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Additional Information

Relevant factors affecting the direction of crack propagation in complete contact problems under fretting fatigue

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Abstract

In fatigue problems, an accurate estimation of the propagation direction is important for life prediction. We identify the most relevant factors that affect the crack orientation during the propagation stage of fretting fatigue cracks, arising from complete contacts. Contrary to what initially expected, parameters such as normal load, cyclic bulk load, etc. do not have a noticeable influence on the orientation. However the relative **Young's moduli** of indenter/specimen materials, the indenter width and the surface coefficient of friction are the most influencing factors. Analyses are performed through the extended finite element method (X-FEM) and an orientation criterion for non-proportional loading proposed by the authors. Experimental fretting fatigue tests confirm the predicted trends. An explanation of this behaviour is also given.

Keywords: Fretting fatigue; complete contact; crack propagation; orientation criterion; extended finite elements.

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5 1. Introduction

6 Fretting problems are found in many mechanical components and are
7 often responsible for accelerating an eventual fatigue failure. They are
8 characterized by the existence of two or more solids in contact that un-
9 dergo relative displacements and they can be broadly classified into fretting
10 fatigue and fretting wear problems. One of the main features of fretting
11 fatigue problems is that cracks initiate next to the contact zone typically
12 under conditions of partial slip [1]. After the initiation stage, cracks usually
13 propagate under the cyclic stress field existing relatively far from the con-
14 tact region. On the other hand, fretting wear situations often hinder crack
15 propagation, as the incipient nucleated cracks are erased by material wear
16 due to gross slip conditions [2] and there is no far field cyclic stress that
17 promotes crack growth. This paper focuses on the study of complete con-
18 tact problems under fretting fatigue in metals, and more specifically, on the
19 crack propagation stage, i.e. the stage when the crack is already nucleated
20 and its length is several times greater than the typical grain size.

21 Compared to the plain fatigue endurance for the same materials, fretting
22 fatigue lives are substantially reduced. The main reason is that the contact
23 region acts as a stress raiser [3, 4] causing crack initiation and subsequent
24 crack propagation until the eventual failure of the component [1]. Hence,
25 the propagation life spans a greater percentage of the total life than in plain
26 fatigue problems, for which crack initiation involves a large part of the total
27 life.

28 When the propagation life of a fretting crack is to be estimated, numer-
29 ical methods such as the extended finite element method (X-FEM) [5–8]
30 or standard FEM with remeshing techniques [9] can be used to model the

31 crack presence under the combined influence of the bulk load and contact
32 stresses. Then, fracture mechanics approaches and crack growth laws can
33 be applied to estimate the remaining life, e.g. [7, 10, 11] in combination
34 with methods that predict the initiation life, such as the variable initiation
35 length approach [12]. Recently, the prediction of crack location and orien-
36 tation in the initiation stage has been addressed in [13] using a cohesive
37 zone model approach (CZM) in conjunction with XFEM. Another proposal
38 to predict the crack initiation direction is given in [14], where a method is
39 developed by calculating the average values of the normal and shear stresses
40 along a critical prospective direction.

41 In this work, a complete contact fretting fatigue configuration as the
42 one sketched in Fig. 1 is considered. In a complete contact, the contacting
43 area is independent of the normal load P due to the abrupt change of the
44 indenter geometry, as opposed to incomplete contacts, such as Hertzian
45 contacts. Some of the parameters affecting the loading conditions are the
46 normal load P , the tangential load Q , the cyclic bulk load σ_{Bulk} and its
47 corresponding stress ratio R (defined as $R = \sigma_{\text{Bulk,min}}/\sigma_{\text{Bulk,max}}$) and the
48 friction coefficient between the contacting solids.

49 A question arises regarding the influence of the different fretting param-
50 eters on the crack orientation. No information was found in the literature
51 regarding this issue. This work makes use of the numerical tools such as
52 X-FEM and the orientation criterion for non-proportional loading condi-
53 tions proposed in [15] to study the relevant parameters that affect crack
54 orientation. The X-FEM enables the parametric study of different fret-
55 ting configurations (loading, materials, etc.) in a straightforward way, as
56 the crack growth is simulated automatically without the need of remeshing

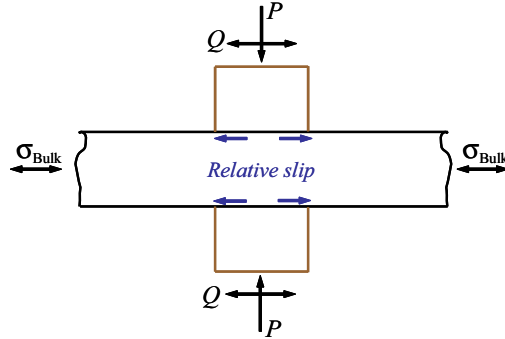


Figure 1: Sketch of the main loads acting in a complete contact fretting fatigue problem, as the one analyzed in this work.

57 [16, 17].

58 As shown in experimental tests carried out by the authors [15] (see
 59 Fig. 2) and in many works in the literature, e.g. [18, 19], cracks emanating
 60 from the edge of a contact pressing onto a surface tend to grow with a
 61 slight deviation inwards beneath the contact and not fully perpendicular
 62 to the applied bulk stress. This slight deviation from the normal direction
 63 cannot be predicted using a conventional orientation criterion, such as the
 64 maximum tangential or hoop stress criterion (MTS) and this is the main
 65 motivation of this research.

66 The objective of the work is to identify the relevant parameters affecting
 67 the crack path orientation. A parametric study of some *a priori* relevant
 68 magnitudes is carried out, such as normal load on the indenters, bulk load
 69 on the specimen, stress ratio, relative stiffness of the indenter and specimen
 70 materials, coefficient of friction and indenter width. This is performed by
 71 benefiting from the main advantage of XFEM, i.e. crack remeshing is not
 72 necessary for simulating crack propagation. Contrary to previous expecta-
 73 tions, it is shown that the relative magnitude of the applied loads has no

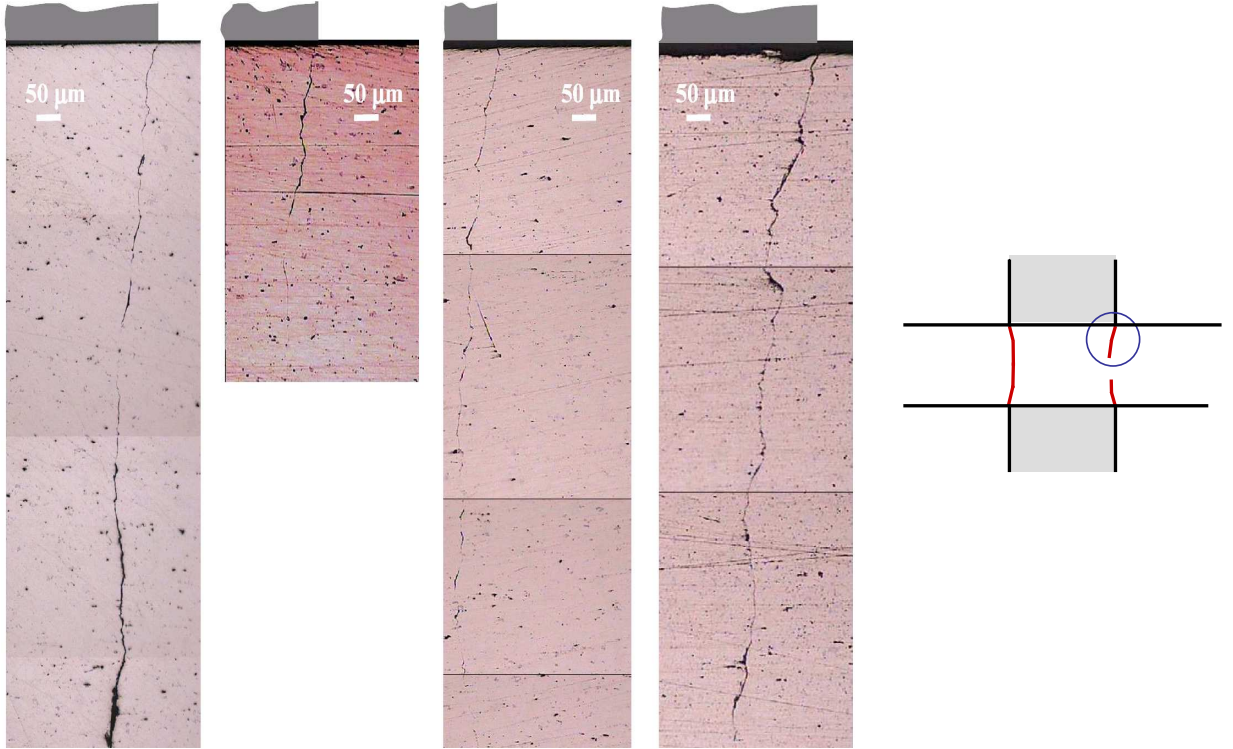


Figure 2: Propagation of non-failure cracks of four tests [15], emanating from the edge of contact. The material of both indenter and specimen is Al 7075-T6. Loads of each test: from left to right, $\sigma_P = 40, 80, 80, 160$ MPa and $\sigma_{\text{Bulk,max}} = 110, 130, 150, 190$ MPa with $R = -1$, load-controlled. Frequency of the fatigue tests is 15 Hz.

74 significant effect on crack orientation although, of course, it does on the
 75 fatigue life. However, it is found that the indenter width, the friction coef-
 76 ficient between the indenter and the specimen and the stiffness ratio of the
 77 indenter with respect to the specimen have a noticeable effect.

78 2. The criterion of the minimum shear stress range

79 In order to predict correctly the path followed by the crack, it is impor-
 80 tant to apply an orientation criterion that considers the nonproportional
 81 evolution of loads in fretting fatigue problems. Usually, the contact loads

82 do not evolve proportionally to the cyclic bulk loads. This means that
83 the principal stress directions and the stress intensity factor ratio K_{II}/K_I
84 change with time, invalidating the application of the classical MTS crite-
85 rion [20]. An orientation criterion for nonproportional loading conditions
86 was proposed by the authors in [15], where a brief review of other criteria
87 for nonproportional loading is also included. It was shown that the correct
88 application of a criterion of this kind is essential to agree with experimental
89 observations and a comparison is provided with the MTS criterion. Indeed,
90 the MTS criterion was applied in one of our first works [5], leading to wrong
91 estimations of the crack path. This was also found in [21].

92 *2.1. Fundamentals of minimum shear stress range criterion*

93 For the geometric and loading configuration considered in this work, the
94 crack remains closed during a large part of the loading cycle, as verified from
95 the numerical analyses. Assuming an elastic behaviour, the stress state un-
96 der crack face contact conditions must be essentially controlled by K_{II} , the
97 only stress intensity factor that can exist for a totally closed crack in 2D.
98 The criterion applied in this work [15] is a generalization for nonproportional
99 evolutions of the so-called criterion of local symmetry, well established for
100 proportional loading by Goldstein and Salganik [22] and Cotterell and Rice
101 [23]. The criterion of local symmetry states that the crack will propagate
102 in the direction where $K_{II} = 0$. For nonproportional loading, the condition
103 $K_{II} = 0$ cannot be reached on the same plane along the whole cycle, and
104 therefore, the proposed criterion seeks the angle for which the range K_{II} is
105 minimized along the cycle. This hypothesis obviously reduces to the con-
106 dition $K_{II} = 0$ when applied to proportional loading problems. It is worth

107 noting that, for proportional loading, $K_{II} = 0$, Nuismer and MTS crite-
 108 ria lead to very similar results [23, 24]. Sumi [25, 26] gives an interesting
 109 comparison of the results provided by different orientation criteria. In prac-
 110 tice, computing K_{II} values under crack face contact must include the effect
 111 of friction tractions on crack faces, as in [27, 28], which can be cumber-
 112 some and prone to inaccuracies when using domain and contour integrals.
 113 Instead, and equivalently for the application of the minimum shear stress
 114 range criterion, we search for the angle that minimizes the shear stress range
 115 at the crack tip, $\min(\Delta\tau)$. Shear stresses develop always in two orthogonal
 116 planes and there are two orthogonal planes on which the range is minimum,
 117 $\min(\Delta\tau)$. From these two possible crack growth directions, we choose the
 118 plane with the maximum $\Delta\sigma_n$, because it is the plane where less frictional
 119 energy is lost and there is more energy available for crack propagation. This
 120 approach is in line with the principle that a crack will grow in the direction
 121 which maximizes the strain energy release rate G [23, 24].

122 As verified in [15] and also in this work, the $\min(\Delta\tau)$ direction coincides
 123 with the direction of the maximum range of normal stress, $\max(\Delta\sigma_n)$. This
 124 is due to the in-plane stress tensor transformation that yields both extremes
 125 in the same direction, although this may not be the general case. However,
 126 one should notice that the criterion based on $\max(\Delta\sigma_n)$ is inconsistent,
 127 since compressive stresses (usually present during a large portion of the
 128 fretting load cycle) do not contribute to crack propagation. Moreover, the
 129 direction predicted by the maximum range of the effective normal stress,
 130 $\max(\Delta\sigma_{n,\text{eff}})$, i.e. considering only the positive part of σ_n , does not lead to
 131 good results at least in the problems studied by the authors, despite the
 132 intuitive idea that only the positive normal stresses will govern the crack

133 propagation under an elastic material behaviour.

134 Fig. 3 sketches the convention used in the procedure. For each crack
 135 growth increment, the criterion is applied ahead the current crack tip and
 136 the prospective local direction is searched for which $\Delta\tau$ is minimum (see
 137 example of the estimation of the third increment direction in Fig. 3). In the
 138 results provided in this work, the predicted angle is reported with respect
 139 to a fixed reference: the angle is measured from the specimen surface. This
 140 way, a crack segment growing inwards (with respect to the indenter contact
 141 zone) has an angle $0^\circ < \theta < 90^\circ$ and $-90^\circ < \theta < 0^\circ$ indicates a crack
 142 segment growing outwards.

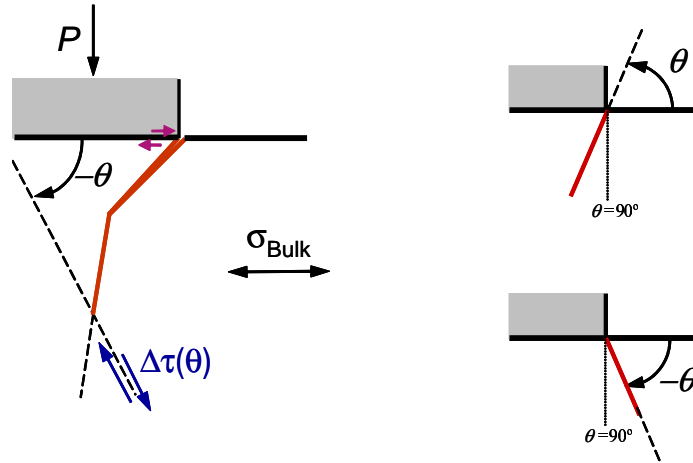


Figure 3: Application of the $\min(\Delta\tau)$ criterion to predict e.g. the third crack-growth increment direction. Sign convention for direction angles of a crack growth increment.

143 *2.2. Application to a crack in a plate subjected to a tension-compression*
 144 *cycle*

145 As an example of simple application of the minimum shear stress range
 146 criterion, a cracked bar of uniform section loaded in tension is analyzed.

147 Our intention with this simple configuration is to illustrate that the cri-
148 terion reduces to the expected angle of 90° for such a simple case under
149 proportional loading. Fig. 4 shows the geometry and loads of the model
150 and a contour plot of the von Mises stress field. This preliminary analysis
151 is performed using standard FEM with ABAQUS (no X-FEM is considered
152 at this stage). Plane strain bilinear elements with four nodes and full in-
153 tegration are used (CPE4 in Abaqus). Inertial effects are not considered
154 in this work and the fatigue crack propagation problem can be regarded as
155 quasi-static. The material is modeled as linear elastic. The bulk load is
156 cyclic with $R = -1$ and no indenter load exists in this simple example. The
157 time evolution of the bulk load is similar to the one shown in Fig. 7 and it
158 is also divided into four load steps.

159 Fig. 5, left, shows the variation of the normal stress σ_n and shear stress τ
160 on a plane forming an angle θ with respect to the horizontal surface. Stresses
161 are evaluated at finite elements located ahead the crack tip and transformed
162 according to the angle of the prospective plane. The successive curves show
163 the variation along time for the last load step (step 4, i.e. between $t = 3.0$
164 and $t = 4.0$). Note that the normal stress is maximum at the end of the step
165 (curves located at the top of Fig. 5, left). This tensile stress is higher than
166 the corresponding compressive stress due to the effect of the crack opening
167 (mode I of fracture), whereas the closing stage does not concentrate such
168 high stresses. The evolution of the shear stresses is analogous. Note that
169 the shear stresses τ are zero for the angles θ where the normal stress is
170 maximum or minimum (for any given increment of time). The fact that all
171 the maxima and minima are attained at the same angles ($\pm 90^\circ$ for σ and
172 $\pm 45^\circ$ for τ) is indicative that the stresses evolve in a proportional way. If

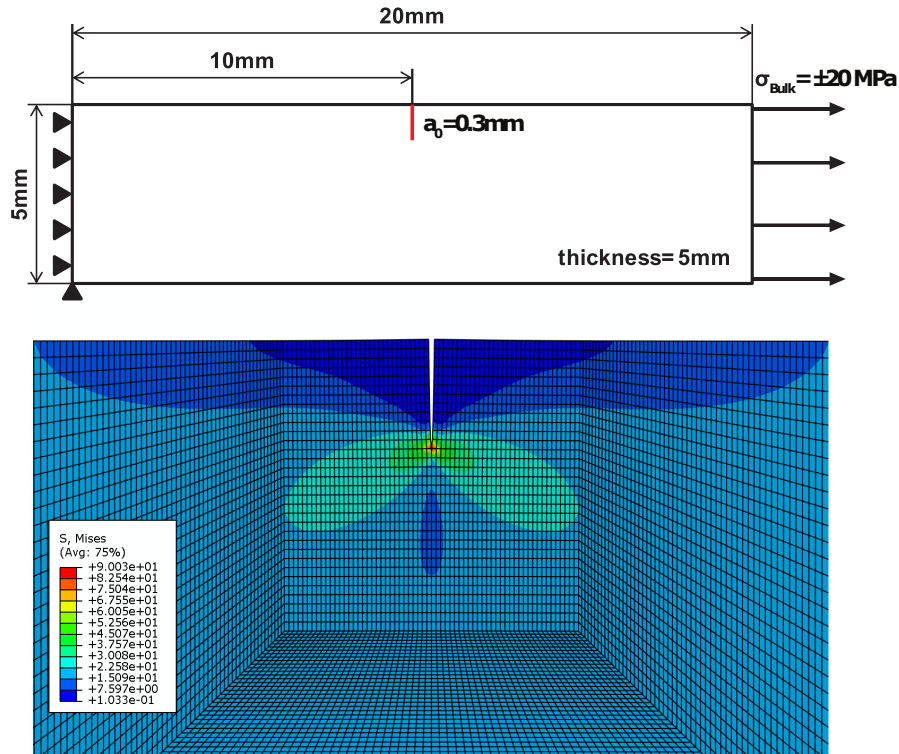


Figure 4: Top, geometry and loads of the model of a cracked bar in tension. Bottom, detailed view of a von Mises contour plot.

173 the time series of maxima and minima are shifted along the θ axis, then
 174 the loads are nonproportional. Fig. 5 right, shows the application of the
 175 $\min(\Delta\tau)$ criterion. The same τ curves of Fig. 5, left, are replotted and the
 176 maximum and minimum with time are marked in black. Then, the range of
 177 variation $\Delta\tau$ is computed simply as $\Delta\tau = \tau_{\text{max}} - \tau_{\text{min}}$. The minimum shear
 178 stress range criterion predicts that the prospective propagation angles are
 179 either 0° or 90° (there are always two prospective angles with a difference
 180 of 90°). The discrimination between both angles is done by choosing the
 181 angle that also leads to the maximum normal stress to that plane. The
 182 predicted angle of propagation for this case is 90° , as expected in such a

183 simple problem.

184 One advantage of the $\min(\Delta\tau)$ criterion over simply choosing the direc-
 185 tion of $\max(\Delta\sigma_n)$ is that the angle is detected sharply, as shown in Fig. 5.
 186 This sharpness of $\min(\Delta\tau)$ is also what governs the mechanics of the prob-
 187 lem, leading to well defined propagation angles.

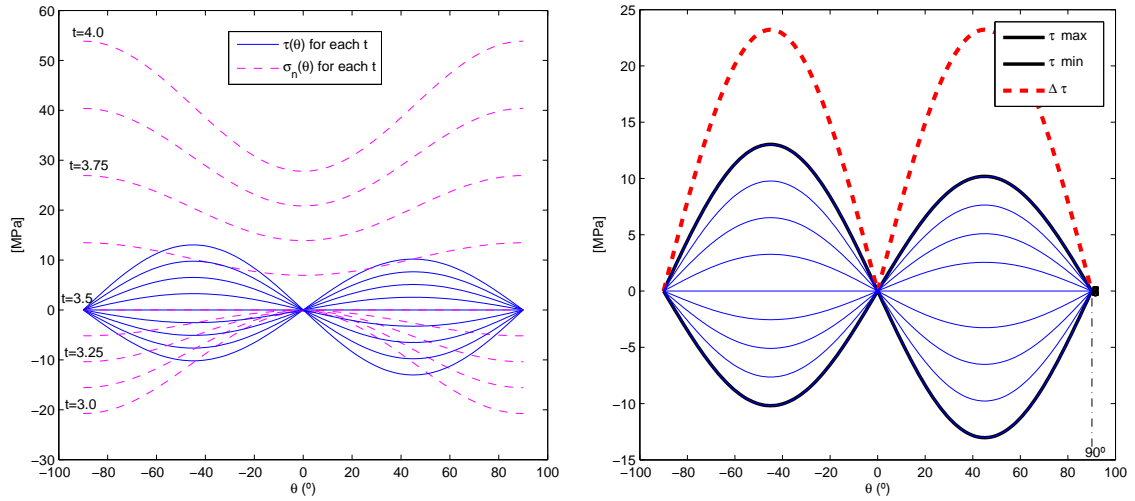


Figure 5: Left, variation of the normal stress σ_n and shear stress τ on planes forming a varying angle θ with respect to the horizontal surface. Stresses are evaluated at elements located ahead the crack tip. Right, application of the $\min(\Delta\tau)$ criterion.

188 3. Numerical model

189 Due to symmetry conditions, a quarter 2D finite element model has been
 190 considered to represent the fretting fatigue tests, as shown in Fig. 6. The
 191 rectangle $L \times b$ corresponds to the portion of the analyzed specimen and
 192 has a length of $L = 4b = 20$ mm, the half width of the indenter c is 5 mm,
 193 and the distance between the contact plane and the point of the indenter at
 194 which loads are applied is $h = 10$ mm. Four node, plane strain quadrilateral
 195 elements with full integration were used with a thickness $t = 5$ mm. The

196 smallest element size considered is $5\mu\text{m}$ at the right end of the contact zone.
197 The friction model assumed for the contact zone is a Coulomb model and
198 the ABAQUS contact formulation based on Lagrange multipliers is used to
199 model the contact between the indenter and the specimen. Unless other-
200 wise stated, the friction coefficient between crack faces (μ_{CF}) and between
201 indenter and specimen (μ_{IS}) is taken as $\mu_{\text{CF}} = \mu_{\text{IS}} = 0.8$ [19]. The material
202 behaviour is assumed linear elastic, despite the high stress concentration
203 at the contact edge. The specimen material is an aluminium alloy 7075-
204 T6, with $E = 72$ GPa and $\nu = 0.3$. For some of the cases analyzed, the
205 indenter material is changed, as explained in Section 5.5. The application
206 of the linear regime is deemed valid, due to the very small edge radius of
207 the indenter and the relative high yield stress of the aluminium alloy. In
208 addition, the loads simulated in this work are typical of high cycle fatigue,
209 and therefore are nominally small (in contrast to high loads typical of low
210 cycle fatigue problems). Therefore, the extent of the plastic zone at crack
211 tip during crack propagation is relatively small and there is no noticeable
212 plastic wake along crack faces. This is confirmed by the observation of
213 the tested specimens, which showed no macroscopic evidence of plasticity
214 (see micrographs in Fig. 2 in which crack faces match very well each other
215 and also a view of the specimen contact surface in Fig. 7 of our previous
216 work [10]). As a consequence, the existing plasticity is very localized and a
217 small scale yielding assumption can be applied, analogous to the small scale
218 yielding assumption admitted in linear elastic fracture mechanics (LEFM)
219 around the crack tip.

220 The loading is considered quasistatic and its sequence is represented in
221 Fig. 7 for one of the examples analyzed (case 3 of Table 1), where four load

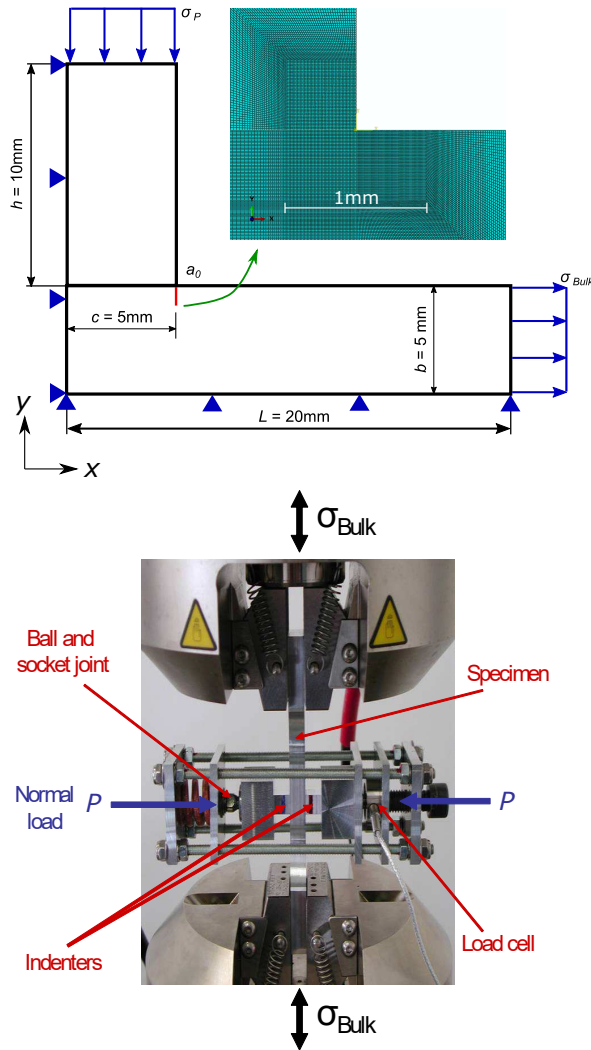


Figure 6: Top, model geometry and detail of the refined mesh at the potential crack growth zone. Bottom, complete contact testing rig, showing the contact elements.

222 steps have been considered in the analysis. Due to the non-linearity of the
 223 contact problem, loads were applied in sufficiently small time increments.
 224 At time $t = 2.00$ (and also at time $t = 4.00$) the maximum σ_{Bulk} is being
 225 applied, which produces a clear opening of the crack. When the bulk load is
 226 decreased in the first half of step 3 ($2.00 < t < 2.50$), mode I is reduced and

227 a clear mixed mode condition appears, which has been observed through FE
 228 analyses. Note that the vertical load due to the indenter is kept constant
 229 during the cycle and mode II increases its dominance over mode I as σ_{Bulk}
 230 is reduced. At approximately $t = 2.50$ crack face contact is produced and
 231 a mode II condition is present at the crack tip. At time $t = 3.00$ the bulk
 232 load is completely reversed (since the stress ratio is $R = -1$) and the load
 233 is transmitted through the crack faces. When the crack is closed, the end
 234 of the contact zone acts now as a strong stress raiser, as the specimen is
 235 compressed against the contact corner. Results in the following section are
 236 presented for the load step 4 ($3.00 < t < 4.00$), when shakedown of the
 237 numerical model response is produced). It has been verified that the stress
 238 states at $t = 3.50$, $t = 4.00$ and those at $t = 2.50$, $t = 2.00$, respectively, are
 239 very similar.

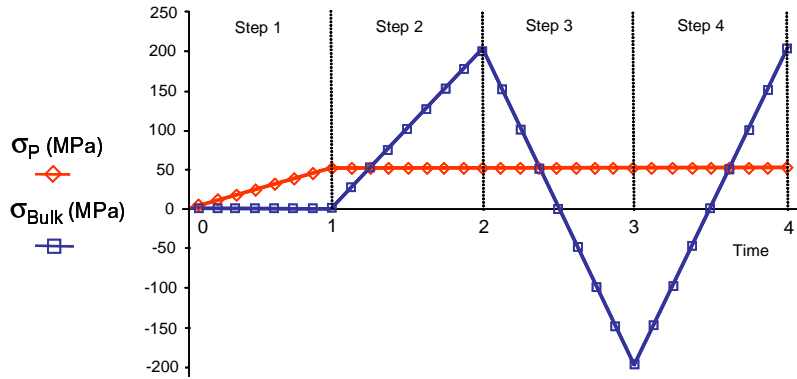


Figure 7: Loads applied to the numerical model for one of the cases studied. Evolution with time.

240 4. Analysis of the loading influence

241 In this section we present an initial study of the loading influence on the
 242 crack orientation. The main parameters considered are the indenter load

243 σ_P (defined as the normal force P divided by the area of application on the
 244 indenter), the cyclic bulk load σ_{Bulk} and the stress ratio R of the cyclic bulk
 245 load. The study is performed for the first propagation angle after the initial
 246 crack shown in Fig. 6 ($a_0 = 0.3$ mm), which is assumed that it is already
 247 present. The analyses were carried out with standard FE models, i.e. no
 248 XFEM is used in this section, as propagation will be considered in the next
 249 Section 5. The stress solution is then postprocessed to estimate the first
 250 propagation angle after application of the $\min(\Delta\tau)$ criterion.

251 The variation of the loading parameters is listed in Table 1. The ge-
 252 ometrical model with an initial crack of length $a_0 = 0.3$ mm and initial
 253 orientation of $\theta = 90^\circ$ has been analyzed under 13 different cases. The
 254 cases consider different combinations of normal load P applied on the in-
 255 denter, the variable bulk load on the specimen σ_{Bulk} and the stress ratio
 256 R . The material stiffness is 72 GPa in all cases, both for the indenter and
 257 specimen. The last column indicates the predicted angle using the $\min(\Delta\tau)$
 258 criterion. Contrary to what initially expected, the first fact that draws at-
 259 tention is that there is no practical variation of the predicted angle, since
 260 all cases lead to an orientation angle of 78° – 79° . Even for the cases with
 261 negligible contacting normal load, $P = 10^{-6}$, the prediction leads to angles
 262 pointing inwards. The influence of the wide ranges tested for σ_{Bulk} and R is
 263 also negligible. This is in full agreement with the experimental evidence col-
 264 lected by the authors [10, 15], summarized in Fig. 2, with growing directions
 265 about 79° .

266 Fig. 8 shows the variation of $\Delta\tau$ versus the prospective crack orientation
 267 angle θ for the last step of the loading cycle. This enables the application
 268 of the minimum shear stress range criterion. Fig. 8, left, shows the results

Table 1: Predicted orientation angles for different load cases, generated by variation of σ_P , σ_{Bulk} and R .

Case	σ_P (MPa)	$\sigma_{\text{Bulk,max}}$ (MPa)	R	θ ($^\circ$)
1	10^{-6}	200	-1	79
2	10^{-6}	200	0	79
3	50	200	-1	79
4	50	200	0	79
5	100	200	-1	78
6	100	200	0	79
7	200	200	-1	78
8	200	200	0	79
9	10^{-6}	200	-0.5	79
10	50	200	-0.5	79
11	100	200	-0.5	79
12	200	200	-0.5	79
13	200	10	-1	79

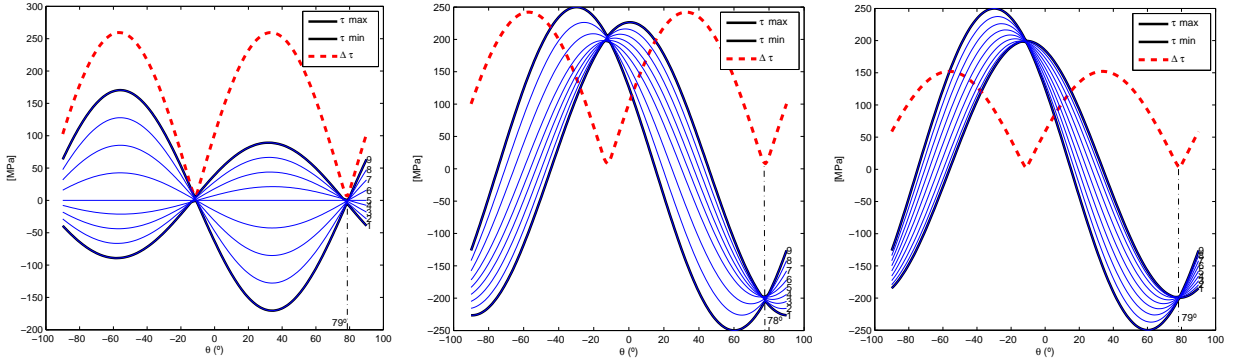


Figure 8: Application of the $\min(\Delta\tau)$ criterion for cases 1, 7 and 8 of Table 1, leading to predicted angles of 79° , 78° and 79° , respectively.

269 for case 1. The high proportionality of the loads is demonstrated by the
 270 same location of the maxima and minima (no shifting of the curves). The
 271 load proportionality is caused by the extremely low value of σ_P considered
 272 in this case 1. However, even under this situation, the effect of the indenter
 273 causes the deflection of the crack to 79° , given the ideal contact conditions
 274 of the numerical model. Results for case 7 are presented in Fig. 8, centre.
 275 Here the nonproportionality is evident due to the high value of σ_P , which is

276 equal to σ_{Bulk} , being the curves shifted ones with respect to each other. Due
277 to the high value of the constant normal load, the effect of the cyclic load is
278 less evident and the range between τ_{min} and τ_{max} is not so important. This
279 range is even less for case 8 (Fig. 8, right), due to the change in R from -1 to
280 0. In all cases, the predicted angle is approximately 79° . We remark that by
281 application of a conventional orientation criterion at the instant of maximum
282 bulk load, such as MTS, an incorrect prediction of the crack direction is
283 obtained (pointing outwards, see [15]). Other examples of inaccurate growth
284 orientations using the MTS criterion under nonproportional fretting loading
285 can be found in Figs. 6 and 7 of [5] and in [21].

286 **5. Study of relevant factors and prediction of propagation paths** 287 **using XFEM**

288 In the previous section, the direction for the first crack growth increment
289 has been estimated for different loading conditions. In this section, the ex-
290 tended finite element method X-FEM [16] is used in combination with the
291 $\min(\Delta\tau)$ criterion to model propagation for successive crack growth incre-
292 ments. The objective is to study other factors that can be relevant for the
293 crack orientation and compare the numerical estimations with experimental
294 tests. In these problems, the initial crack is $a = 0.05$ mm and the crack
295 growth increment is set as $\Delta a = 0.05$ mm. The initial crack orientation is
296 based on a critical plane analysis [29, 30]. For further details, please refer
297 to [5] and [10].

298 *5.1. Crack propagation using X-FEM*

299 The great advantage of the X-FEM method is that the crack faces do not
300 need to conform to the element sides of a mesh. Therefore, a single mesh
301 can be used for virtually any arbitrary crack intersecting the mesh. This
302 avoids remeshing and it becomes especially useful when modeling crack
303 propagation in fatigue problems. This is accomplished through a special
304 mathematical formulation of the FE method that includes the enrichment
305 of the standard finite elements with additional degrees of freedom (DOFs)
306 at the nodes. These additional DOFs are associated with the nodes of the
307 elements that are geometrically intersected by the crack (called enriched
308 nodes and elements, respectively). Thus, the discontinuity is included in
309 the numerical model without modifying the discretization. The X-FEM
310 formulation allows for a further type of enrichment for the nodes next to
311 the crack-tip. These nodes are enriched with additional DOFs to represent
312 the first term of the classical Williams series expansion in linear elastic
313 fracture mechanics in terms of the displacement field. Further details can
314 be found in [15] and references therein.

315 The analyses have been carried out using the X-FEM implementation
316 developed by the authors [17, 31] by means of a user's subroutine linked to
317 the commercial code ABAQUS. This implementation can take into account
318 crack face contacts along the loading cycle, which have been proved to be
319 essential for the correct crack prediction. Fig. 9 (left) shows paths obtained
320 experimentally and numerically for the following parameters: $E_{\text{indenter}} =$
321 $E_{\text{specimen}} = 72 \text{ GPa}$ (Al 7075-T6), $P = 160 \text{ MPa}$, $\sigma_{\text{Bulk}} = 190 \text{ MPa}$, $R = -1$,
322 indenter width $2c = 10 \text{ mm}$ and $\mu_{\text{CF}} = \mu_{\text{IS}} = 0.8$. A good agreement
323 between the experimental path and the crack path predicted using X-FEM

324 is obtained. Fig. 9 (right) shows a von Mises contour plot at the first steps
 325 of the crack.

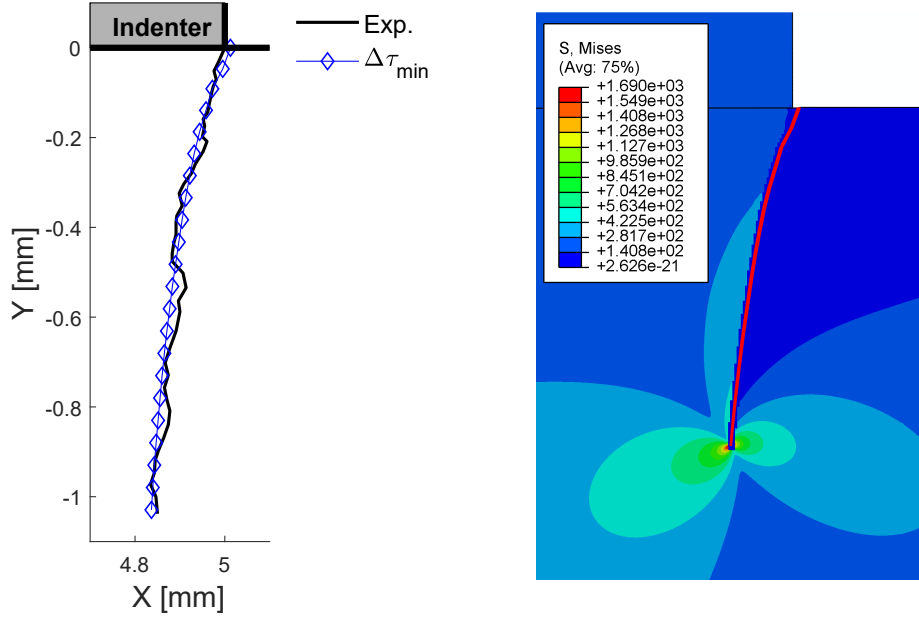


Figure 9: Left, comparison between the crack paths obtained experimentally (rightmost micrograph shown in Fig. 2) and obtained through X-FEM in combination with the $\min(\Delta\tau)$ criterion. The parameters of this problem are: $E_{\text{indenter}} = E_{\text{specimen}} = 72$ GPa, $P = 160$ MPa, $\sigma_{\text{Bulk,max}} = 190$ MPa, $R = -1$, indenter width $2c = 10$ mm and $\mu_{\text{CF}} = \mu_{\text{IS}} = 0.8$. Right, von Mises contour plot at one of the stages of the numerical simulation of crack propagation.

326 In this section, the influence of these factors will be analyzed: indenter
 327 normal load P , coefficient of friction, indenter width and indenter stiffness.
 328 In general, and unless otherwise stated, the following values are considered
 329 $E_{\text{indenter}} = 72$ GPa (Al 7075-T6), $E_{\text{specimen}} = 72$ GPa, $P = 40$ MPa, $\sigma_{\text{Bulk}} =$
 330 110 MPa, $R = -1$, indenter width $2c = 10$ mm and $\mu_{\text{CF}} = \mu_{\text{IS}} = 0.8$. This
 331 configuration is considered as the reference configuration.

332 5.2. Influence of the indenter normal load σ_P

333 Fig. 10 (left) shows the propagation paths estimated for different values
 334 of the indenter load σ_P after 14 crack growth increments. Note that a new
 335 set of loads is considered in this and following subsections because of the
 336 limitations of the testing rig shown in Fig. 6.

337 The results confirm the analysis given in Section 4 because it is verified
 338 that the amount of indenter load does not significantly affect the crack
 339 orientation. This is in line with the experimental evidence of Fig. 2. This
 340 unexpected behaviour was one of the motivations of this work. Although
 341 the crack path tends to grow inwards, normal loads higher than $\sigma_P = 80$
 342 MPa do not further affect the crack orientation and converge to very similar
 343 crack paths.

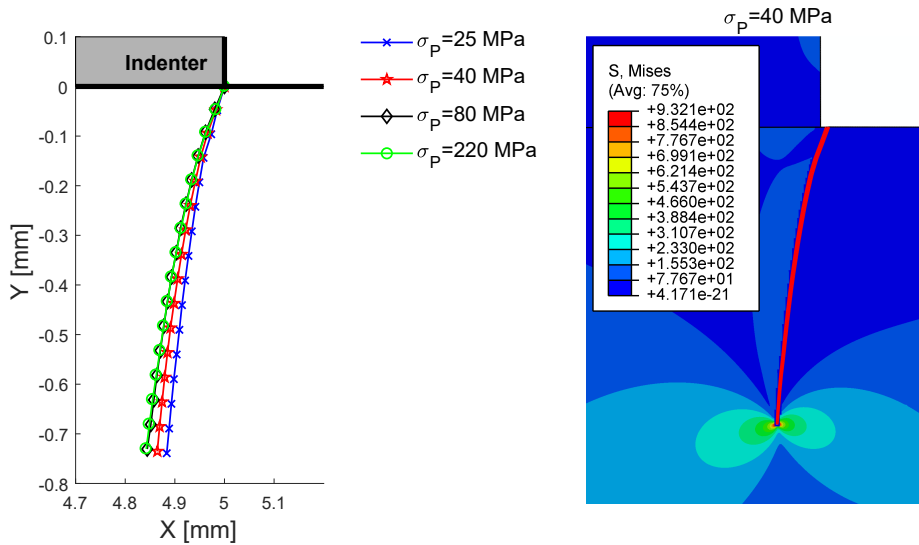


Figure 10: Left, influence of the indenter load σ_P . Crack propagation after 14 increments using X-FEM and the $\min(\Delta\tau)$ criterion. Right, von Mises contour plot and crack propagation for $\sigma_P=40$ MPa (reference problem).

344 *5.3. Influence of the friction coefficient μ_{CF} and μ_{IS}*

345 The potential influence of the friction coefficient between crack faces
346 and between the indenter and specimen is addressed in this subsection. A
347 friction coefficient of $\mu = 0.8$ reported in [19] has been considered in the rest
348 of analyses of this work for both contacts. However, a sensitivity analysis
349 of these parameters was deemed necessary due to the uncertainty of their
350 estimation.

351 Fig. 11 (left, **top**) shows that a wide range variation of μ_{CF} between crack
352 faces does not lead to any relevant modification of the crack path. However,
353 Fig. 11 (left, **bottom**) displays a relevant influence of μ_{IS} between indenter
354 and specimen. The rest of parameters are the same as in Section 5.2, with
355 $\sigma_P = 40$ MPa. Values close to $\mu_{IS} = 0.8$ and greater converge to very
356 similar crack paths. On the other hand, there is no shear stress along the
357 contacting plane for the limiting and ideal case of $\mu_{IS} = 0$. Therefore, the
358 directions 0° and 90° are directions of $\min(\Delta\tau)$ and, accordingly, the crack
359 grows in the 90° direction.

360 *5.4. Influence of the indenter width $2c$*

361 The standard width of the indenter used in this work and previous stud-
362 ies [15] is $2c = 10$ mm, which implies a relative ratio to specimen **height** $2b$
363 of 1.0, see Fig. 6. Different analyses were carried out changing the indenter
364 width and keeping the rest of parameters as in Section 5.2 with $\sigma_P = 40$
365 MPa. The results shown in Fig. 12 (left) reveal that there is a large influ-
366 ence of the indenter width on the inclination of the crack path: the larger
367 the indenter width, the larger the inclination of the crack path up to a point
368 at which this inclination reaches a limit. This behaviour will be explained

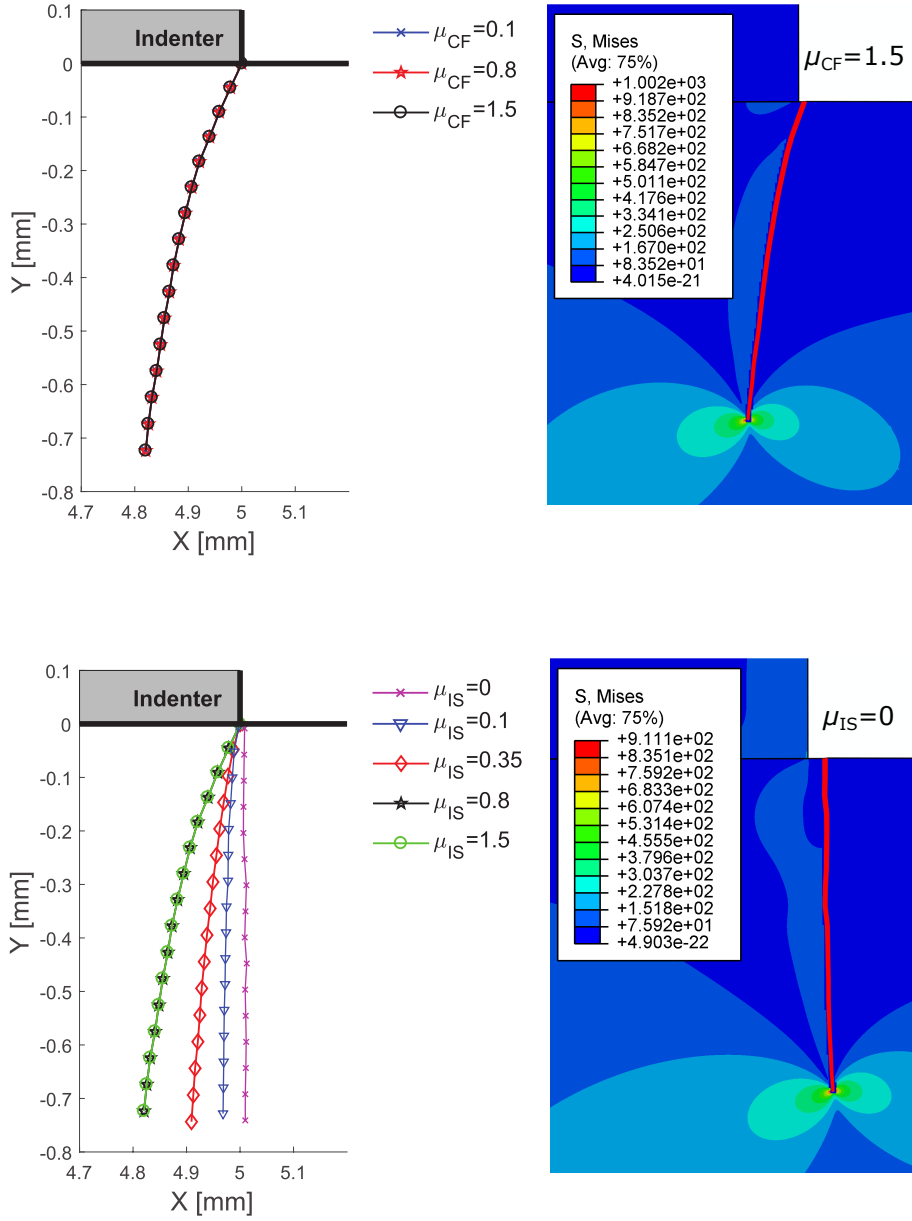


Figure 11: **Top**, influence of the friction coefficient μ_{CF} between crack faces and μ_{IS} between indenter and specimen (**bottom**). Crack propagation after 14 increments using X-FEM and the $\min(\Delta\tau)$ criterion. **Figures on the right show von Mises contour plots and numerical crack propagations for one case of both comparisons.**

369 in Section 5.7.

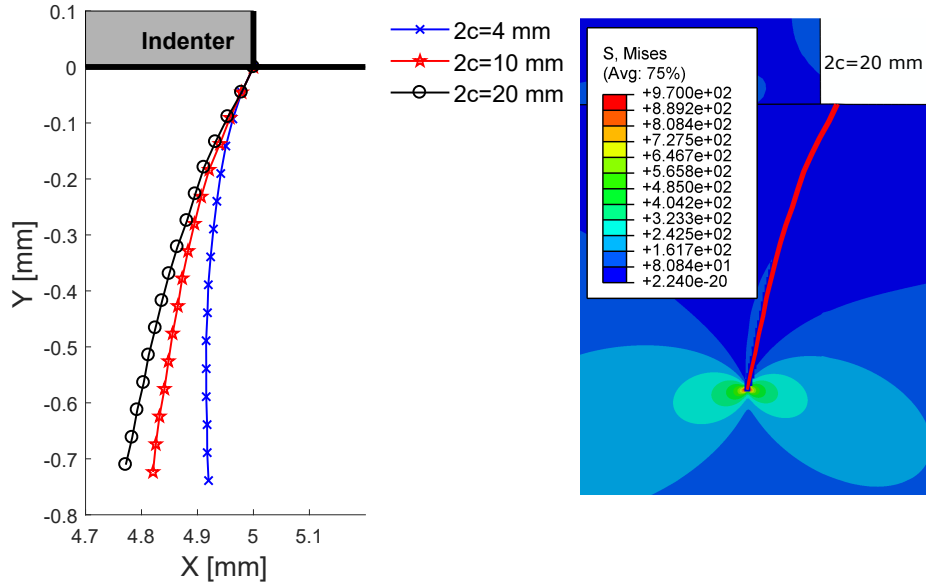


Figure 12: **Left**, influence of the indenter width $2c$. Crack propagation after 10 increments using X-FEM and the $\min(\Delta\tau)$ criterion. **Right**, von Mises contour plot and crack propagation for the case $2c=20$ mm.

370 5.5. Influence of the indenter *Young's modulus* E_{indenter}

371 Fig. 13 (**left**) shows the results obtained when changing the **Young's**
 372 **modulus** of the indenter, i.e. considering dissimilar materials for indenter
 373 and specimen. It can be seen that the relative stiffness of the indenter with
 374 respect to the specimen has an influence on the crack deflection inwards the
 375 contact zone. This effect, together with the influence of the indenter width,
 376 enabled us to gain insight into the mechanisms that cause the inclination
 377 of the the crack path, as explained in Section 5.7.

378 It can be observed that for the case of a negligible **Young's modulus**
 379 the predicted angle after the initial crack is close to $\theta = 90^\circ$. The larger

380 the indenter **Young's modulus**, the larger the inclination of the crack path,
 381 reaching a limit which corresponds to an angle of about $\theta = 75^\circ$.

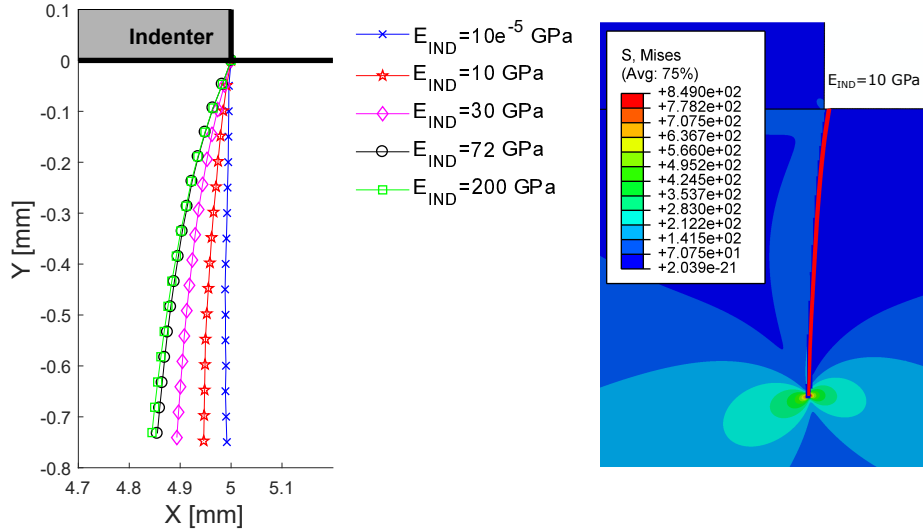


Figure 13: **Left**, influence of $E_{indenter}$. Crack propagation after 14 increments using X-FEM and the $\min(\Delta\tau)$ criterion. $E_{specimen} = 72 \cdot 10^3$ MPa for all cases. **Right**, von Mises contour plot and crack propagation for the case $E_{indenter} = 10$ GPa.

382 5.6. Experimental verification

383 Figs. 14 and 15 show experimental paths found for tests with different
 384 indenter widths and different indenter **Young's moduli**, respectively. The
 385 loads are defined in Section 5.2, with $\sigma_P = 40$ MPa. As expected, the
 386 micrographs show that the crack inclination is slightly greater for a steel
 387 indenter than for an aluminium indenter. Fig. 14 shows a greater inclination
 388 of the path, which was also observed in the previous numerical results.

389 5.7. Discussion

390 It has been shown that the most relevant parameters affecting the crack
 391 orientation are the indenter width, the relative **value of the indenter Young's**

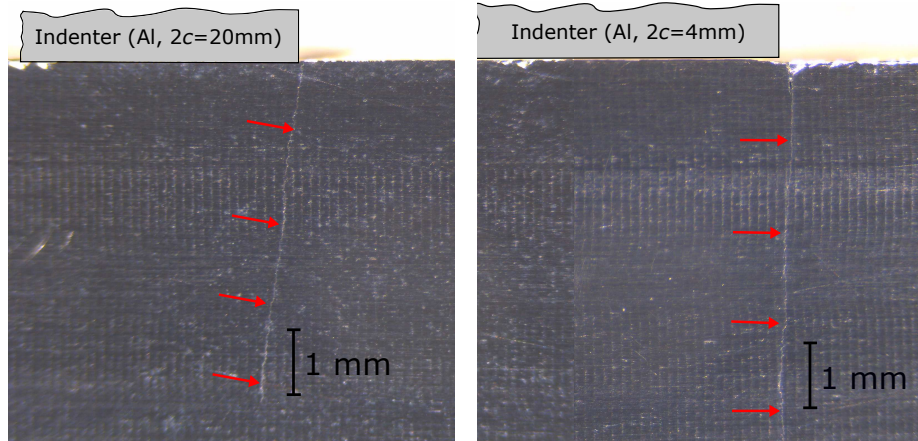


Figure 14: Influence of indenter width. Experimental paths found for two tests with aluminium indenters, width $2c = 20$ mm (left) and width $2c = 4$ mm (right).

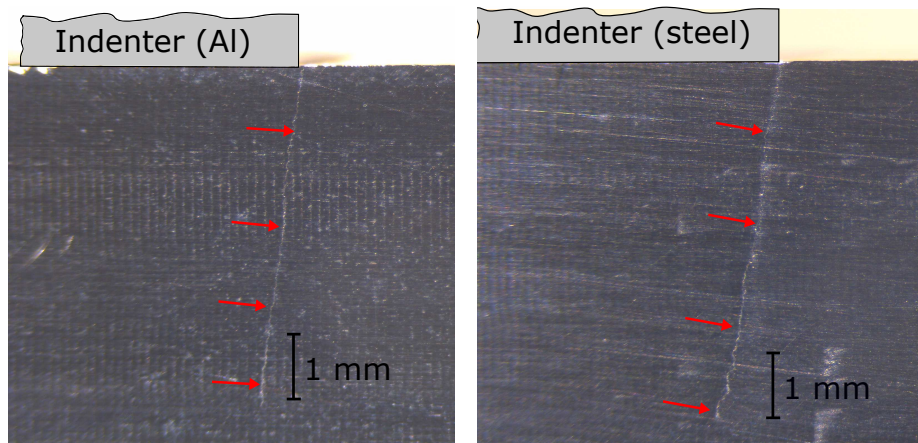


Figure 15: Influence of E_{indenter} . Experimental paths found for two tests with aluminium (left) and steel (right) indenters, both of width $2c = 20$ mm.

392 modulus with respect to the specimen and the coefficient of friction between
 393 indenter and specimen. In what follows, we provide a simple explanation for
 394 this behavior: the indenter acts as a contacting solid next to the specimen
 395 through which the force lines deviate. This is due to its stiffness and geom-
 396 etry, since a stiff solid tends to transfer a higher load than a compliant solid

397 (assuming a parallel configuration). Similarly, a large indenter width allows
 398 for an easier force line deviation than for a small indenter width (relative to
 399 the specimen height). This can be visualized by the directions followed by
 400 the maximum principal stresses shown in Fig. 16 for one of the analysis. It
 401 can be seen that the principal directions (that can be assimilated to local
 402 force lines) tend to divert to the indenter just behind the crack. Therefore,
 403 it is expected a growth path approximately normal to the directions of the
 404 force lines in this region. The amount of deflection reaches a limit, despite
 405 a high increase of E_{indenter} , due to the geometric configuration of the model
 406 that does not allow for further deviation of the force lines.

407 For the case $\mu_{\text{IS}} = 0$ shown in Fig. 11, the absence of shear stresses along
 408 the contacting surface implies that the directions 0° and 90° are principal
 409 stress directions. Therefore, the line forces in the specimen are parallel to
 410 the contacting surface and do not divert to the indenter, leading to a crack
 411 growth in the 90° direction.

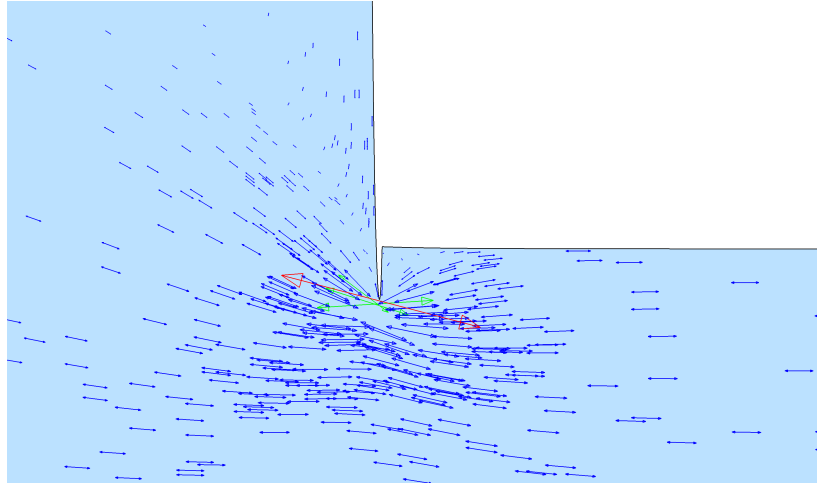


Figure 16: Deviation of the maximum principal stress directions near the contact zone. Enlarged view around the crack tip.

412 **6. Conclusions**

413 In this work, a parametric study of the main factors affecting a fretting
414 fatigue problem under complete contact conditions has been performed.
415 Several parameters have been varied, such as the normal load on the inden-
416 ter, the cyclic bulk load on the specimen, the stress ratio, the coefficient of
417 friction, the indenter width and the elasticity modulus of the indenter. It
418 has been shown that the parameters related to the loading have very little
419 effect on the crack deflection, whereas changes of the indenter width, the
420 indenter stiffness or the friction coefficient between indenter and specimen
421 have a more significant effect on the crack direction.

422 The crack path prediction has been performed numerically using XFEM
423 including a formulation that allows for crack face contact, which is essential
424 to take into account the effects during the compressive part of the cycle.
425 The approach combines XFEM with the criterion of the minimum shear
426 stress range along the whole cycle of loading, since this type of problem
427 is subjected to nonproportional loading, which invalidates the application
428 of conventional orientation criteria, such as the maximum tangential stress
429 (MTS). The numerical results are in good agreement with the experimental
430 observations, confirming the relevant factors affecting the crack orientation.
431 The work provides insight into the mechanisms governing crack orientation
432 in fretting fatigue and can lead to more accurate fatigue life estimations
433 once the crack path is predicted using the proposed procedure.

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