adjustable parameter in each controller, which can be monotonically tuned to meet a good trade-off between the control performance and robust stability. Illustrative examples from the literature, along with a practical application to the temperature control system of a laboratory reactor, have well demonstrated the effectiveness and advantage of the proposed predictor-based control scheme.

### VIII. ACKNOWLEDGMENT

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### REFERENCES

- [1] J. E. Normey-Rico, E.F. Camacho, *Control of Dead-Time Processes*. London, UK, Springer, 2007.
- [2] K. J. Åström, T. Hägglund, "Advanced PID Control", ISA-The Instrumentation, Systems, and Automation Society, 2006.
- [3] O. J. M. Smith, "Closer control of loops with dead time", *Chem. Eng. Prog.*, vol.53, no.5, pp.217-219, 1957.
- [4] M. Morari, E. Zafiriou, *Robust Process control*, Prentice Hall, New Jersey, 1989.
- [5] M. R. Mataušek, A. D. Micic, "On the modified Smith predictor for controlling a process with an integrator and long dead-time", *IEEE Trans. Autom. Control*, vol.44, no.8, pp.1603-1606, 1999.
- [6] S. Majhi, D. P. Atherton, "Obtaining controller parameters for a new Smith predictor using autotuning", *Automatica*, vol.36, no.11, pp.1651-1658, 2000.
- [7] A. Ingimundarson, T. Hägglund, "Robust tuning procedures of dead-time compensating controllers", *Control Eng. Pract.*, vol.9, no.11, pp.1195-1208, 2001.
- [8] P. García, P. Albertos, T. Hägglund, "Control of unstable non-minimum-phase delayed systems", *J. Process Control*, vol.16, no.10, pp.1099-1111, 2006.
- [9] J. E. Normey-Rico, E. F. Camacho, "Dead-time compensators: A survey", *Control Eng. Pract.*, vol.16, no.4, pp.407-428, 2008.
- [10] P. García, P. Albertos, "Robust tuning of a generalized predictor-based controller for integrating and unstable systems with long time-delay", *J. Process Control*, vol.23, no.8, pp.1205-1216, 2013.
- [11] K. Kirtania, M.A.A. S. Choudhury, "A novel dead time compensator for stable processes with long dead times", *J. Process Control*, vol.22, no.3, pp.612-625, 2012.
- [12] R. Sanz, P. García, Q. C. Zhong, et al. "Predictor-based control of a class of time-delay systems and its application to quadrotors," *IEEE Trans. Ind. Electron.* vol. 64, no. 1, pp. 459-469, 2017.
- [13] W. Michiels, S. I. Niculescu, "On the delay sensitivity of Smith predictors", *International Journal of Systems Science*, vol.34, no.8-9, 543-551, 2003.
- [14] L. Sun L, D. Li, Q. C. Zhong, et al. "Control of a class of industrial processes with time delay based on a modified uncertainty and disturbance estimator," *IEEE Trans. Ind. Electron.* Vol. 63, no. 11, pp. 7018-7028, 2016.
- [15] T. Liu, W. Zhang, D. Gu, "Analytical design of two-degree-of-freedom control scheme for open-loop unstable processes with time delay", *J. Process Control*, vol.15, no.5, pp.559-572, 2005.
- [16] A. S. Rao, M. Chidambaram, "Analytical design of modified Smith predictor in a two-degrees-of-freedom control scheme for second order unstable processes with time delay", *ISA Transaction*, vol.47, no.4, pp.407-419, 2008.
- [17] W. Zhang, J. M. Rieber, D. Gu, "Optimal dead-time compensator design for stable and integrating processes with time delay", *J. Process Control*, vol.18, no.5, pp.449-457, 2008.
- [18] J. E. Normey-Rico, E. F. Camacho, "Unified approach for robust dead-time compensator design", *J. Process Control*, vol.19, no.1, pp.38-47, 2009.
- [19] A. Visioli, Q. Zhong, "Control of integral processes with dead time", Springer, 2010.
- [20] M. R. Mataušek, A. I. Ribić, "Control of stable, integrating and unstable processes by the modified Smith Predictor", *J. Process Control*, vol.22, no.1, pp.338-343, 2012.
- [21] Q. B. Jin, Q. Liu, "Analytical IMC-PID design in terms of performance/robustness tradeoff for integrating processes: From 2-Dof to 1-Dof", *J. Process Control*, vol.24, no.3, pp.22-32, 2014.
- [22] Y. Chen, T. Liu, P. Albertos, P. García, "Analytical design of a generalized predictor-based control scheme for low-order integrating and unstable systems with long time delay", *IET Control Theory & Appl.*, vol.10, no.8, pp. 884-893, 2016.
- [23] W. Tan, C. Fu, "Linear active disturbance-rejection control: analysis and tuning via IMC," *IEEE Trans. Ind. Electron.* vol. 63, no. 4, pp. 2350-2359, 2016.
- [24] Chen, Wen-Hua, et al. "Disturbance-observer-based control and related methods—An overview," *IEEE Transactions on Industrial Electronics*, vol.63, no.2, 1083-1095, 2016.
- [25] T. Liu, F. Gao, *Industrial Process Identification and Control Design: Step-test and Relay-Experiment-Based Methods*. London, UK, Springer, 2012.

(The authors' vita will be provided herein if accepted for publication)