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CANALES Y PUERTOS

# TRABAJO DE FIN DE MASTER

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Structural robustness assessment of the Pirelli Tower in  
Milan (Italy). Consequences of the local failure of central  
columns.

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*Presentado por*

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# Trabajo de Fin de Máster

**Título:**

Evaluación de la robustez estructural de la Torre Pirelli de Milán. Consecuencias del fallo local de las columnas centrales.

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Avaluació de la robustesa estructural de la Torre Pirelli de Milà. Conseqüències de la fallada local de les columnes centrals.

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**Idioma elaboración:**

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**Resumen (español):**

La Torre Pirelli de Milán es uno de los iconos europeos de la construcción en altura. Se trata de un edificio de 127 metros de altura, cuya estructura fue diseñada por los ingenieros Pier Luigi Nervi y Arturo Danusso. Tras su construcción, a finales de los años 50, el edificio sufrió una renovación en 1978 y un refuerzo estructural tras el impacto de un avión en 2002. El edificio permanece actualmente en servicio y es, sin lugar a dudas, uno de los edificios más reconocidos y emblemáticos de Milán.

Ante la situación actual de cambio climático, amenazas terroristas y envejecimiento de las estructuras existentes, se producen, cada vez más, eventos extremos como: explosiones de gas, impactos de vehículos, deslizamientos de laderas, tornados, riadas, ataques terroristas, etc. Estos eventos habitualmente dan lugar a fallos locales en los edificios que pueden llegar a propagarse como un efecto dominó, dando lugar a lo que se denomina colapso progresivo. Las normas actuales más avanzadas tienen en cuenta el diseño frente al colapso progresivo atendiendo al concepto de robustez. De acuerdo con este concepto, un edificio debe ser capaz de resistir un fallo local en algunos de sus elementos, sin llevar a un colapso total o desproporcionado.

En este Trabajo Final de Máster (TFM) se evalúa la robustez estructural de la Torre Pirelli, de acuerdo con las especificaciones de los Eurocódigos. En aquellos casos en que los Eurocódigos no presentan herramientas aplicables al caso de estudio, se consideran las especificaciones de las normas norteamericanas de la General Service Administration y del Department of Defense.

Dada la complejidad y magnitud del trabajo a realizar, este TFM se ha llevado a cabo de forma conjunta con otro estudiante. La principal particularidad de este TFM está en el tratamiento de los fallos locales de las columnas centrales del edificio.

El trabajo llevado a cabo ha consistido en:

1. Estudio e investigación sobre la estructura de la Torre Pirelli;
2. Análisis del estado del arte sobre colapso progresivo y robustez de edificios;
3. Simulación computacional detallada del edificio;
4. Análisis de las consecuencias de diversos escenarios de fallos locales;
5. Evaluación de la robustez estructural y conclusiones.

El trabajo llevado a cabo ha permitido evaluar, a través de complejos modelos computacionales, la robustez de un edificio que, en su momento, no había sido diseñado para resistir eventos extremos como los definidos en las normas actuales.



**Palabras clave (español):**

Edificación; Rascacielos; Robustez; Colapso Progresivo; Evaluación estructural

**Observaciones:**

Este trabajo lo han realizado conjuntamente los alumnos Geremia Brusafarro y Francesco Da Rif. Las partes en las que han trabajado juntos son:

1. Estudio e investigación inicial sobre la Torre Pirelli,
2. Modelo de cálculo inicial con SAP2000.

Cada alumno se ha responsabilizado de analizar la robustez estructural de la torre ante diferentes escenarios de fallo local. Mientras el Sr. Brusafarro se ha encargado de analizar las consecuencias del fallo local en las columnas centrales del edificio, el Sr. Da Rif ha hecho lo propio con el fallo local en los muros laterales.

**Tiempo de dedicación estimado:**

360 horas



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# Abstract

The structures are designed to resist to normal actions (wind, snow, etc.), but sometimes they are subjected also to extreme events (explosions or terrorist attacks, etc.), that could cause local damages and could lead to the progressive collapse of a large part or all of the structure. In the last 30 years the interest in the events that could cause progressive collapses has been growing, especially after some terrorist attacks that caused a lot of deaths.

The aim is now to obtain structure resistant enough to avoid excessive damages, and to maintain the operativity; this is fundamental for the structures with strategic importance for the country, whose collapse would cause enormous economic damage and, above all, numerous losses of human life.

The purpose of this work is to introduce the concept of structural robustness and describe the design recommendations of the codes; then a study of the structural behavior of an important Italian building will be carried out, using the software SAP2000.



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# INTRODUCTION

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As established by the design codes, the structures are designed and checked to support the loads acting on them. Currently, the designer evaluates ordinary loads (wind, snow, overload) and extraordinary actions (explosions, earthquakes). This procedure is carried out to obtain a set of load combinations that are used to design the structural elements, the aim is to limit the probability of failure under a predefined value and also optimize the costs of the structure.

The set of events that generally act on a structure are defined on a statistical basis, but there is a family of events, which are able to act on the works, that are not predictable. According to the formulation of Taleb's theory [43], events not statistically definable are identifiable with the term "Black Swans"; this definition was first used in the economic field and was subsequently extended to other areas, such as structural engineering.

To avoid the consequences of these "Black swan", the man has tried to imitate the nature, with the purpose to elaborate some techniques to increase the robustness of the structure, or the resistance against these extreme events. The modern building codes provide for the adoption of suitable design techniques, in order to achieve a robust structure and limit the consequences of unforeseen events, as the lack of application of these techniques could lead to progressive collapses.

Some of the most famous collapses of this type are illustrated below:

- Ronan Point Tower [31] (London, May 16, 1968):

During the sixties many identical structures were built, made of prefabricated elements; the Ronan Point Tower was a complex of apartments that was part of this category of buildings, and it was 22 stories high. On May 16, 1968, in the early hours of the day, there was an accident in apartment 90 on the 18th floor. The occupant of the apartment lit a match to light the stove, to prepare tea as every morning. However, due to a gas leak, the match caused an explosion that knocked out the tenant and blew up the apartment walls. In this structural system the floors were supported by the underlying load-bearing walls, gravity load transfer occurred only through them, so the explosion caused the collapse of the

structure, which resulted in four deaths and seventeen injuries. The collapse was attributed to the displacement of the walls due to the explosion which initiated a progressive collapse of the upper part and subsequently of the lower part, the partially collapsed structure is shown in Figure 1.1. After the accident, the tower was repaired and reinforced; years later, at the time of dismantling, it was found that the work on the tower was of poor quality. The collapse of the Ronan Point Tower was an example of how existing codes did not yet have provisions relating to structural robustness, and therefore led to changes in the codes of various countries such as the United States or the United Kingdom.



**Figure 1.1:** The Tower after the collapse of the angle. Source: Ronan Point Apartment Tower Collapse and its Effect on Building Codes [31].

- Alfred P. Murrah Federal Building [9] (Oklahoma, April 19, 1995):

On April 19, 1995, at approximately 9:02 a.m., a Ryder rental truck was used in a terrorist attack on the Federal Building. This truck contained more than 3,000 kg of ammonium nitrate fertilizer, plus nitromethane and diesel; it was detonated in the north side of the building, and due to the damage of the closest column to the

explosive, a progressive collapse of a large part of the structure occurred. The collapse occurred seconds after the bomb exploded, which destroyed a third of the building and damaged some nearby buildings (Figure 1.2). The death toll was disastrous, there were 168 deaths of which 19 were children, and over 800 people were injured; it is still the domestic terrorist attack on American soil with the greatest number of casualties and economic damage.



**Figure 1.2:** The north side of the Alfred Murrah Federal Building in Oklahoma City after the bombing on April 19, 1995. Source: <https://www.nytimes.com> [29].

- World Trade Center [11] (New York, September 11, 2001):

The World Trade Center was a complex of seven buildings, famous above all for the exceptional importance of the Twin Towers, whose structural typology was that of a tube within another tube [6]. On September 11, 2001, four airliners were hijacked by four Al-Qaeda terrorist squads, and two of them crashed against the buildings (Figure 1.3). The impact caused the rupture of several perimeter columns of the towers, and ignited several liters of jet fuel; due to the high temperatures reached there was a softening of the steel of the structural elements, and unfortunately the consequent reduction of resistance of them led to a progressive collapse of the structures. This was the most famous and disastrous attack on American soil, the economic damage to the complex of buildings forming the World Trade Center was enormous and there were 2763 victims of which 2192 civilians, 343 firefighters, 71 law enforcement officers, and all the people present in the hijacked planes.



**Figure 1.3:** The towers after the impact. Source: <https://www.thesun.co.uk> [45]

As can be seen from the cases illustrated, the consequences of these events are often catastrophic, in fact they can lead to the loss of many human lives, but also to high-level economic, environmental and social damages, which could have repercussions for years.

As said before, the ability of a structure to avoid or limit this type of collapse is called structural robustness, and today represents a further requirement to be considered in the framework of the basic characteristics of a structure, which becomes particularly important and stringent in the case of critical structures or strategic with regard to the Civil Protection.

In recent years, structural civil engineering has shown increasing interest in structural robustness: there have therefore been many experimental and numerical studies, and most of the technical standards have indicated design criteria and procedures to be implemented in order to strengthen the structures.

In this work, several analyzes will be carried out on one of the most important building in the history of Italy constructed during the sixties, the Pirelli building in Milan.

Chapter 2 describes the different types of actions that can affect the structures, in particular the extreme actions and their characteristics.

Chapter 3 illustrates the general theory of robustness, first describing the concept and definitions present in the literature, then providing information about the typologies

of progressive collapses and possible consequences. After that, an overview on the design approaches will be explained, followed by the description of the design method to gain robustness, and at the end the different methods to evaluate it.

Chapter 4 talks about the design method proposed by the Eurocode.

Chapter 5 presents the building that has been analysed from an historical point of view, describing it from the born to the restoration interventions and also giving some structural information. At the end of the chapter it is possible to read about the investigation process, thanks to which fundamental information was found for carrying out this work.

Chapter 6 provide a description of the building only from the structural point of view, talking about the structural system, the materials, and the loads.

Chapter 7 is about the creation of the Finite Element model, made using the software SAP2000. The process is described in detail for every part of the structure, allow possible future readers to replicate it for other cases.

Chapter 8 shows the analyzes carried out with SAP2000; static analyzes were initially made to verify the correctness of the model, after which both static and dynamic analyzes were made to study the robustness of the building.

At the end, the conclusions show a summary of the results obtained and the final considerations regarding the robustness of the building and the writing of this work.



# EXTREME EVENTS ON STRUCTURES

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As already said before, the structures are designed and checked to support the loads acting on them. Different types of loads have to be evaluated, there are the ordinary loads, for example the snow or the wind, and also some extraordinary actions as explosions, these are the so called “Black Swans” [43].

## 2.1 Black Swans

Before the discovery of Australia in 1606, it was believed that only white swans existed, as empirical observation had not provided contrary evidence. Following the discovery of the southern continent, there was the one of the black swans. The discovery of this animal species highlighted that scientific observation alone did not provide complete and irrefutable data, but incomplete and false. This observation was the incipit for the formulation of the Black Swans Theory developed by Nassim Nicholas Taleb [43], initially used in the economic field and subsequently in other fields of study.

The Black Swan Theory states that the Universe is governed by events with greater probability of occurrence, but also by unknown and unpredictable extreme events. The nature of Black Swans makes it impossible, therefore, to develop tools and theories capable of predicting them (both as single rare events and as their concatenation) and mitigate their effects; despite this, the world scientific community is gearing up in order to draw up methodologies that are concerned with the consequences of the effects of such events.

The most famous example of a Black Swan, sadly known to all, probably is the terrorist attack of September 11, 2001, in which the World Trade Center complex collapsed, despite being designed to withstand natural actions of considerable magnitude.

Alongside these man-made events there are the natural Black Swans, the catastrophic and unpredictable events that are typical of Nature, such as earthquakes, floods or other natural disasters, which are characterized by a significant social and economic impact and are manifest in particular ways.

In addition to black swans, Taleb [43] classifies the remaining events into two

categories according to the probability of ascending occurrence and the consequent knowledge of the same:

- Gray Swans: these are events that have a low probability of occurrence, but also limited consequences. They are foreseeable with adequate equipment. An example of a Gray Swan is the tsunami of March 11, 2011, which occurred off the Japanese coast. This earthquake caused a series of chain damage such as a tsunami and releases of radioactive materials resulting from the failures of some nuclear power plants.
- White Swans: they are predictable events in every aspect, with a well-defined cadence, of which there is a large number of cases in the literature. Examples of the White Swans are the ordinary actions as the loads of wind and snow.

## 2.2 Black Swans in civil engineering

One of the most used mathematical tools for studying and analyzing real world phenomena is the normal distribution. Underlying this, there is the Central Limit Theorem, which states that the arithmetic mean of a fairly large number of iterations of independent random variables, each with a well-defined mean and variance, assumes a distribution that is approximately normal [16]. Consequently, the best tool capable of describing real-world phenomena, and therefore also of analyzing them, appears to be the normal distribution. This turns out to be true in most cases, for which good predictions can be made; however, there are cases where the use of the normal distribution is not adequate and gives misleading results, for example for the Black Swans, for which three main characteristics can be identified [16]:

- (i) First of all, the main feature of this type of event is that there must have been no possibility of predicting it in the past; an event or situation with this characteristic is defined as "singular". Taking again as an example the attack on the Twin Towers of 2001, it can be defined as a singular event, as the hijacking and crash of airliners and the subsequent fuel fire can be considered a unique event among all those that they could happen during the life of a building.
- (ii) Then, the effects of this type of events have important consequences both from the economic point of view and from the social point of view; again considering the previous example, the consequences were the large number of victims and the collapse of the towers with the damage to the adjacent buildings.



(iii) Finally, after the occurrence of some events, by thinking about them, a solution could have been adopted to limit their consequences without too much effort. This is the third feature that transforms an extreme event into a Black Swan; therefore, terrorist attacks can be considered to belong to this category. Considering the attack on the Alfred Murrah Federal Building, it can be argued that the damage caused by the bomb is disproportionate to the cause. In fact, the breaking of a column caused the collapse of a large part of the building, this happened because the building lacked the ability to limit the propagation of damage, or to transmit the loads on that column to the ground through other mechanisms. In this case it would have been possible to limit the damage and the casualties if adequate structural arrangements had been used.

According to what has been stated about the characteristics and intrinsic properties of Black Swans, in the field of structural engineering, they are defined as phenomena such as [16]:

- Exceptional natural events: the use of the term "exceptional" serves to convey the idea of the quantities behind these phenomena, in this case the return period. These types of events are characterized by great return periods, and it is not possible with certainty to have statistics on events with a return period greater than about 200 years, due to the short period of observation. This makes it necessary to implement mathematical strategies to obtain the magnitude of such an event. The theory of extreme values is a branch of statistics particularly dedicated to the study of natural phenomena, it deals, as the name implies, with the extreme deviations from the median of probability distributions.
- Unexpected combinations of events: the analyzes carried out with current design methods consider a set of situations that can occur while the building is in operation. The situations taken into account are the most probable ones, but sometimes unexpected situations that are not considered during the project can occur, the result of which can be a Black Swan. Common actions acting on constructions (wind, snow, crowd, vehicle loading on bridges) are always considered in the design phase, but for exceptional actions specific situations are not considered a priori. However, the designer has the possibility to consider even heavier load combinations in favor of safety; but even then, a properly designed structure could be prone to Black Swans due to the uncertainty and unpredictability of the extreme combination of events.
- Anthropogenic actions: such as terrorist attacks on structures or crowds moving after an unexpected event. Their main feature is that they do not follow random

distributions, this and also the singularity of the event make this type of event impossible to predict. Therefore, since no statistics relating to the event are available, it is not possible to study it using traditional approaches, as already mentioned above. For example, terrorist attacks are not a random process, but man acts in order to maximize economic and social damage.

# STRUCTURAL ROBUSTNESS

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As seen in previous chapters, important structures must be able to limit the spread of damage when a load-bearing element such as a column at the base breaks. This property of structures is called robustness; the aim is to limit the consequences in the case of extreme events, thus avoiding a progressive collapse or a disproportionate collapse of the entire structure or part of it.

The study of the extreme events that can affect a structure began after World War II, when the behavior of many buildings hit by bombs was analyzed. An important step was taken after the Ronan Point Tower accident, since 1968 in fact some codes began to take into consideration the risk that could derive from some extreme events. Later, after the terrorist attacks on the Alfred P. Murrah Federal Building (1995) and the Twin Towers (2001), interest in studying this topic grew, leading to the introduction of new requirements and recommendations in the codes [7].

## 3.1 Some definitions from the literature

During the years of research and studies on the subject, numerous definitions of progressive collapse, disproportionate collapse and robustness have been developed; despite this, no agreement has yet been reached regarding formulations and techniques.

### 3.1.1 Disproportionate and progressive collapse

First, the concepts of progressive collapse and disproportionate collapse should be introduced. The two terms seem to have the same meaning, and are often misused as synonyms, so it is interesting to explain the difference. We can speak of progressive collapse when, starting from the breakage of a single element (such as a column) or a few of them, the entire structure or a large part of it collapses, due to the propagation of damage through the structural scheme of the building [7]. Starossek defines six different types of it [41], which will be discussed later.

**Table 3.1:** Different definition of the two collapses. Source: Research and practice on progressive collapse and robustness of building structures in the 21st century [7].

Source	Definition
Allen and Schriever	Progressive collapse [...] can be defined as the phenomenon in which local failure is followed by collapse of adjoining members which in turn is followed by further collapse and so on, so that widespread collapse occurs as a result of local failure.
Gross and McGuire	A progressive collapse is characterized by the loss of load-carrying capacity of a relatively small portion of a structure due to an abnormal load which, in turn, triggers a cascade of failure affecting a major portion of the structure.
GSA guidelines	Progressive collapse is a situation where local failure of a primary structural component leads to the collapse of adjoining members which, in turn, leads to additional collapse. Hence, the total damage is disproportionate to the original cause.
ASCE 7-05	Progressive collapse is defined as the spread of an initial local failure from element to element resulting, eventually, in the collapse of an entire structure or a disproportionately large part of it.
Ellingwood	A progressive collapse initiates as a result of local structural damage and develops, in a chain reaction mechanism, into a failure that is disproportionate to the initiating local damage.
Canisius et al.	Progressive collapse, where the initial failure of one or more components results in a series of subsequent failures of components not directly affected by the original action is a mode of failure that can give rise to disproportionate failure.
NISTIR 7396*	Progressive collapse – The spread of local damage, from an initiating event, from element to element resulting, eventually, in the collapse of an entire structure or a disproportionately large part of it; also known as disproportionate collapse.
Agarwal and England	Disproportionate collapse results from small damage or a minor action leading to the collapse of a relatively large part of the structure. [...] Progressive collapse is the spread of damage through a chain reaction, for example through neighbouring members or storey by storey. [...] Often progressive collapse is disproportionate but the converse may not be true.
Krauthammer	Progressive collapse is a failure sequence that relates local damage to large scale collapse in a structure.
Starossek and Haberland	Disproportionate collapse. A collapse that is characterized by a pronounced disproportion between a relatively minor event and the ensuing collapse of a major part or the whole of a structure. Progressive collapse. A collapse that commences with the failure of one or a few structural components and then progresses over successively affected other components.
Kokot and Solomos	Progressive collapse of a building can be regarded as the situation where local failure of a primary structural component leads to the collapse of adjoining members and to an overall damage which is disproportionate to the initial cause.
Parisi and Augenti	Progressive collapse [...] is a chain reaction mechanism resulting in a pronounced disproportion in size between a relatively minor triggering event and resulting collapse, that is, between the initial amount of directly damaged elements and the final amount of failed elements.

Instead, we can speak of disproportionate collapse when the final collapse is, as the name implies, disproportionate in size compared to the initial cause, thus involving a large part of the structure. In the Table 3.1 various definitions of the different collapses can be found.

In some cases, the two types of collapses coincide, for this reason it is sometimes believed that the two terms have the same meaning. However, as can be deduced from the definitions, when we speak of progressive collapse we refer to the propagation of damage through the structure, therefore to the response of the system against a certain type of event. Instead, when we speak of disproportionate collapse we refer to the comparison between the initial damage and the final collapse of the structure, without referring to the response of the system; unlike progressive collapse which can be qualitatively described, more measures are needed to identify it.

To facilitate the understanding of the meaning of the two terms and the difference between them, the following example may be useful: with the appropriate precautions, a progressive collapse can be proportionate to the initial damage if it is possible to contain the propagation of damage to a small part of the structure; similarly, a disproportionate collapse may not be a progressive collapse, it means that it can occur without the propagation of damage, for example when a structural element of an isostatic structure is damaged.

### 3.1.2 Robustness

Having introduced the concepts of progressive collapse and disproportionate collapse, it is therefore important to set resistance objectives against extreme events or abnormal loads, that are actions not normally considered a priori during the design phase; these objectives will depend on the economic and social importance of the work.

A quantitative approach to reduce the potential damage due to these two types of collapse is based on reducing the probability that they will occur, providing the structure with robustness. In this way, a new or an existing structure subject to an extreme event may be able to limit the consequences of it.

Even if no agreement has been reached regarding the definition of robustness, it is generally defined as the ability of a structure to avoid or limit the consequences of an event, so that the result is not disproportionate to the cause; the structure must therefore be able to transmit to the ground, through other mechanisms, the loads weighing on an initially intact structural element [7].

Some of the definitions available in the literature are listed in Table 3.2.

**Table 3.2:** Definitions of robustness. Source: Research and practice on progressive collapse and robustness of building structures in the 21st century [7].

<b>Source</b>	<b>Definition</b>
GSA guidelines	Robustness – Ability of a structure or structural components to resist damage without premature and/or brittle failure due to events like explosions, impacts, fire or consequences of human error, due to its vigorous strength and toughness.
EC1 – Part 1–7	Robustness: The ability of a structure to withstand events like fire, explosions, impact or the consequences of human error, without being damaged to an extent disproportionate to the original cause.
Bontempi et al.	The robustness of a structure, intended as its ability not to suffer disproportionate damages as a result of limited initial failure, is an intrinsic requirement, inherent to the structural system organization.
Agarwal and England	Robustness is [...] the ability of a structure to avoid disproportionate consequences in relation to the initial damage.
Biondini et al.	Structural robustness can be viewed as the ability of the system to suffer an amount of damage not disproportionate with respect to the causes of the damage itself.
Vrouwenvelder	The notion of robustness is that a structure should not be too sensitive to local damage, whatever the source of damage.
JCSS	The robustness of a system is defined as the ratio between the direct risks and the total risks (total risks is equal to the sum of direct and indirect risks), for a specified time frame and considering all relevant exposure events and all relevant damage states for the constituents of the system.
Starossek and Haberland	Robustness. Insensitivity of a structure to initial damage. A structure is robust if an initial damage does not lead to disproportionate collapse.
Fib Model Code 2010	Robustness is a specific aspect of structural safety that refers to the ability of a system subject to accidental or exceptional loadings (such as fire, explosions, impact or consequences of human errors) to sustain local damage to some structural components without experiencing a disproportionate degree of overall distress or collapse.
Brett and Lu	[...] ability of a structure in withstanding an abnormal event involving a localized failure with limited levels of consequences, or simply structural damages.

Adequate robustness can be achieved through redundancy, which is the ability to transmit to the ground through other mechanisms the loads initially borne by one or more elements that have been subsequently damaged. There are several ways to achieve the desired strength, for example:

- Through Alternative Load Paths (ALPs)
- Dividing the structure into isolated parts (segmentation)
- Or by designing some specific elements in such a way that they can withstand extreme events.

It must be remembered that when a structure is robust it does not mean that it is oversized, but that, in the case of extreme events, it is able to activate resistance mechanisms other than those relating to normal loads.

## 3.2 Typology of progressive collapse

Starossek identifies 6 types of progressive collapse [41]:

- (I) Pancake;
- (II) Zipper;
- (III) Domino;
- (IV) Section;
- (V) Instability;
- (VI) Mixed-type collapses.

The classification is based on how the damage propagates through the structural system, the initial action from which the collapse originates, and the final damage to the building.

### 3.2.1 Pancake-type collapse

The best sadly known example of this type of collapse is the one of Twin Towers, of the World Trade Center. Local damage was caused by the impact of the planes and the ignition of the fuel, which resulted in a decrease in the vertical bearing capacity over the entire cross section of the two towers. When the bearing capacity was no longer sufficient, the upper part of the two structures collapsed, starting to move downwards with increasing kinetic energy. The lower part of the two structures, which was still

intact, was not able to withstand the forces resulting from the impact of the upper part, the two structures therefore collapsed completely. As it can be seen, this type of collapse has the following characteristics:

- Initial damage to the vertical bearing elements, which can be of any type and have any cause.
- The separation of all or part of the structure, resulting in a fall as a vertical movement of a rigid body.
- Increasing in the drop rate due to the transformation of potential energy into kinetic energy.
- Impact of falling elements on the lower part of the structure, the energy of which depends on their size and height of fall; if it is sufficiently high, the affected elements will be damaged.
- Progression of the vertical collapse due to the rupture of other vertical elements, because of the forces deriving from the impact of the upper part of the structure.
- The main forces in the elements, the action triggering the collapse, and the direction of propagation are parallel, they are all vertical.

### **3.2.2 Zipper-type collapse**

An example of a progressive collapse of the zipper-type could be that of a cable-stayed bridge, in which there is a sudden failure of a cable. In this case there would be a redistribution of the actions, and if the overload of the adjacent cables is excessive they can break, causing a ripple effect that would eventually collapse the bridge. Another example of this could be that of a retaining wall in which there is a progressive breakage due to the damage of one or more anchors. Or, considering a continuous beam, the failure of a column could trigger the progressive collapse of the entire structural element. The characteristics of this type of collapse are the following:

- There is initially the failure of one or more elements, for any cause.
- The failure causes an equal and opposite force that acts in the point of failure.
- Consequent redistribution of the actions previously supported by the broken elements.



- The initial failure of the elements is sudden, therefore there is an impulsive increase in the load in the adjacent elements; a dynamic response of the structure follows.
- Due to the combination of static and dynamic effects, there could be an overload of the alternative load transfer paths, meaning that there is failure of elements close to those that initially failed.
- There is a progressive collapse of the structure in a direction approximately perpendicular to that of the principal forces in the elements, unlike the pancake-like collapse in which they were parallel.

### 3.2.3 Domino-type collapse

Like the game with the same name, in which an overturned block gives rise to a chain reaction that causes all the others to fall, the Domino-like collapse consists of the initial overturning of one element which in a similar way causes others to fall, thus generating a progressive collapse. It is typical of structures with a repetitive horizontal arrangement, sufficiently slender and not braced. The mechanism that characterizes this type of collapse is the following:

- Initial overturning of an element, the fall of which consists in a rigid rotation motion around a lower edge of it.
- Increase in the rate of fall due to the transformation of potential energy into kinetic energy.
- Deceleration due to the action of other discrete elements.
- The element exerts a horizontal force on another adjacent to it, the transmission line can be parallel or orthogonal to the direction of overturning.
- The horizontal force is both of static origin, because it depends on the inclination, and of dynamic origin, because it depends on the movement.
- Overturning of the element on which the horizontal force acts.
- Progressive collapse in the horizontal direction.

As in the Pancake-type, collapse can occur due to impact forces; and as for the Zipper-type, it is a parallel load transfer system and the forces exerted by the elements during the failure are orthogonal to its propagation. Instead, what distinguishes this type of collapse from the previous ones is the overturning of individual elements, and

the fact that the propagation action does not act in the direction of the main forces transmitted by the elements before the collapse, thus highlighting the weakness of the system towards forces different from the main ones. Furthermore, the collapse can also occur by traction due to the connecting elements. Lastly, another difference from the previous types is that the transmission action acts on discrete elements, this involves a more concentrated force, but it makes it easier to predict the behavior of the system.

### 3.2.4 Section-type collapse

This type of failure is not usually called progressive collapse (but rapid fracture), but Starossek considers it useful to include it among the various types in order to exploit similarities and analogies. An example of the Section-type collapse can be made by considering a beam subject to a bending moment or a bar subject to axial action. If a part of the section is cut, there is an internal redistribution of the forces in the remaining part of the section. This causes an increase in stresses, which can give rise to a failure along the entire section or breakage of further parts of it. Comparing this type of collapse with the previous ones, it resembles the Zipper-type collapse, it is possible to make a brief comparison to analyze its characteristics:

- Initially, part of the section breaks, instead of one or more structural elements.
- In both types there is a consequent redistribution of stresses.
- The initial failure is sudden, consequently the load increase is impulsive, and this causes dynamic effects.
- There could be an overload of the alternate path of load transfer (the remaining section in the first case, the adjacent structural elements in the second), which could therefore generate a total or partial collapse of them.

The difference with the Zipper-type collapse is that, while a system is structured and formed by discrete elements, a transverse section is amorphous and homogeneous; however the similarities are sufficient to allow the application to Zipper-type collapse of methods usually used for section failure. This analogy is therefore applicable for structures such as cable-stayed bridges, cable networks or membrane structures; while it is not significant as regards the study of buildings, as they are more prone to pancake-like collapses.

### 3.2.5 Instability-type collapse

Due to imperfections in the material, or transverse loads, some structural elements could undergo large deformations and be damaged. This phenomenon is called instability, and structures are always designed so that it doesn't happen.

Instability could occur for example in reticular structures or beams where elements in compression are stabilized by bracing elements. Or the instability of a plate could occur, in the case of the failure of its stiffening. In both cases, a small triggering cause, i.e. the breakage of an element, could cause a collapse of the whole structure or a large part of it, thus causing a progressive collapse. The disproportion between the triggering cause, which is small, and the final collapse, is typical of the Instability-type collapse.

The characteristics of this type of collapse are the following:

- Initial settlement of stiffening elements of other elements subject to compression.
- Instability of the compressed elements, resulting from the breakdown of the stiffener.
- Sudden failure of the destabilized elements.
- Progression of the collapse in the rest of the structure.

Collapse can spread in several ways. If the element in which the instability occurs is a primary element, the entire structure could collapse immediately. If, on the other hand, the element affected by the instability is small, it could subsequently cause the consecutive failure of other elements. Propagation of destabilization could also occur, in which some stabilizing elements yield due to the failure of other elements, this could happen for example when the load-bearing elements are at the same time stiffening elements. During the collapse of the elements compressed by instability there is the transformation of potential energy into deformation energy. Dynamic effects can be negligible or significant.

The propagation action can vary depending on the case:

- In the case of instability of a primary element there is no propagation, but a total collapse.
- Most of the time it is a destabilizing action, such as for pipelines.
- In a few cases it can also be a force.

In both cases, the compression causing instability is a static action already present before the collapse. This is a property of the type-Instability collapse, in fact if the propagation action is not destabilizing, the type of collapse is different even if some elements have yielded due to instability.

### 3.2.6 Mixed-type collapse

Some collapses of structures that have occurred in the past are not clearly classifiable in one of these categories, for which easily recognizable collapses are described. Sometimes the collapse is produced by the interaction of two or even more mechanisms, in these cases the collapse is said to be of a mixed type.

## 3.3 Consequences of failure

Consequences are defined as the results of an event, if they can be considered beneficial then the event is defined as desired, otherwise the event is defined as undesired; in this section only the latter will be considered, because when talking about structural robustness, the consequences of a collapse are disastrous. The "*Structural Robustness design for practicing engineers*" instructions [35] deal with the topic in detail, therefore a summary will be reported.

### 3.3.1 Consequence analysis

The consequences can be of various types, they depend on the structural system of the buildings, and a detailed list is given in table 3.3. In order to describe them, a procedure called "consequence analysis" is used. These can have various aspects, however in some cases they can be described by some parameters such as number of victims, structural damage, or damage resulting from a certain period of disruption. The starting point of the consequences analysis is to identify the characteristics of the structure of interest, such as:

- The structural system
- The intended use
- The planned activities
- The number of people affected.

The strategic importance of the building for the territory must also be taken into consideration.

Some consequences do not depend on structural behavior, such as smoke poisoning when a fire breaks out. On the other hand, when the structural response is important, those elements whose behavior is inadequate or insufficient are defined as "vulnerable". The failure of the vulnerable elements can cause a collapse of the remaining part of the

structure, if this happens the building is considered not very robust, for this reason the structural system is considered very important and may require advanced analyzes.

**Table 3.3:** Types of consequences of an undesirable event. Source: Structural Robustness design for practicing engineers [35].

<b>Consequence type</b>	<b>Consequence</b>
Human safety	Fatalities Injuries Damage of vital facilities Delayed long term effects Psychological
Economic/Property	Damage to the building/structure Damage to surrounding properties Damage to contents
Business Continuity	Loss of income Loss of customers Inability provide vital services and/or activities Costs of detours and delays Costs to the economy of a region
Environmental	Reversible environmental damage Irreversible environment damage Effect on wildlife
Social and Political	Loss of reputation Increase of public fears Loss of political support Enforcement of stringent new measures “Blight”/long-term evacuation

Above all, large and strategically important structures should be endowed with robustness. Therefore, a convenient way to decide how to design structures against accidental actions, may be to classify them according to the consequences of an extreme event. A qualitative categorization is provided by Eurocode EN 1991-1-7 [38], as will be seen in a following chapter.

### 3.3.2 System representation

As already specified above, the robustness of a building and therefore the consequences of an extreme event, are greatly influenced by the type of structural system. The damage of the individual elements is considered a direct consequence, and can cause human, monetary and environmental losses, or indirect consequences, some examples can be:

- A fire or explosion after an earthquake
- A fire after a bomb or gas pipe explodes
- A fire after a storm
- A deterioration of some elements, following the damage due to an extreme event.

It is very important to consider both types of consequences for the risk assessment.

The consequences can also be expressed in terms of economic losses associated with the failure of individual elements and the change in the system due to them, however, it could be an ethical problem for some people to economically quantify human lives.

### **3.3.3 Formal scenario approach**

In the risk assessments, necessary for the design of a structure, an approach based on the following steps can be used:

Step 1 → hazard modeling

Step 2 → direct damage assessment

Step 3 → subsequent evaluation of indirect consequences, and total consequences.

The consequences could be measured with a monetary unit, as mentioned above, or with the number of victims, when the economic damage is not significant.

## **3.4 Structural design approaches**

In this section will be presented the different design approaches historically used in civil engineering, up to today's approaches used against black swans [10].

### **3.4.1 Working Stress Design and Ultimate Strength Design**

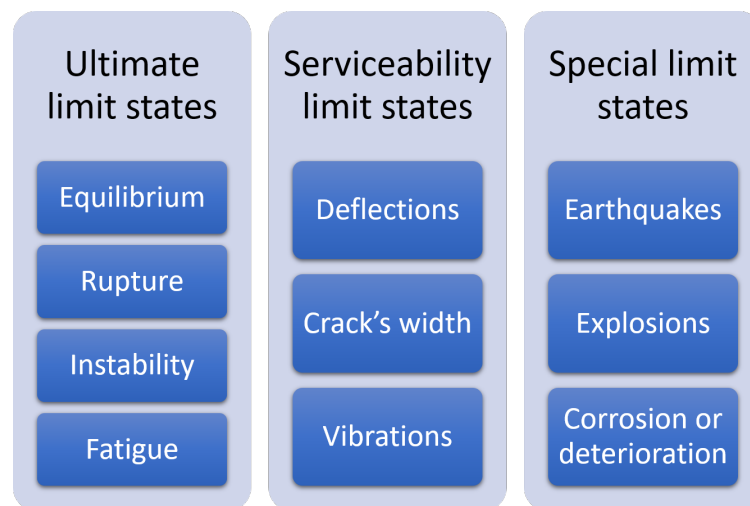
Working Stress Design was the traditional method for designing metal, concrete or wood structures. The hypotheses underlying the method are the elastic behavior of the material, and that it is possible to obtain an adequate degree of safety by reducing the tensions acting on the materials, and since they are kept very low the first hypothesis is justified. However, these hypotheses do not take into account some secondary effects such as creep and shrinkage, stress concentrations, or others; due to the redistribution of the stresses, large increases in tension can therefore occur in certain sections. This

approach is very conservative, and usually results in relatively large sections with excellent performance under service loads.

With increasing knowledge of the behavior of reinforced concrete, the Ultimate Strength Design was born as an alternative to Working Stress Design. This considers the stresses of the structure at the moment of impending collapse and the non-linearities in the constitutive bonds of the materials, and to reduce the loads it uses an appropriate load factor which can be different for the different loads in the combinations. The result is more slender sections than the Working Stress Design, however sometimes the service performance is not sufficient due to excessive deflections or cracks.

### 3.4.2 Limit State Design

Working Stress Design and Ultimate Strength Design are based only on service load conditions and ultimate load conditions, respectively. An evolution of the two methods is represented by the Limit State Design, which takes into account both situations: in fact, the term "Limit State" refers to a condition of imminent failure, both as regards the achievement of safety objectives and regarding the achievement of performance goals. The basic idea is therefore to identify all the different failure modes and ensure the structure adequate resistance against each of them. Figure 3.1 shows several examples for each limit state.



**Figure 3.1:** Different limit states. Source: Progression of Structural Design Approaches: Working Stress Design to Consequence - Based Engineering [10].

To take into account the uncertainties of the loads, the characteristic values are multiplied by a safety factor that increases the value, and to take into account the uncertainties on the resistance, the characteristic values are multiplied by resistance factors usually lower than unity.

This process is essentially based on force, however for loads very close to the capacity of the structure there could be significant inelastic deformations; hence the need to use a process based on displacements.

### **3.4.3 Performance - Based Design**

For small lateral loads, the structures are generally designed to remain in the elastic range, however for very intense loads this is uneconomical, so the structure is designed to allow plastic deformations. In these cases, in some elements the elastic demand exceeds the capacity, however the deformations of the structure are acceptable and there are no operational problems. For this reason the elements that reach the plasticization are designed taking as a reference the deformations and no longer the stresses. The Deformation - Based Design provides clearer results from a physical point of view, and the Performance - Based Design develops from it.

In seismic design, the performance objectives to be achieved are considered implicitly, so the need for a new strategy based on achieving certain performances rather than rules to follow that implicitly allow to reach the required level of functionality has begun to be noticed. The design criteria of the Performance - Based Design, as the term implies, are based on the required performance, and thus allow the structure to meet the demands both from the point of view of resistance and from the point of view of operation.

The basic idea of the approach is to associate the damage of a structure with measurable parameters, such as the displacements that can be considered a good indicator, even if sometimes the damage also depends on other factors. In recent years this approach has gained popularity among engineers, especially among those involved in seismic design, therefore methods have been studied to also take into account the characteristics of the ground motion and other uncertainties in quantifying the damage level. This was also made possible by the development of computer technology and computational capacity, which allow to do models and simulations of existing buildings.

From a practical point of view, the use of this approach can be summarized in three phases:

1. Analysis of a linear elastic model for the project with the loads indicated by the codes
2. Subsequently, a non-linear model of the structure is analyzed, to verify the real behavior taking into account also possible plasticizations
3. Finally, the last phase consists in the interpretation of the numbers associated



with a certain level of damage, in order to obtain useful information for making design decisions.

### 3.4.4 Consequence - Based Design

This approach was developed from Performance - Based Design, and is becoming popular for designing against unexpected and dangerous events. This is because the Black Swans are practically impossible to predict, and the Consequence - Based Design allows to minimize the consequences regardless of the cause of the damage to a structure.

Generally the various structural elements are designed with similar resistances according to the codes; instead the elements designed with this approach will have different reliability depending on the importance of the consequences due to their breaking, the goal is to avoid the collapse of the entire structure or a large part of it.

Consequence - Based Design enters the design phase with some coefficients called consequence factors: they have values between 0 and 1, and are applied to the strength of the structural elements. Their value depends on the contribution of the considered element to negative consequences, because the whole structure depends on the safety of the single elements and on the response of the system in case of local failure.

## 3.5 Design methods to gain robustness

There are still no universal rules to provide adequate robustness to a structure, however the Eurocode and other international standards propose four methods:

- The use of horizontal and vertical ties
- Alternative load paths (ALPs)
- Key elements design
- Risk-based methods.

The Eurocode allows the use of all the four methods, other regulations do not. Finally, there would be a fifth method, which consists in the compartmentalization of the structures to limit the collapses only to parts of them, but it is not mentioned in the regulations [7].

### 3.5.1 Horizontal and vertical ties

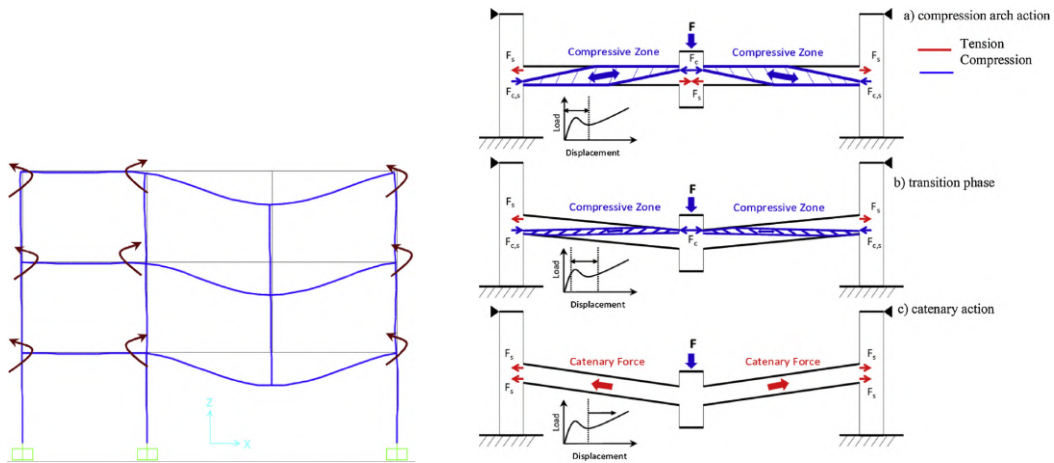
This method consists in providing minimum levels of tying, continuity and ductility to a structure, in such a way as to provide it with sufficient strength to avoid a progressive

collapse, for this reason it is defined as an indirect design method. The use of this method is recommended for structures where the risk of progressive collapse is low, and consists in applying horizontal and vertical ties whose minimum required tensile force is established by the standard. Some standards also indicate limitations to rotations in connections, since if too high the system is not able to stop the progressive collapse, as shown by some research. The contribution to robustness given by this method is not quantifiable, but it is accepted by all that the effect of its application is beneficial [7].

### **3.5.2 Alternative load paths**

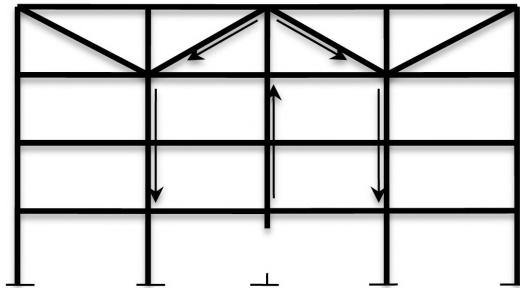
The use of ALPs consists in providing the structure with the ability to transmit loads through alternative paths, after the failure of a structural element such as a column. To carry out an analysis of the ALPs, numerous simplifications and assumptions can be made, such as considering or not the cause of the damage, considering the non-linearity of the materials, or considering the dynamic nature of the event. Since the assumptions that can be made are a lot, it is possible to have numerous results with different robustness; the refinement of the analyzes depends on the importance of the structure [7].

The load transmission mechanisms are: the Vierendeel action, the compressed arc action, and the catenary action [1]. After the damage of a vertical element, the Vierendeel action occurs when the columns or walls remain "hanging" on the horizontal elements that assume the typical double-curved configuration with a point of inflection in the middle, this particular shape provides the shear force necessary for the redistribution of the load. The manifestation of this resistant mechanism can occur when the structure is equipped with a high degree of hyperstaticity and flexural strength, such as for example tall buildings with vertical and horizontal structural elements with rigid nodes. The compressed arch action is the main mechanism resistant to small displacements of beams or slabs supported by columns or walls when these are removed. To be able to develop, this mechanism requires high lateral restraints and that the deflections are small, which means a small ratio between the length of the horizontal element and the height of the section. These conditions allow the formation of a compressed area which thins as vertical displacements increase. When these exceed about the thickness of the horizontal element, the arc action is converted into the catenary action, and the system resists due to the traction of the reinforcing bars. This mechanism develops with large displacements and with large rotations at the joints, in fact the greater the deflection of the horizontal element, the greater the vertical component of the traction force is, therefore ductile structural elements are required. The three main mechanisms are showed in Figure 3.2.



**Figure 3.2:** Principal mechanism of load redistribution, Vierendeel action on the left and transition from arch action to catenary action on the right. Source: A review on building progressive collapse, survey and discussion [1].

Then there is also another type of mechanism called Direct ALP (Figure 3.3), and as the name implies it consists in direct transmission through a system of trusses or by the membrane action of the walls [36].



**Figure 3.3:** Direct transmission of the load. Source: An Investigation into Tensile Membrane Action as a Means of Emergency Load Redistribution [36].

### 3.5.3 Key elements design

Then there is the method of designing key elements, especially recommended when the previous method is not applicable or is not sufficient. This method consists in identifying the structural elements whose failure could cause a progressive collapse, and designing them to withstand even accidental actions. Unlike the previous two methods, therefore, the use of key elements aims to avoid local failures.

Some codes propose 34 kPa as the design load of the key elements, which corresponds to the estimated pressure of the explosion that caused the collapse of a corner of the Ronan Point Tower; however, the project depends a lot on the extreme event considered, so in some cases this value may not be sufficient [7].

### 3.5.4 Risk-based methods

The last method proposed in the various regulations is to use risk-based approaches. Risk can be considered implicitly for example by classifying structures into categories, and various regulations recommend different methods to reduce it. Some regulations also provide limits regarding the risk of collapse, such as that on the maximum damage area of the Eurocode 1-1-7 [38].

However, some codes do not consider the risk adequately and this could represent a problem for the most important structures, in this case it is advisable to use methods characterized by maximum precision; the Eurocode, for example, for the categories belonging to the highest consequence classes proposes in Annex B a procedure to analyze the risk in a systematic way [7].

### 3.5.5 Compartmentalization

This method, also called isolation by segmentation, aims to avoid the progression of damage throughout the structure following a local failure. As previously mentioned this method is not mentioned in the codes, however it is very useful, especially for bridges [7].

## 3.6 Evaluation of robustness

To decide whether a structure is sufficiently robust or if additional measures are necessary to make it safer, methods are needed to take the decision. Starossek [42] proposes a qualitative classification, listing the general requirements to consider robust a structure, they are:

- Expressiveness: a structure possesses this characteristic when the measures are easily associated with an increase in robustness, and allow a clear distinction between the robust and non-robust structure.
- Objectivity: the structure must be equipped with measures that do not depend on the user.
- Simplicity: the measures taken to gain robustness should be simple.
- Calculability: the measures used in the structure must be able to be derived from the behavior of the structure, the parameters to be entered must be quantifiable; furthermore, the analyzes of the reinforced structure must be sufficiently precise and must not require a too high computational power.

- Generality: the structure must be equipped with general measures, applicable to any structure.

Some of these characteristics may be in conflict, so in order for a structure to be robust it is not necessary for it to have all of them.

After that, in the literature there are many other ways to quantify the robustness of a structure and they are divided into three types. The first are the threat-dependent reliability/risk-based measures, they take into account the event that caused the damage to the structure, and are based on different probability of damage or risk analysis. The second types are threat-dependent deterministic measures, which also consider the cause of failure. Finally, the third types are the threat-independent deterministic measures, these do not consider what caused the damage, but are generally based on a comparison between the intact structure and a predetermined damage scenario. A detailed description of the measures present in the literature for each of the three types can be found in "*Research and practice on progressive collapse and robustness of building structures in the 21st century*" [7].

## 3.7 Evaluations of failures with numerical models

The continuous improvement of informatic technology and the exponential increase in the computational power of modern computers, combined with the growth of the efficiency of the calculation algorithms, allow to perform very detailed analyzes even for complex structures and scenarios. The use of numerical models therefore proves to be very useful for the study of the progressive collapse of a structure, because they are capable of faithfully reproducing various extreme events (such as vehicle impacts and explosions) and their consequences. The precision of the numerical models has been validated on several occasions with a comparison with experimental models, and the main advantage of the former is the possibility of not using the latter, thus being able to save money and avoid the dangers associated with them. The main techniques used to study the progressive collapse of structures will be described below [7].

### 3.7.1 Finite Element Method (FEM)

This is the most used method for the study of progressive collapse and for numerical models in general. The accuracy of the analyzes can vary greatly depending from case to case, and depends on numerous factors.

The most developed type of models are the Macromodels, they are often used to reproduce entire structures and threat-dependent scenarios, and are composed of mono-

dimensional (beams) or two-dimensional (shells) elements. The computational cost of them is much lower than that of Micromodels, therefore used only to study parts of the structure (such as beam-column joints), and composed of solid elements.

Depending on the required precision, it is then possible to carry out static or dynamic analyzes, clearly the former require greater computing power and longer times. The same can be said for linear or nonlinear analyzes.

It is also possible to choose whether to create two-dimensional or three-dimensional models, however some regulations only allow the use of the latter.

Finally, implicit or explicit calculations can be performed, the latter providing more precise results.

This will be the method used in this final master's paper, as it was decided to deepen the knowledge of the SAP2000 calculation software, introduced during the last academic year, as this is widely used in the world of work.

### 3.7.2 Discrete Element Method (DEM)

This method is used for problems in which the materials are discontinuous, they are considered as a set of discrete elements whose interactions are determined on a statistical basis, thus making it possible to determine their behavior in the macroscopic scale. Because of this, the computational cost is high but the results are very precise, being able to represent well large deformations, nonlinearity, and discontinuities; for this reason their use is appropriate to study progressive collapse. To reduce the computational cost it is possible to use Discrete Elements together with Finite Elements.

### 3.7.3 Applied Element Method (AEM)

In this method, the modeling is similar to that of Finite Elements, each object is divided into smaller elements, the difference is in the way these are joined: each element is attached to the others at the point of contact with springs that simulate the behavior of the material. For this reason, the Applied Element Method is very useful for studying the behavior of structures when there are large displacements and rotations, or fractures. In particular, thanks to this method, it is possible to study the collapse process in each of its phases (Figure 3.4).



**Figure 3.4:** Phases of collapse. Source: own elaboration.

The computational cost is quite high, however with the recent development of technology these analyzes can be carried out in a not too long time.

#### **3.7.4 Cohesive Element Method (CEM)**

The CEM method has been used extensively to study fractures. In fact, the cracks are represented with nonlinear elements of zero thickness while the other elements are linear. In this way it is possible to represent the nonlinear behavior of each element using cohesive elements placed in the critical areas where damage is expected.

This method has been used very little for the study of progressive collapses, however the comparison with some experimental models of multi-storey reinforced concrete buildings showed better results than other nonlinear analyzes.





# ROBUSTNESS IN CURRENT REGULATIONS

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## 4.1 EN 1990 Eurocode: Basis of structural design

The Eurocode [37] defines the reliability of a structure or part of it as the ability to meet the requirements with which it was designed throughout its useful life. After that, it defines accidental actions as actions of short duration and of low probability of occurrence, but of great magnitude. As mentioned above, the consequences of them could be very serious if the proper measures are not taken.

In section "2.1 Basic requirements", the Eurocode [37] prescribes that the design of a structure must take into account possible explosions, impacts, or human errors; the facility must be able to limit the damage so that the consequences are not disproportionate to the cause.

The events that need to be assessed in the design phase must be decided for each individual project, and the consequences of them can be reduced or limited with one or more of the following measures:

- By reducing the risk to which the structure is exposed
- By choosing a geometry of the structure for which the risk of the actions considered is minimal
- Designing the structure so that it can withstand the failure of an element or damage to a small part
- By tying together different elements
- Structural systems that could collapse without warning are to be avoided.

The design requirements can be met with an adequate choice of materials, with an adequate design, or with specific controls on procedures.

According to the verification by the partial factor method, the structures must satisfy the requirements for the ultimate limit states (ULS) and the service limit states (SLS). There are several combinations of actions to consider during the design for both limit states, one of them is the combination for accidental design situations, which belongs to ULS. It can be expressed in the following way:

$$\sum_{j \geq 1} G_{j,k} + P + A_d + (\psi_{1,1} \text{ or } \psi_{2,1}) \cdot Q_{k,1} + \sum_{i > 1} \psi_{2,i} \cdot Q_{k,1} \quad (4.1)$$

Depending on the specific accidental situation, for each specific case it will be chosen whether to use  $\psi_{1,1} \cdot Q_{k,1}$  or  $\psi_{2,1} \cdot Q_{k,1}$ . This combination can also be used to consider a situation following an extreme event, thus considering  $A_d = 0$ .

## 4.2 EN 1991 Eurocode 1: Action on structures – Part 1-7: General actions – Accidental actions

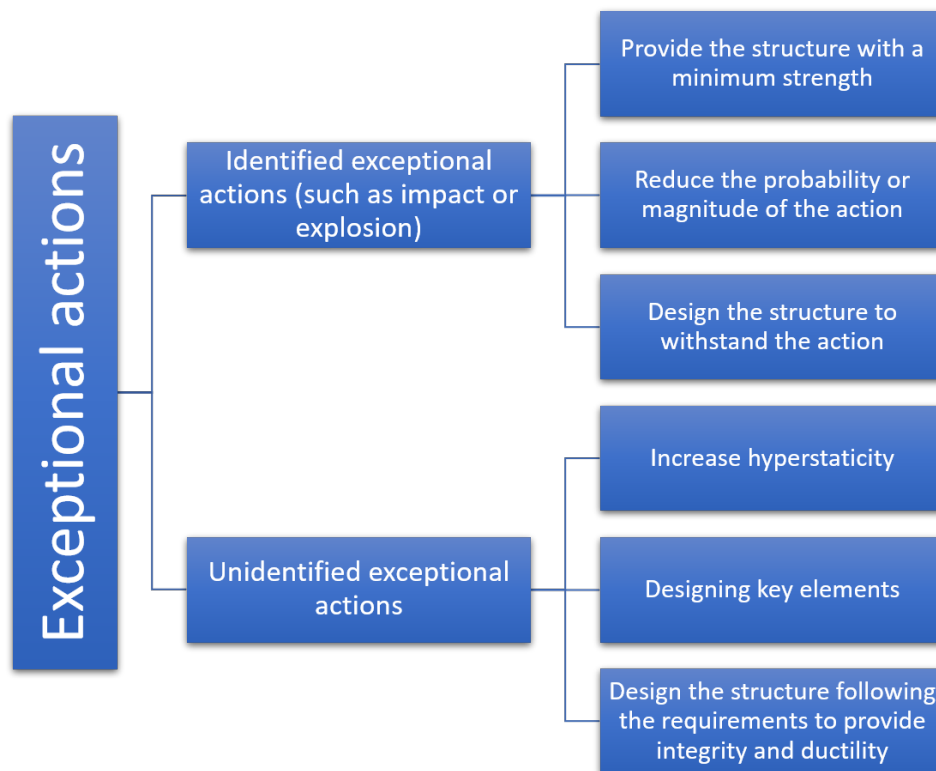
Recommendations regarding progressive collapse can be found in the “Part 1-7: General actions – Accidental actions” of the Eurocode 1 [38]; the code recommends designing structures taking into account the exceptional actions described in EN 1990, point 3.2 (2) P [37].

Exceptional actions to be taken into consideration must be agreed with the client and the competent authority. With them, it can also be decided to accept the fact that some extreme events can cause the total collapse of the structure, if the consequences are negligible and there is no loss of life. The exceptional actions to be taken into consideration can be of two types, identified actions and unidentified actions, the strategies to be adopted are different depending on the case, and are summarized in the Figure 4.1.

The Eurocode [38] also defines classes of consequences, on which the measures to be used will depend:

- CC1, low consequences, no specific measures are needed
- CC2, average consequences, exceptional actions must be taken into account at least in a simplified way, or prescriptions on construction details can be applied
- CC3, high consequences, specific analyzes and more refined methods are needed.

This classification will be seen in more detail later.



**Figure 4.1:** Strategies for the control of exceptional actions by type, from Eurocode 1-1-7 [38]. Source: own production

### 4.2.1 Identified exceptional actions

According to the code, the actions to be taken into account depend on various factors:

- The probability of occurrence
- The consequences they could cause
- The measures taken against them
- Public perception
- The level of risk; this cannot be obtained to be null, so acceptable values are given in the national appendix.

In case that the structure remains stable after an extreme event, or if it maintains the overall bearing capacity, some possible localized breakages can be considered acceptable. The strategies against the exceptional actions identified are the following, and can be applied individually or simultaneously:

- Reduce the likelihood of occurrence or the intensity of actions

- Protect the structure by reducing the effects of actions, using for example barriers
- Provide the structure with strength in different ways, such as designing key elements, respecting the requirements on ductility, or providing it with sufficient hyperstaticity.

#### **4.2.2 Strategies against unspecified causes**

The strategies used against the unspecified causes are based on limiting the extent of localized ruptures, to avoid the propagation of damage and the collapse of the structure. As in the previous case, there are several methods, which can be used individually or simultaneously:

- Design key elements with high resistance
- Design the structure to prevent the damage diffusion
- Provide the structure with adequate robustness.

#### **4.2.3 Design against unspecified causes**

Eurocode 1-1-7 provides in Appendix A [38] the rules for designing a structure with sufficient strength, to avoid the propagation of damage due to an unspecified cause, which could result in progressive collapse. This strategy is based on consequence classes, therefore the Eurocode [38] provides a more precise classification, as can be seen in Table 4.1. As mentioned previously, the measures to be taken during the design phases depend on the class accordingly, and they are the following:

- For Class 1 buildings: if the building has been designed following the rules present in the standards from EN 1990 to EN 1999, no specific measures against extreme events are necessary.
- For Class 2a buildings: the use of effective horizontal ties for framed structures or effective anchoring of the floors in load-bearing wall structures is recommended.
- For Class 2b buildings: as for the previous class, it is recommended to use horizontal tie rods or effective anchors depending on the type of structure, with the addition of vertical tie rods; furthermore, it is necessary to verify that the breaking of a structural element does not compromise the stability of the structure or cause excessive damage, to prevent that "key elements" can be used.
- For Class 3 buildings: Eurocode recommends a more detailed assessment of the structure through a risk analysis.

**Table 4.1:** Categorization of consequences classes. Source: Eurocode 1-1-7, Table A.1 [38].

Consequence class	Example of categorization of building type and occupancy
1	Single occupancy houses not exceeding 4 storeys. Agricultural buildings. Buildings into which people rarely go, provided no part of the building is closer to another building, or area where people do go, than a distance of 3/2 times the building height.
2a	5 storey single occupancy houses. Hotels not exceeding 4 storeys. Flats, apartments and other residential buildings not exceeding 4 storeys. Offices not exceeding 4 storeys. Industrial buildings not exceeding 3 storeys. Retailing premises not exceeding 3 storeys of less than 1000 $m^2$ floor area in each storey. Single storey educational buildings. All buildings not exceeding two storeys to which the public are admitted and which contain floor areas not exceeding 2000 $m^2$ at each storey.
2b	Hotels, flats, apartments and other residential buildings greater than 4 storeys but not exceeding 15 storeys. Educational buildings greater than single storey but not exceeding 15 storeys. Retailing premises greater than 3 storeys but not exceeding 15 storeys. Hospitals not exceeding 3 storeys. Offices greater than 4 storeys but not exceeding 15 storeys. All buildings to which the public are admitted and which contain floor areas exceeding 2000 $m^2$ but not exceeding 5000 $m^2$ at each storey. Car parking not exceeding 6 storeys.
3	All buildings defined above as Class 2 Lower and Upper Consequences Class that exceed the limits on area and number of storeys. All buildings to which members of the public are admitted in significant numbers. Stadia accommodating more than 5000 spectators. Buildings containing hazardous substances or processes.

#### 4.2.4 Measures to gain robustness

The damage diffusion limit following the breakage of an element may vary according to the structure, however Eurocode [38] recommends the value corresponding to the minimum between 15% of the floor and 100  $m^2$ ; for buildings belonging to classes of greater consequence it is necessary to take measures to ensure that this limit is respected.

For framed structures in class 2a or higher, the use of effective horizontal ties is recommended, the purpose of which is to connect the vertical elements to the building structure. These ties must be placed around the perimeter of each floor and also inside, in two orthogonal directions. The tie rods can be made up of profiles, reinforcements, nets, steel sheets, or a combination of them; the internal and perimeter tie rods are

respectively designed to withstand the following design tractions:

$$T_i = \max(0, 8 \cdot (g_k + \psi q_k) \cdot sL; 75 \text{ kN}) \quad (4.2)$$

$$T_p = \max(0, 4 \cdot (g_k + \psi q_k) \cdot sL; 75 \text{ kN}) \quad (4.3)$$

where

- $g_k$  and  $q_k$  are the characteristic loads
- $s$  and  $L$  are respectively the center distance and the length of the ties
- $\psi$  is the combination coefficient used in 4.1.

Structures with load-bearing walls must have a cellular shape in plan and an effective anchoring of the floor to the walls. For class 2b load-bearing walls it is also necessary to have internal and perimeter horizontal tie rods, designed using the following tensile forces:

$$T_i = \max\left(F_t; \frac{F_t \cdot (g_k + \psi q_k) \cdot z}{7,5 \cdot 5}\right) \quad (4.4)$$

$$T_p = F_t \quad (4.5)$$

where:

- $F_t = \min(60 \text{ kN/m}; 20 + 4n_s \text{ kN/m})$
- $n_s$  is the number of floor
- $z$  is the minimum value between five times the floor height and the bigger distance in the direction of the horizontal ties between vertical elements.

For class 2b structures it is also necessary to use vertical ties, to chain the vertical elements along their entire length. For framed buildings, the vertical elements must therefore be designed to withstand an exceptional tensile design action equal to the greatest vertical reaction applied to a structural element, this action should not be considered acting concurrently with permanent and variable loads. for buildings with load-bearing walls, on the other hand, the Eurocode [38] prescribes conditions for considering the anchoring effective.

Furthermore, for some structures, in order to limit the damage caused by the breakage of an element, it will be necessary to use "key elements". These elements and their connections must be designed to withstand an exceptional design load  $A_d$ , concentrated or distributed, using the combination of actions 4.1; the value recommended by the Eurocode [38] is  $A_d = 34 \text{ kN/m}^2$ .

## 4.2.5 Risk assessment

The risk analysis consists in the identification of the dangers, in the evaluation of the consequences, and finally in the application of measures so that the risk is reduced; all the important things for the evaluation such as the context, the objectives, the technical and environmental conditions, or even the hypotheses and simplifications made, must be described in detail.

### 4.2.5.1 Qualitative analysis

In the qualitative part, first of all it is necessary to identify all the dangers, this is a very important phase and therefore the engineer has at his disposal numerous techniques to carry out a detailed job. The possible dangers to a structure can arise from:

- Normal loads too high
- Resistances too low
- Environmental conditions other than those used for the project
- Exceptional actions identified
- Other unidentified causes.

To find the risk scenarios it is necessary to know well the structure, the project and the materials and actions that act on it, its weakest points, and also its main use in order to identify the consequences of certain extreme events and define acceptable risks.

### 4.2.5.2 Quantitative analysis

When practicable, risk analysis also has a quantitative part. This part consists in identifying the dangers for the structure, estimating the probabilities of occurrence and the potential consequences; the probabilities could be different from the actual frequencies of occurrence, because they are also based on judgment. Through this analysis, the risk can be expressed as the average probability of occurrence of certain consequences, which must be lower than limit values. The information to consider during the quantitative risk analysis are following:

- The availability of data and their accuracy
- The potential consequences of extreme events
- Previous analyzes

- The objectives and decisions to be made.

At the end of the analysis it is advisable to reconsider the hypotheses and protection measures adopted.

#### **4.2.5.3 Mitigation measures**

After identifying the risk, if this is deemed unacceptable, it is necessary to mitigate it with appropriate measures. At the base of these choices there is the "As Low As Reasonably Practicable" (ALARP) principle, which can be summarized as follows:

1. First, an acceptable risk range is identified, between a lower limit and an upper limit
2. If the risk is lower than the lower limit, no measures are necessary
3. If the risk is higher than the upper limit, it is not tolerable and measures must be taken
4. If the risk is within the tolerable zone, an economically favorable alternative is sought.

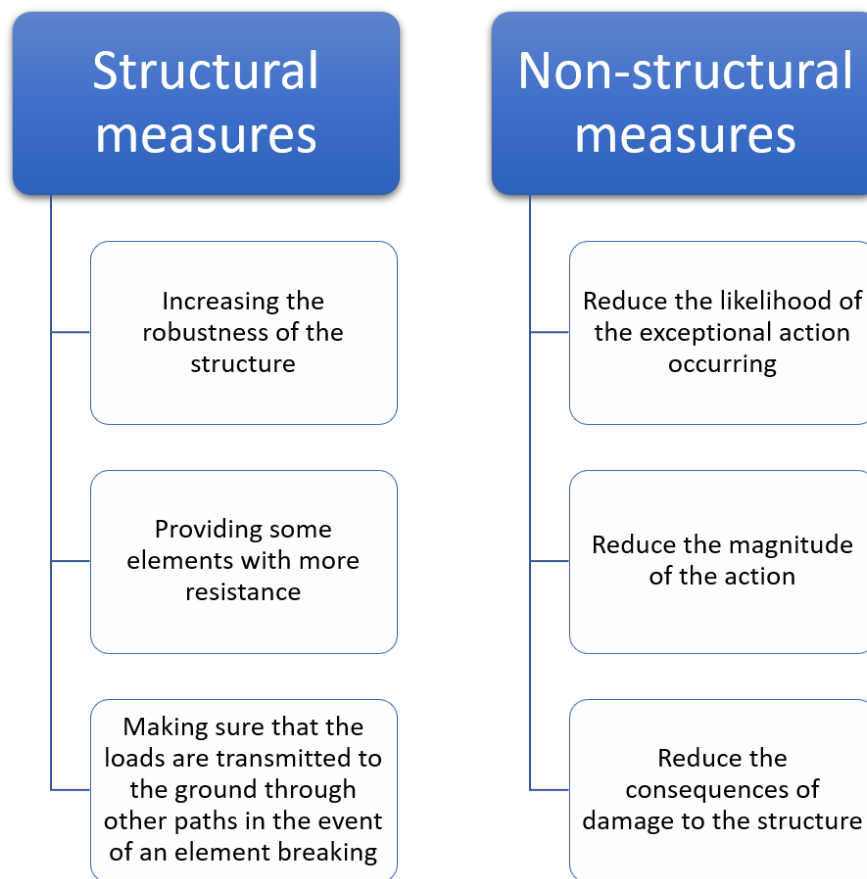
To determine when the risk can be considered acceptable, regulations can be used as a reference, but also experience; for qualitative analysis the consequences of an extreme event and the final objectives can be considered, because a good compromise must be found between risk mitigation and cost minimization. When the risk is deemed unacceptable, one or more of the following measures can be implemented:

- Avoid the risk, for example by changing the project
- Reduce the risk, for example by modifying the design and providing the structure with adequate protective measures
- Risk control, for example with monitoring
- Reduce the risk by providing the structure with adequate strength, resistance and hyperstaticity
- Allowing controlled collapse, so as to reduce the number of potential victims.

#### **4.2.5.4 Application to civil engineering**

In civil engineering the measures that can be used to reduce unacceptable risks can be broadly divided into two types, a scheme of them is shown in figure 4.2.





**Figure 4.2:** Typologies of mitigation measures, from Eurocode 1-1-7 [38]. Source: own production.

#### 4.2.5.5 Final recommendations

Risk assessment is very important for the design of a structure, as if done hastily the consequences could be disastrous; therefore, the Eurocode [38] recommends that each phase be carried out with meticulousness and precision. The consequences and their probabilities deriving from the qualitative and quantitative analyzes must be communicated to all those involved. In addition, the data used for the analysis and their sources should be listed, as well as the assumptions made. And finally, the conclusions of the analysis and the recommended measures to reduce the resulting risk must be specified.



# THE CASE OF STUDY: THE PIRELLI BUILDING

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After introducing the topic of structural robustness, the aim of this thesis work is to verify the robustness of a building by using advanced computational modelling. The Pirelli skyscraper was chosen as a case study, as it is one of the most important tall buildings not only in Italy but also in Europe.

The Pirelli skyscraper, commonly called “Pirellone” (that means “Big Pirelli”) by the Italian people, is the headquarter of the Lombardy Regional Council. It is located on the southwest corner of Piazza Duca d’Aosta, next to the Milano Centrale railway station (Figure 5.1) [20].



**Figure 5.1:** Piazza Duca d’Aosta in Milano, with the Pirelli building, and the railway station on the right. Source: <https://www.ordinearchitetti.mi.it> [33].

## 5.1 The history of the building

### 5.1.1 The birth

In 1943, due to the bombing of Milan, many offices of the Pirelli company in front of the Central Station were destroyed. This area was important for the company because in 1872 the engineer Giovanni Battista Pirelli opened the first plant [27].

In 1952 the architect Gio Ponti together with the engineer Giuseppe Valtolina developed a scheme of the skyscraper, which the Pirelli executives liked, and then decided in 1953 to present the project to the municipality to obtain the necessary modification to the Town Plan. A few months later, the two were officially assigned to design the building [27].

As it can be seen, the engineers Pier Luigi Nervi and Arturo Danusso are not yet working in the project, they began to participate in 1954. Nervi's contribution can be seen in the structural system of the building; while that of Danusso consists in the tests carried out through the use of models at the "Istituto Sperimentale Modelli e Strutture" (ISMES) of Bergamo, which he strongly desired [27].

The initial scheme envisaged by Ponti had a more "box-like" shape, a structural system more consistent with expectations was achieved after some changes made with the study of Nervi and Danusso, whose passages are not known for the inexistence of drawings [27]:

- Elimination of reinforced concrete perimeter pillars, to minimize vertical load-bearing elements
- Elimination of the corner of the triangular tips, and lengthening of them to show that they were separated
- Detachment of the cover so that it looked like a halo.

In February 1955 the first phases of analysis and design were completed, and a very detailed experimental study began on a 1:15 scale model. The construction of the model took place between 1 March and 15 May of the same year, and the concrete was left to mature for the following two months, also to allow the assembly of the loading and measuring equipment. Both static and dynamic tests were carried out to evaluate the effects of oscillations due to potential earthquakes and wind. Danusso also speaks of an experimentation of a minor model in the wind tunnel at the Politecnico di Milano, unfortunately there are no traces or publications of this test. Also a 1:5 floor slab element was built, which was then tested between September and October of the same year [27].

In 1956 further tests were made, and then the model was brought to break by increasing its own weight and wind action; the result was that the structure showed unexpected reserves of resistance [27].

Then, after the experimental tests were completed, construction work began in 1956 and ended in 1960, and the skyscraper was inaugurated [27].

### 5.1.2 The restoration in 1978

The skyscraper housed the offices of Pirelli until 1977, later in 1978 it was purchased by the Lombardy region, which currently owns it, thus becoming the main office of the regional council [30]. The negotiations were led by Leopoldo Pirelli and Cesare Golfari, the President of the Region in that period. To adapt the building for the new function, various renovations were carried out:

- The transformation of the data processing center into the new council chamber
- The restructuring of the representative entrance, the presidency and the vice presidency
- The renovation of the offices and services as well as a general adaptation of the systems under the supervision of the architect Bob Noorda.

### 5.1.3 The plane crash

The day April 18, 2002, is sadly known in the history of the building because, at 5:47 pm, a small Commander 112 Tc single-engine tourist plane hit the facade of the building, between the 26th and 27th floors. The accident was probably due to a distraction of the pilot, the planes of the time had automatic pilot but did not control the altitude; the plane had to land nearby, and as the pilot had to solve problems with the landing gear not opening, he did not notice that the plane was no longer at 1000 feet but at 700. The pilot attempted a turn maneuver at the last moment but it was not enough to avoid the accident [22]. The plane penetrated into the building and after the impact, the two tanks positioned on the wings exploded, and the engine detached from the fuselage and then came out from the opposite facade of the building, in via Fabio Filzi [4].

In addition to the pilot of the aircraft Luigi Marco Fasulo, two regional employees died in the impact: the lawyers Annamaria Rapetti and Alessandra Santonocito; there were also 60 wounded [30]. Furthermore, the explosion caused damage especially to the two floors, and minor damage to the rest of the building, however there was no collapse of the supporting structure, thanks to the "Nervian" conception of the work.

For the restoration, a working group was set up consisting of historians, architects, engineers and experts in specific subjects, to decide the intervention criteria to be adopted: a conservative restoration was opted for, taking up the ideas of Giò Ponti. The restoration began in the spring of 2003, the building was inaugurated and reopened on April 18, 2004, and finally on May 31, 2005, once the internal works were completed, the building was once again occupied by the regional offices, in particular by the Regional Council [30].

#### **5.1.4 Some curiosities**

The building holds several records [20]:

- It was the tallest building in the European Union from 1960 to 1966, the year of construction of the Tour du Midi in Brussels.
- It was the tallest building in Italy from 1960 to 1995, when the Telecom Italia Tower was built.
- It was the tallest building in Milan from 1960 until 2009, when it was then surpassed by the Lombardia palace.

Then there is another curiosity, this time not concerning the structural aspect. Since 2017, a pair of peregrine falcons have been nesting on top of the Pirelli Skyscraper. The birds of prey were discovered during some works on the building, it was therefore decided to build an artificial nest for them to host the hatching, and a webcam was installed that observes the life of the hawks 24 hours a day. The hawks were called Giò and Giulia, in homage to Giò Ponti and his wife Giulia Vimercati, and it has become customary for many citizens to wait for the return of the birds to the nest every year. Also in 2020 the hawks returned to the Pirellone, and from the three eggs laid in the nest between 9 and 10 April three falcons were born, two males and one female, whose names chosen by the readers of the facebook page are Luna, Gino and Guido [18].

## **5.2 Structural system of tall buildings**

An aspect that distinguishes tall buildings is represented by the loads to which they are subject [6]. For normal buildings the main loads are represented by their own weight and almost always by the overload of use, both are vertical actions; on the other hand, horizontal actions can also be important for this type of building, they are:

- The wind: which consists of pressures perpendicular to the exposed surface

- The earthquake: which consists of movements of the ground, that generate inertia forces in the elements with greater mass.

What these two types of actions have in common is that they are both random and dynamic, they depend on the location of the structure, and on its height.

When horizontal actions play an important role in the design, it is important for tall buildings to limit the maximum deformations, both as regards the total displacement measured at the top of the building, and as regards the relative displacements between one floor and the next; moreover it is necessary to limit the accelerations in the upper floors for reasons of comfort of the users [6].

To achieve this, there are several possible strategies: the value of the actions can be reduced with some measures such as seismic isolators at the base or an aerodynamic shape, or stiffness can be provided to the building by choosing the structural system appropriately, and it is precisely this argument that will be presented in this paragraph.

A tall building subject to wind and earthquake behaves like a cantilever embedded in the ground, therefore subject to bending and shear actions. There are several structural systems that can be used to design a building with adequate stiffness, of which a brief description will be made [6]:

**Frame:** it is formed by joining columns and beams using rigid nodes, an example of this structural system can be seen in the Figure 5.2.



**Figure 5.2:** Lever house in New York, SOM, 1952. Source: <https://www.world-architects.com> [24].



This system works well for structures up to 15-20 floors high, buildings with this type of structural system deform like a Vierendeel beam in the face of horizontal actions, the problems that can arise are high moment and shear stresses and high horizontal deformations.

**Frame joined to walls:** the walls work as a diaphragm, i.e. they work in their plane, absorbing a large amount of shear force and bending moment. The walls can be internal or in the facades of the building, this is a structurally better choice because having walls in the perimeter it gives greater inertia to the structure. As regards the interaction between the two systems, we have that the walls deform mainly by bending while the frames deform both by bending and by shearing, it follows that in the lower part of the building it is the walls that reduce the deformations and the stresses of the frame, and vice versa in the upper part, the frame reduces the deformations and stresses in the walls. It is advisable to always adopt symmetrical wall distributions with regard to geometry and stiffness, to avoid unpleasant torsional effects. In the Figure 5.3 it is possible to see an example, in which the lateral walls are evident.



**Figure 5.3:** Grand Hotel Bali in Benidorm, 2002. Source: <https://www.pinterest.it> [19].



**Core:** this type of structural system can be considered a variant of the wall system, a core is in fact a joint of walls that form a closed perimeter, this is a three-dimensional system that provides great architectural possibilities. If the core is completely closed then it is not possible to exploit the spaces inside, if instead it is totally or partially open it could have torsional effects. The use of cores can be combined with a perimeter frame and floors stiffened by outrigger systems. One of the most famous example of this type of buildings is represented in the Figure 5.4.



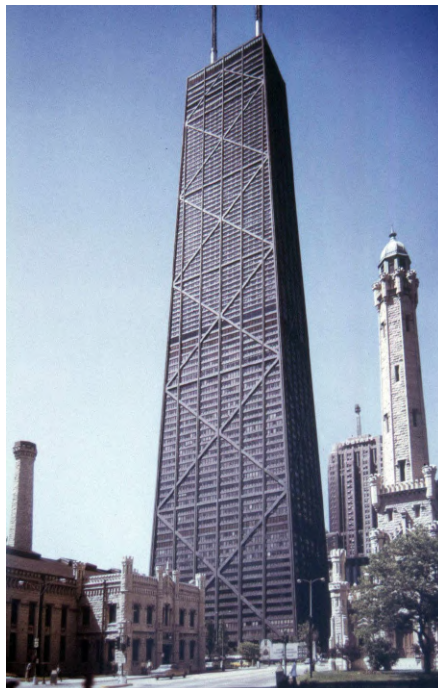
**Figure 5.4:** Turning Torso in Malmo, Calatrava, 2005. Source: <https://www.archilovers.com> [44].

**Tube type system:** this type of structure consists of many pillars close together in the perimeter of the structure, the advantages are the great inertia possessed by the building and the large amount of internal space; the disadvantages, however, are that the windows are small, as well as the access to the building. A phenomenon that occurs in this type of building is the so-called "shear lag", which consist in the greater stress on the pillars at the top of rectangular plans. A sadly known example is showed in Figure 5.5.



**Figure 5.5:** Twin Towers of the World Trade Center in New York, Robertson, 1972. Source: <https://it.wikipedia.org> [47].

**Variants of the tube type system:** there are several variations of this type of system, for example the **tube with external grid** (Figure 5.6), the system thus becomes very efficient because it behaves like a truss beam and there is a reduction in "shear lag", the drawbacks are the windows and the construction of the nodes.



**Figure 5.6:** John Hancock Center in Chicago, Khan, 1969. Source: <http://sudnascosto.blogspot.com> [23].

Then there is the system of **bundles of nodes** (Figure 5.7), the tubes share one or more walls and are linked by the floor, therefore they work in a joint shape, thus obtaining a very rigid system.



**Figure 5.7:** Willis Tower in New York, SOM and Khan, 1973. Source: <https://it.wikipedia.org> [46].

The structural system of the Pirelli skyscraper (showed in Figure 5.8) was initially a frame joined to walls, however the project was modified thanks to the intervention of Nervi and Danusso, and the vertical elements were reduced to a minimum. In fact, Nervi immediately realized that the use of both systems would lead to problems in making uniform the deformations under stress and the relative resistance, so he chose to eliminate the perimeter pillars and to keep only the ends and the central pillars-walls. The choice was made easier by the fact that in the two ends there were already elements designed by Ponti to resist "with gravity", and also by the need to have large free surfaces inside the building [27].

The term "with gravity" is also used to define a certain type of dam, this is because the behavior of the large pillars-walls and of the elements in the tips is similar to them: a vertical element with a large base is stable if the resultant of the forces acting on it remains included within the central core of inertia, a condition favored therefore by



greater loads. If this were not verified, it would be necessary to block the vertical element at the base, the disadvantages would have been the higher cost, greater deformability and consequently greater oscillations in the upper part of the building. This consideration therefore led to the confirmation of the desire to eliminate any other vertical element, which would have had a counterproductive effect in relation to vertical loads and no benefit for transverse stability [27].



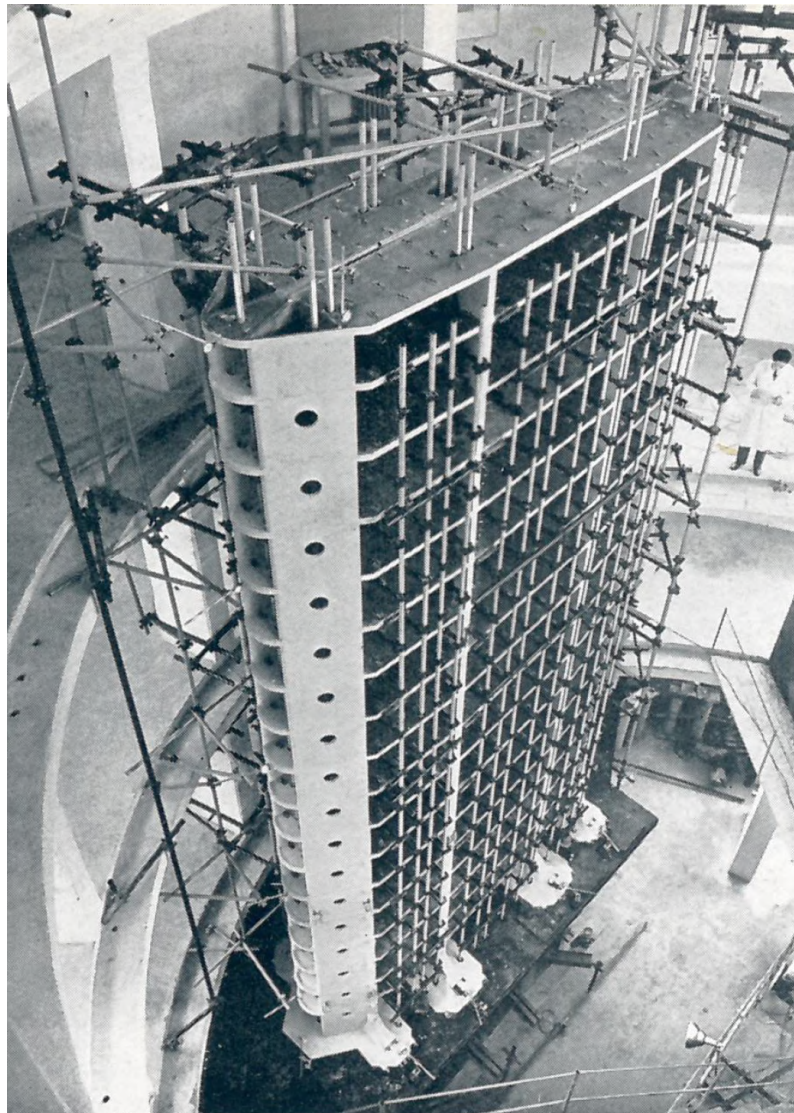
**Figure 5.8:** Pirelli building in Milano, Ponti and Nervi, 1960. Source: own shoot.

The facades, on the other hand, were made of aluminum and glass, according to Ponti's initial idea: with these curtain walls he wanted to be able to show the internal organism of the building and at the same time provide it with a resistant and lasting coating [32].

With reference to the foundations, the geotechnical investigations were very accurate and reached up to 50 meters deep. Following these, it was decided to reclaim the first 10 meters below the foundation using cement injections. The total load acting on the foundations was about 600,000 kN; during construction the lowerings were kept under constant observation, a total settlement of 10 millimeters was recorded, while the differential settlements were negligible [28].

### 5.3 Experimental studies at ISMES

Experiments on models at the ISMES (Experimental Institute for Models and Structures) began in 1955, once the first phases of analysis and design were completed [27]. Due to the importance of the work, these experiments were very much desired by Danusso, who defines a model as "*the most perfect calculating machine available*". The model was in 1:15 scale, about 9 meters high, it was the largest model made in Italy until then (Figure 5.9).



**Figure 5.9:** Model of the Pirelli building in 1:15 scale. Source: Edilizia moderna n. 71 - L'ossatura [28].

Often the material of the models is different from that of the proto-type, in order to preserve the elastic properties of the materials; in some cases, however, as for Pirelli, a material similar to the real one was used to allow the study of the structure even

beyond the elastic range, and the determination of the global safety coefficient bringing the model to failure. The main innovation brought by this model was in fact this: the attention had been directed towards the study of the ultimate capacity of the structure, for a more realistic assessment of the degree of safety, due to the plastic adaptations of the materials. The model was made with some approximations [27]:

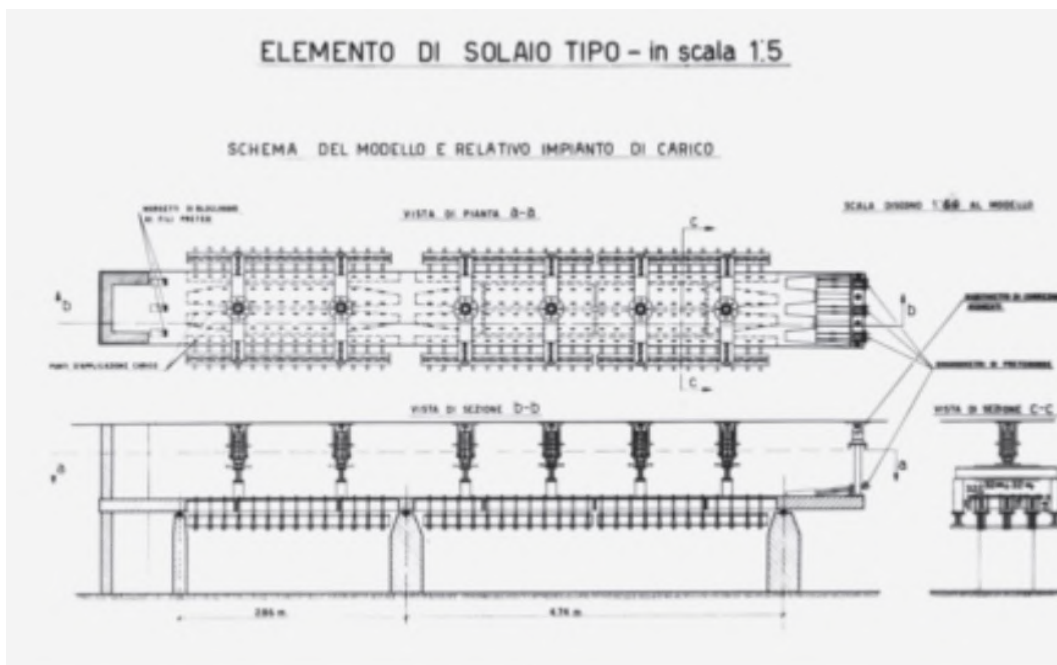
- The geometric elements that made up the tips were represented by closed triangles with the internal legs slightly more inclined than in reality, and without the small internal longitudinal septa
- The two central box-like elements were represented as vertical H-shaped septa
- Only one every two floors was made
- Other simplifications in the tips and walls, preserving the sections and moments of inertia
- The roof was not built as it was not considered an important static detail for the structure
- In the model there are also circular openings, probably added later to lighten the structure, because in the photos taken at the time of packaging they were not present yet.

The concrete of the model had an efficiency ratio of 6, which means that to maintain the unitary deformations it was necessary to induce unitary stresses in the model 6 times smaller than the real ones. The sections of the armor were made with iron wires and nets. The modeling of the terrain was not considered very important, because the soil of Milan was alluvial with fairly uniform characteristics, so under the foundation slabs they placed a layer of rubber 8 millimeters thick [27].

By mid-1955 the model was completed, and the concrete had finished maturing, so analysis could begin. The loads were applied using a system of tubular metal elements linked to the structure with rubber rings. Initially, preliminary load cycles were made for a settling of the model, then the first vertical load tests were made with self-weight cycles on the pillars, followed by self-weight cycles plus the overload on the floors. Then static tests were carried out in an elastic regime to evaluate the effects of the wind on the structure. Dynamic analyzes were also performed to verify the oscillatory behavior of the structure due to earthquakes and wind. This series of tests concluded at the end of the year [27].

At the same time as the building model, the floor model was tested (Figure 5.10); in fact one of the main concerns of Nervi and Danusso was the central span of the

building, 24 meters long by the will of Ponti. This problem was solved with a height of 75 centimeters of the T-beams, with wider webs in correspondence of the supports, to avoid excessive deformations in the span and unwanted bending effects on the internal partition walls. Various analyzes were carried out on this model with the load of the shoring of the upper floors, also verifying the need for prestressing. In fact, prestressing would have represented an executive problem due to the poor regulation of the implementation techniques and the poor preparation of the executing companies, but also from the structural point of view due to the risk of sudden failures. However, the analyzes carried out at ISMES showed that the resistance of the floors was sufficient, so it was possible to avoid using prestressing [27].



**Figure 5.10:** Scheme of the floor model. Source: Capolavori in miniatura [27].

In 1956 other tests were carried out in elastic regime on the model, after having made some changes: the pillars were connected up to the top, and the openings were made in the walls of the tips. New dynamic analyzes were also carried out to determine the periods of oscillation of the structure, and in the end the model was brought to breakage by increasing the fundamental actions: the own weight and the wind. The results confirmed Nervi's intuitions, the model showed resistance reserves in an elasto-plastic regime beyond expectations. Moreover, thanks to the model, it was also determined up to what height the vertical elements had been sufficiently deformable to compensate for the thermal deformations of the floors, and with the results obtained it was decided to make the ends of the lower floors hinged [27].



## 5.4 The restoration after the plane crash

The crash of the small tourist plane on April 18, 2002 caused the death of 3 people and more than 60 injured, but also several damage to the building [22], as shown in the Figure 5.11.



**Figure 5.11:** The building after the impact. Source: <https://milano.fanpage.it> [26].

The explosion of the fuel tanks caused an arching of the floor 27 upwards, and of the floor 26 downwards: in the first, the center of the span rose 6 centimeters from the deformed position, in the second, the center of the span lowered by 25 centimeters [5], which became 40 when the floor was flooded with water from the fire extinguishing system [30]. There was also minor damage to the lower floors, but fortunately there was no collapse. In addition, the building was already deteriorating: in the blind areas due to the detachment of the ceramic tiles, and in the two facades as the aluminum and glass frames caused water infiltration and heat dispersion [32].

For the restoration, due to the importance of the work, experts in different fields (history, architecture, engineering) were chosen to analyze the documentary sources and the characteristics of the building, and finally decide how to operate. Since the skyscraper was already an important work at the time of its construction, it was decided to restore it, although in any other situation the attic 26 would have been demolished and rebuilt. The only parts that it was decided to replace were those destroyed in the impact, the missing ceramic tiles, glass and gaskets [32].

The restoration was entrusted to Renato Sarno Group and Corvino Multari Architetti Associati, with the collaboration of the engineers Antonio Migliacci and Maurizio Acito,



the architect Adriano Crotti and the professor Giorgio Torraca; the works began in the first months of 2003 [30].

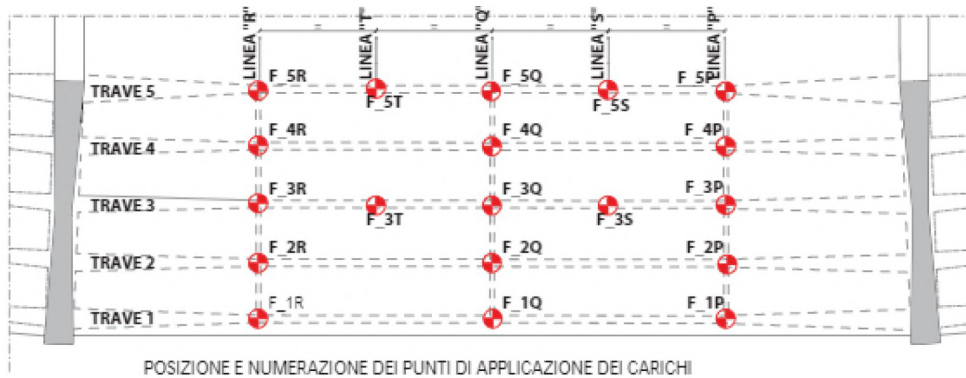
For the work on the outside of the building, a mapping of the missing ceramic tiles was made (represented in Figure 5.12), which were thus restored with different and more efficient technologies than those of the time, while the tiles still present were either consolidated or replaced depending on of their state of decay. The main aluminum structure of the facades was disassembled, cataloged, straightened where necessary and re-anodized, the pieces destroyed in the impact were reproduced starting from matrices obtained from the original parts, instead the glasses and sealants were replaced because they were no longer able to guarantee a efficient comfort [32].



**Figure 5.12:** Mapping of missing ceramic tiles. Source: Il restauro del grattacielo Pirelli a Milano [32].

Regarding the restoration of the 26th floor, a realignment using jacks was opted, giving a plastic deformed shape in the opposite direction to that caused by the explosion. For the study, a numerical model of the single beam with forced settlements was created with the code STRAUS7, and a non-linear static analysis was performed; a second model was also built with an imposed load history, which confirmed the load required for the previously obtained realignment. Finally, to consider the boundary conditions and the interactions between the beams, a third model of the entire deck was used, whose results confirmed the estimates of the total forcing, of about 3500 kN [4].

The realignment was carried out using adjustable props and jacks, additional props were placed in the 3 lower floors to have a distribution of the counter load of the jacks. Beam by beam forcing was applied, the lifting phase lasted about 30 hours, and all movements were monitored in real time. Figure 5.13 represents the numbering and position of the jacks [4].



**Figure 5.13:** Position of the jacks. Source: Procedura numerico-sperimentale della fase di riallineamento delle travi dell'impalcato del 26° piano del grattacielo Pirelli a Milano [4].

For the internal beams, in the points where a plastic hinge had formed following the accident (an example is shown in the Figure 5.14), they were cut and replaced with pieces of FeB44k steel  $\Phi 26$  bars and the damaged concrete was rebuilt, to restore the continuity of the section. Finally, in order for the sections to return to their original strength, reinforcement systems were built, consisting of post-tensioned cables external to the beams [4] [5].



**Figure 5.14:** Plastic hinges. Source: Procedura numerico-sperimentale della fase di riallineamento delle travi dell'impalcato del 26° piano del grattacielo Pirelli a Milano [4].

Unlike the 26th floor, the one above was arched upwards. The reinforcements had all remained in the elastic range as one might think, apart from the center line of the innermost beam; there were also cracks or other damages to the concrete sections. Compared to the intact floors, the behavior of the 27th was less rigid, therefore a reinforcement with FRP strips was provided, applied to the intrados of the beams, to have a suitable stretch current in the span. The restoration procedures were similar to those for floor 26, but without the use of props: the sections of concrete were rebuilt where necessary and then the tapes were applied. Finally, the linoleum floor coverings were reinstated, according to Ponti's original design [5].

With the restoration the following spaces were also renovated [30]:

- The Memorial: the 26th floor was left empty in the central part, in memory of the three victims of the serious accident.
- The Giorgio Gaber Auditorium: as it had been in disuse for more than 20 years, it was renovated to be used for institutional events or to be rented to external parties; it consists of a large room that can accommodate about 350 people, and has been adapted with the necessary technological and logistical equipment to accommodate congresses and shows.
- The Belvedere: Figure 5.15 shows the top floor of the building.



**Figure 5.15:** The top floor the Pirelli building. Source: Milano – Belvedere 31° piano Grattacielo Pirelli – cemento-vetro-acciaio [25].

This is known as the “Belvedere” for the particular and evocative 360 degree

panoramic view over the whole of Lombardy, in fact from the 31st floor of the skyscraper it is possible to admire the Prealps, Monte Rosa, the Grigne, but also some of the characteristic places of the city such as the Central Station, the Duomo, San Siro or the Torre Velasca. The floor is open to the public only on the occasion of some particular events.

## **5.5 Investigation for the informations**

This section will describe the search for information that made it possible to study the subject in detail. The important technical information has therefore been summarized in tables made with AutoCAD, which can be found in Appendix A.

### **5.5.1 Computer searches**

The articles on structural robustness found in scientific publication sites such as "ScienceDirect", "Elsevier", and "ResearchGate" were very useful, other more or less relevant information was obtained from other research on the internet, finally some fundamental information was obtained from regulations.

### **5.5.2 On-site research**

To find precise information regarding the Pirelli building, it is not enough to search on the web. In fact, with a thorough search on the internet it is possible to find information regarding the history, but only some general data regarding the structure.

Therefore, to have the necessary information in order to model the building with a calculation software, it is necessary to request the structural plans directly from the building owners. The procedure to obtain the authorization and subsequently the documents takes many days, and the timing would have been further extended to about two or three months due to the problem of COVID-19 that afflicts the year 2020, and which in this case would slowed down the bureaucracy. Fortunately, the Pirelli building is more than forty years old, so the documentation has become historical, and it has no longer been necessary to request it from a private individual. So, after a long search on the internet and after having contacted the offices of the Lombardy region several times, it was possible to discover that the documents of interest could be found in the historical archives "Cittadella degli Archivi" in Milan.

Therefore, after having obtained the necessary information on the procedures and having contacted the archives by telephone and e-mail, it was possible to discover that in order to consult the documents it is first necessary to obtain an authorization that



can be provided by the Direction of the Cittadella degli Archivi for exclusive study reasons. Consequently, the author of this work Geremia Brusaferrò, and his colleague Francesco Da Rif, proceeded to fill in the form to obtain authorization by attaching the necessary documents, including a letter of presentation written by Professor Marco Ballerini of the University of Trento. In about ten days it was then possible to obtain an appointment at the Cittadella degli Archivi, in Via Ferdinando Gregorovius, 15.

The public opening hours of the archives were from 9:00 to 12:00, and since it took 4 hours by train to reach the site, it was necessary to take it at 4:00 to be able to arrive in Milan on time, and subsequently go by subway to the archives. Once on site, it was possible to go to the consultation room accompanied by office workers, who also provided personal protective equipment such as gloves and masks.

PROF. ING. GUIDO OBERTI DEL POLITECNICO DI TORINO	Relazione Calcoli	
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VERBALE DI COLLAUDO DELLE OPERE IN CONGLOMERATO CEMENTIZIO  
 ARMATO DEL GRATTACIELO PIRELLI DI PIAZZA DUCA D'AOSTA IN  
 MILANO

Il sottoscritto prof. ing. GUIDO OBERTI, ordinario del Politecnico di Torino, iscritto all'albo professionale della Provincia di Milano al n. 2183, dichiara di aver proceduto in collaborazione con l'arch. Carlo Barbieri, assistente presso l'Istituto di Scienza delle Costruzioni del Politecnico di Milano, per incarico della Soc. Pirelli di Milano alle operazioni di collaudo relative ai lavori di cui trattasi.

DATI RELATIVI AI LAVORI ED ESAME DEI DOCUMENTI

Proprietà : Soc. p.a. PIRELLI - Milano - V.le Abruzzi, 94  
 Dati generali: Stabile a destinazione civile; uffici della Soc. Proprietaria.

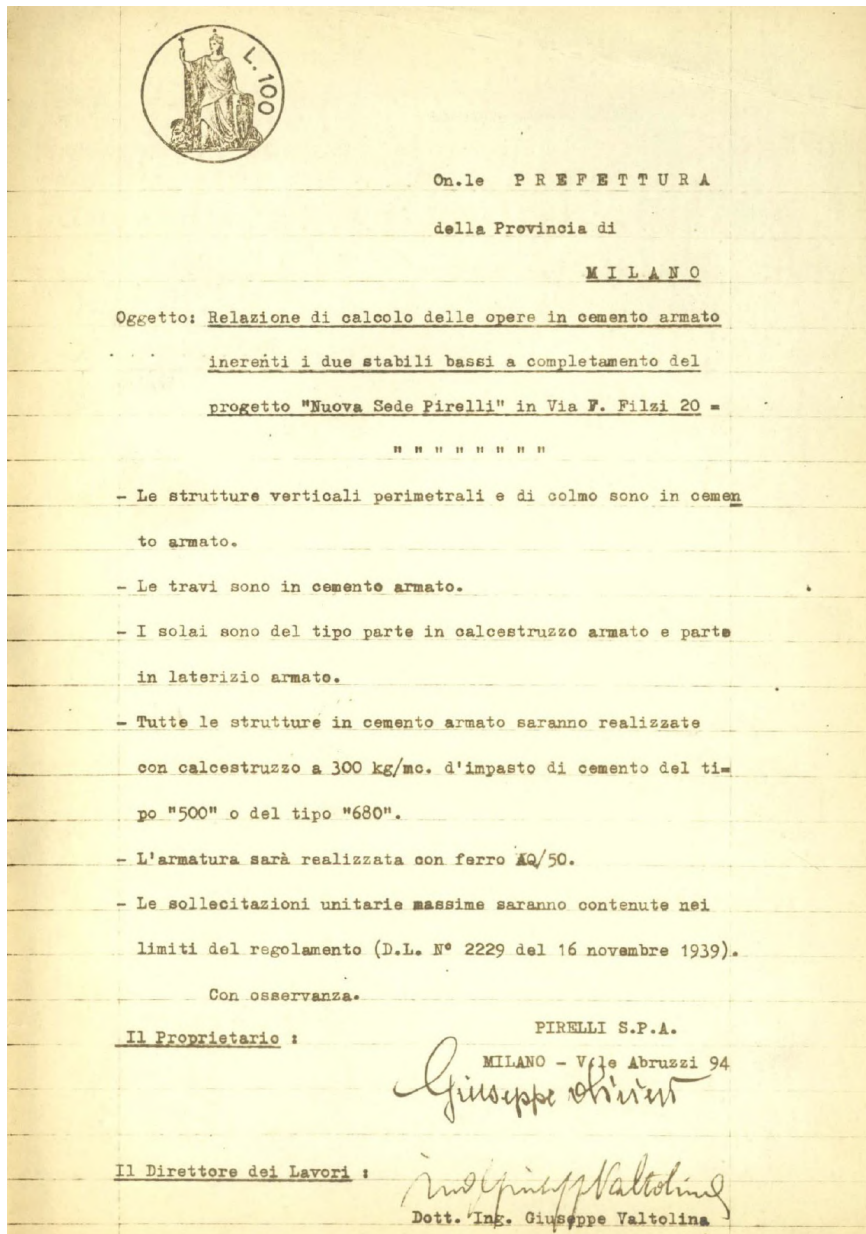
Tecnici responsabili : prof. ing. Arturo Danusso - via Andrea Doria 7 Milano - ing. Pier Luigi Nervi - lungotevere Marzio 3 Roma - progettisti delle opere in c.a.

- ingg. Giuseppe Valtolina e Antonio Forneroli - direttori dei lavori con la collaborazione dell'ing. D'Event Teodoro per l'assistenza in cantiere
- Impresa Bonomi, Comoli - Silce S.p.a. Via Poerio 2 Milano - esecutrice del c.a.

Le visite di controllo alle opere in c.a. per conto della Prefettura vennero eseguite dall'ing. Casanighian.

**Figure 5.16:** Cover of the test report of the lower buildings adjacent to the Pirelli building. Source: Atti di fabbrica a conservazione perpetua: Grattacielo Pirelli [13].

After that, it was possible to consult the documents relating to the Pirelli building under close surveillance. There were some text documents, but most of them were authorizations, therefore nothing useful for the study of the building; the only technical text documents found that proved useful were a test report (Figure 5.16), containing some useful information on the concrete used, on the loads, and on the deflections of some floors, and a calculation report of the adjacent low buildings (Figure 5.17), also containing information on materials.

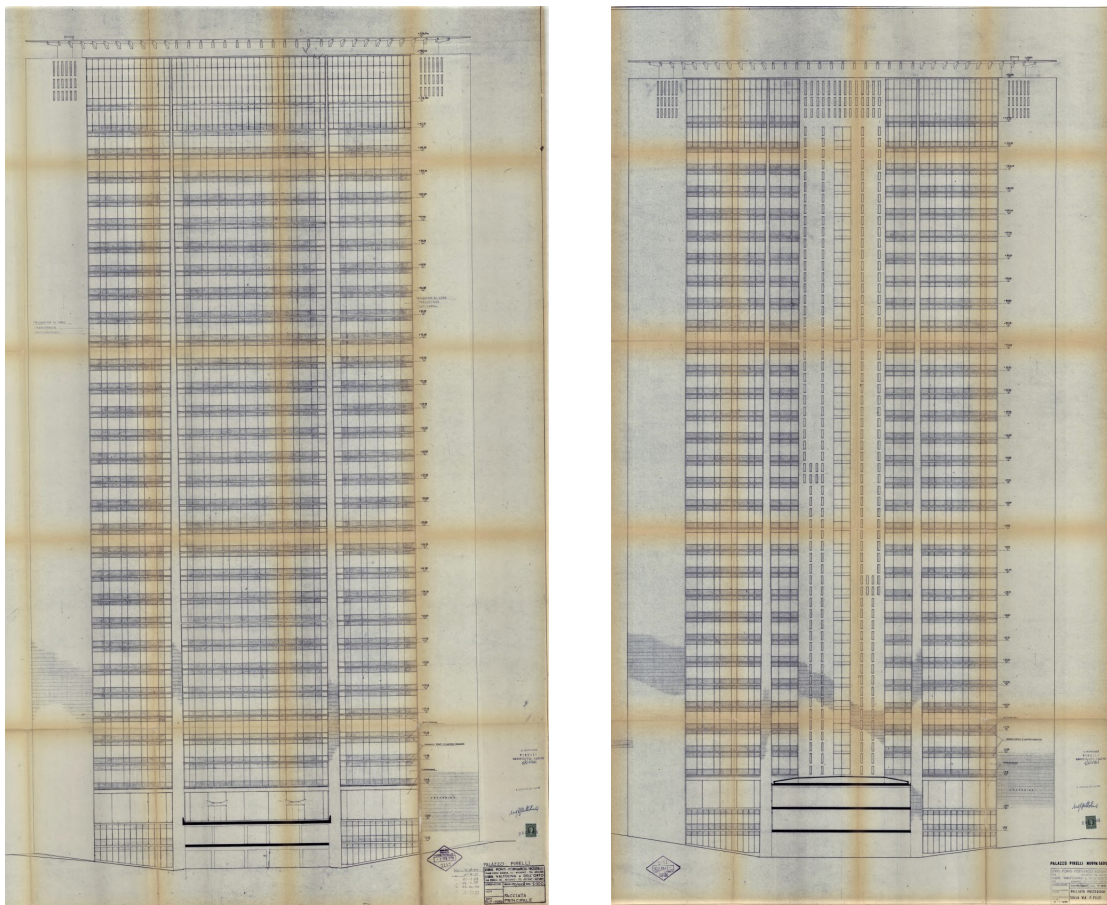


**Figure 5.17:** Cover of the calculation report. Source: Atti di fabbrica a conservazione perpetua: Grattacielo Pirelli [13].

Most of the documents, on the other hand, were technical drawings, made between



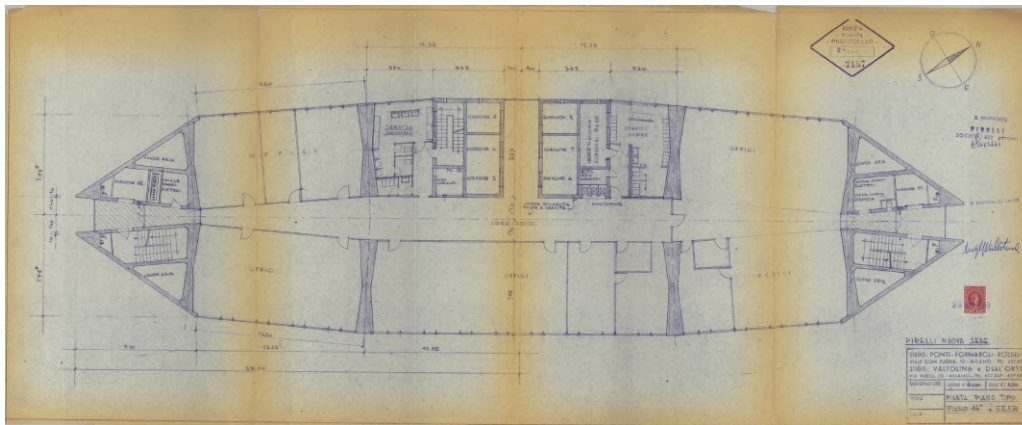
1955 and 1959, they were so many that it was only possible to take a quick look to verify the usefulness of the content for the purpose, the only three hours available were not enough to be able to stop and admire them. The technical drawings were all handmade, some with impressive precision and care, yet the paper showed signs of aging, such as tearing and yellowing. The dimensions of the designs were variable, ranging in size from A2 to A0 format, the latter being the majority, so special care was required to open and fold without damaging them. The scale of the drawings was also variable, there were drawings representing the whole neighborhood, drawings with a larger scale of the whole complex of buildings, and more detailed drawings of the individual buildings. From the research, many structural plans with different versions of the skyscraper over the years were found, from which it was possible to obtain the measurements to be able to reproduce the building in the calculation software, some example will be showed in the next pages. The two facades of the building are represented on the Figure 5.18.



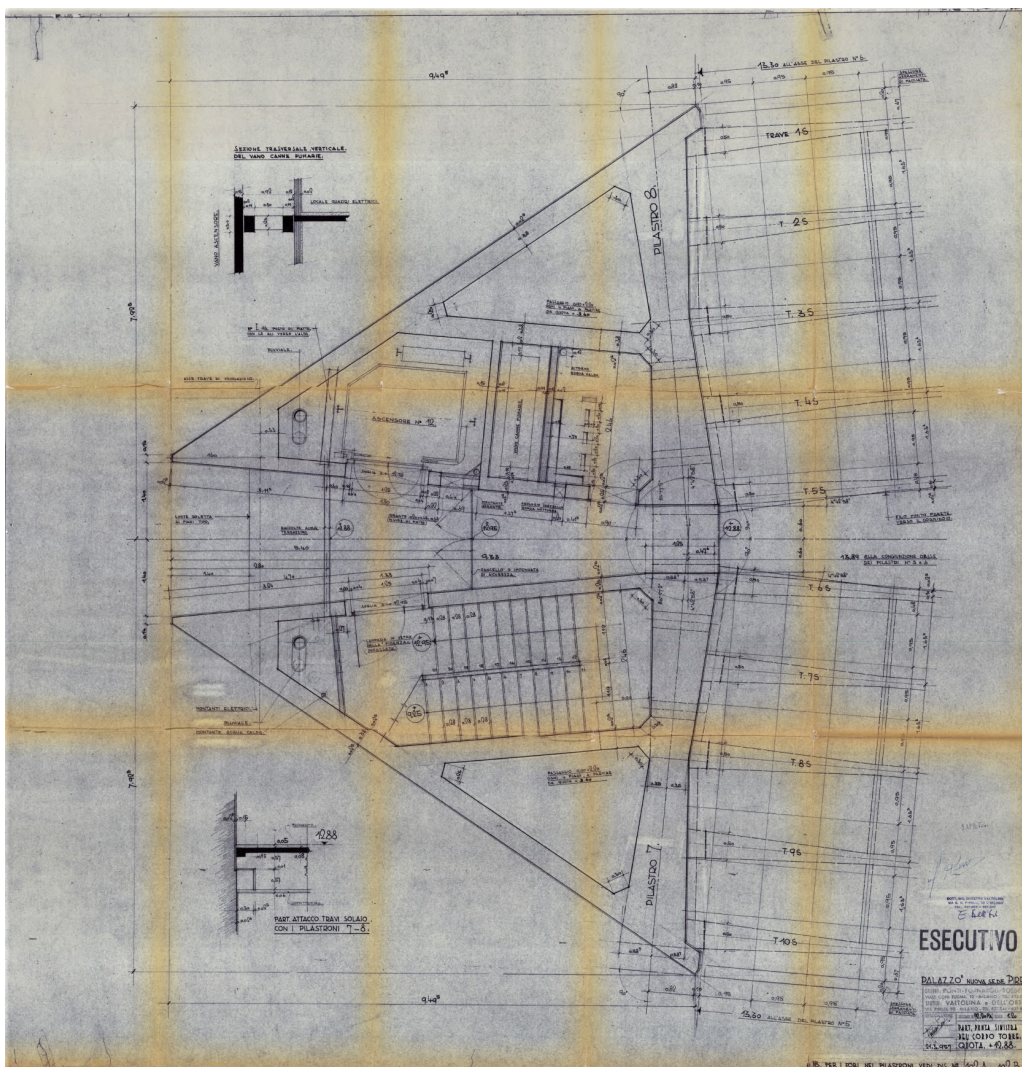
**Figure 5.18:** Front facade on the left, and rear facade on the right, where the central box-shape elements are visible. Source: *Atti di fabbrica a conservazione perpetua: Grattacielo Pirelli* [13].

In Figure 5.19 we can see a generic plant, and in Figure 5.20 a particular.





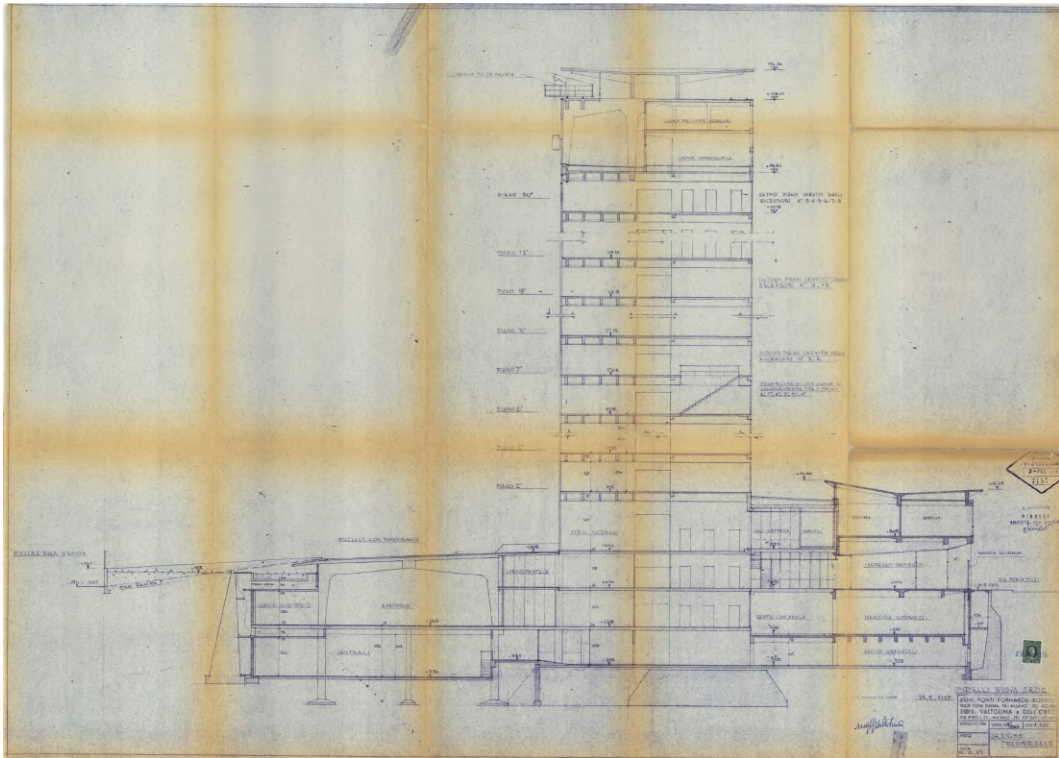
**Figure 5.19:** Plant of the fourteenth floor. Source: Atti di fabbrica a conservazione perpetua: Grattacielo Pirelli [13].



**Figure 5.20:** Particular of a lateral triangular element. Source: Atti di fabbrica a conservazione perpetua: Grattacielo Pirelli [13].

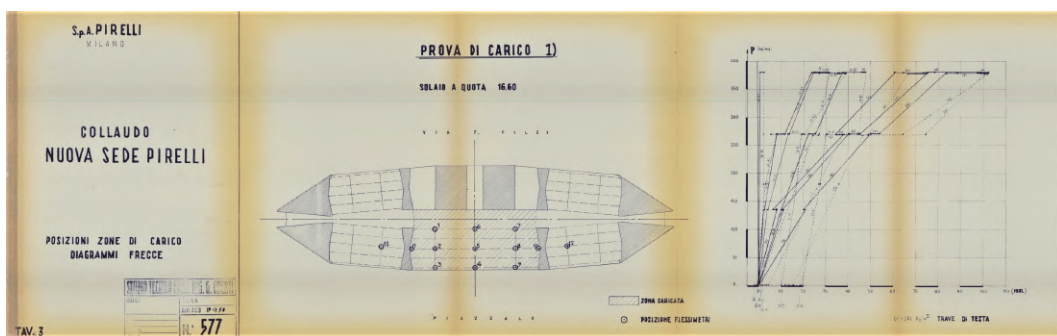


And in the last, Figure 5.21, a lateral view is represented, with a cut in the middle of the building.



**Figure 5.21:** Lateral view. Source: Atti di fabbrica a conservazione perpetua: Grattacielo Pirelli [13].

Technical tables were also found regarding the building's testing, showing information on the positions of load tests carried out on the slab floors, and the results (Figure 5.22).



**Figure 5.22:** Example of results of a load test. Source: Atti di fabbrica a conservazione perpetua: Grattacielo Pirelli [13].

Unfortunately, it was not possible to find a calculation report regarding the building, this probably because many documents have been destroyed or lost over the years, so all information was obtained from the documents mentioned above.

The time available would not have been sufficient to consult all the documents and choose the useful ones, however the staff of the offices were very helpful, allowing the consultation to be completed after the closing time. Eventually, the documents were scanned with special tools, capable of producing very high resolution PDFs without damaging the paper, reaching a size of 1 gigabyte for 32 files.

After that the employees of the Citadel of the Archives shared the files through the Wetransfer service, while the author of this work and his colleague returned to their respective cities with another 4-hour trip.

It would have been very interesting to enter the building during the day in Milan, to be able to see it closely, but due to the Coronavirus emergency the access was forbidden to unauthorized persons; so it was only possible to admire it from the square.

# STRUCTURAL CHARACTERIZATION

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Thanks to the valuable documents provided by the Cittadella degli Archivi [13] and some information obtained from other sources, it was possible to model the building. The purpose of this chapter is to summarize and schematize these data.

## 6.1 Structural system

The Pirelli skyscraper is 127,10 meters high, and consists of thirty two floors above ground and two underground, in which there are parking lots. The structural system of the building is composed of walls, these have massive dimensions and therefore allow to minimize the amount of vertical elements, which consist of triangular side elements, four internal pillars-walls, and two box-shaped elements in the middle. The main floors are made up of large T-beams, the lateral ones have a length of about 12 meters while the central one reaches 24 meters; other floors are composed by only a slab. The facades consist of curtain walls made of glass and sheet metal, with aluminum frames; the roof instead is a concrete slab stiffened by ribs and supported by walls.

## 6.2 Materials

This section will describe the materials used to construct the building. Some of the information was found in the original documents of 1960 [13], but the most important data are those reported by the Engineer and Professor Maurizio Acito [3], who was involved in the renovation of the building after the plane crash.

### 6.2.1 Concrete

As indicated in the documents of the time, two types of concrete were used [13], different for the type of cement and its dose; in the Table 6.1 are represented some characteristics of them, the value of the resistances were obtained between 1956 and 1958 with tests on samples extracted from the foundations and the walls [3].

**Table 6.1:** Characteristics of the two types of concrete. Source: Le indagini sperimentali come strumento di verifica sismica [3].

Concrete	Building	Foundations
Cement type	680	500
Cement dose [ $kg/m^3$ ]	400	300
$R_{cm}$ [ $MPa$ ]	26,9	38,1
Standard deviation [ $MPa$ ]	5,1	4,6

The resistance of the two types of material will then be evaluated as it was done at the time of the accident, following the Eurocode 2-1-1 [40], thus taking into account the variation in the characteristics of the concrete over time. The procedure is the following: first, it is necessary to have the cylindrical strength of the concrete and its evolution over time:

$$f_{cm} = R_{cm} \cdot 0,83 \quad (6.1)$$

$$f_{cm}(t) = f_{cm} \cdot \exp\left(s\left(1 - \sqrt{\frac{28}{t}}\right)\right) \quad (6.2)$$

where  $s = 0,38$  is a value that depends on the concrete and  $t = 22174$  is the number of passed days; so now it is possible to calculate the resistances of the materials:

$$f_{ck}(t) = f_{cm}(t) - 8 \quad (6.3)$$

$$f_{cd}(t) = \alpha_{cc} \cdot \frac{f_{ck}}{\gamma_c} = 0,85 \cdot \frac{f_{ck}}{1,5} \quad (6.4)$$

Then, once obtained  $f_{cm}(t)$  it is possible to proceed with the calculation of the elastic modulus of the concrete as a function of time, considering  $E_{cm} = 22000 MPa$ .

$$E_{cm}(t) = E_{cm} \cdot (f_{cm}(t)/10)^{0,3} \quad (6.5)$$

The results can then be seen in Table 6.2.

**Table 6.2:** Parameters of the two materials.

Parameters	Building	Foundations
$f_{cm}$ [ $MPa$ ]	22,3	31,6
s	0,38	0,38
t [days]	22174	22174
$f_{cm}(t)$ [ $MPa$ ]	32,2	45,6
$f_{ck}(t)$ [ $MPa$ ]	24,2	37,6
$f_{cd}(t)$ [ $MPa$ ]	13,7	21,3
$E_{cm}$ [ $MPa$ ]	22000	22000
$E_{cm}(t)$ [ $MPa$ ]	31244	34687

## 6.2.2 Steel

The calculation report [13] of the slab floors indicates the type of steel used for the reinforcement of the beams, but does not give any information regarding the reinforcements contained in the walls, which are instead indicated in the documents relating to the restoration by Maurizio Acito [3]. For the beams of the slab floors it was used the semi-hard steel RUMI LU3, an high resistance reinforcement steel characterized by a special section showed in Figure 6.1, the bars used were all  $\Phi 26$ ; for the walls it was used a steel indicated into the standards of the time [21], the Aq 50, the diameter of the bars of the internal element was  $\Phi 30$ , for the triangular lateral elements  $\Phi 14$   $\Phi 20$   $\Phi 22$  and  $\Phi 30$  were used.



**Figure 6.1:** Image of the RUMI LU3 steel. Source: Le indagini sperimentali come strumento di verifica sismica [3].

From the tensile tests carried out at the time, the characteristics of the two types of steel are known so it is possible to obtain the design resistances as indicated by Eurocode 2-1-1 [40]:

$$f_{yd}(t) = \frac{f_{yk}}{\gamma_s} = \frac{f_{yk}}{1,15} \quad (6.6)$$

$$f_{ud}(t) = \frac{f_{uk}}{\gamma_s} = \frac{f_{uk}}{1,15} \quad (6.7)$$

The Table 6.3 contains the characteristics of the steels.

**Table 6.3:** Characteristics of the steels.

Parameters	Beam bars	Wall bars
Steel type	RUMI LU3 4400	Aq 50
$f_{yk}$ [MPa]	440	270
$f_{uk}$ [MPa]	600	500
Average elongation [%]	12	16
$f_{yd}$ [MPa]	382,6	234,8
$f_{ud}$ [MPa]	521,7	434,8
$E_s$ [MPa]	200000	200000

## 6.3 Load analysis

### 6.3.1 Permanent structural loads $G_1$

The permanent structural loads are represented by the weight of the beams and shells elements, which the calculation software automatically takes into account, therefore a self-weight of  $\gamma_c = 25 \text{ kN/m}^3$  is considered for them, as recommended by the Eurocode 1-1-1 [39]; for the steel it was considered  $\gamma_s = 76,97 \text{ kN/m}^3$ , as the default option of SAP2000.

However, not all the structural elements will be represented in the model, so it is necessary to manually calculate the weight of the  $G_1$  load relative to the stairs for each storey with a different height. In Table 6.4 are calculated the weights of the stairs located in the lateral triangular elements.

**Table 6.4:**  $G_1$  weight of the stairs of the lateral triangular elements for every different storey.

Elements	Thickness [m]	Width [m]	Length [m]	$\gamma$ [kN/m <sup>3</sup> ]	n°	$G_1$ [kN]
Ramp slab	0,2	1,2	2,8	25	2	33,60
Steps	0,185	1,2	0,28	25	20	31,08
Landing	0,2	3	2,5	25	1	37,50
TOT ( $h = 3,70 \text{ m}$ )						102,18
Ramp slab	0,2	1,2	2,8	25	2	33,60
Steps	0,175	1,2	0,28	25	20	29,40
Landing	0,2	3	2,5	25	1	37,50
TOT ( $h = 3,50 \text{ m}$ )						100,50
Ramp slab	0,2	1,2	2,8	25	2	33,60
Steps	0,18	1,2	0,28	25	20	30,24
Landing	0,2	3	2,5	25	1	37,50
TOT ( $h = 3,60 \text{ m}$ )						101,34
Ramp slab	0,2	1,2	2,8	25	2	33,60
Steps	0,1875	1,2	0,28	25	20	31,50
Landing	0,2	3	2,5	25	1	37,50
TOT ( $h = 3,75 \text{ m}$ )						102,60
Ramp slab	0,2	1,2	2,8	25	4	67,20
Steps	0,14	1,2	0,28	25	40	47,04
Landing	0,2	3	2,5	25	2	75,00
TOT ( $h = 5,60 \text{ m}$ )						189,24
Ramp slab	0,2	1,2	2,8	25	4	67,20
Steps	0,1925	1,2	0,28	25	40	64,68
Landing	0,2	3	2,5	25	2	75,00
TOT ( $h = 7,70 \text{ m}$ )						206,88

And Table 6.5 shows the weights of the central box-shaped elements.

**Table 6.5:**  $G_1$  weight of the stairs of the central box-shaped elements for every different storey.

Elements	Thickness [m]	Width [m]	Length [m]	$\gamma$ [kN/m <sup>3</sup> ]	n°	$G_1$ [kN]
Ramp slab	0,2	0,8	2,8	25	2	22,40
Steps	0,185	0,8	0,28	25	18	18,65
Landing	0,2	2,2	2,2	25	1	24,20
TOT ( $h = 3,70$ m)						65,25
Ramp slab	0,2	0,8	2,8	25	2	22,40
Steps	0,175	0,8	0,28	25	18	17,64
Landing	0,2	2,2	2,2	25	1	24,20
TOT ( $h = 3,50$ m)						64,24
Ramp slab	0,2	0,8	2,8	25	2	22,40
Steps	0,18	0,8	0,28	25	18	18,14
Landing	0,2	2,2	2,2	25	1	24,20
TOT ( $h = 3,60$ m)						64,74
Ramp slab	0,2	0,8	2,8	25	2	22,40
Steps	0,1875	0,8	0,28	25	18	18,90
Landing	0,2	2,2	2,2	25	1	24,20
TOT ( $h = 3,75$ m)						65,50
Ramp slab	0,2	0,8	2,8	25	4	44,80
Steps	0,14	0,8	0,28	25	36	28,22
Landing	0,2	2,2	2,2	25	2	48,40
TOT ( $h = 5,60$ m)						121,42
Ramp slab	0,2	0,8	2,8	25	4	44,80
Steps	0,1925	0,8	0,28	25	36	38,81
Landing	0,2	2,2	2,2	25	2	48,40
TOT ( $h = 7,70$ m)						132,01

### 6.3.2 Permanent non-structural loads $G_2$

All permanent non-structural loads will not be represented in the model, so they must be manually calculated and applied to the structure. Little data was found regarding the finishes of the structure, so reasonable materials and thicknesses were assumed. Tables 6.6 and 6.7 represent the supposed materials and the weight of the floor slabs and the roof.

**Table 6.6:** Permanent non-structural loads of the floor slabs.

Material	$\gamma$ [kN/m <sup>3</sup> ]	Thickness [m]	$g_2$ [kN/m <sup>2</sup> ]
Subfloor	18	0,05	0,9
Linoleum	/	/	0,03
Plaster	20	0,01	0,2
TOT			1,13

**Table 6.7:** Permanent non-structural loads of the roof.

Material	$\gamma$ [ $kN/m^3$ ]	Thickness [ $m$ ]	$g_2$ [ $kN/m^2$ ]
Insulating	0,3	0,2	0,06
Subfloor	18	0,06	1,08
Waterproofing	/	/	0
Pavement	/	/	0,5
Plaster	20	0,01	0,2
TOT			1,84

For the load of the internal partitions, it is assumed that it is  $g_2 = 0,8 kN/m^2$ .

For the facades, it can be seen from the technical drawings [13] that they are made of glass and sheet metal, the area covered by each of them for each different floor is not indicated, therefore it has been assumed. Table 6.8 indicates the weights per linear meter of the elements placed between the two box-like elements of the rear facade, while Table 6.9 indicates the weights of the remaining part.

**Table 6.8:** Weights per linear meter of the elements placed between the two box-like elements of the rear facade.

Element	$h_{net}$ [ $m$ ]	Thickness [ $m$ ]	$\gamma$ [ $kN/m^3$ ]	$A_{covered}$ [%]	$g_2$ [ $kN/m$ ]
Glass	3,42	0,02	25	0,5	0,86
Sheet metal	3,42	0,01	27	0,5	0,46
TOT ( $h = 3,70 m$ )					1,32
Glass	3,22	0,02	25	0,5	0,81
Sheet metal	3,22	0,01	27	0,5	0,43
TOT ( $h = 3,50 m$ )					1,24
Glass	3,32	0,02	25	0,75	1,25
Sheet metal	3,32	0,01	27	0,25	0,22
TOT ( $h = 3,60 m$ )					1,47
Glass	3,47	0,02	25	0,75	1,30
Sheet metal	3,47	0,01	27	0,25	0,23
TOT ( $h = 3,75 m$ )					1,54
Glass	5,32	0,02	25	1	2,66
Sheet metal	5,32	0,01	27	0	0,00
TOT ( $h = 5,60 m$ )					2,66
Glass	7,42	0,02	25	0,83	3,09
Sheet metal	7,42	0,01	27	0,17	0,33
TOT ( $h = 7,70 m$ )					3,43



**Table 6.9:** Weights per linear meter of the elements of the facades.

Element	$h_{net}$ [m]	Thickness [m]	$\gamma$ [kN/m <sup>3</sup> ]	$A_{covered}$ [%]	$g_2$ [kN/m]
Glass	3,52	0,02	25	0,5	0,88
Sheet metal	3,52	0,01	27	0,5	0,48
TOT ( $h = 3, 70$ m)					1,36
Glass	3,32	0,02	25	0,5	0,83
Sheet metal	3,32	0,01	27	0,5	0,45
TOT ( $h = 3, 50$ m)					1,28
Glass	3,42	0,02	25	0,75	1,28
Sheet metal	3,42	0,01	27	0,25	0,23
TOT ( $h = 3, 60$ m)					1,51
Glass	3,57	0,02	25	0,75	1,34
Sheet metal	3,57	0,01	27	0,25	0,24
TOT ( $h = 3, 75$ m)					1,58
Glass	5,42	0,02	25	1	2,71
Sheet metal	5,42	0,01	27	0	0,00
TOT ( $h = 5, 60$ m)					2,71
Glass	7,52	0,02	25	0,83	3,13
Sheet metal	7,52	0,01	27	0,17	0,34
TOT ( $h = 7, 70$ m)					3,47

Instead, for the stairs, the permanent non-structural loads represented in the following table were assumed.

**Table 6.10:** Weights of the stairs of the lateral triangular elements and the central elements.

Element	$\gamma$ [kN/m <sup>3</sup> ]	Thickness [m]	Area [m <sup>2</sup> ]	$G_2$ [kN]
Plaster	20	0,01	15,5	3,10
Pavement	15	0,02	15,5	4,65
TOT (lateral)				7,75
Plaster	20	0,01	11	2,20
Pavement	15	0,02	11	3,30
TOT (central)				5,50

There would be also the weight of the lifts, but being very small it can be neglected.

### 6.3.3 Imposed loads Q

From the information on the test report [13], the building is assigned to offices, therefore according to the Eurocode [39] the overload to be applied is  $q_{catB} = 2$  kN/m<sup>2</sup> and  $q_{catB} = 4$  kN/m<sup>2</sup> on the stairs; while the roof belongs to cat H since it is not practicable, so  $q_{catH} = 0,5$  kN/m<sup>2</sup>. The wind and snow loads have been calculated, but are not

reported for the sake of brevity and because in the combination they enter with a zero coefficient.

## 6.4 Load combination

The purpose of this work is to analyze the behavior of the Pirelli building after the destruction of a load-bearing element due to an extreme event, such as an explosion or a terrorist attack. For this reason, the load combination that will be used is the accidental [37]. The approach used is threat-independent so the accidental load is zero, and for the first variable load the  $\psi_{2,i}$  coefficient is selected as the National Annex of the Eurocode recommend, so the 4.1 becomes:

$$\sum_{j \geq 1} G_{j,k} + \sum_{i \geq 1} \psi_{2,i} \cdot Q_{k,1} \quad (6.8)$$

Hence, the factors for variable actions assume the following values:

- $\psi_{2,1} = 0,3$  for the overload of  $q_{catB}$
- $\psi_{2,i} = 0$  for the all others.

# FINITE ELEMENT MODEL

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Thanks to the many information collected, resumed in the drawings of Appendix A, it was possible to reproduce the Pirelli building with the calculation software SAP2000. This software was introduced during the last academic year in several courses, and it is a versatile and widely used software in the world of work, so it was chosen with the aim of deepening the knowledge.

The modeling was very demanding and required many days of work, in fact it was organized in different phases:

1. Schematization of information regarding the building and identification of missing data
2. Assumptions on geometry, and graphical representation using AutoCAD software to produce the drawings in Appendix A
3. Calculation of the weights of the structural elements not represented in the model, using the Excel software
4. Calculation of the loads acting on the structure, always with Excel
5. Internet search for information regarding the use of the software, very useful for this purpose were the CSI America website [14], the YouTube channel [12], and the manuals
6. Simplification of some structural elements still using Excel, in order to make their insertion in SAP2000 practical
7. Then there was the true modeling phase, where the Pirelli building was represented with mono-dimensional and bi-dimensional elements
8. And finally, the last phase was the one of the analysis, which will be described in detail in the next chapter.

## **7.1 The Finite Elements**

The building was represented using two types of finite elements: one-dimensional or frames, and two-dimensional or shells. A brief description of both is given below.

### **7.1.1 The Frame element**

The frame element can be used to effectively represent beams or columns. It consists of a segment defined by two joints that represents the length, and a section perpendicular to it, that can have a simple or complex shape and can be constant or variable along the element. The frame element is usually represented by the segment between the two joints, which is called the neutral axis and passes through the centroid of the section. However, for a more correct approximation of reality, it is possible to specify insertion points to locate the element with respect to the joints and the segment, as will be shown later. The frame element has a system of local axes, the 1 is directed along the length and the other two are perpendicular to it, and activates all six degrees of freedom at the two joints. Distributed or concentrated loads can be applied to it, and takes into account the effects of biaxial shear and bending, axial deformation, and torsion.

### **7.1.2 The Shell element**

The element shell can be used to model two-dimensional elements, with the behavior of a membrane, a plate, or a shell. It is a surface defined by three joints, or by four joints, even not co-planar. The latter is the more accurate formulation of the two, and for both cases there are geometry recommendations in the manual for more accurate results. The reference surface is then characterized by a thickness, and it is normally located in the middle of the element; but as for frames its position can be changed to move the element relative to the joints. Each element has local axes, the first two in the plane of the surface while the 3 is perpendicular; and it also activates all six degrees of freedom at the joints. Only distributed loads can be applied to the element, and for each joint the stresses, internal forces and moments are calculated. It is also possible to define elements composed of several layers, to obtain a particular behavior.

## **7.2 The modeling of the building**

The calculation of the loads and weights acting on the structure was seen in the previous chapter, therefore this section will describe the modeling phase and the simplifications made to facilitate the insertion of the structural elements in the calculation software.

The structure was created by modeling only the resistant portions, the frames as beam elements while the walls as shell elements; the foundations were not modeled because they were not of interest for the purpose of this work, consequently the building was considered to be embedded at the base.

Before proceeding with the insertion of the structure, the grid was added at the height of each floor and on the roof, with the command "*Define - Coordinate Systems/Grid - Add new system...*" (Figure 7.1).

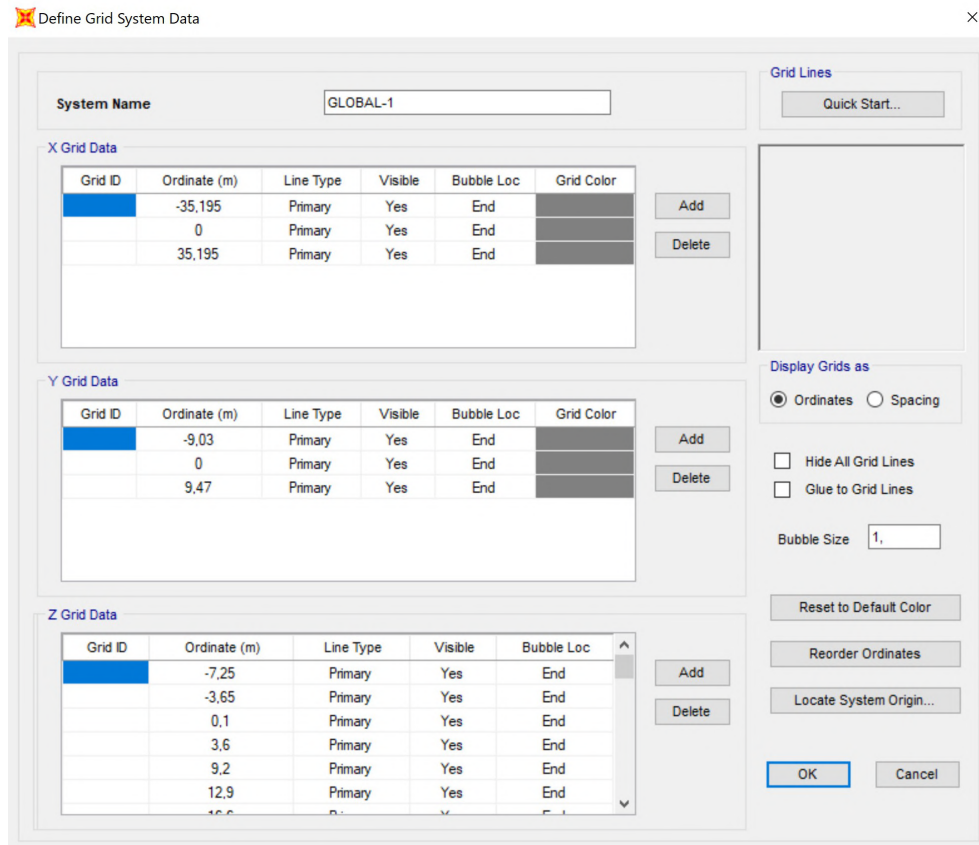


Figure 7.1: Adding of the grid.

To draw the elements in the precise position in which they are, a special layer was created on AutoCAD, in which the axes of each of them were represented. The file was therefore saved in .dxf format, and when drawing any element in SAP2000, it was imported with the command "*File - Import - AutoCAD .dxf File...*", in order to have the necessary nodes in the correct coordinates.

### 7.2.1 Materials

First of all, the materials Concrete 400 and steel RUMI LU3 were inserted in SAP, using the command "*Define - Materials... - Add New Material...*" (Figure 7.2), even if only the

first of the two would be used.

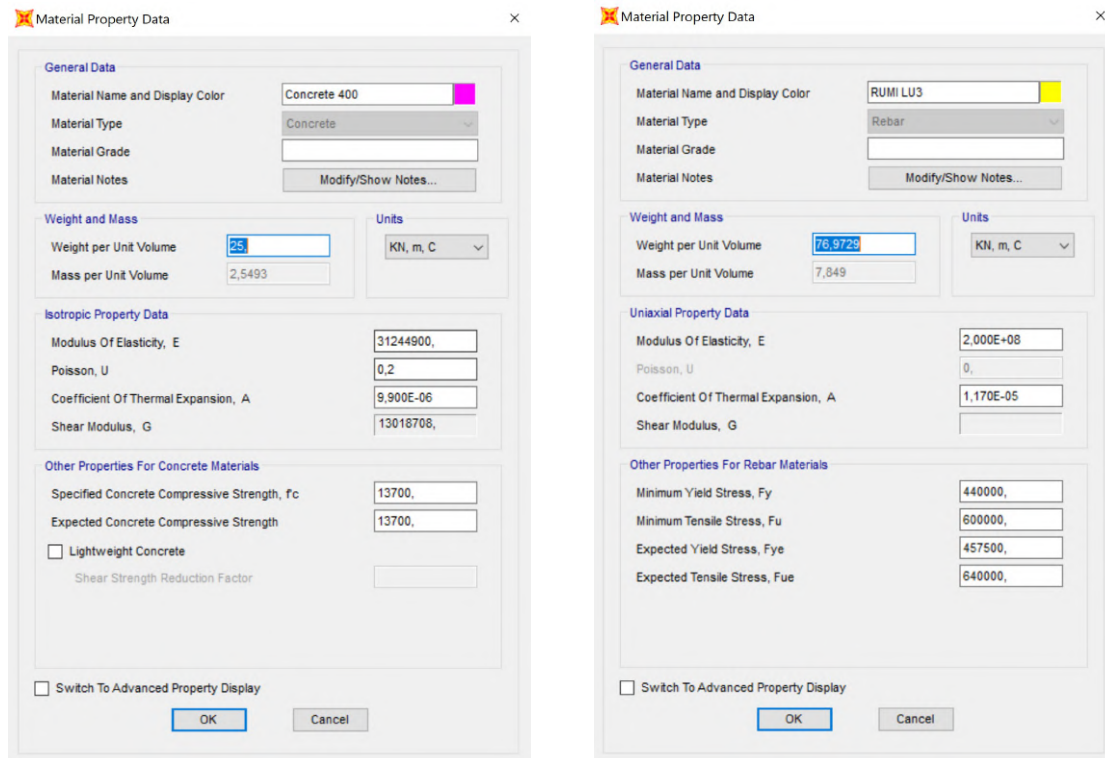


Figure 7.2: Materials.

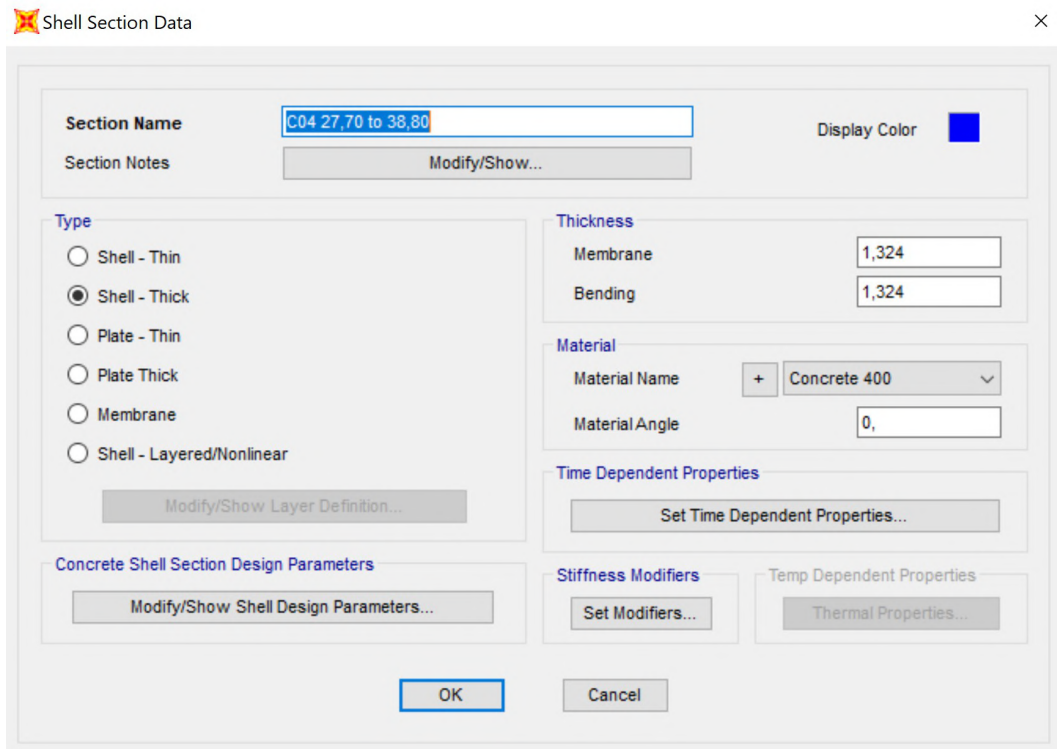
## 7.2.2 Central elements

The central elements are four enormous pillars-walls, which have a butterfly shape, as can be seen from the drawings in Appendix A, and are divided into two trapezoidal elements starting at 3,60 meters.

No precise information was available on the actual dimensions of the elements but only some plants of the base, the middle, and the top of the building, from which it can be seen that the variation of the sections with the height has a linear trend [13]; therefore all the other dimensions have been deducted from the available measurements. The section at the base, at the point of maximum thickness measures about 2 meters in width, and has constant dimensions until the height of 0,1 m, then it narrows with increasing height; after the division into the trapezoidal elements, both the thickness and the dimension perpendicular to it decrease, shortening on the internal side.

To be able to insert these elements easily in SAP2000, it was necessary to simplify their geometry. The section was approximated from butterfly shape to rectangular, keeping the same area. The thickness was made to vary every three floors, so twelve types of shell elements were defined with the command "Define - Section Properties -

*Area Sections... - Add New Section...*" (Figure 7.3), each having the average thickness of the three corresponding floors.



**Figure 7.3:** Example of a section of the four pillars-walls.

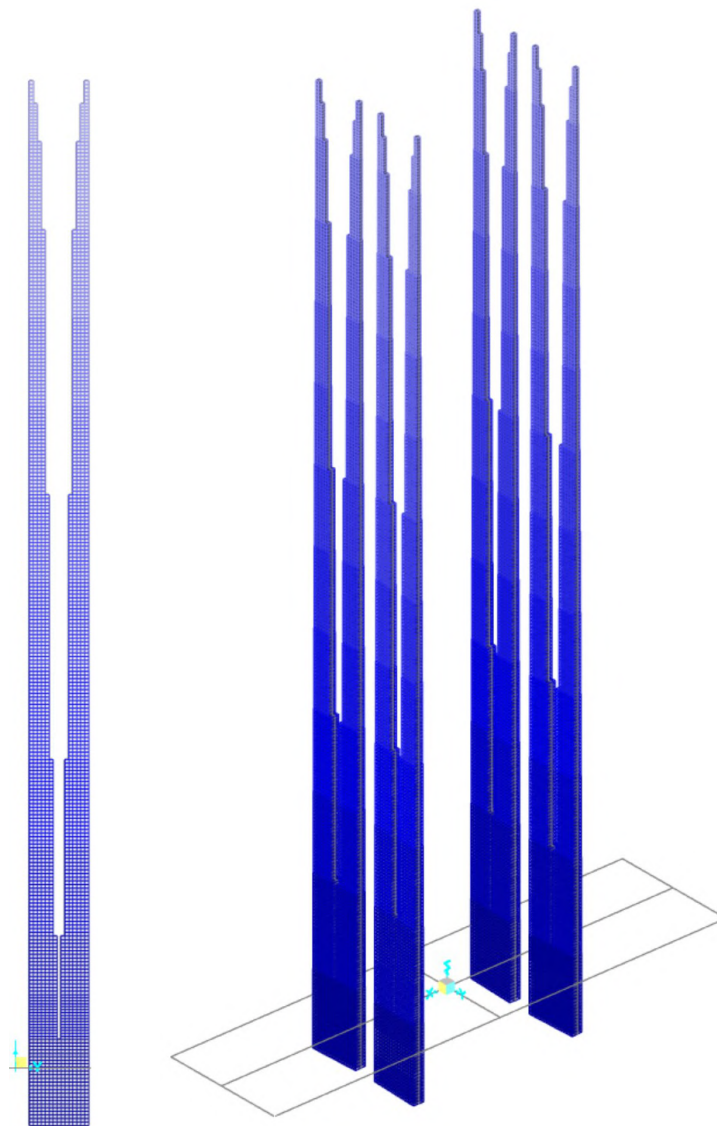
The finite elements have been drawn with the command "*Draw Poly Area*", all starting from the upper left vertex and continuing counterclockwise, drawing them in this way is very important to have the local axes all directed in the same directions, so the same procedure will be followed for any other two-dimensional structural element. Initially each wall was designed with a length of 7,33 meters, the bifurcation in the middle was made later.

The shortening of the walls on the inside after the division to 3,60 meters is also linear with the height, in order to simulate it with sufficient precision it was therefore decided to divide the finite elements of the walls into fifteen along the length with "*Edit - Edit Areas - Divide Areas...*", and to delete one or two at certain heights in order to approximate reality as closely as possible. Table 7.1 shows the lengths of the walls and the height up to which they are maintained.

Finally, after drawing the first pillar, it was copied with the "*Edit - Replicate - Mirror*" command to obtain the other four (Figure 7.4).

**Table 7.1:** Length of the walls in the SAP2000 model.

Dimension FE [m]	n° EF	Real wall length [m]	$h_{building}$ [m]
0,489	2	0,489	124,2
0,489	4	0,977	121,3
0,489	6	1,466	116,5
0,489	8	1,955	105,4
0,489	10	2,443	72,1
0,489	12	2,932	38,8
0,489	14	3,421	16,6
0,489	15	7,330	0,1

**Figure 7.4:** The four pillars-walls completed.

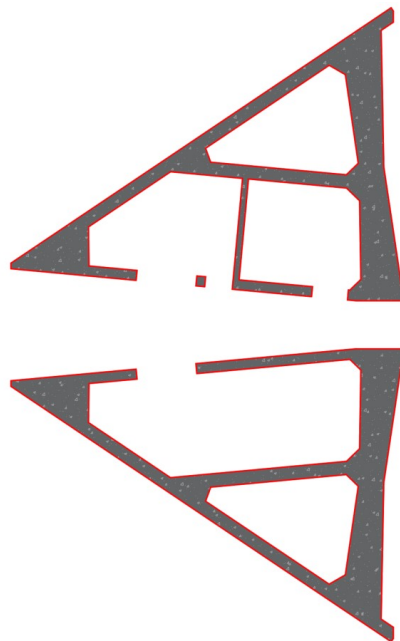


### 7.2.3 Lateral triangular elements

On each of the two sides of the building there are two triangular elements, inside which there are stairs and elevators; they basically consist of:

- A very massive wall, with the same shape as the central pillars-walls, comparable in size, and approximately parallel to them
- An external wall, next to the facades
- A wall on the inside, thinner, and separated from the other triangular element by means of a corridor parallel to the largest dimension of the plant
- Internal walls to the triangular elements.

Figure 7.5 represents an example of the two triangular side elements on the left side of the building.

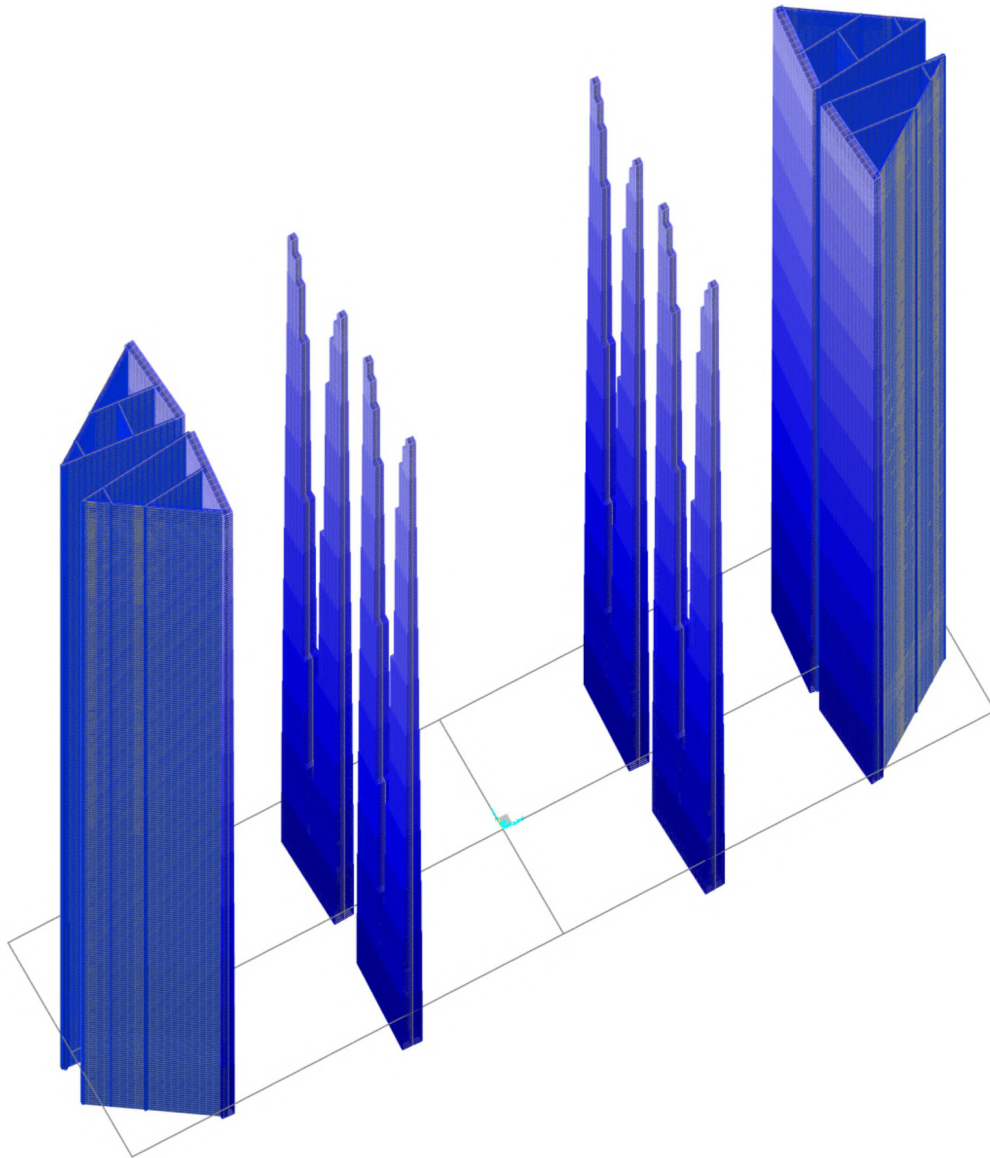


**Figure 7.5:** Two lateral triangular elements.

Also in this case, the information regarding the geometry was incomplete, therefore it was assumed that the variation in thickness with height had a linear trend, except for the thinnest walls, which were kept constant from the base to the top.

In the walls of these elements there are no variations in length, so the geometric simplifications concern only the thicknesses. For the walls similar to the pillars the same procedure was performed, which consists in calculating a rectangular section with the same area, and making it vary every three floors, thus obtaining twelve sections.

For the other larger walls, to approximate the shrinkage with increasing height, the thickness was made to vary from 40 centimeters to 25 centimeters with a step of 5 centimeters, trying to keep the likelihood with reality as much as possible, So four other sections have therefore been defined. As mentioned above, the thickness of the thinnest walls was kept constant.



**Figure 7.6:** Vertical elements.

After drawing the first lateral triangular element, the finished elements were divided into smaller parts as was done for the four pillars-walls, and in the same way it was mirrored to make the other three. In this case, however, it was possible to mirror only the larger walls, since the position of the smaller ones was different for each case. Figure 7.6 represents the lateral triangular elements, together with the central elements previously created.

Finally, the last operation to make the triangular side elements more similar to reality concerns the external walls: the thickness variation does not occur around the axis which therefore changes position with the height, but it is the external edge that maintains its position unchanged. To make this happen also in the model, it is therefore necessary to set for the sections concerned that the point that must keep its position unchanged is not the axis but the outer edge, this can be done with the command "Assign - Area - Area Thickness Overwrites (Shells)..." (Figure 7.7).

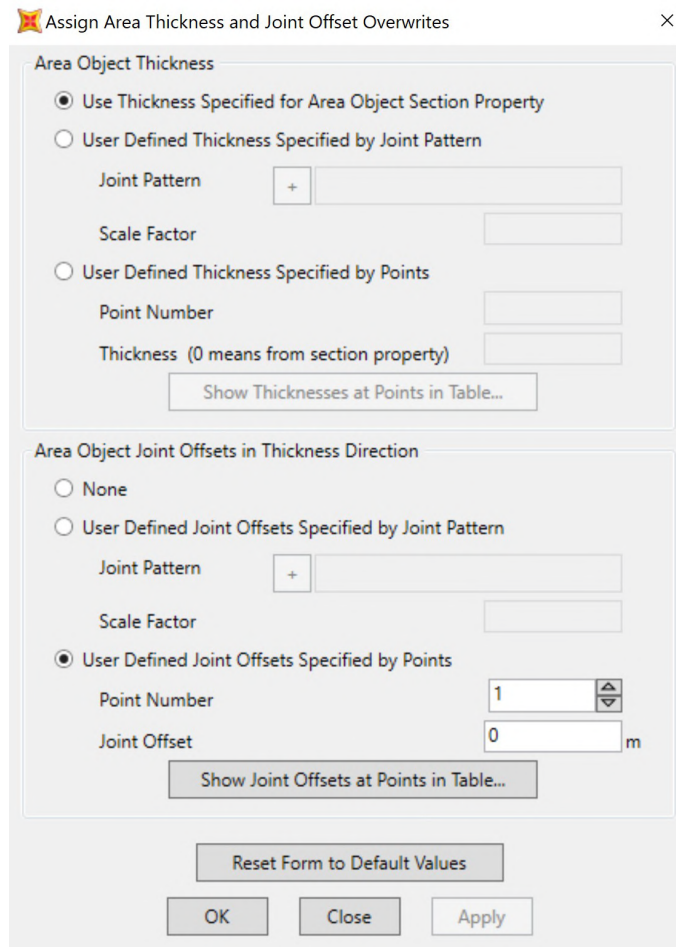
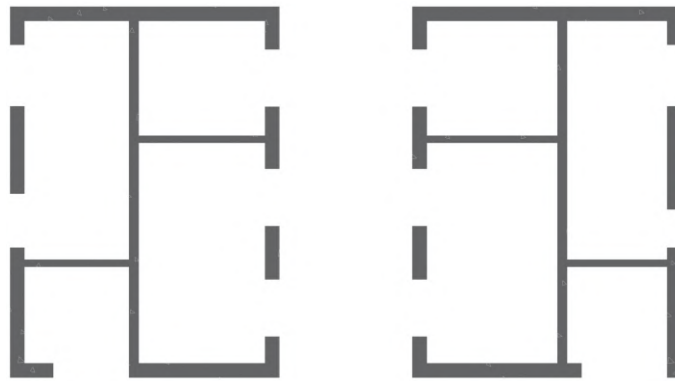


Figure 7.7: Modification of the insertion point of the sections.

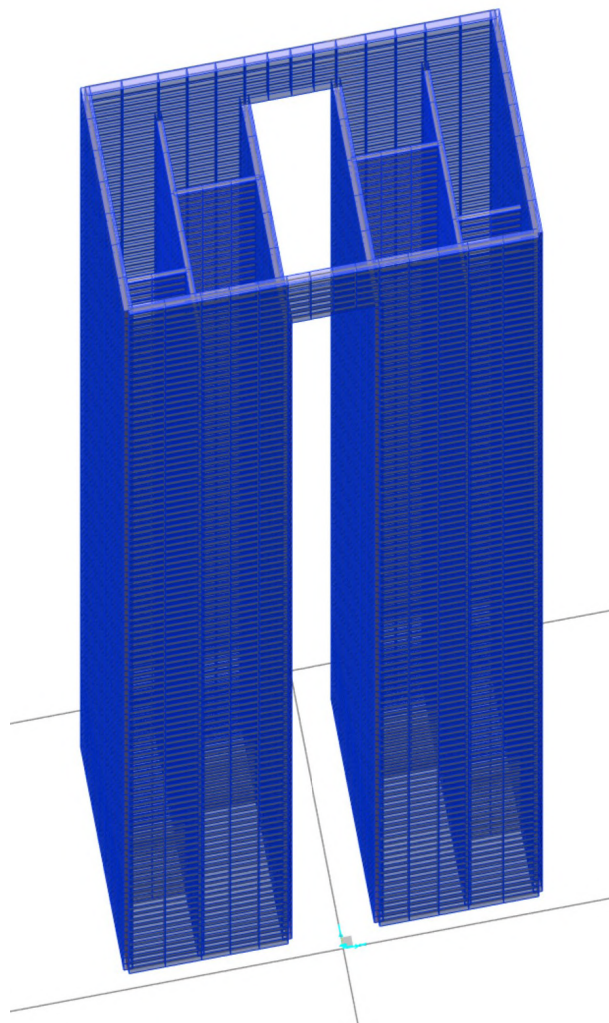
## 7.2.4 Box-shaped elements

The section of these elements consists of external walls with a greater thickness, and thinner internal walls (Figure 7.8); the thicknesses remain constant with the height, so for these elements it was sufficient to define a single type of section along the height. The two box-shaped elements contain the stairs and the elevators, and join the top floor to house the elevator machinery.



**Figure 7.8:** Plant of the box-shaped elements.

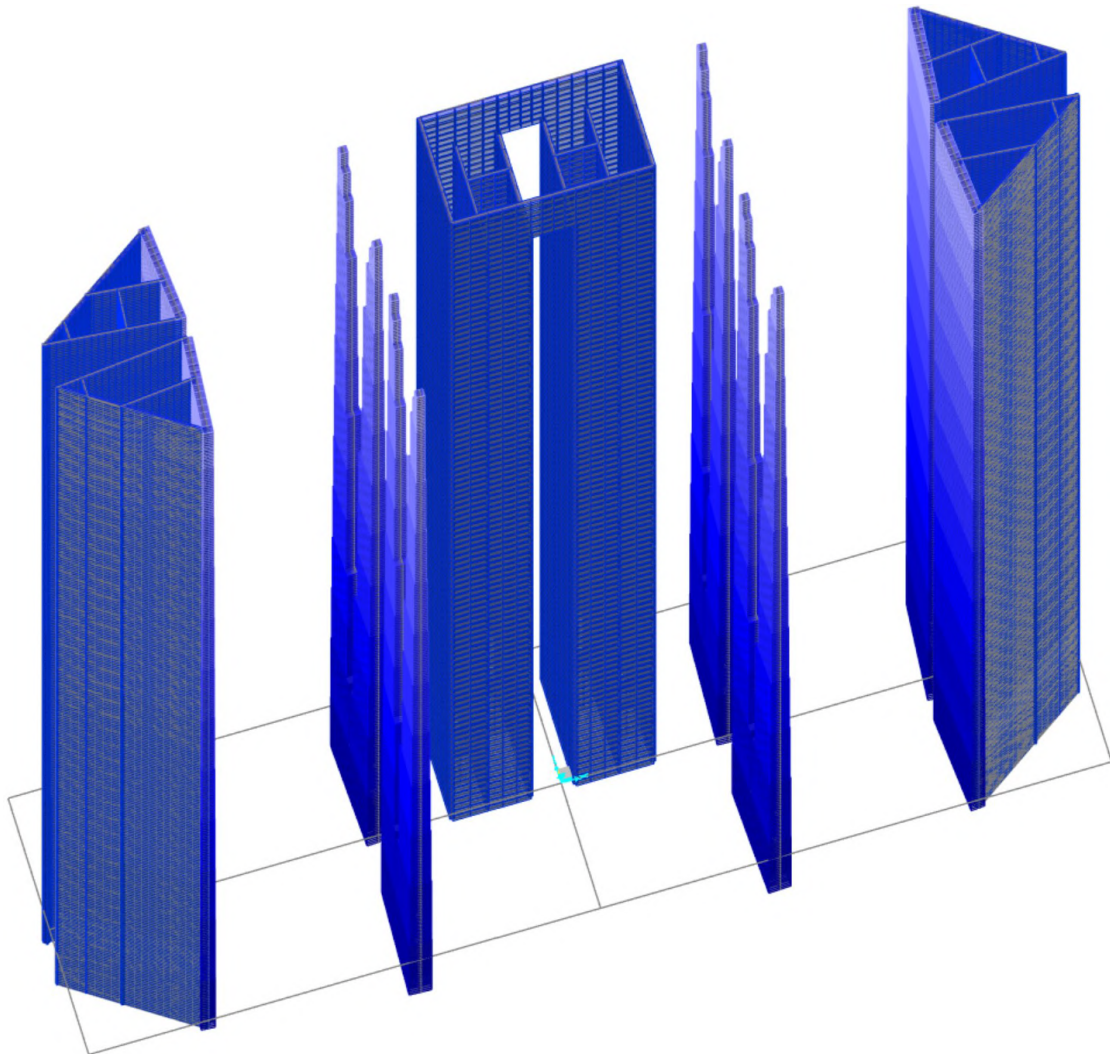
Also in this case, the finite elements have been drawn and then divided into several parts, thus obtaining the result in Figure 7.9, the openings corresponding to the doors were not represented in the calculation software.



**Figure 7.9:** Box-shaped elements.



Now all the resistant vertical elements have been defined, and the final result can be appreciated in the following figure.

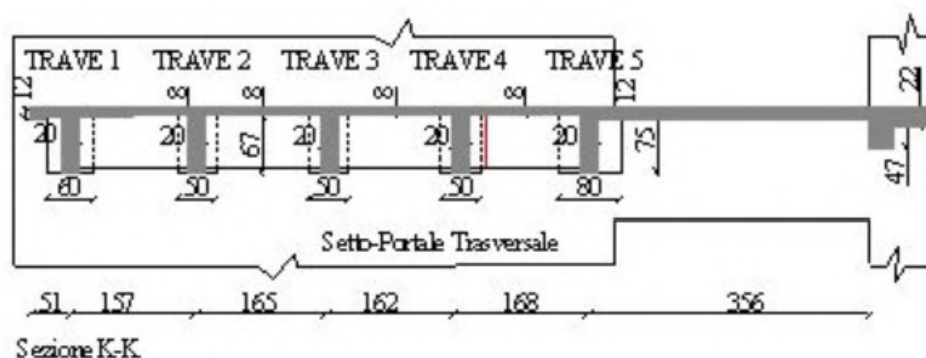


**Figure 7.10:** Resistant vertical elements.

### 7.2.5 Floor slabs

After defining all the necessary vertical elements, the next step was to define the horizontal ones.

In the documents obtained at the Cittadella degli Archivi of Milano [13], not sufficiently detailed information regarding the floor was found, but only the indications of the position of the T-beams in some drawings. An important detail was instead found in a technical report by the engineer Maurizio Acito [2] who is, as already said, one of the people who worked on the restoration of the building after the plane crash, in fact Figure 7.11 shows the thickness of the slab in the different parts.

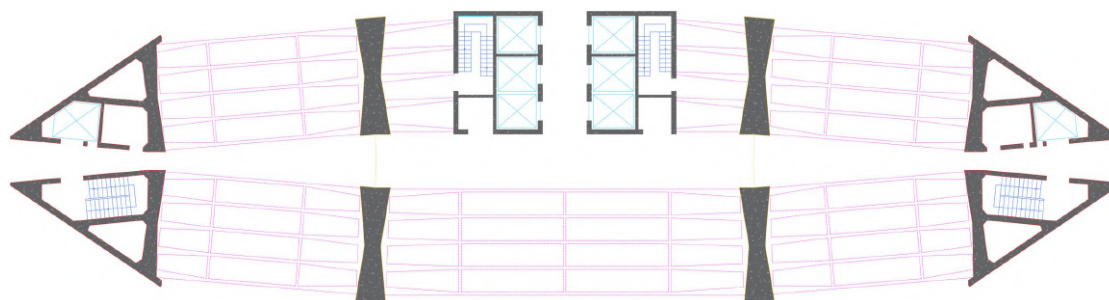


**Figure 7.11:** Resistant vertical elements. Source: Recupero statico delle strutture del 26.mo e 27.mo piano del grattacielo Pirelli. La dinamica dell'incidente. [2].

It therefore appears that the floor is divided into three types:

- The first type is included between the bigger vertical elements and it is formed by T-beams
- The second type is a 12 centimeters thick slab that can be found in the central corridor that goes from one part of the structure to the other, and over the external beams
- The third type is the one between the two box-shaped elements, consisting of a 22 centimeters thick plate.

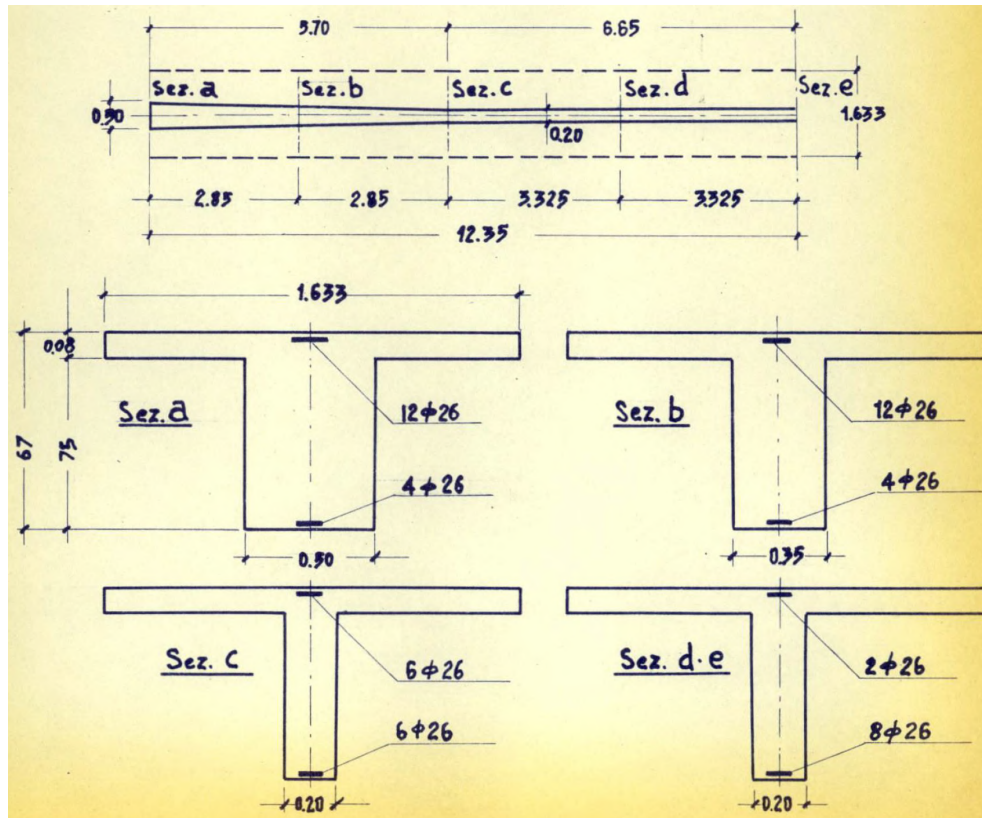
In the following figure, the floor made of T-beams, the central corridor, and the part of the floor between the two central box elements can be clearly identified.



**Figure 7.12:** Representation of the typical plant.

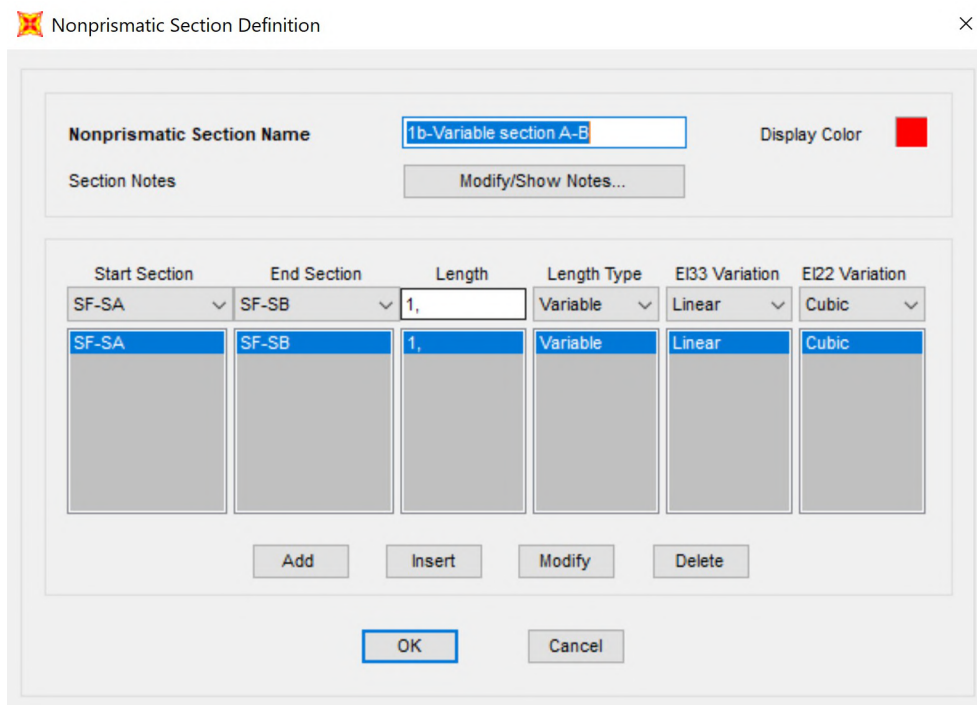
To create the parts with a constant slab, sections of 12 and 22 centimeters thickness respectively were defined, and after that the finite elements were drawn and divided into smaller ones. For floors made of T-beams, the dimensions are known thanks to a drawing in the test report, in Figure 7.13 can be seen the representation of a typical

beam of the central span. Moreover, the width of the web of the beams is variable, in the central part of the span it remains constant, and then grows linearly until it reaches the maximum width in correspondence with the constraint. The geometry was wanted in this way by Nervi, in order to reduce the deflections in the span, and to be able to resist the negative bending moment near the constraints [27].



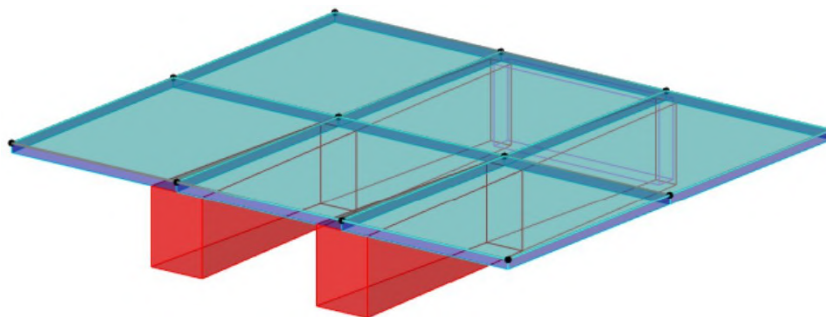
**Figure 7.13:** Particular of the T-beams. Source: Atti di fabbrica a conservazione perpetua: Grattacielo Pirelli, parte 1 e 2 [13].

The initial idea of replacing this part of the floor with an orthotropic plate was soon abandoned, because it turned out to be more expensive than representing the floor as it is in reality. So, in the end the floor was made with a combination of shell elements and beam elements: the flanges of the T-beams were made with two-dimensional elements as for the other parts of the floor, the webs were made using one-dimensional elements with "Draw Frame/Cable" command. Even the change in the width of the webs has been faithfully represented: the command "Define - Section Properties - Frame Sections... - Add New Property..." must be used and in the "Frame Selection Property type" option it is necessary select "Other - Nonprismatic". This will open a window like the one in Figure 7.14, in which the initial and final sections must be inserted, and also the trend of the variation of the moment of inertia in the two bending directions of the beam.



**Figure 7.14:** Example of nonprismatic section definition.

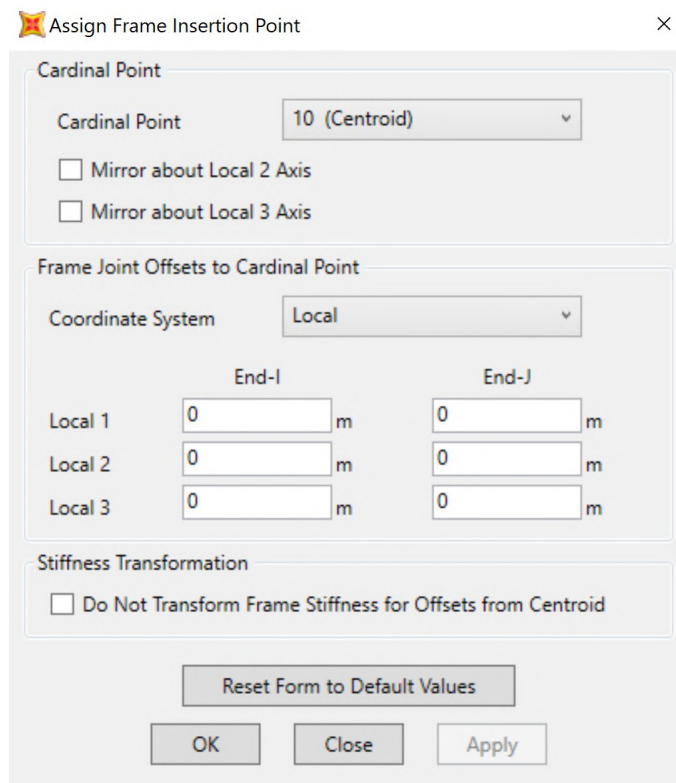
The last thing to do is move the slabs and beams relatively lower than the nodes, as in Figure 7.15.



**Figure 7.15:** Particular of the floor.

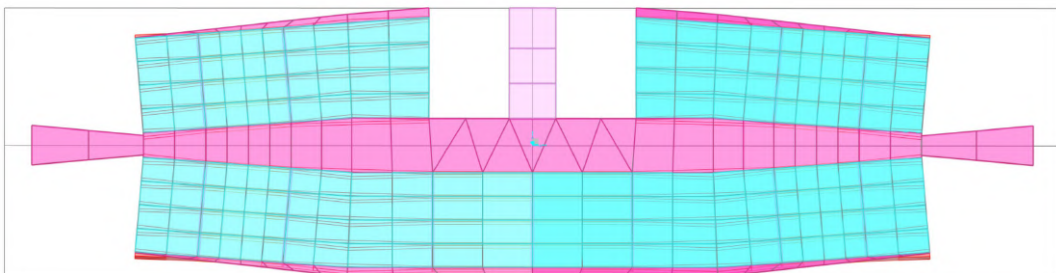
The slabs can be moved down with the command "*Assign - Area - Area Thickness Overwrites (Shells)...*" previously shown in Figure 7.7, so the upper side will be at the actual height of the floor where the nodes were placed. The beams can be moved down with the command "*Assign - Frame - Insertion Point...*" shown in Figure 7.16, in this way the section of the beam will not be defined around the segment delimited between the nodes but in a lower position, thus allowing the correct representation of the T-shape.





**Figure 7.16:** Modification of the insertion point of a beam.

Finally the slab was completed (Figure 7.17), and it was copied with the "*Edit - Replicate*" command to define all the others.

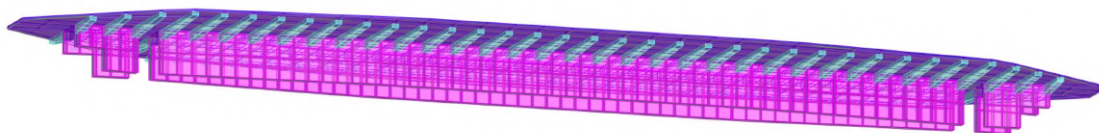


**Figure 7.17:** A complete floor slab.

## 7.2.6 Roof

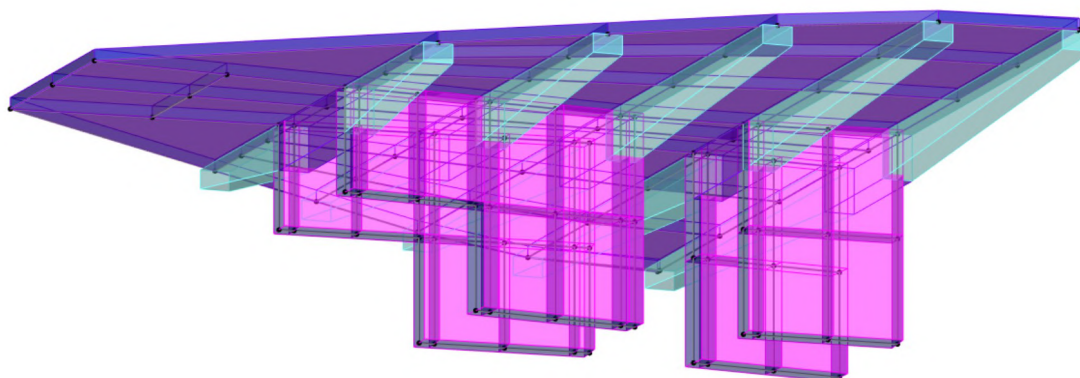
Regarding the roof, from the drawings [13] it was possible to understand that it is a concrete slab supported by walls and reinforced by ribs. Unfortunately no measurements were present, so the representation in SAP2000 is based on assumptions to make the roof look as close to reality as possible, the details can be found in Appendix A.

The slab and the walls were made with shell elements, a section was then created for each of them; the ribs, on the other hand, were made with beam elements to which a variable section was assigned in the same way as was done for the floor. Figure 7.18 represents the finished roof seen from below, so the walls and ribs are also visible.



**Figure 7.18:** The roof.

As was done for the sections of the shell and beam elements of the floors, the relative position respect to the insertion joints has been moved, as shown in Figure 7.19. In this way, the ribs are located exactly under the concrete slab and there is no interpenetration of matter even if the elements share the joints.



**Figure 7.19:** Left part of the roof.

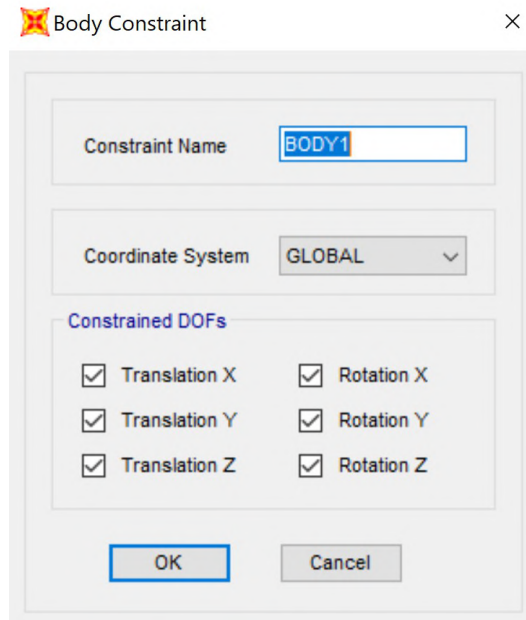
## 7.2.7 Additional measures

### 7.2.7.1 Constraints

Each structural element of the model has been created, but it is not yet ready to be loaded and analyzed. The last thing to define remains the connection between horizontal and vertical elements, because they have no joints in common.

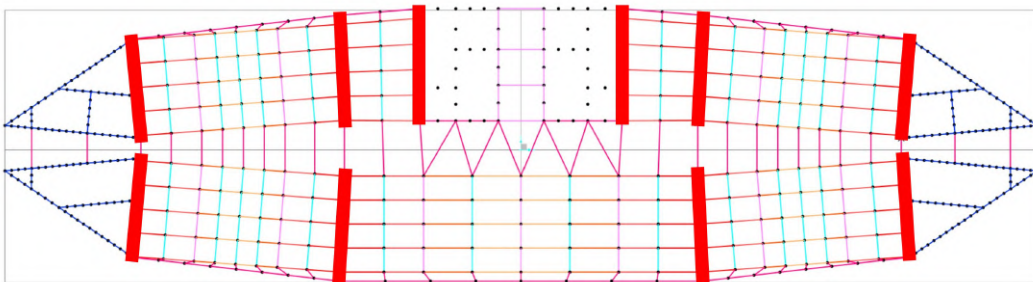
It is therefore necessary to define nodes of finite dimension, which is a group of joints to which the rigid behavior is applied to some Degrees of Freedom, this can be done with the command "*Assign - Joint - Constraints... - Define Joint Constraints... - Add New Constraint...*" (Figure 7.20), selecting the "Body" type. For the connection between floors and vertical load-bearing elements, all the Degrees of Freedom were blocked

in order to simulate an interlocking, except for the first four floors where only the translation along Z was blocked and the shell elements separated to simulate the simple support scheme.



**Figure 7.20:** Example of the definition of a body type constraint.

This is because, as indicated in "*Capolavori in miniatura*" [27] and in "*L'ossatura*" [28], Nervi wanted the floors supported and not embedded to avoid thermal deformation problems. The position of the constraints defined for each floor is highlighted with red lines in Figure 7.21.

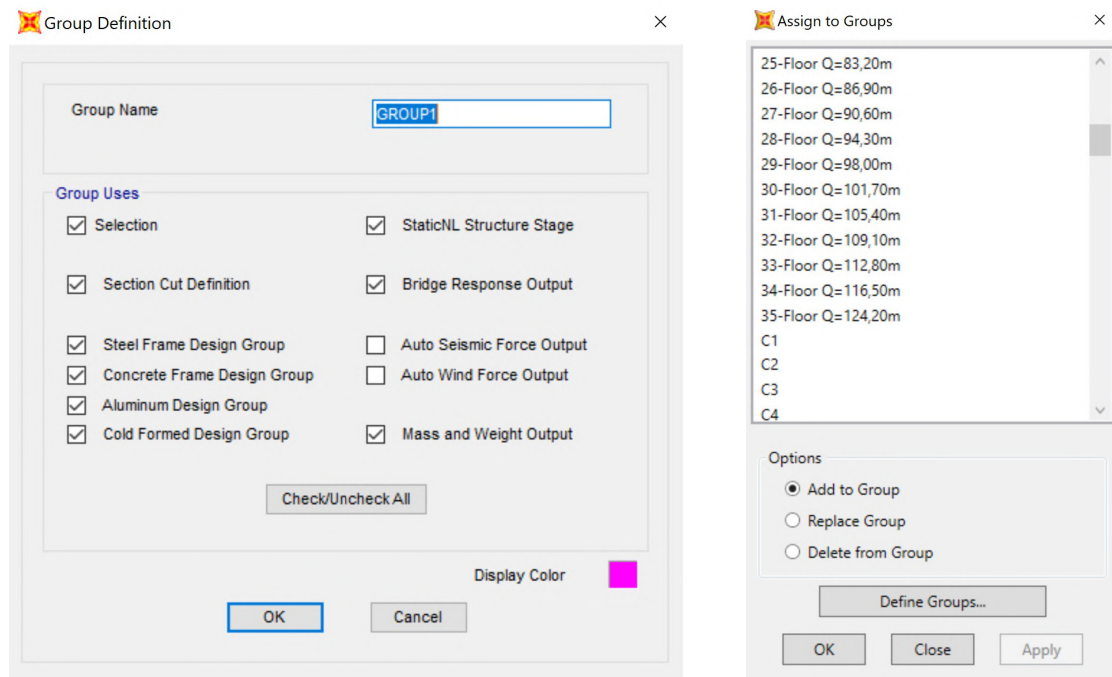


**Figure 7.21:** Position of floor constraints.

To check the correct definition of the constraints, the displacements were subsequently checked, first by blocking every degree of freedom and then leaving the rotations free: according to the Science of Construction, the deflection in the span with the hinges was five times greater. The same thing was done for the roof, in order to connect the base of the retaining walls to the rest of the structure as a rigid body.

### 7.2.7.2 Groups

In some cases it is useful to be able to easily select only some elements of the model, in order to view and perform operations only on them. For this purpose, groups can be defined and elements assigned to them with the "*Define – Groups...- Add New Group...*" and "*Assign – Assign to Group...*" commands (Figure 7.22).



**Figure 7.22:** Definition and assignment of groups.

Several groups have been defined, either just after creating some elements, or after completing the model, for example:

- One group for each floor and for the roof
- A group for each vertical resistant element and its internal parts
- A group for the elements imported from AutoCad and necessary for the drawing, to delete them when they would no longer be needed.

The nomenclature is very important for this purpose, it must be simple and tidy in order to easily understand which elements are assigned to the respective groups, in this way their use will be useful and effective.

## 7.3 Load definition

After the creation of the structure was completed, the loads defined in the previous chapter were added.

The procedure consists of creating some Load Patterns with the command "*Define - Load patterns...*" as shown in Figure 7.23, the corresponding load will be assigned to each one.

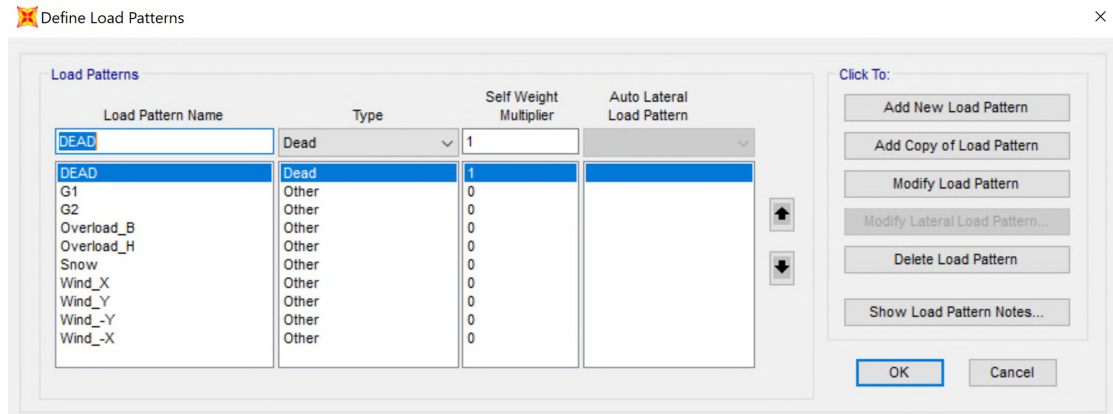


Figure 7.23: Load patterns definition.

Some have been assigned with the command "*Assign - Area Loads - Uniform (Shell)...*" (Figure 7.24), selecting the areas affected, they are  $q_{snow}$ ,  $g_1$ ,  $g_2$  and  $q_{catB}$  of the floor, and  $q_{catH}$ .

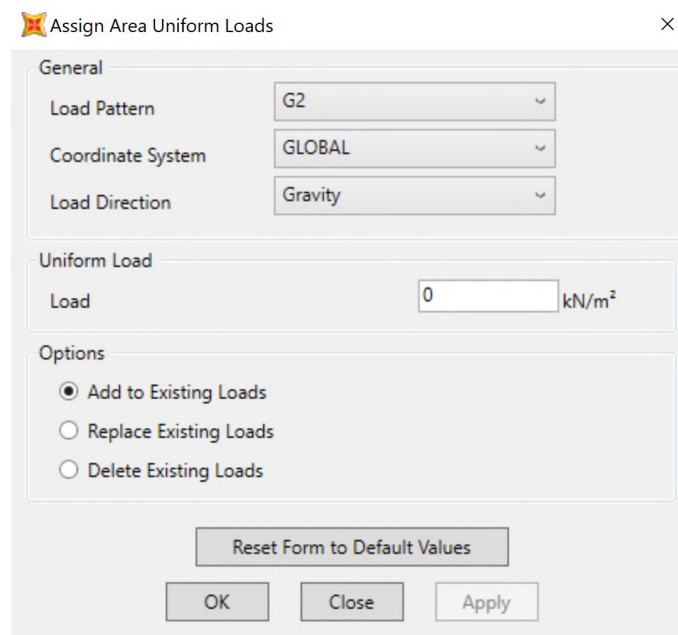


Figure 7.24: Area loads assignment.

For the loads acting on the stairs ( $g_1$ ,  $g_2$ ,  $q_{catB}$ ) or on the facades ( $g_2$ ,  $q_{wind}$ ) there was no element on which to apply the load directly, so the procedure was different. The manually calculated load was multiplied by the area or length in which it would be applied, and then it was divided by the number of joints in that area, in order to apply

a part of the load in each of them. The concentrated loads applied to each joint of the stairs are shown in Table 7.2.

**Table 7.2:** Concentrated loads of the stairs.

Element	Lateral elements		Central elements	
	n° joints	Load [ $kN$ ]	n° joints	Load [ $kN$ ]
$G_1$ ( $h = 3, 70 m$ )	38	2,69	13	5,02
$G_1$ ( $h = 3, 50 m$ )	38	2,64	13	4,94
$G_1$ ( $h = 3, 60 m$ )	38	2,67	13	4,98
$G_1$ ( $h = 3, 75 m$ )	38	2,70	13	5,04
$G_1$ ( $h = 5, 60 m$ )	38	4,98	13	9,34
$G_1$ ( $h = 7, 70 m$ )	38	5,44	13	10,15
$G_2$	38	0,20	13	0,42
$Q_{catB}$	38	1,63	13	3,38

The concentrated loads applied to each joint of the facades are shown in Table 7.3.

**Table 7.3:** Concentrated loads of the facades.

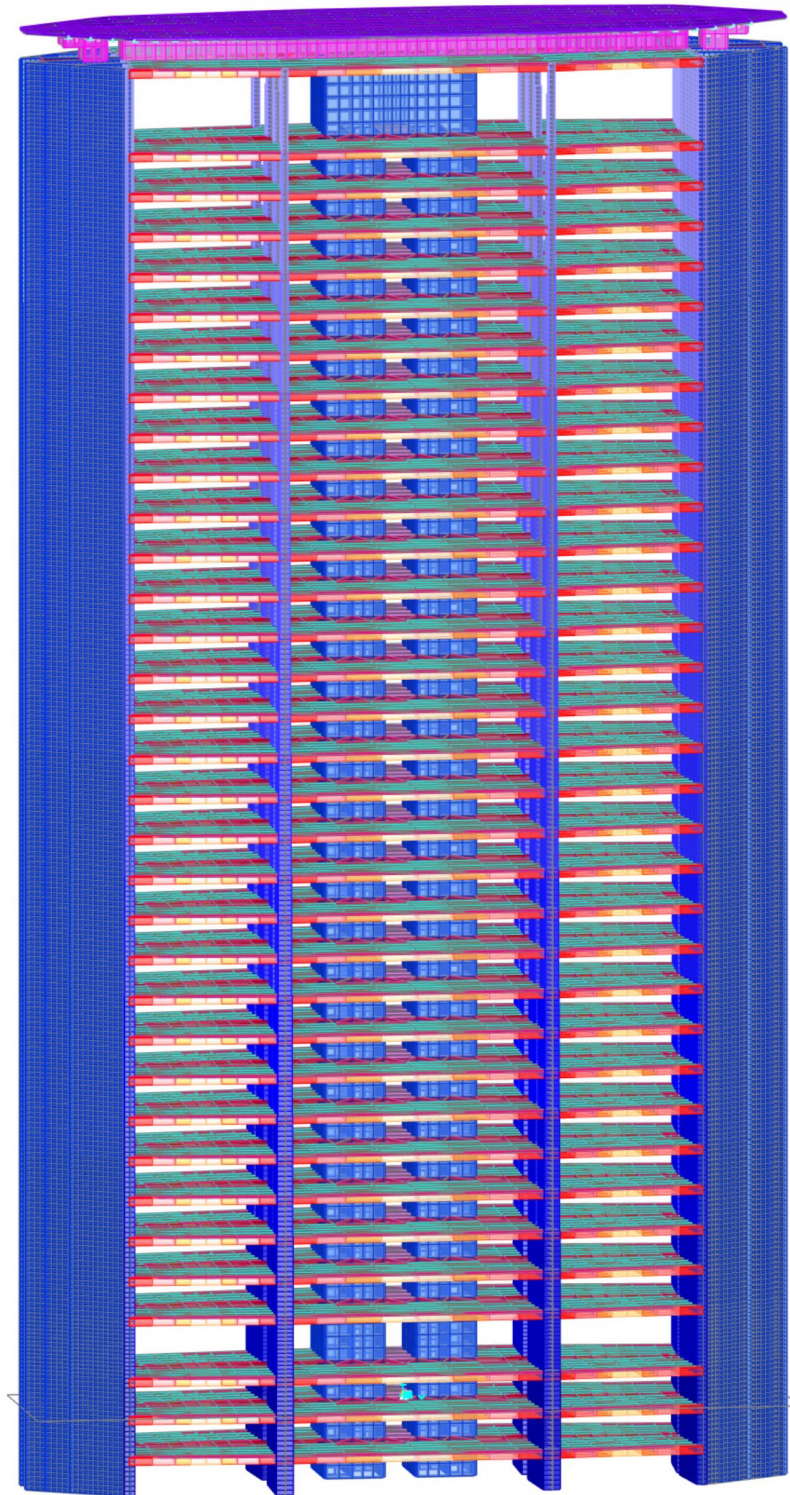
Element	Supported floors		Embedded floors	
	n° joints	Load [ $kN$ ]	n° joints	Load [ $kN$ ]
$G_2$ front ( $h = 3, 70 m$ )	31	2,32	25	2,88
$G_2$ front ( $h = 3, 50 m$ )	31	2,19	/	/
$G_2$ front ( $h = 3, 60 m$ )	31	2,59	/	/
$G_2$ front ( $h = 3, 75 m$ )	31	2,71	/	/
$G_2$ front ( $h = 5, 60 m$ )	31	4,65	/	/
$G_2$ front ( $h = 7, 70 m$ )	/	/	25	7,38
$G_2$ middle rear ( $h = 3, 70 m$ )	2	2,04	2	2,04
$G_2$ middle rear ( $h = 3, 50 m$ )	2	1,92	/	/
$G_2$ middle rear ( $h = 3, 60 m$ )	2	2,28	/	/
$G_2$ middle rear ( $h = 3, 75 m$ )	2	2,38	/	/
$G_2$ middle rear ( $h = 5, 60 m$ )	2	4,12	/	/
$G_2$ middle rear ( $h = 7, 70 m$ )	/	/	2	5,31
$G_2$ lateral rear ( $h = 3, 70 m$ )	30	1,78	22	2,43
$G_2$ lateral rear ( $h = 3, 50 m$ )	30	1,68	/	/
$G_2$ lateral rear ( $h = 3, 60 m$ )	30	1,99	/	/
$G_2$ lateral rear ( $h = 3, 75 m$ )	30	2,07	/	/
$G_2$ lateral rear ( $h = 5, 60 m$ )	30	3,56	/	/
$G_2$ lateral rear ( $h = 7, 70 m$ )	/	/	22	6,21

The same was done for the wind, but since in the load combination it has a zero coefficient, for the sake of brevity it was decided not to report the tables with the loads.



## 7.4 Complete model

The model is then finished, and can be appreciated in Figure 7.25. It is composed by 114667 joints, 10487 frame elements and 113850 shell elements.



**Figure 7.25:** The finished model.





# ANALYSIS AND RESULTS

Once the model is finished, after numerous checks, it is possible to proceed with the analyzes; this chapter will describe in detail the procedures used.

Each time a Load Pattern is created, the software automatically defines a corresponding Load Case; the difference between the two is that the former are used to define type magnitude and direction of the applied forces, while the latter are used to choose the type of analysis to be carried out. The Load Cases are also fundamental to see the results of the analyzes, of the single load or of their combinations. Due to limited computer power, only linear analyzes will be performed for this job.

For the purpose of this work, the necessary combination is the accidental one, with the coefficients as well as in 6.8. To define it, the command "*Define - Load Combinations... - Add New Combo...*" was used, the Load Cases are then inserted with the respective coefficient and the "*Linear Add*" type is selected (Figure 8.1).

Load Combination Data

Load Combination Name (User-Generated): Accidental

Notes: Modify/Show Notes...

Load Combination Type: Linear Add

Options: Convert to User Load Combo, Create Nonlinear Load Case from Load Combo

Define Combination of Load Case Results

Load Case Name	Load Case Type	Scale Factor
Overload_B	Linear Static	0,3
DEAD	Linear Static	1,
G1	Linear Static	1,
G2	Linear Static	1,
Overload_B	Linear Static	0,3

Buttons: Add, Modify, Delete

Buttons: OK, Cancel

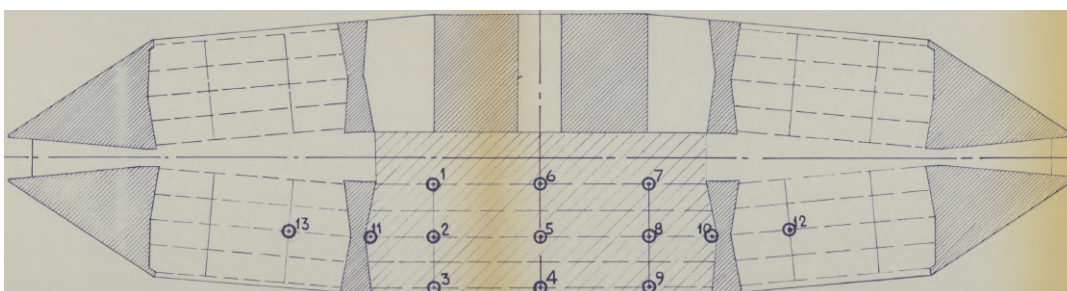
**Figure 8.1:** Accidental combination definition.

## 8.1 Static analyses

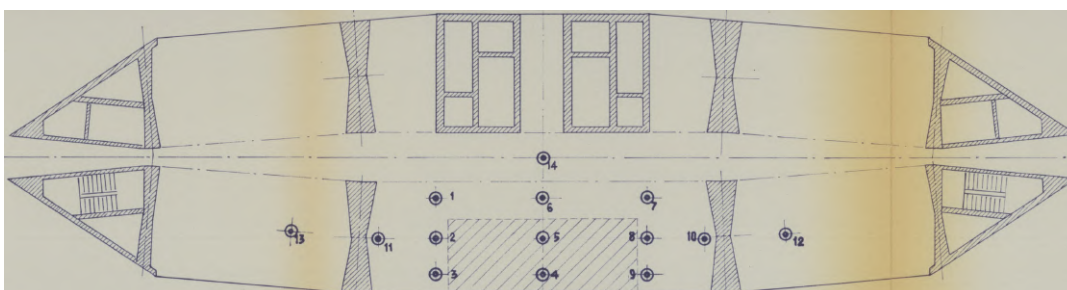
During the construction of the new Pirelli headquarters, load tests were carried out to verify the lowering of the floors, the ones carried out on the Pirelli building were:

- Load test 1, on floor at  $h = 16,60\text{ m}$ , with a load of  $380\text{ kg/m}^2 = 3,73\text{ kN/m}^2$
- Load test 4, on floor at  $h = 90,60\text{ m}$ , with a load of  $600\text{ kg/m}^2 = 5,89\text{ kN/m}^2$ .

The following figures show the loaded zone for each floor, and the position of the fleximeters.



**Figure 8.2:** Load test 1. Source: Atti di fabbrica a conservazione perpetua: Grattacielo Pirelli [13].



**Figure 8.3:** Load test 4. Source: Atti di fabbrica a conservazione perpetua: Grattacielo Pirelli [13].

Also to check the correct functioning of the model, it was considered useful to reproduce these tests. The application of the load was slow and not sudden, so a static analysis is fine to verify the results; it was not necessary to modify the Load Cases because those created automatically by the software have static calculation as default option, and it was not necessary to define any combination because only the contribution of the individuals is of interest.

After inserting a new Load Pattern for each case, the analysis was started with the command "*Run Analysis - Run Now*", and the results were compared in the following tables with those that were present in the test report of the time [13]. As it can

immediately be seen, due to some imperfection of the real structure, the lowerings of the fleximeters are not symmetrical. Furthermore, the results of the software are slightly different from the real ones, probably due to the approximations made to enter the data on SAP2000 or due to the lack of information.

**Table 8.1:** Comparison between the results of load test 1.

Position	$\Delta Z_{fleximeter}$ [mm]	$\Delta Z_{SAP2000}$ [mm]
1	-1,95	-2,53
2	-3,90	-4,46
3	-2,55	-5,60
4	-7,85	-11,81
5	-8,85	-9,49
6	-6,45	-6,73
7	-3,05	-2,53
8	-4,15	-4,46
9	-2,85	-5,60
10	-0,10	-0,03
11	-0,15	-0,03
12	0,00	0,04
13	0,00	0,04

Then especially for the Load Test 4, the joints of the model are not exactly in the same position as the fleximeters, so a bit of error in the lowering is also due to this.

**Table 8.2:** Comparison between the results of load test 4.

Position	$\Delta Z_{fleximeter}$ [mm]	$\Delta Z_{SAP2000}$ [mm]
1	-1,50	-3,24
2	-3,94	-4,59
3	-5,21	-5,81
4	-12,07	-12,71
5	-8,59	-10,02
6	-3,20	-7,13
7	-1,41	-3,24
8	-4,17	-4,59
9	-5,26	-5,81
10	-0,04	-0,09
11	-0,06	-0,09
12	0,31	0,20
13	0,34	0,20

In the end, the results obtained from the model can be considered satisfactory because the displacements are all of the order of millimeters.

## 8.2 Dynamic analyses

When an extreme event such as a vehicle collision or an explosion hits a structure, it can cause severe damage to the load-bearing elements that can spread to the rest of the structure and lead to progressive collapse. Eurocode 1-1-7 [38] recommends several strategies to provide sufficient robustness to a structure; with regard to the Pirelli building, the project can only be evaluated, as it is an existing structure.

The verification of the robustness consists in analyzing a threat-independent scenario by removing a column or part of a wall; in the real structure such damage would be instantaneous, therefore it is necessary to carry out a dynamic analysis in which the damaged bearing element is made to disappear suddenly. It has to be clarified from the outset that the damage caused by an exceptional action is not limited to the removal of a load-bearing element, but the aim of these analyzes is to evaluate the robustness of the building and for this reason the regulations require the use of this procedure.

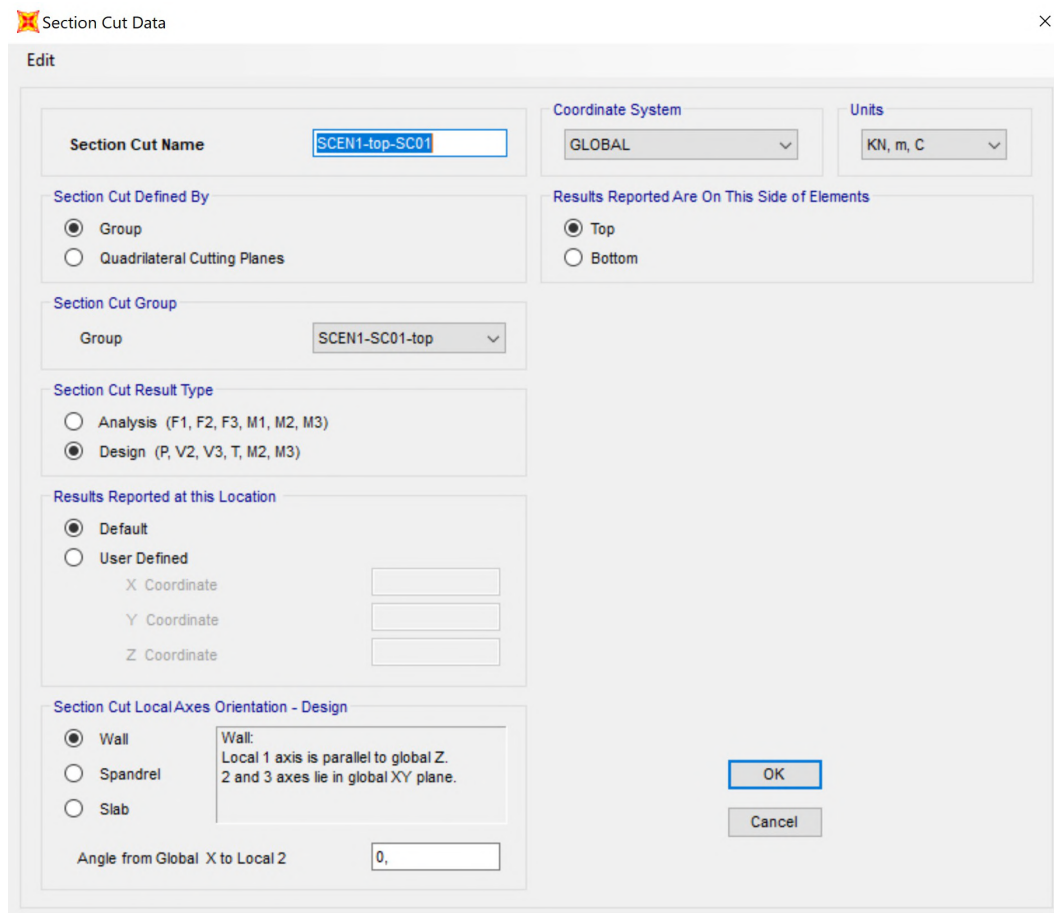
The Eurocode [38] only indicates to remove a portion of the wall with a maximum length of 2,25 times the height of the floor, so for more detailed information it was decided to refer to the American regulations of the General Services Administration [8] and the Department of Defense [17]. It was found that the entire wall from the base to the top must be eliminated and the minimum length of the removal is equal to the height of the floor. Furthermore, the walls to be eliminated are those at ground level or in other critical points; it was therefore decided that this work will focus on the central pillars-walls, while some scenarios concerning the lateral walls have been studied by Francesco Da Rif [15], a colleague of the author.

### 8.2.1 Procedure

To carry out the linear dynamic analyzes, the procedure recommended by the producers page of the software [34] was followed: it consists in replacing a structural element with equivalent forces, which will then be canceled by equal forces with opposite direction to simulate sudden damage.

The procedure required knowing the internal forces at the point where the element has to be removed, and controlling them is not as easy for shell elements as it is for beam elements, it was therefore necessary to use section cuts. The section cuts allow to see all the forces acting on any side of a two-dimensional finite element, and have been defined in the following way: the finite element and the two joints on the side of interest must be assigned to a group, and then with the command "*Define - Section Cuts... - Add Section Cut...*" the corresponding section cut was created (Figure 8.4). After repeating this for each finite element, a static analysis of the initial model was started,

and the results were exported in tabular form with the command "File - Export - SAP2000 MS Excel Spreadsheet .xls File...".



**Figure 8.4:** Creation and setting option of a section cut of a wall.

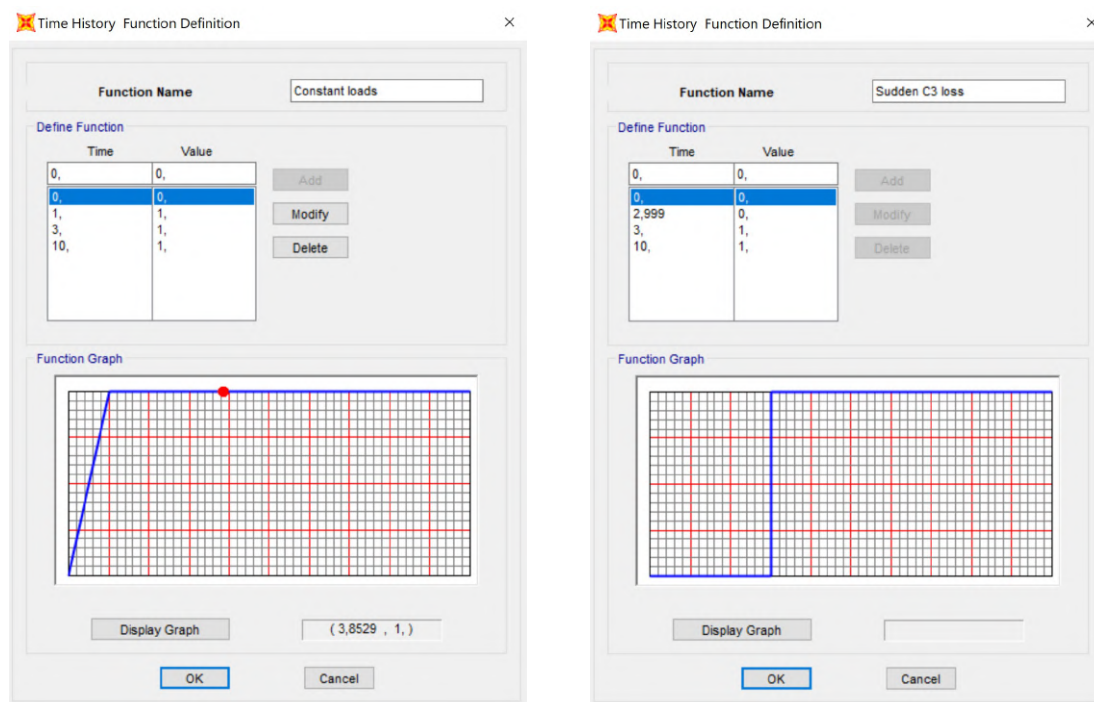
Subsequently, copies of the model were made, in one of them the finite elements of the wall were removed and a static analysis was performed; the results of it are not realistic because in reality the disappearance of the carrier is sudden, the purpose is only to make a comparison with the results of the dynamic analysis.

To perform the dynamic analysis another copy of the model was used, as already mentioned above the column and its disappearance were represented with equivalent forces equal in module but directed in the opposite direction, therefore two new Load Patterns ("G1-equivalent C3" and "Sudden-C3 loss") were created and added to the accidental combination with a unitary coefficient.

It was decided to enter the concentrated forces found with section cuts as forces per unit of length, so they were divided by the size of the finite elements. It is not possible to apply forces per unit of length directly on two-dimensional elements, for this purpose "fictitious" beams of zero mass and negligible stiffness have been created; inserting the forces per unit of length directly from SAP2000 would have taken a long

time, so they were entered in an Excel file which was then imported with the "File - Import - SAP2000 MS Excel Spreadsheet .xls File..." command.

Then, a fundamental step was the creation of functions that represent the trend of the forces over time with the command "Define - Functions - Time history... - Add New Function..."; as recommended on the CSIAmerica web page [34] two functions were created, represented in Figure 8.5.



**Figure 8.5:** Time history functions.

The first function, "Constant loads", grows up to one, and represents the structure and its loads and the forces equivalent to the removed wall; the second function, "Sudden C3 loss", grows from zero to one in a thousandth of a second, and permit to add in an instant the forces that cancel the "G1-equivalent C3", thus simulating the sudden collapse of the load-bearing element.

After that, it was necessary to modify the Load Cases in order to set the dynamic calculation, with the command "Define - Load Cases... - Modify/Show Load Case...", as showed in Figure 8.6. It was necessary to change the type of Load Case from "Static" to "Time History" and as solution the "Modal" type was chosen; a test was also made with the "Direct Integration" type, but the necessary time was much higher and the calculation was aborted before the end because the size of the result files could no longer fit on the computer's SSD. Instead, the "Frequency Domain" type solution was recently added to the software for this type of Load Case, the manual still needs to be updated and no articles describing its use have been found, so it was discarded a priori.



Load Case Data - Linear Modal History

Load Case Name: Sudden - C3 loss [Set Def Name] [Modify/Show...]

Load Case Type: Time History [Design...]

Initial Conditions:
 

- Zero Initial Conditions - Start from Unstressed State
- Continue from State at End of Modal History

 Important Note: Loads from this previous case are included in the current case

Modal Load Case: Use Modes from Case [MODAL]

Analysis Type:
 

- Linear
- Nonlinear

 Solution Type:
 

- Modal
- Direct Integration

 History Type:
 

- Transient
- Periodic

Load Type	Load Name	Function	Scale Factor	Time Factor	Arrival Time	Coord Sys	Angle
Load Pattern	Sudden - C3	Sudden C3 loss	1,	1,	0,	GLOBAL	0,
Load Pattern	Sudden - C3 loss	Sudden C3 loss	1,	1,	0,	GLOBAL	0,

Show Advanced Load Parameters [Add] [Modify] [Delete]

Time Step Data:
 

- Number of Output Time Steps: 1000
- Output Time Step Size: 0,01

 Mass Source: Previous (MSSSRC1)

Other Parameters:
 

- Modal Damping: None [Modify/Show...]

 [OK] [Cancel]

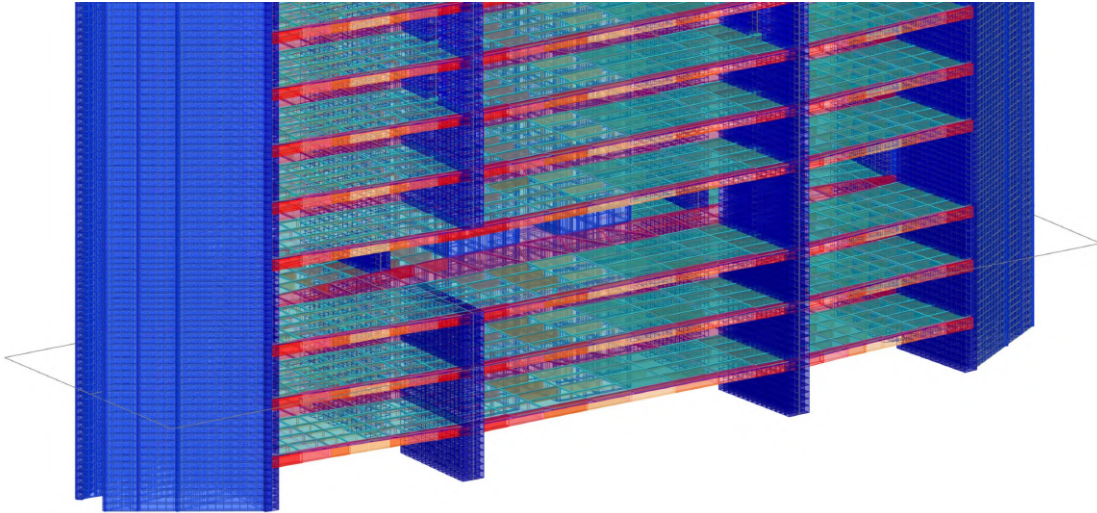
**Figure 8.6:** Setting options of the time history Load Cases.

After that, for each case the correct temporal function must be set, the "Constant loads" has been selected for all, except for the "Sudden-C3 loss" for which has been selected the homonymous one. For the output data after analysis, the smaller the step size is, the more precise the results will be, in this case an interval of 0,01 seconds has been chosen; and since the time functions last 10 seconds,  $10/0,01 = 1000$  was chosen as the number of steps. Then, the software web page [34] recommends setting a damping lower than 1 % or even zero for tall buildings, the latter value has been chosen in favor of safety. Finally, the more modes considered in the "Modal" Load Case calculation, the more precise the dynamic analysis will be, so 200 were set because when trying to consider more the results were practically equal. At the end, the analysis was started and lasted only an hour, using a computer with an Intel(R) Core(TM) i7-8705G CPU @ 3.10GHz, and the size of the result files was just under 10 gigabyte.

### 8.2.2 Scenario 1

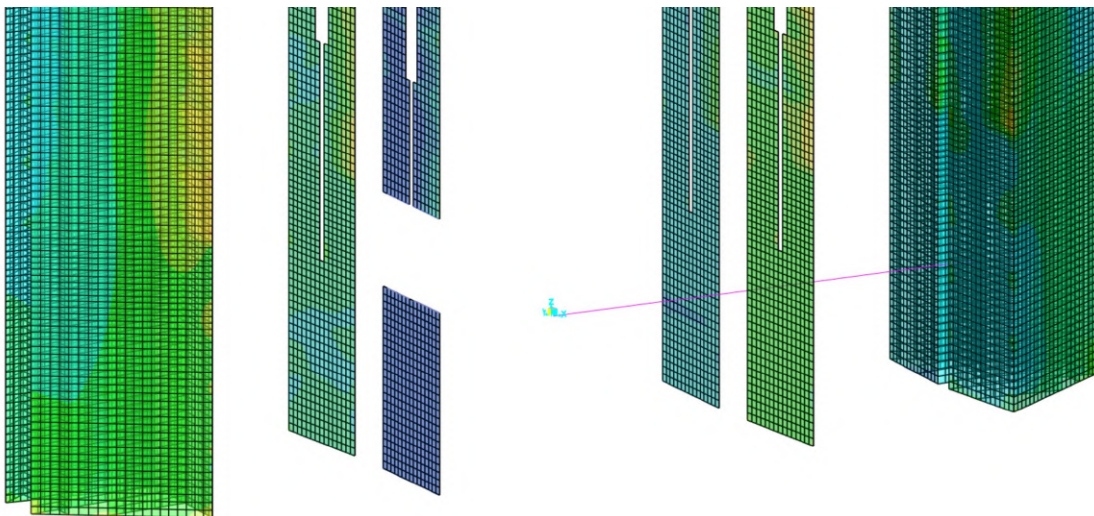
As already mentioned, the scenarios analyzed are both related to columns. The first taken into consideration consists into the loss of a wall at ground level due to a terrorist attack: the height of the floor is 5,60 meters, the limits on the length to be removed are  $L_{max} = 12,60 m$  from Eurocode [38], and  $L_{min} = 5,60 m$  from the two American standards [8] [17]. The removed wall, showed in Figure 8.7 is 6,84 meters long and

therefore respects both limits.



**Figure 8.7:** Scenario 1: removal of the anterior left wall.

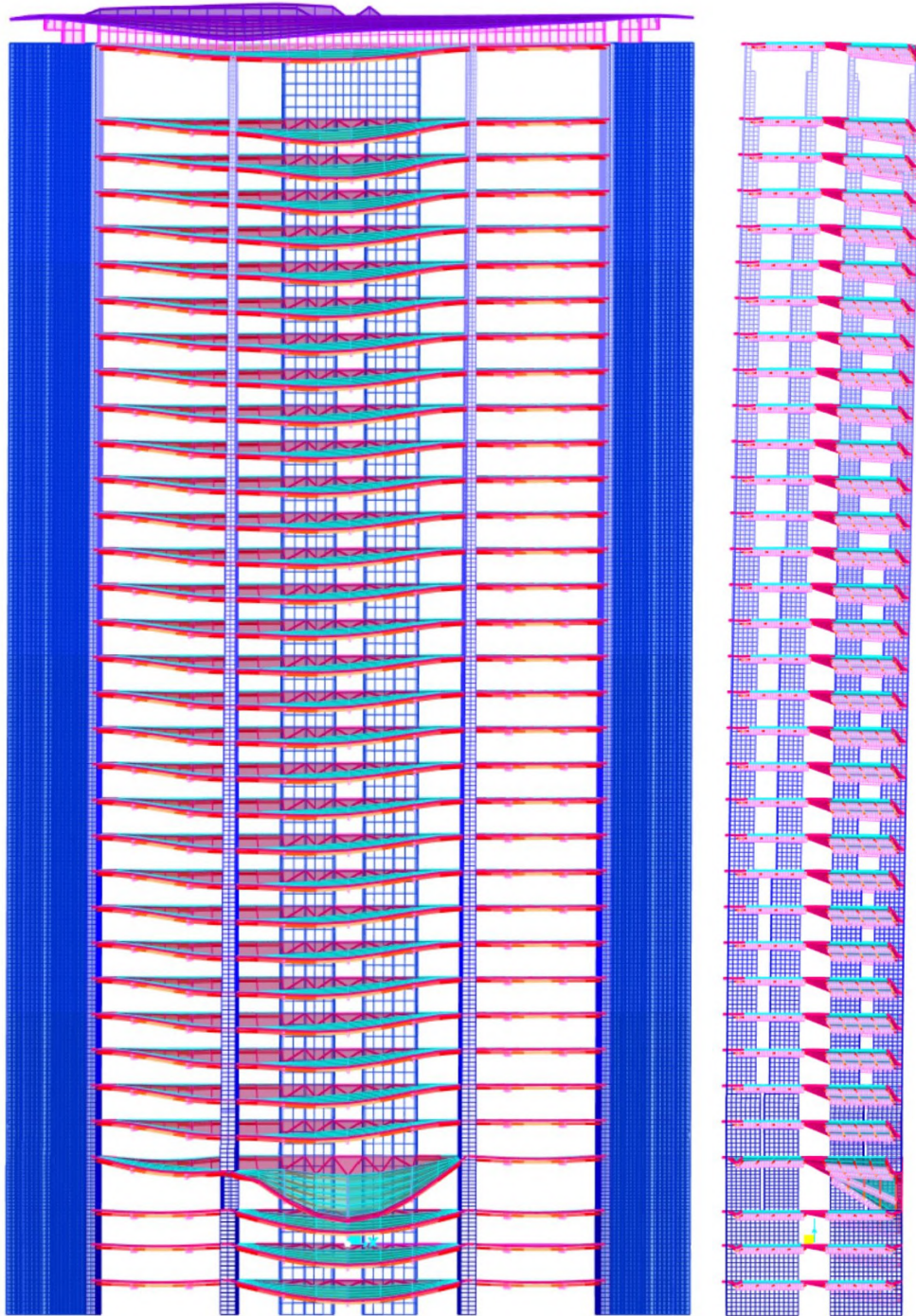
The first things to check are the stresses in the concrete to identify possible damage to the structure. With the command "*Display - Show Tables...*" it is possible to visualize any input and output data of the structure, therefore it was used to check the stresses for the accidental combination, the "*Envelope*" option was set to be able to see the maximum and minimum values reached during the 10 seconds analyzed. From the tables, it is obtained that in the shell elements the relevant stresses are those of membrane type. With the command "*Show Forces/Stresses - Shells...*" it is also possible to have a graphic representation of the stresses, which helps to identify the critical areas. For example, in Figure 8.8 it can be seen the stress distribution of the vertical elements, and the increase of the compression can be clearly identified in the elements beside the damaged wall.



**Figure 8.8:** Scenario 1: stresses of the vertical elements.



For vertical walls, stresses are lower than the characteristic strength of concrete except in a few localized areas of the two anterior pillars-walls, but this is admissible as the failure of an entire wall is an exceptional situation. The floors also show good behavior along the warping direction, while in the central slab there are some areas where the resistance of concrete is exceeded. The roof shows no problems, but only some damage in the area above the missing wall.



**Figure 8.9:** Scenario 1: enveloped displacements.

To check how the load is transmitted over the missing wall it is useful to see the deformation of the building with the command "*Show Deformed Shape...*", it is possible to check the results at any of the 1000 instants but to see the maximum displacements the "*Envelope*" option has been selected. As can be clearly seen in Figure 8.9 the loads are transmitted thanks to the Vierendeel action, this was predictable because it is a typical mechanism in tall buildings with a high degree of hyperstaticity and high flexural strength as the Pirelli building. The resisting action is not visible from the side, this is probably because the 12 centimeters central slab is not strong enough to develop a load redistribution mechanism. Then the fourth floor is supported, as already explained above, so it does not contribute to the support of the wall and shows much greater displacements than the others due to the dynamic effects.

Some vertical displacements were compared in Table 8.3 with those of the static cases in two points considered important: the external joint of the suspended wall, and the external joint of the central span. For the quasi-permanent combination (equal to the accidental one in this case) the current deflection limit is  $L/250 = 24/250 = 0,096 m$  [40], considering the age of the building, deflection is considered more than acceptable in the case without damage; then it is clearly visible how the displacements of both points increase after the damage, and from the comparison between the static and dynamic case it can be seen that the dynamic effects cannot be neglected.

**Table 8.3:** Comparison between displacements.

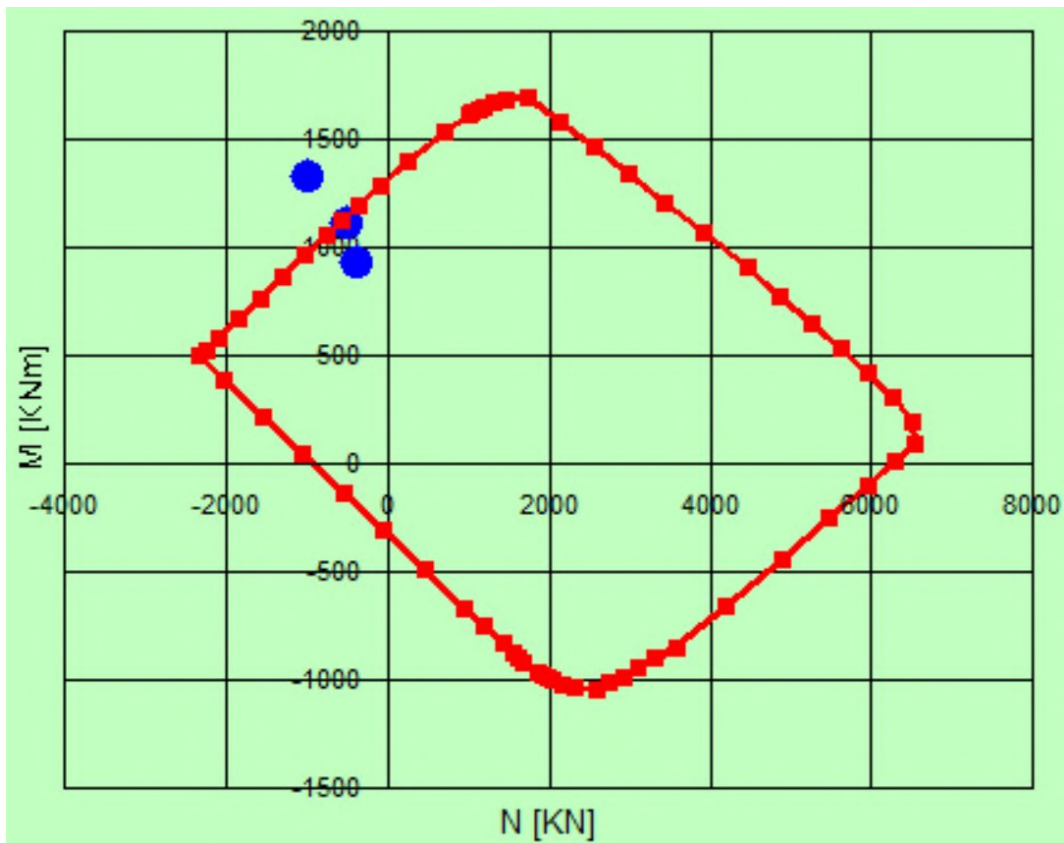
Position	Case	Settlement [m]
External point of damaged column 18676	Without damage	-0,003
	Static	-0,062
	Dynamic (envelope)	-0,110
External point of fourth central floor slab 98068	Without damage	-0,128
	Static	-0,164
	Dynamic (envelope)	-0,455

Then, as can be seen from the previous image, the damage to the wall caused a slight slope of the pillar-wall, resulting in a lateral displacement of 14 centimeters at the top.

Finally, the forces in the floors were checked. It was not possible to obtain a graphical representation of the moments and shear forces because the T-beams are composed of finite elements shell and frame, and the diagrams can be seen only for the latter type. Consequently, the values of the forces of the two types of elements were obtained with the command "*Display - Show Tables...*" and were summed together in order to obtain the total stresses in the points of interest. Since the main contribution from the tables is that of the frame elements, the forces in the instant in which the bending

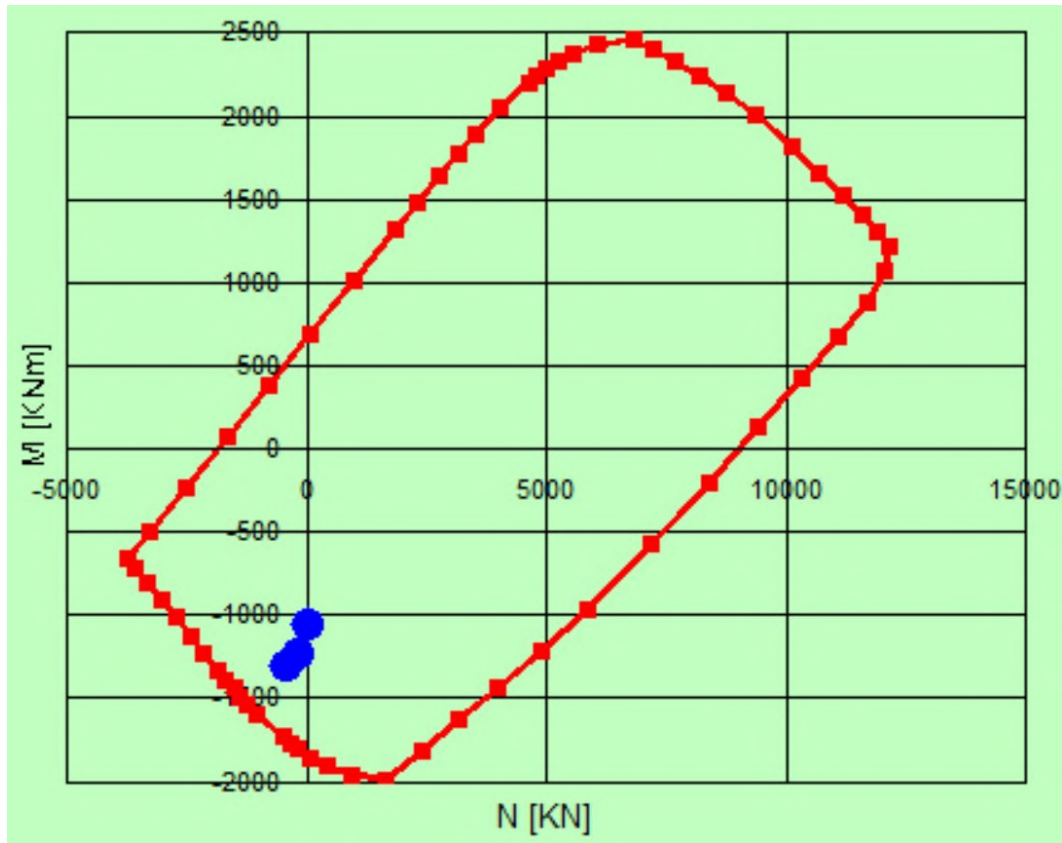
moments of these are maximum have been used for the verification. The resistance values have been obtained with the VCASLU software, thanks to which it is possible to find the resistance domain for any type of section and with any arrangement of reinforcement. The geometry of the section used in the various cases is that of Figure 7.13, and the resistances inserted in the program are the characteristics, because as already said before the situation considered is exceptional.

As previously mentioned, the fourth floor is constrained with hinges, therefore the positive bending moment in the middle of the span has been verified. All five beams have been checked, but Figure 8.10 shows the stresses of only three of them due to a software limit, together with the resistance domain; it turns out that only the outermost beam is not strong enough.



**Figure 8.10:** Resistance domain of the central section of the T-beams.

For the interlocked floors negative bending moments and shear stresses in the extremes were checked; the controlled section was the right extreme of the central span of the fifth floor, because it result to be the most stressed section from the SAP2000 tables. The Figure 8.11 represents the resistance domain obtained with VCASLU and the forces for the three most stressed beams, it appears that all the beams are sufficiently resistant.



**Figure 8.11:** Resistance domain of the right extreme section of the T-beams.

While the shear stresses were compared to the shear resistance for elements without shear reinforcement, as indicated by the Eurocode 2-1-1 [40] with the formula:

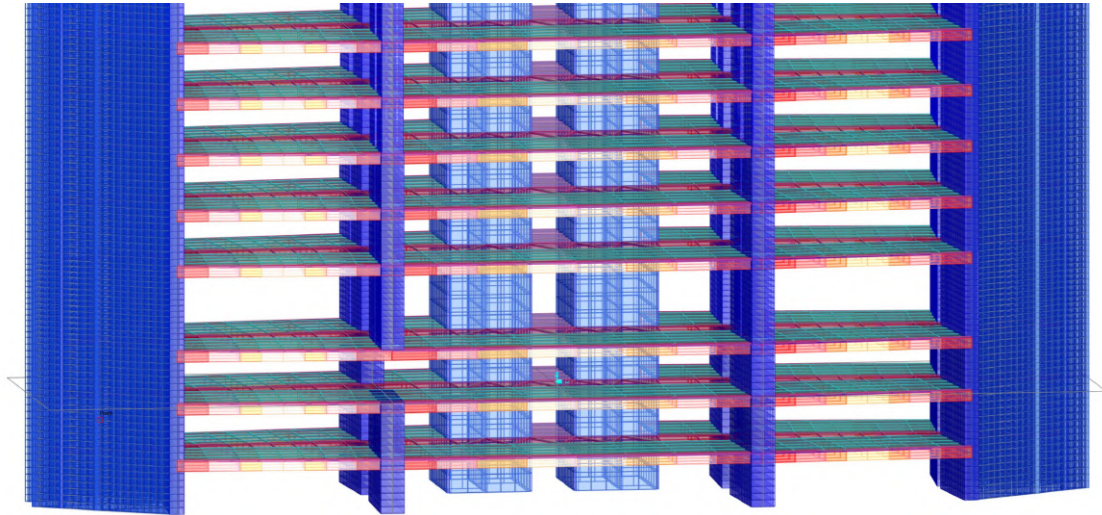
$$V_{Rd,c} = \max \begin{cases} (C_{Rd,c} \cdot k \cdot (100 \cdot \rho_L \cdot f_{ck})^{1/3} + k_1 \cdot \sigma_{cp}) \cdot b_w d \\ (0,035 \cdot k^{1,5} \cdot f_{ck}^{0,5} + k_1 \cdot \sigma_{cp}) \cdot b_w d \end{cases} \quad (8.1)$$

The result is  $V_{Rd,c} = 393,63 \text{ kN}$ , and it is bigger than all the shear forces acting on the five T-beams.

### 8.2.3 Scenario 2

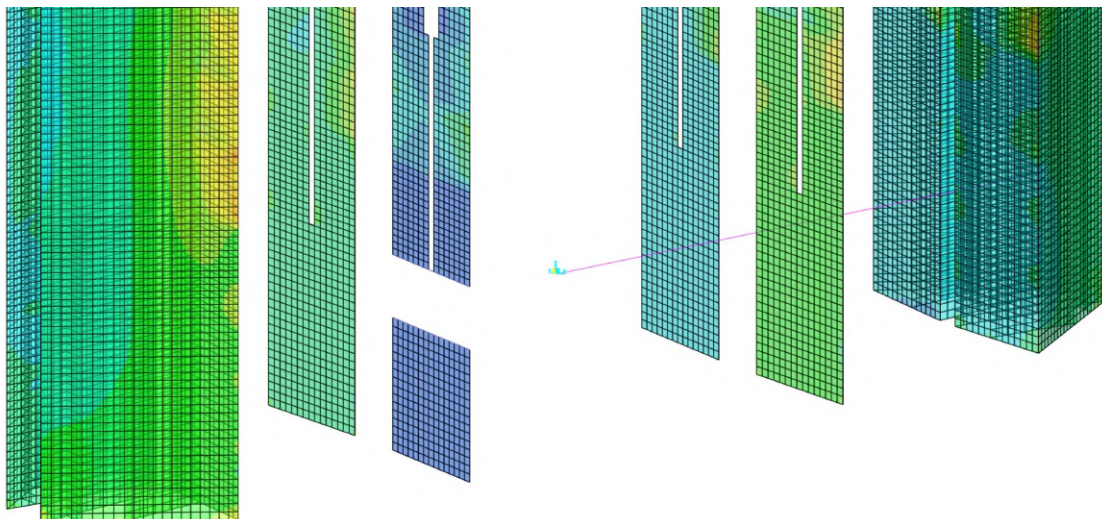
The floor below the ground level of the Pirelli building is intended for parking lots and it can be considered a critical area by American regulations [8] [17], so as a second scenario the same wall has been eliminated but on the lower floor respect to the previous case; a bomb in a car parked there is assumed as cause of the damage (Figure 8.12). The wall has not yet divided, so the finite elements removed were fifteen, for a total length of 7,33 meters; the floor height is  $h = 3,50 \text{ m}$ , so the removed length respects both limits, which are  $L_{min} = 3,50 \text{ m}$  and  $L_{max} = 7,88 \text{ m}$ .





**Figure 8.12:** Scenario 2: removal of the anterior left wall.

As for Scenario 1, the stresses in the elements have been controlled and exceed the characteristic strength of concrete only in a few points. The example of Figure 8.13 confirms the previous results, showing bigger compression on the elements beside the damage and a little increase of the stresses in the pillar-wall behind; in general, the values are slightly higher than in the previous case, this is because the load to be redistributed is bigger. The box-shaped elements in the middle did not show relevant results from this point of view, so they were not included in the image for a cleaner view of the other vertical elements.

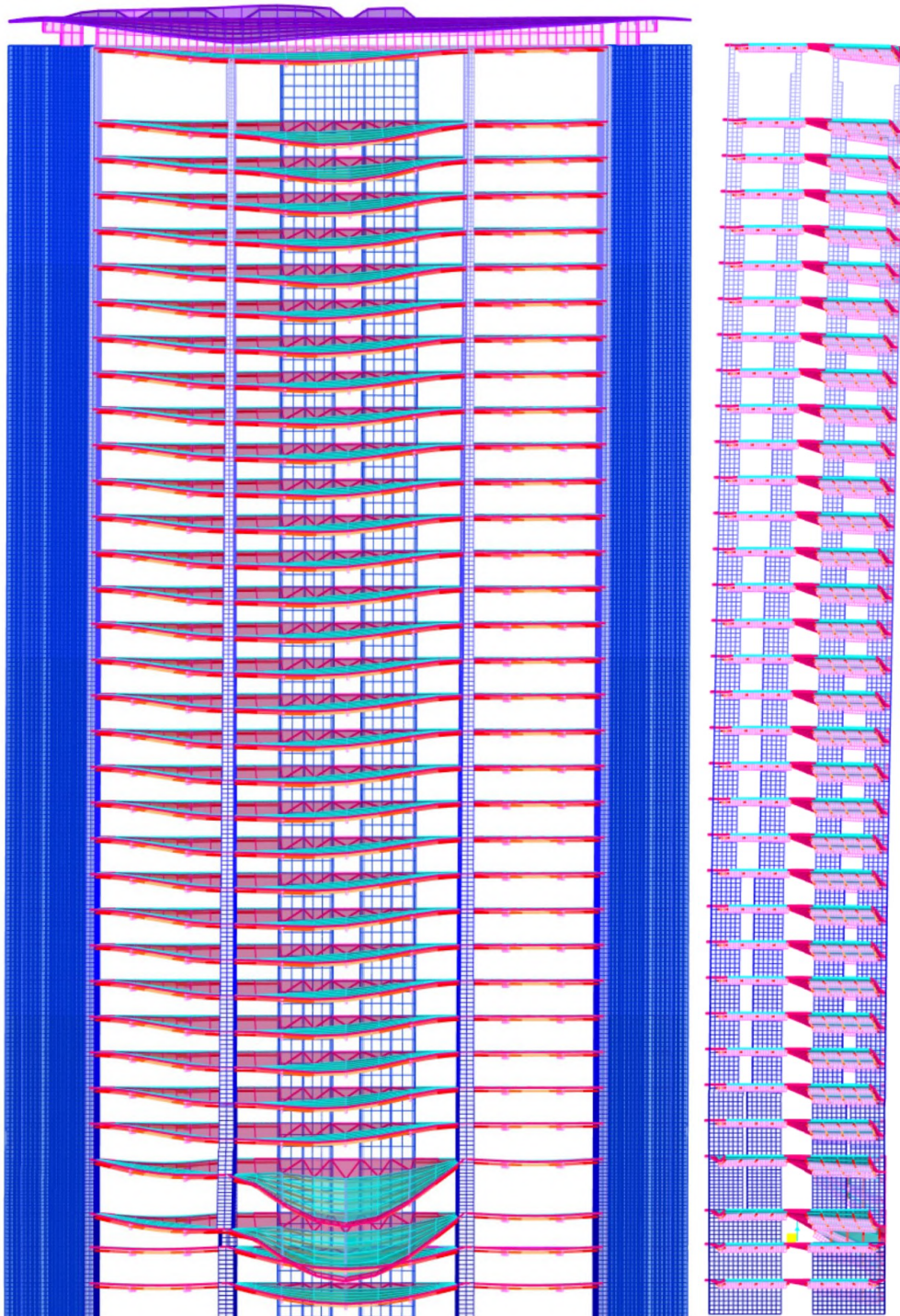


**Figure 8.13:** Scenario 2: stresses of the vertical elements.

Then, the deformations were checked and also in this case the building resists thanks to the Vierendeel action (Figure 8.14), visible only from the front view. In this case also the third floor shows much larger deformations than the others, due to the



different type of constraint and the dynamic effects.



**Figure 8.14:** Scenario 2: enveloped displacements.

Again, the vertical displacements in the different cases of a point of the column and of the central point of the slab of 24 meters were compared in Table 8.4, and the displacement of the top of the damaged wall was checked, resulting in 14 centimeters horizontally. Very similar to Scenario 1, the dynamic effects increase the displacements

with not negligible quantities.

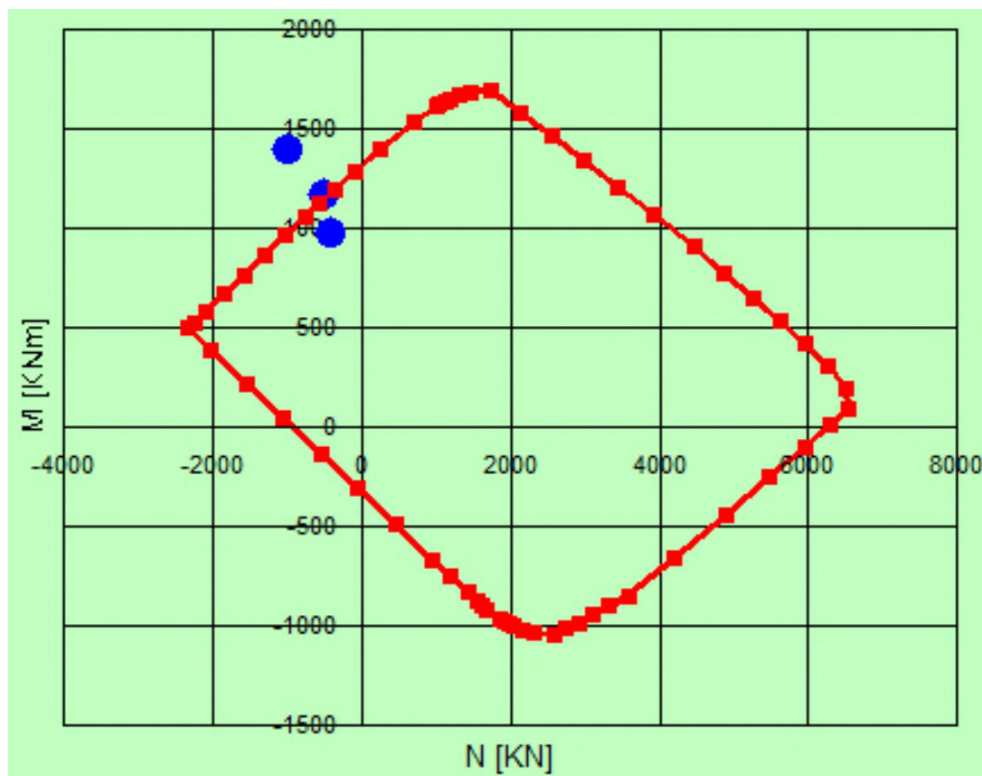
**Table 8.4:** Comparison between displacements.

Point	Case	Settlement [m]
External point of damaged column 249	Without damage	-0,002
	Static	-0,065
	Dynamic (envelope)	-0,113
External point of third central floor slab 97798	Without damage	-0,130
	Static	-0,168
	Dynamic (envelope)	-0,472

Also in this case the forces in the floor slab were checked as previously done:

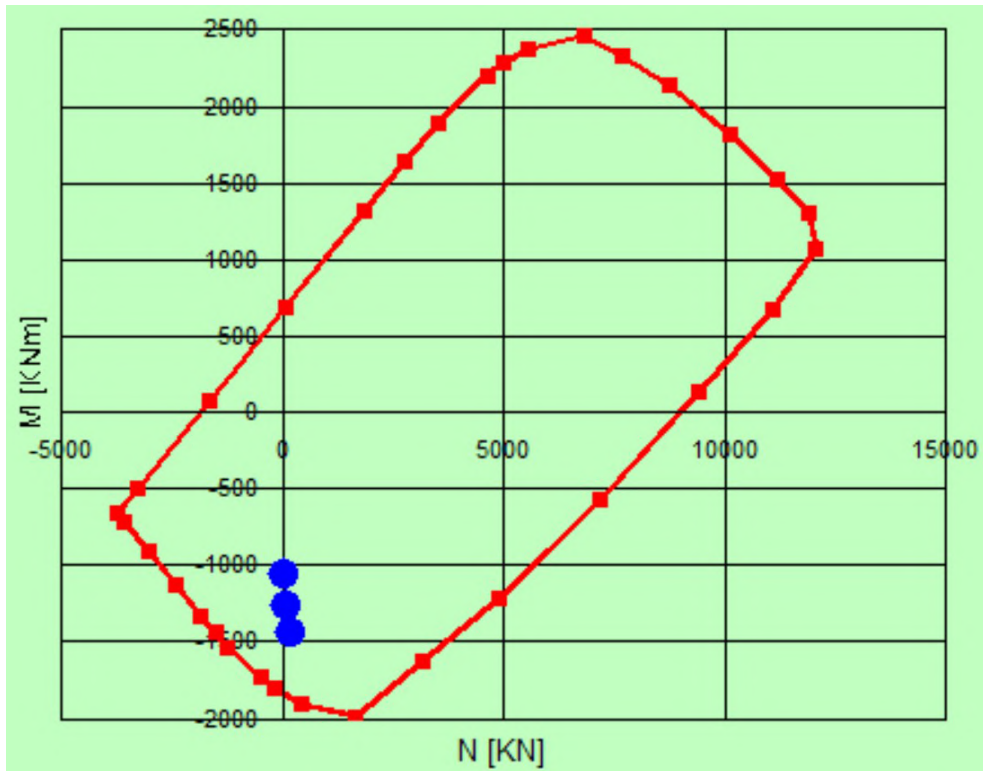
- The positive moment, in the middle span of the third floor
- The negative bending moment and the shear stress, in the right extreme of the fifth floor.

As expected, the results are similar to those of Scenario 1. For the supported floor the resistance of the two outermost beams is not sufficient (Figure 8.15).



**Figure 8.15:** Resistance domain of the central section of the T-beams.

For the locked floor, all the beams have enough flexural resistance (Figure 8.16).



**Figure 8.16:** Resistance domain of the right extreme section of the T-beams.

But not all the beams have enough shear resistance, the shear force in the outermost beam is  $V_{Ed} = 399,64 \text{ kN}$ , slightly bigger than  $V_{Rd,c} = 393,63 \text{ kN}$ .

## 8.2.4 Results

The results obtained show that after the damage of a front pillar-wall, the building resists mainly thanks to the Vierendeel action of the floors. This load transmission mechanism is activated when the structure has a great degree of continuity between the various elements and considerable flexural strength; this is the case of the Pirelli building which satisfies this requirement, thanks to the number of floors and the dimensions of the vertical and horizontal structural elements.

In fact, in the previous images it is clearly visible the wall hanging from the floors, which assumes the typical S shape that characterizes this resistance mechanism. The resistant action, on the other hand, is not visible from the side, this is probably because the 12 centimeters thick slab is not as rigid as the floors made from T-beams, and also because the damaged wall and the one behind it are very close and have different vertical displacements. The effect of the Vierendeel action is also confirmed by the distribution of stress in the vertical elements: in fact the walls beside the damaged one show a greater increase in stress, which is smaller for the wall behind.



When the Vierendeel action develops, in some points of the horizontal elements the bending moment increases while in others it changes sign. Therefore it is necessary to provide a continuous reinforcement along them, and for the same reason the vertical elements also need it. Since the case study is an exceptional situation, the stresses in the elements were compared with the characteristic resistance of the concrete and not with the design resistance, that is very reduced by some coefficients.

The tensions are always lower than the resistance except in the joints between the vertical and horizontal elements of the two pillars-wall on the top floors, especially in the intact one. This is probably due to the increased stresses because of the Vierendeel action and moreover for the reduction of the size of the vertical elements with the height. The 12 centimeters thick plate shows excessive tensile stresses, this probably due to the excessive difference in displacements between the damaged wall and the one behind. To better understand its behavior, a non-linear analysis should be used to take into account the reinforcement, and the result would probably be the formation of plastic hinges at the constraints with the two pillars due to large rotations. Finally, there is damage to the cover, in the area above the damaged wall.

From the control of the forces of the floors it has resulted in both cases that the external beams of the hinged floors break due to the excessive positive bending moment, while for the embedded floors it results that the bending resistance is sufficient and only in one beam the characteristic shear resistance is exceeded by 6 kN.

The localized damages highlighted in the stress control are accepted because the few points where this happens are in critical areas for which an adequate reinforcement would nowadays be envisaged in the design phase. Furthermore, all the calculations performed in this work are linear due to the limited computational power, therefore they do not take into account the contribution of the reinforcements in the elements; to obtain more likely results, it would be necessary to perform nonlinear analyzes. The same can be said for the results obtained from the control of the forces in the floors, the resistances are exceeded slightly and only in a few cases so an accurate result would be obtained with a non-linear analysis. Furthermore, the shear forces were calculated at the end of the beams and therefore in the axis of the walls, if they had been calculated taking into account the thickness of the latter the checks would have been passed.



## CONCLUSIONS

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At the end of this work, numerous considerations can be made.

First of all, the behavior of the Pirelli building was studied in response to the damage to one of the pillars-walls of the front facade. The resistant mechanism was that of the Vierendeel action, evident in the floors made of 75 centimeters high T-beams, and not found in the slab behind which is only 12 centimeters thick, probably because it is not rigid enough.

This alternative load path mechanism was an expected result because it requires structures with a great degree of continuity and considerable resistance to bending, qualities possessed by the structure thanks to the high number of floors, and the massive walls and thick horizontal elements, interlocked among them. If the building had fewer floors, the structure probably wouldn't have behaved the same way.

Stresses were also checked and compared with the characteristic resistance of the material and not the design one, because the analyzed situation was exceptional. They almost all respect the limit, except in some critical areas as highlighted in the previous chapter. This result was considered valid because the analyzes carried out were of a linear type, consequently the presence of bars and any additional reinforcements was not taken into consideration. However, in areas where a high moment is expected or in critical areas such as the joints between vertical and horizontal elements, the presence of adequate reinforcing bars is essential; therefore, to take this into account, it is better to carry out nonlinear analyzes which are very onerous from the computational point of view. In this way it would be possible to evaluate the behavior of these areas and the consequent effects on the rest of the structure.

From the control of the forces of the floors it was seen that the outermost beams of the hinged floors suffer damage due to too high positive moments. A non-linear analysis would allow to obtain more precise results, however the damage is considered acceptable because it is located in a small number of beams and moreover because the hinged floors do not contribute to the Vierendeel action. Negative moments do not cause problems. And the shear strength is exceeded in only one beam, but by a negligible amount, the verification would be certainly passed calculating the shear force

at the edge of the wall, taking into account its thickness.

The final judgment on the Pirelli building is that it can be considered robust; the fact that it was built in the sixties when the concept of robustness was not yet known and therefore not foreseen by the regulations, highlights today more than the time the extraordinary nature of the structure and the genius of Pier Luigi Nervi.

Thanks to this work it was also possible to acquire potentially useful experience in the professional field. Before this moment, the knowledge concerning the calculation software SAP2000 was trivial and superficial, but now it is sufficient to perform even more advanced operations. Other things learned concern the research of the material, for example knowing which websites are dedicated to scientific research and from which it is possible to obtain information. However, the research was very demanding and made it possible to understand that it is not enough to use only the Internet, but it is also necessary to telephone to offices or archives and even private entities; and other times it is necessary to travel to distant places, spending more time on the journey than on finding the information.

Finally, the aim of this work was not only to investigate a current "hot topic" which is structural robustness. The author's intentions were also to learn more about tall buildings, having found the subject fascinating during the "Conceptual Design of Singular Structures" course at the Universidad Politécnica de Valencia. The ultimate goal would be to turn this interest into a job in the future; consequently, despite the difficulties, this work was tackled with great enthusiasm, and on a personal level the result is considered satisfactory.

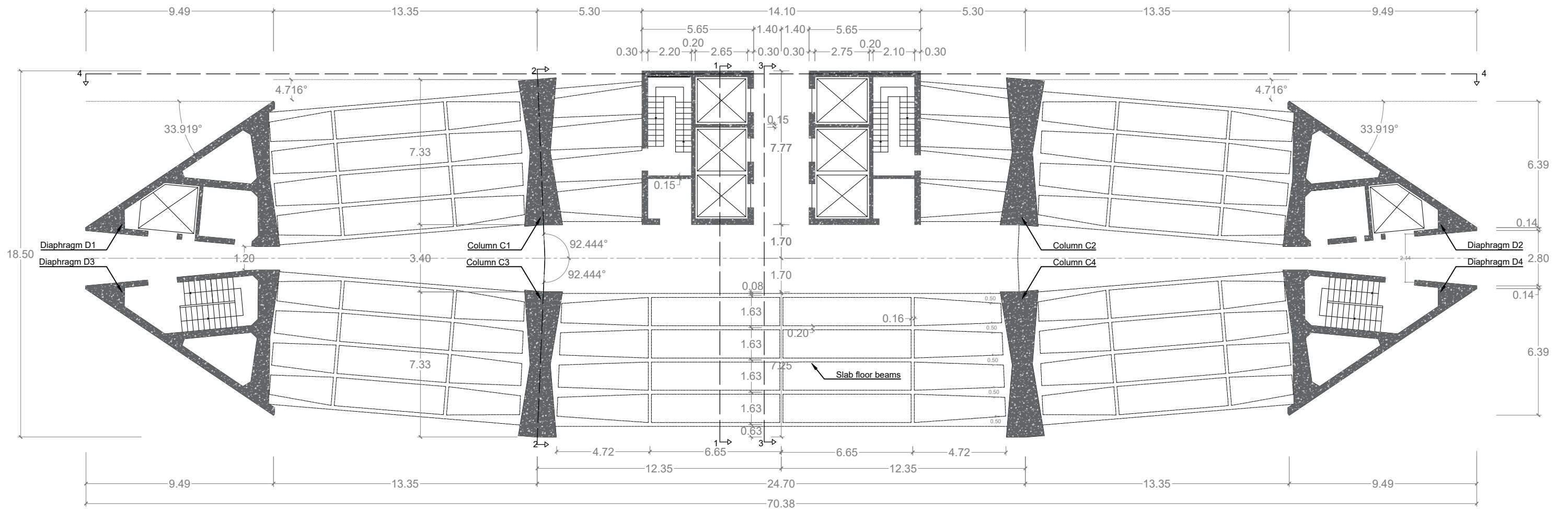
## APPENDIX

# DRAWINGS

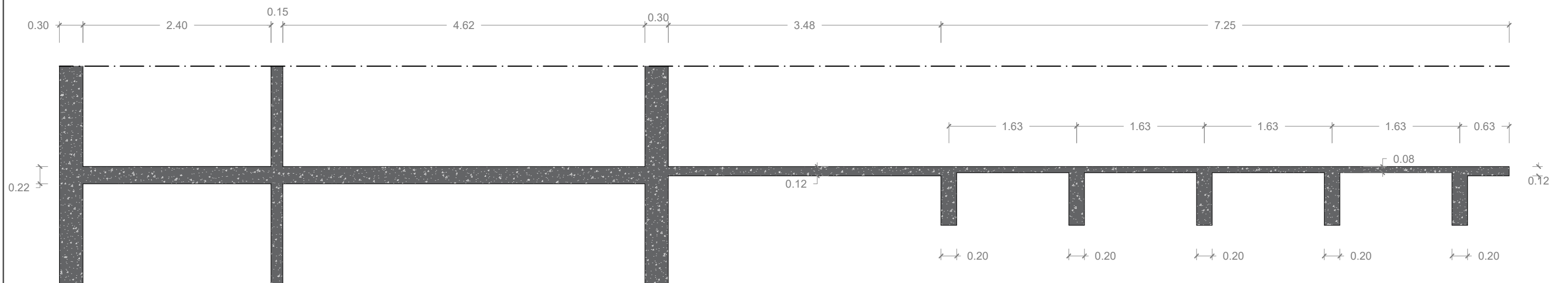
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This appendix shows all the geometric information relating to the structural elements of the Pirelli building. As already described above, assumptions were made for some elements, as no information was found. The technical drawings were realized using the software AutoCAD.

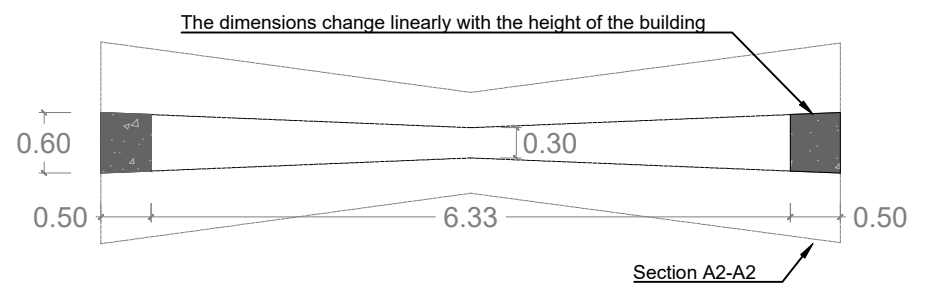
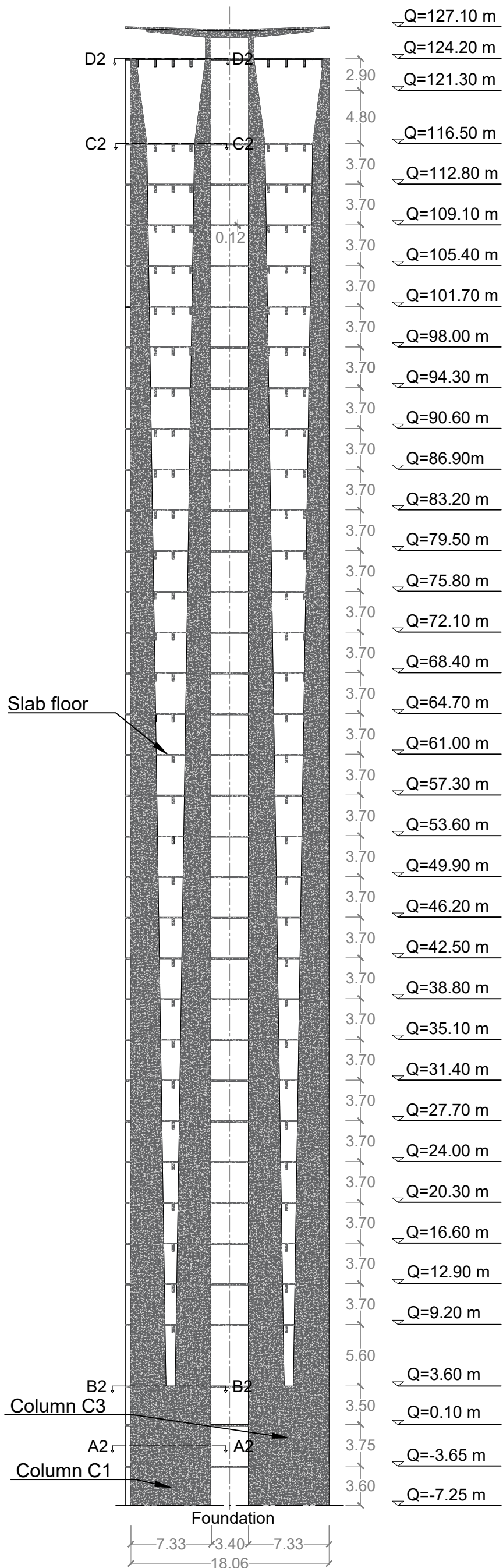
### Plan view of a generic storey (scale 1:200)



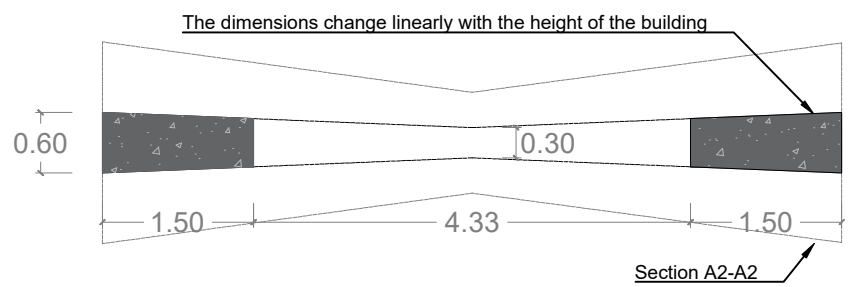
### Transversal section 1-1 of the floor slab (scale 1:50)



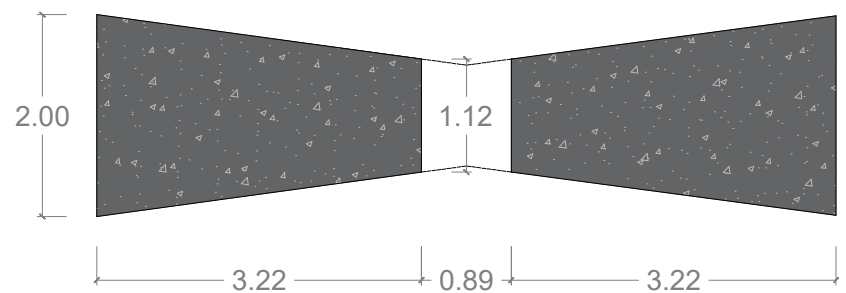
# Transversal section 2-2 (scale 1:400)



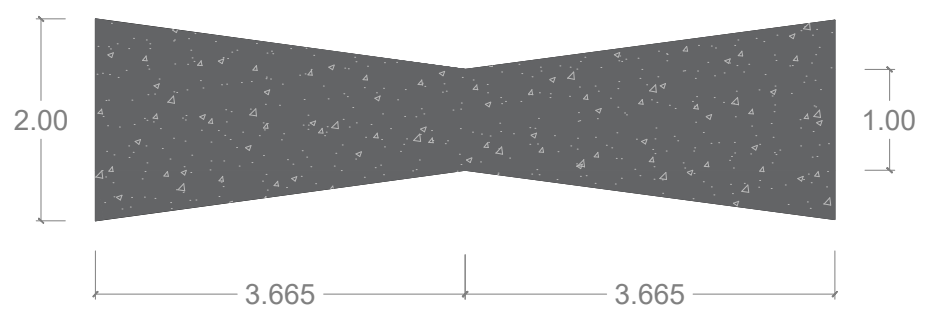
Column C1: section C2-C2 at 116,50 m (Scale 1:75)



Column C1: section B2-B2 at 3,60<sup>+</sup>m (Scale 1:75)

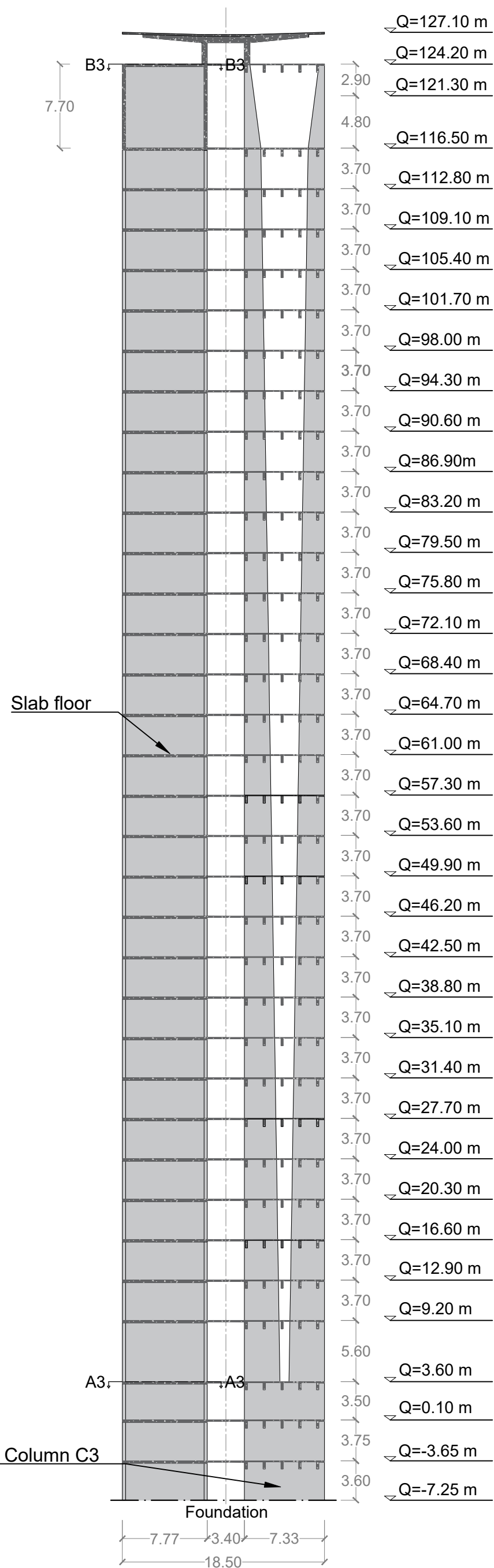


Column C1: section A2-A2 from -7,25 m to 3,60<sup>-</sup>m (Scale 1:75)

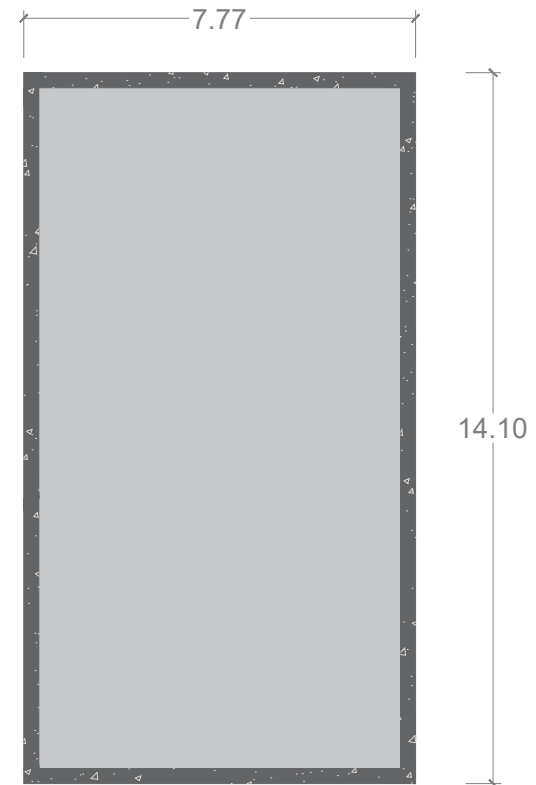




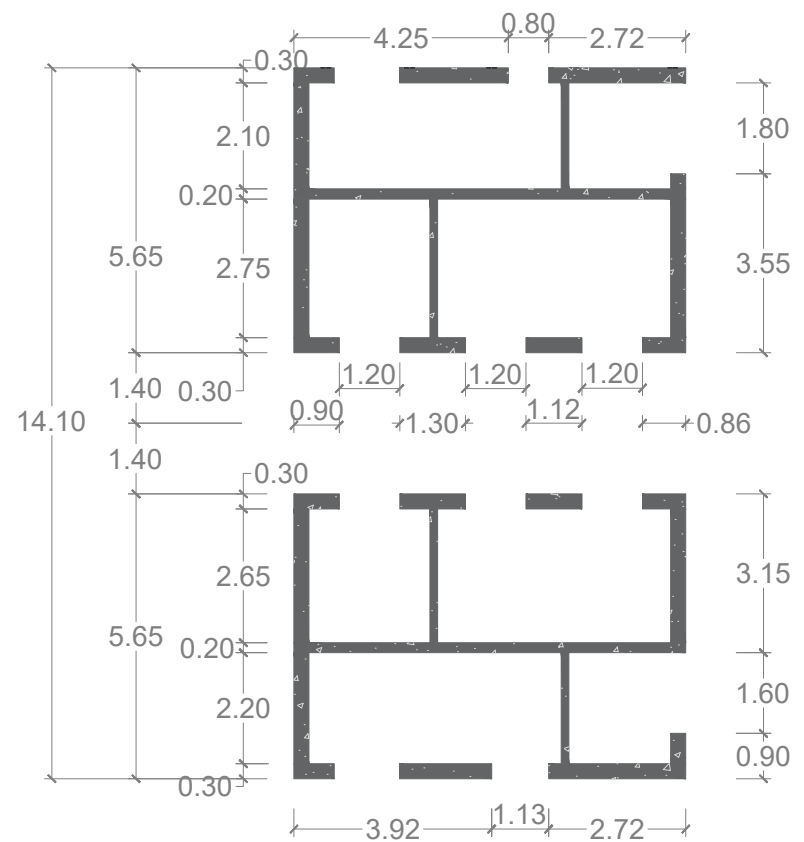
# Transversal section 3-3 (scale 1:400)



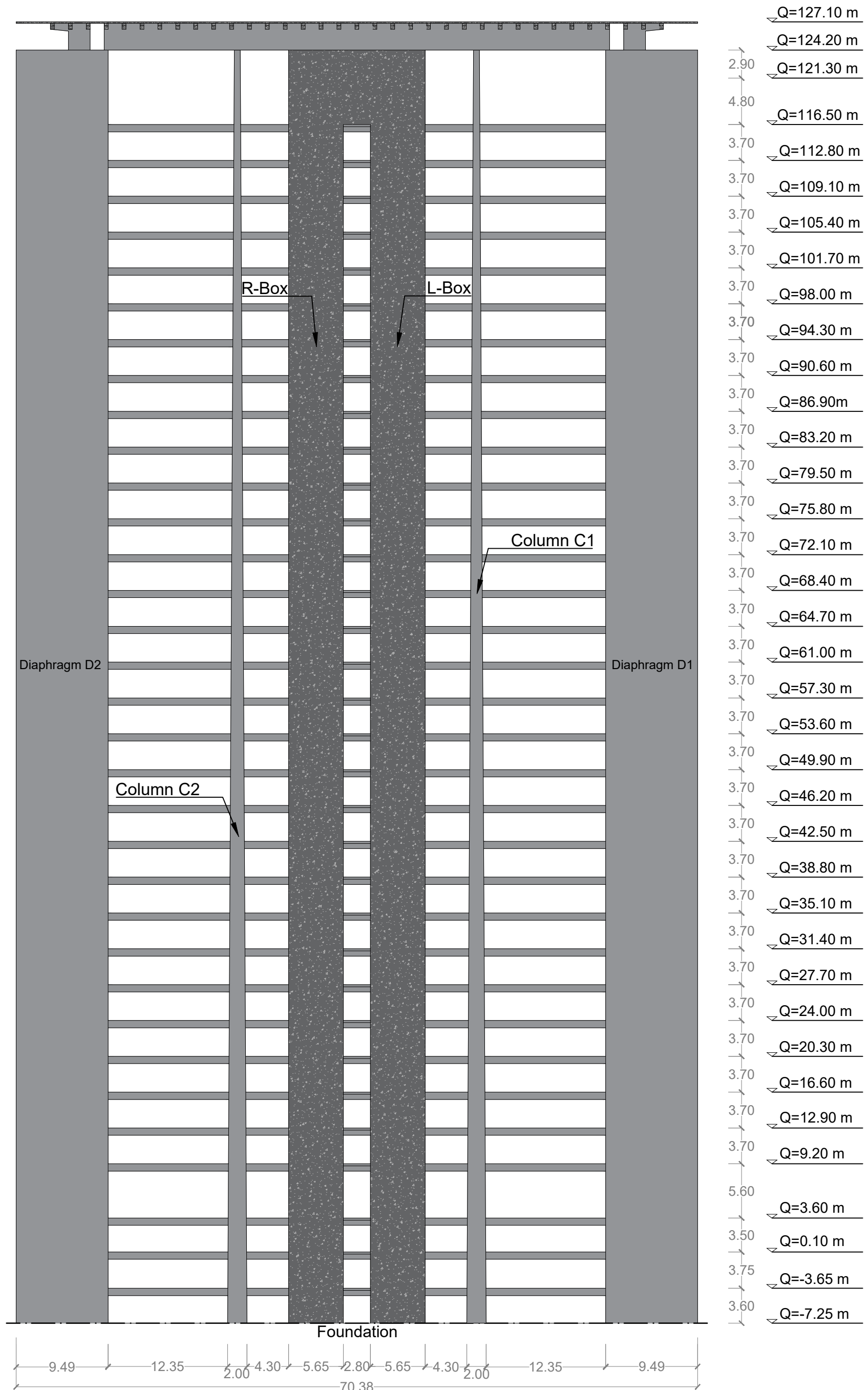
## Section B3-B3 between 116,50<sup>+</sup>m and 124,20 m (Scale 1:150)



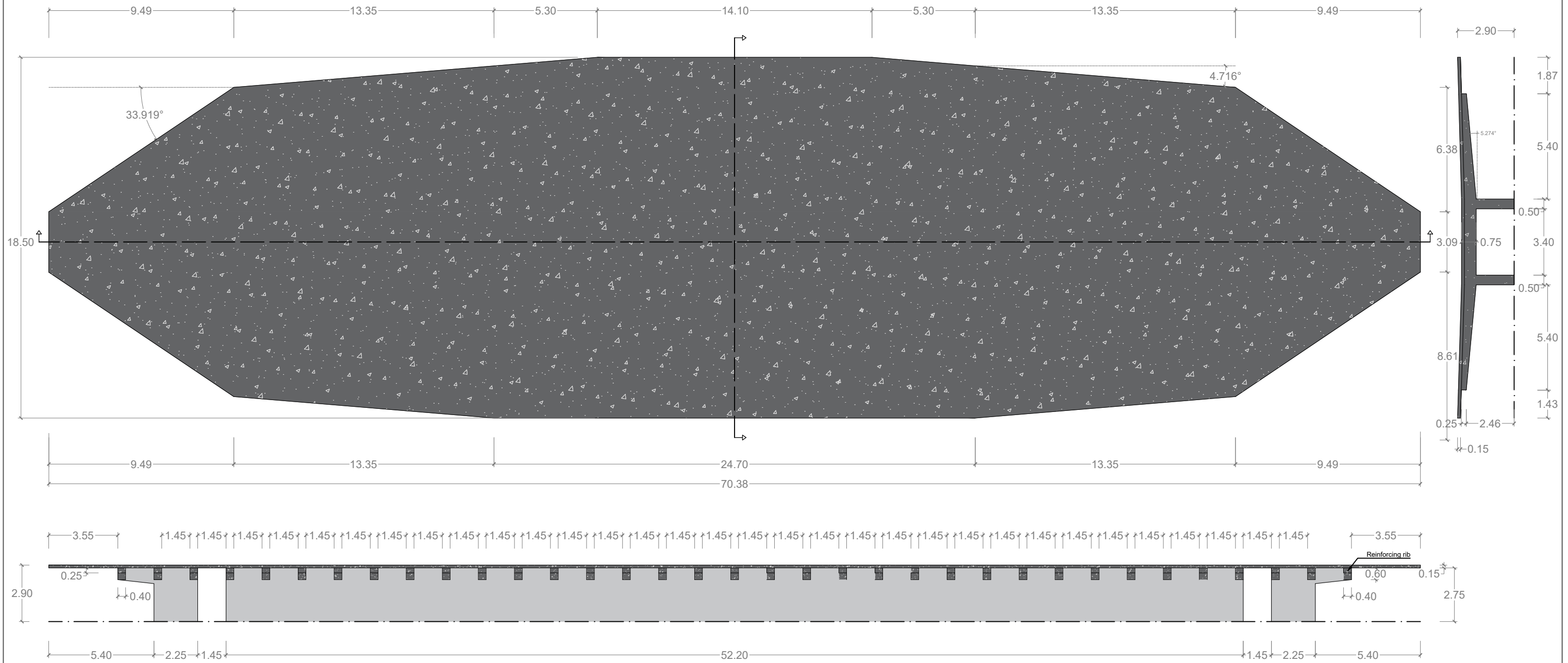
## Section A3-A3 between -7,25 m and 116,50<sup>-</sup>m (Scale 1:150)



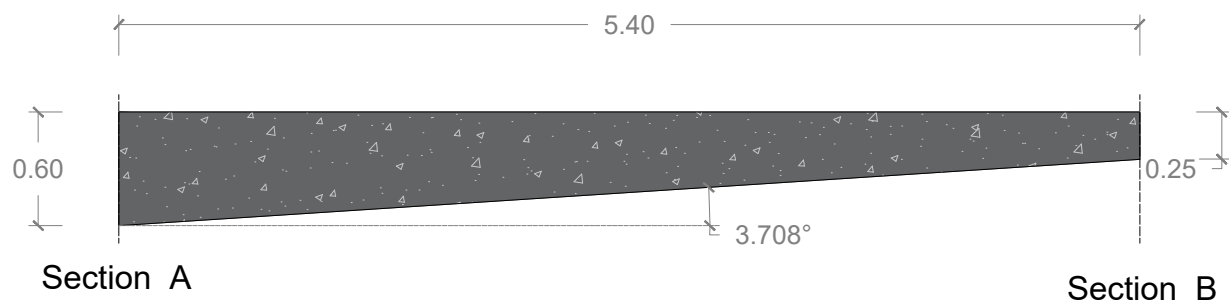
# Longitudinal section 4-4 (scale 1:400)



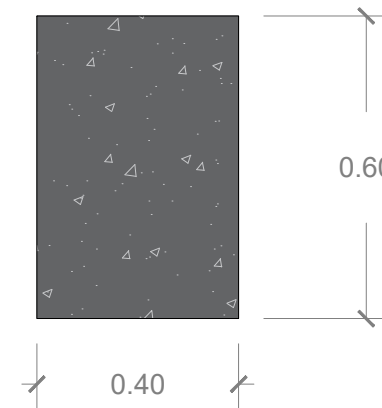
# Plan view of the roof and sections (scale 1:200)



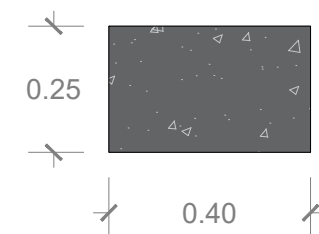
Longitudinal section of a reinforcing rib of the roof  
(Scale 1:40)



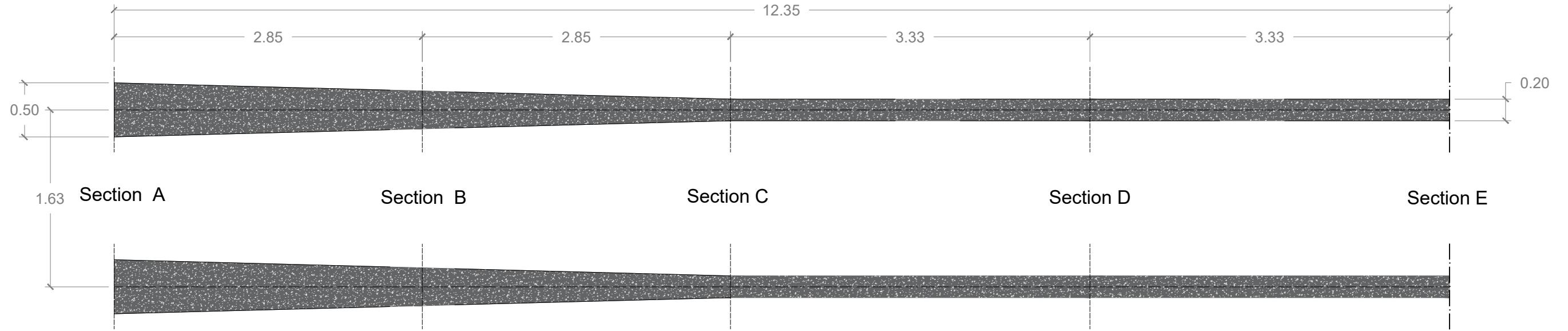
Transversal section A of a reinforcing rib of the roof (Scale 1:15)



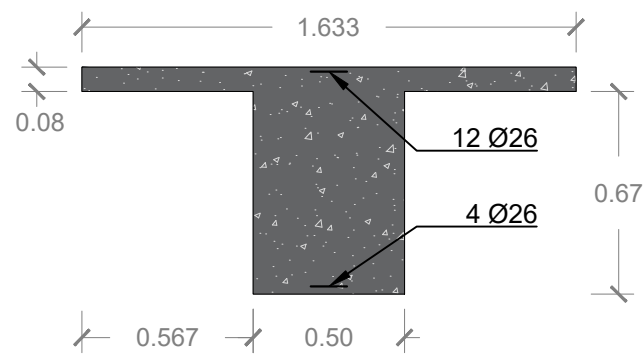
Transversal section B of a reinforcing rib of the roof (Scale 1:15)



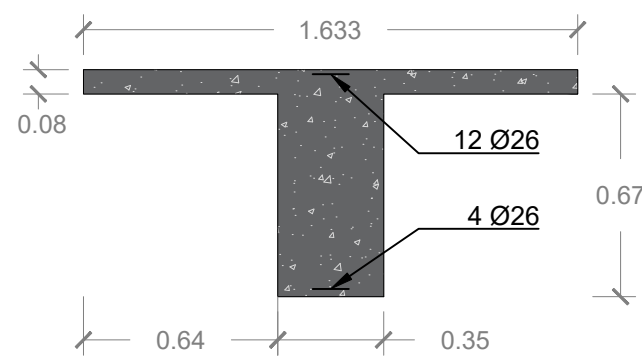
# Horizontal section of the floor slab beams (scale 1:40)



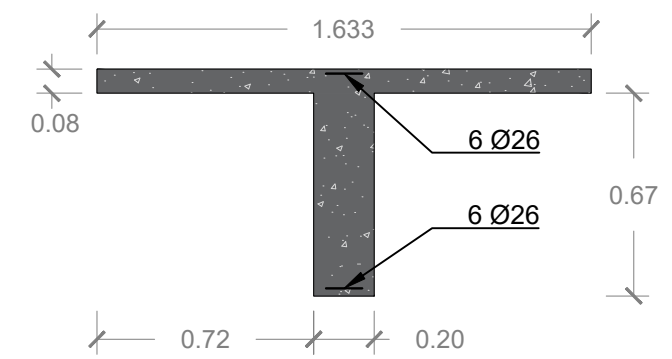
Section A (scale 1:25)



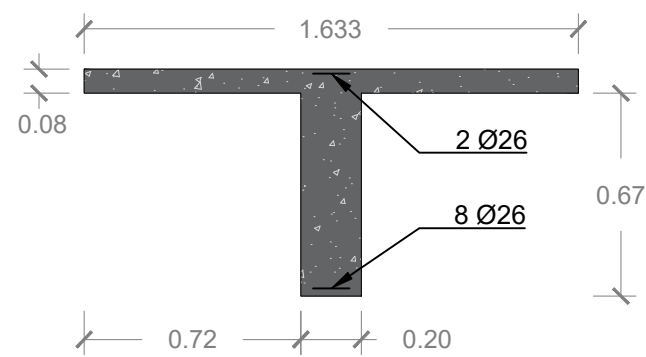
Section B (scale 1:25)



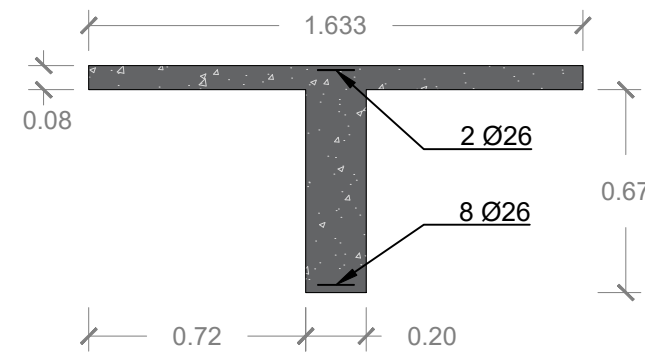
Section C (scale 1:25)



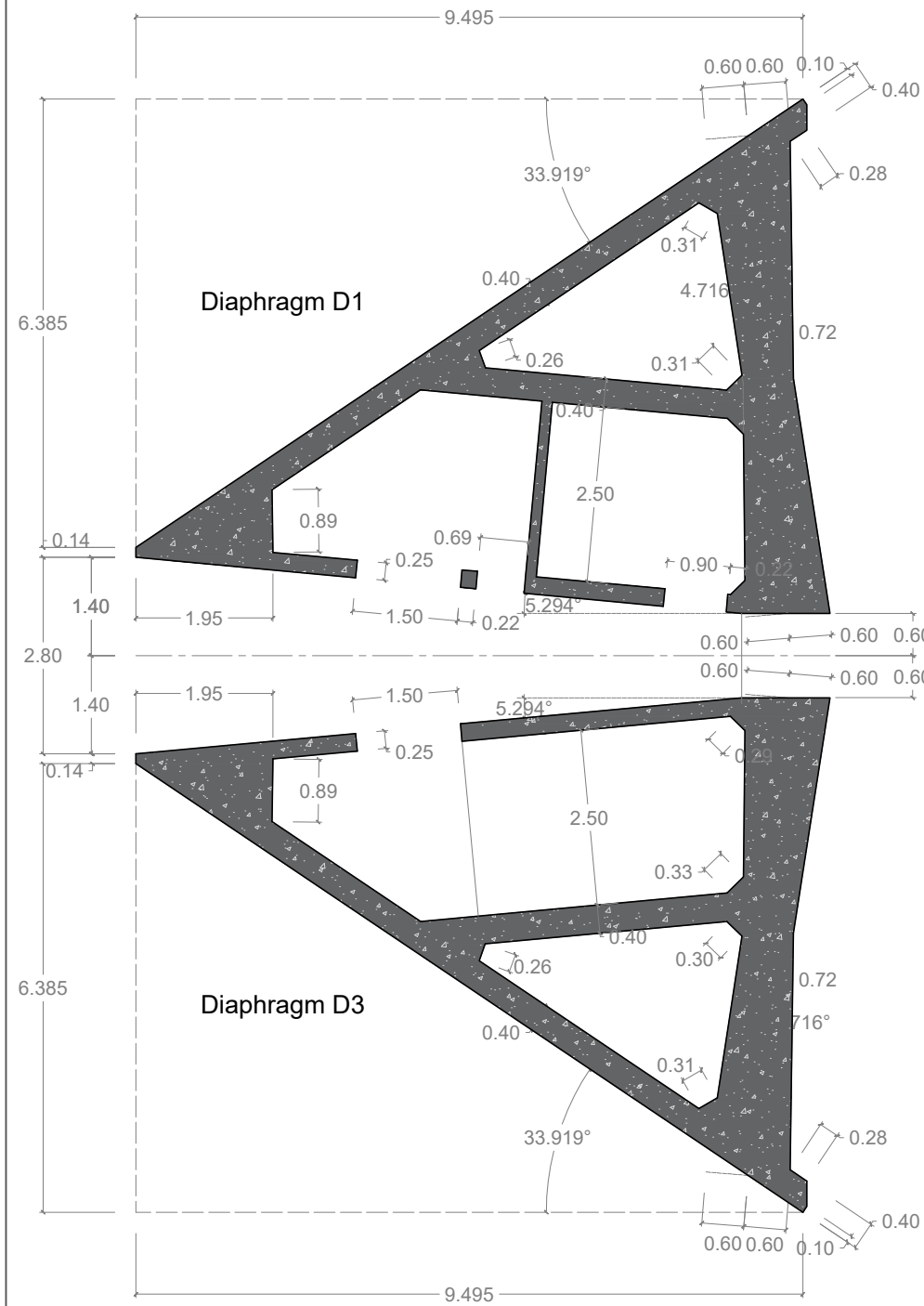
Section D (scale 1:25)



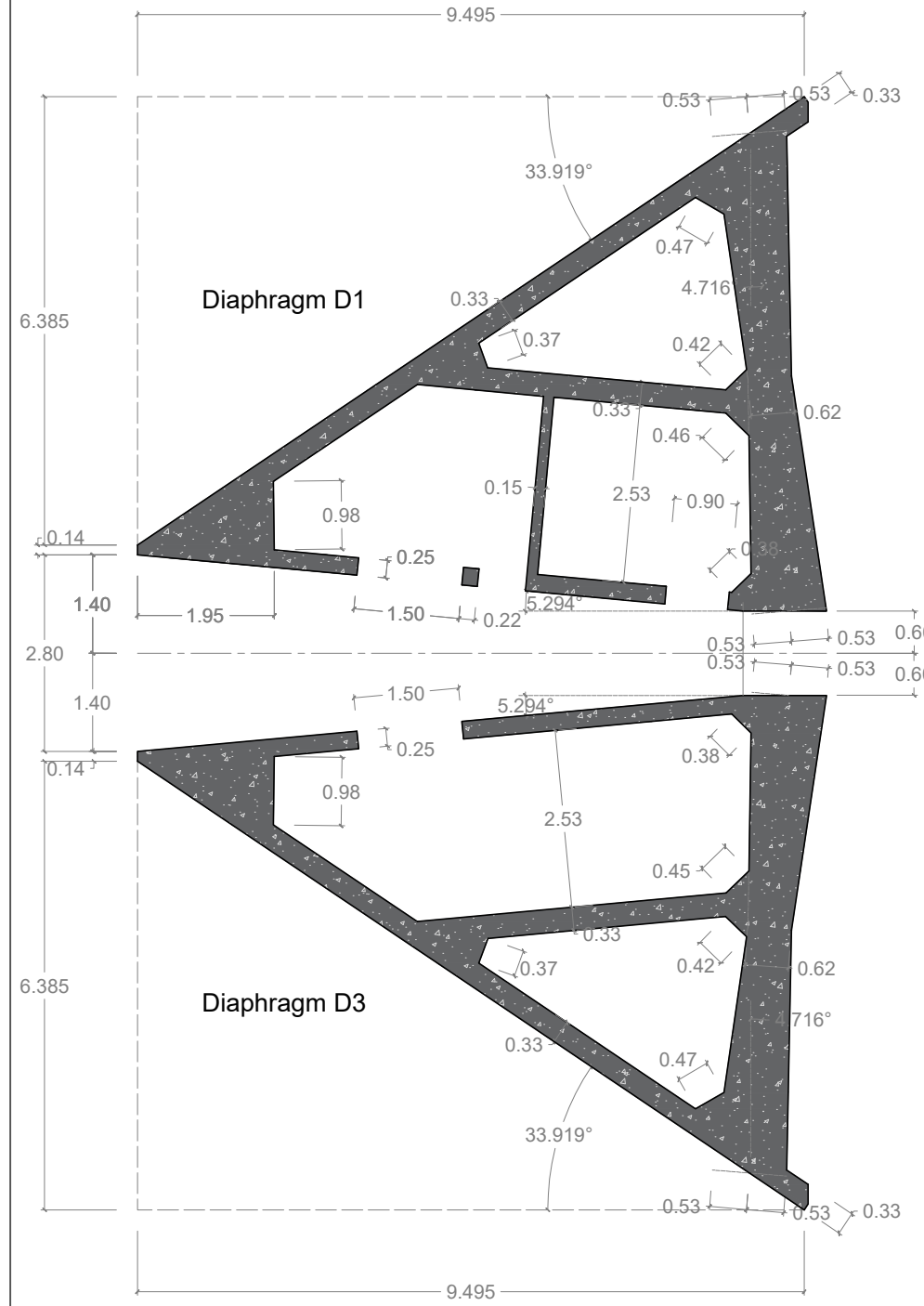
Section E (scale 1:25)



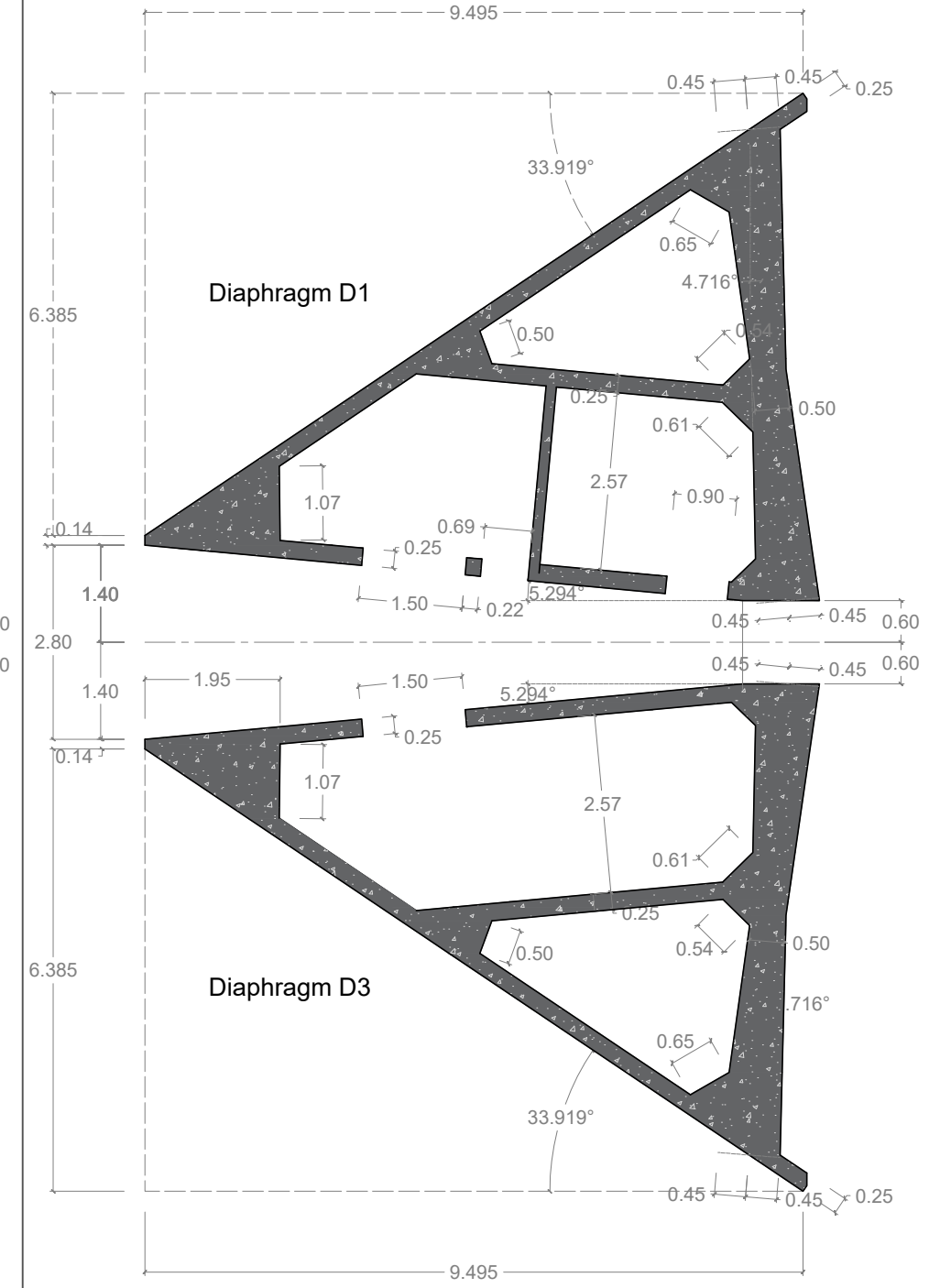
Section diaphragms D1 and D3. Q=3,60 m



Section diaphragms D1 and D3. Q=61,00 m

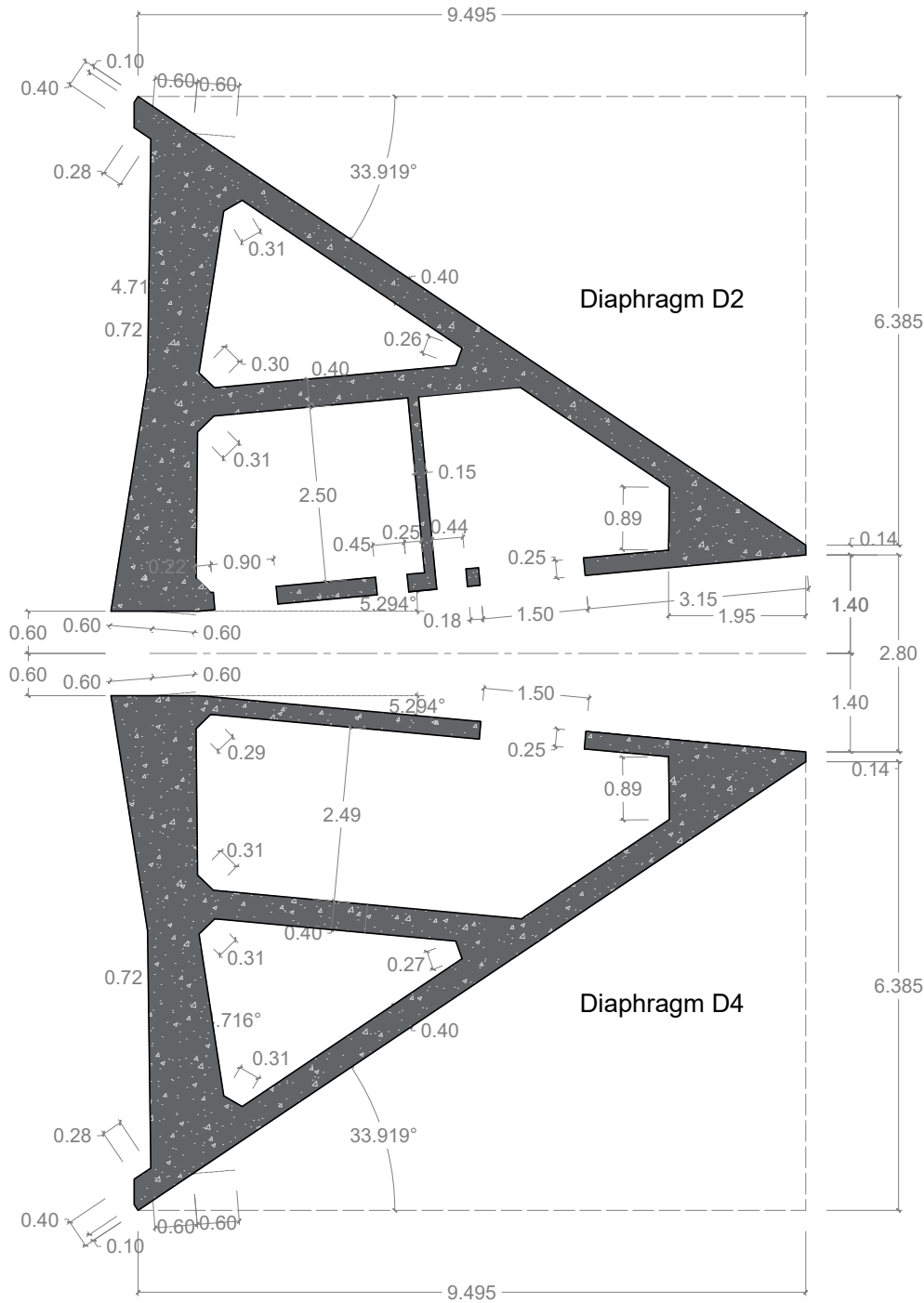


Section diaphragms D1 and D3. Q=124,20 m

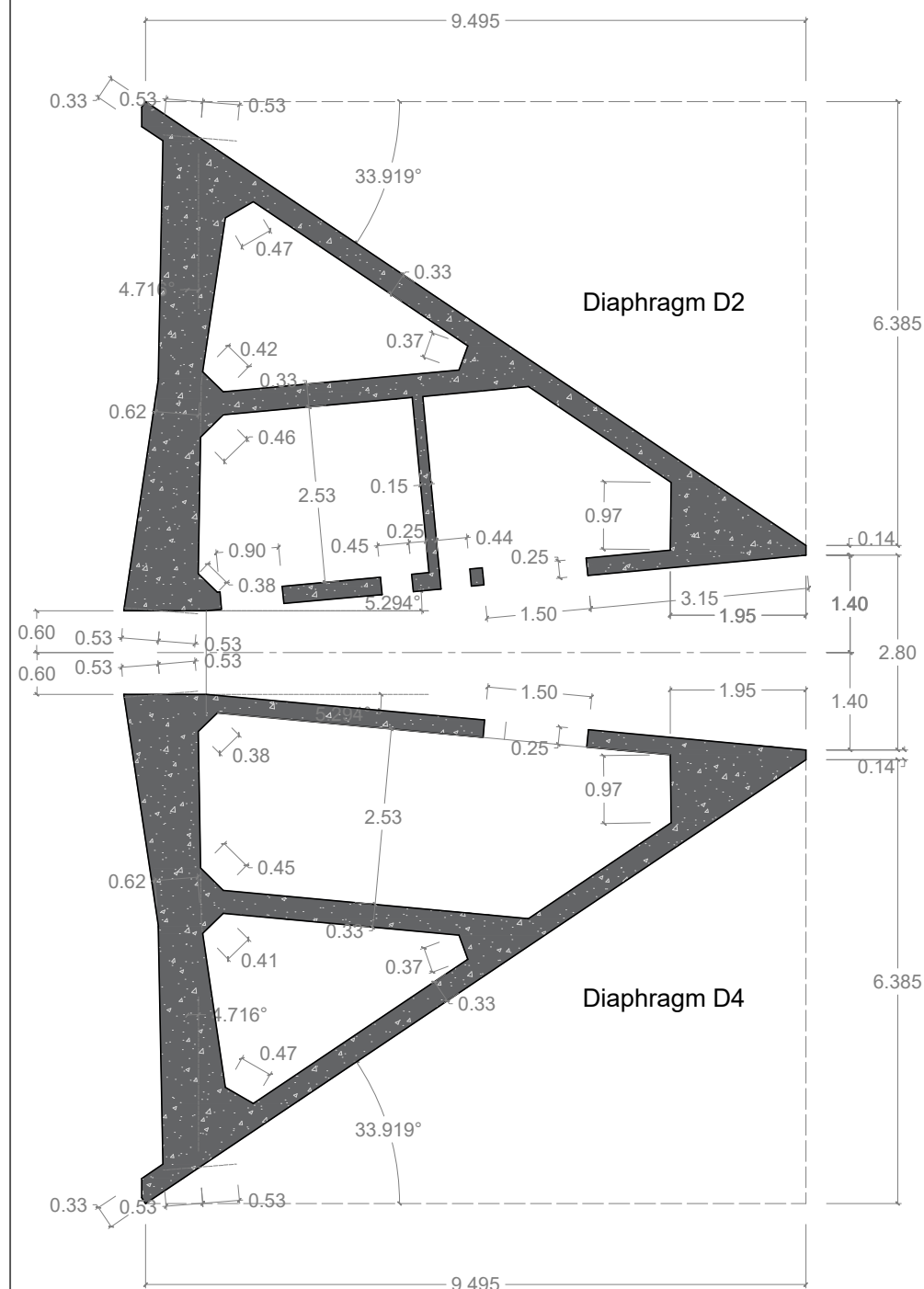




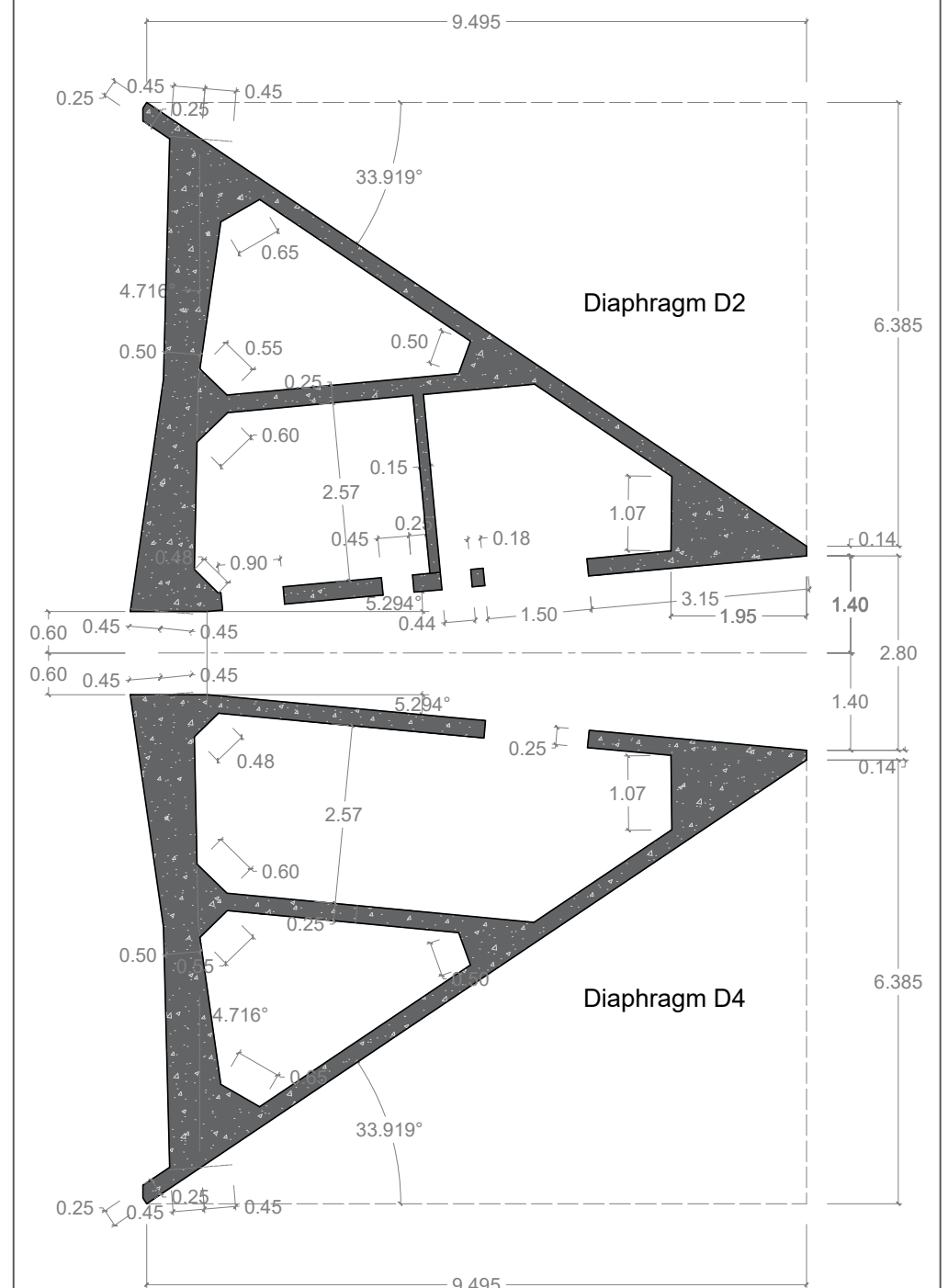
Section diaphragms D2 and D4. Q=3,60 m



Section diaphragms D2 and D4. Q=61,00 m



Section diaphragms D2 and D4. Q=124,20 m







# SUSTAINABLE DEVELOPMENT GOALS



UNIVERSITAT  
POLITECNICA  
DE VALÈNCIA



COMPROMETIDA CON LOS OBJETIVOS DE DESARROLLO SOSTENIBLE

## Anexo al Trabajo Fin de Grado/Máster

**Relación del TFG/TFM “Evaluación de la robustez estructural de la Torre Pirelli de Milán. Consecuencias del fallo local de las columnas centrales.” con los Objetivos de Desarrollo Sostenible de la Agenda 2030.**

Grado de relación del trabajo con los Objetivos de Desarrollo Sostenible (ODS).

Objetivos de Desarrollo Sostenibles	Alto	Medio	Bajo	No Procede
ODS 1. <b>Fin de la pobreza.</b>				X
ODS 2. <b>Hambre cero.</b>				X
ODS 3. <b>Salud y bienestar.</b>				X
ODS 4. <b>Educación de calidad.</b>				X
ODS 5. <b>Igualdad de género.</b>				X
ODS 6. <b>Agua limpia y saneamiento.</b>				X
ODS 7. <b>Energía asequible y no contaminante.</b>				X
ODS 8. <b>Trabajo decente y crecimiento económico.</b>				X
ODS 9. <b>Industria, innovación e infraestructuras.</b>		X		
ODS 10. <b>Reducción de las desigualdades.</b>				X
ODS 11. <b>Ciudades y comunidades sostenibles.</b>		X		
ODS 12. <b>Producción y consumo responsables.</b>				X
ODS 13. <b>Acción por el clima.</b>		X		
ODS 14. <b>Vida submarina.</b>				X
ODS 15. <b>Vida de ecosistemas terrestres.</b>				X
ODS 16. <b>Paz, justicia e instituciones sólidas.</b>				X
ODS 17. <b>Alianzas para lograr objetivos.</b>				X

Descripción de la alineación del TFG/M con los ODS con un grado de relación más alto.

Mi trabajo describe algunos análisis estructurales de un importante edificio en Italia para evaluar su robustez estructural, que es un tema en desarrollo en el campo de la ingeniería civil, y por lo tanto se relaciona con los objetivos 9 y 11. Además, evitando el colapso de edificios, se reducen las emisiones de CO<sub>2</sub> asociadas a la construcción de un edificio nuevo, entonces se relaciona también con el ODS 13.



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