

Contents

Abbreviations

Notation

Resumen	i
---------	---

Abstract	iii
----------	-----

Resum	v
-------	---

Agradecimientos	vii
-----------------	-----

Dedicatoria	ix
-------------	----

1 Introduction and Objectives	1
--------------------------------------	----------

1.1 Introduction	1
1.1.1 Modeling Multi-Physical Systems	1
1.1.2 Incorporating Uncertainties	3
1.1.3 Power Systems Modeled as SDAEs	4
1.1.4 Lyapunov Exponents in SDAEs	4
1.2 Thesis Objectives	5
1.2.1 The Road Towards the Objectives' Achievement .	6
1.3 Outlines of the Thesis	7

2 Modeling Dynamics of Constrained Systems under Uncertainty	9
---	----------

2.1 Differential-Algebraic Equations	9
2.1.1 The Index of DAEs	11
2.1.1.1 Tractability Index	11
2.1.1.2 Differentiation Index	12
2.1.1.3 Strangeness Index	13
2.1.2 Some Classes of DAEs	14

Contents

2.2	Stochastic Differential Equations	15
2.3	Stochastic Differential-Algebraic Equations	18
2.4	Numerical Integration Methods	21
2.4.1	Euler-Maruyama Method	22
2.4.2	Milstein Method	22
2.4.3	Runge-Kutta Method	23
3	Random Dynamical Systems and Lyapunov Stability	25
3.1	Random Dynamical Systems	26
3.1.1	RDSs generated by SDEs	28
3.2	Stability of Dynamical Systems	30
3.2.1	Lyapunov Stability Theory	30
3.2.2	Lyapunov Stability Assessment	31
3.2.2.1	Lyapunov First Method and LEs	32
3.2.2.2	Lyapunov's Second Method	34
3.3	LEs of Ergodic RDSs	35
4	Numerical Methods for computing LEs	39
4.1	Discrete QR Method	40
4.2	Continuous QR Method	41
4.3	Computational Considerations	43
4.4	Numerical Examples	44
4.4.1	Example 1	44
4.4.2	Example 2	49
5	Application to Power Systems	55
5.1	General Structure of Power Systems	55
5.2	Power System Stability	57
5.2.1	Definition of Stability	57
5.2.2	Definition of Reliability	59
5.2.3	Definition of Security	60
5.2.4	Tools for Dynamic System Analysis	61
5.3	Applying LEs as DSA Method	61
5.4	Modeling Power Systems through SDAEs	63
5.4.1	Modeling Stochastic Perturbations	64
5.5	Study-Cases	65
5.5.1	Case 1: SMIB with stochastic load	66
5.5.2	Case 2: SMIB with regulator perturbed by noise .	68
6	Final Discussion	75
6.1	Conclusions	75

Contents

6.2	Future Developments	77
6.3	List of Publications	78
6.3.1	Journal Publications	78
6.3.2	Conference Publications	78
	Bibliography	79