UNIVERSITAT POLITÈCNICA DE VALÈNCIA

ESCOLA POLITÈCNICA SUPERIOR DE GANDIA

Master en Ingeniería Acústica

UNIVERSITAT POLITÈCNICA DE VALÈNCIA

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"Evolutionary optimization processes for acoustic applications where size matters"

TRABAJO FINAL DE MASTER

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GANDIA, 2021

RESUMEN

El presente trabajo fue realizado por Daniel Benítez Aragón para obtener el título de Máster Universitario en Ingeniería Acústica por la Universitat Politècnica de València. El trabajo se basó en definir procesos de optimización evolutivos para aplicaciones acústicas, en las que el tamaño y la geometría de las mismas determinan los resultados.

Se desarrolló un algoritmo e, introduciendo una cierta cantidad de muestras, se obtenían de manera iterativa individuos con mejores resultados acústicos, reduciendo progresivamente su tamaño y geometrías.

Cada iteración eliminaba aquellos individuos con los resultados más desfavorables, siendo reemplazados por nuevas muestras. Estas eran generadas combinando pares de individuos óptimos, adaptando de manera aleatoria características geométricas de ambos "predecesores", de una forma similar a un proceso de evolución genética.

Palabras clave: *acústica*, *optimización*, *tamaño*, *geometría*, *algoritmo*.

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ABSTRACT

This thesis was carried out by Daniel Benítez Aragón in order to obtain the University Master's Degree in Acoustic Engineering of the Polytechnic University of Valencia. The project was based on defining evolutionary optimization processes for acoustic applications, in which their size and geometry directly influence the results.

An algorithm was developed, and by introducing a certain number of samples, subjects with better acoustic results were iteratively obtained, progressively reducing their sizes and geometries.

Each iteration eliminated those individuals with the most unfavourable results, being replaced by new samples. These were generated by combining pairs of optimal individuals, adapting randomly geometric characteristics of both, similar to a process of genetic evolution.

Key words: *acoustics*, *optimization*, *size*, *geometry*, *algorithm*.

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Acknowledgements

Firstly, I would like to say thank you to Jaime Galiana Nieves for being a side-collaborator and for all the support, dedication, availability and help he has provided to me. It has been extremely valued.

Also, to all and each member of my family who understood all the time I invested during this Master's Degree, and who respected it and supported me.

Thanks to you all.

1. Introduction

1.1. Purpose

The purpose of this document focuses on defining optimization processes for acoustic applications whose behaviour are **directly influenced by their size and geometry**, since they are the most critical aspects of this kind of applications in the majority of the cases.

These optimization processes shall be carried out with an **evolutionary algorithm**, in which new cases are generated iteratively based on the results of the previous cases, i.e. using **genetic implementations**.

This evolutionary algorithm was implemented on a **MATLAB** code, which is applicable to any acoustic implementation that lies on geometrical properties. This code, based on multi-objective genetic algorithms (hereafter, **MOGA**), allowed to find the best individuals for a two cost function problem, which usually are in conflict between each other. By defining the equations that describe the phenomenon, and the physical limits for which it is applicable, the optimization could subsequently be applied.

In regards to this, several cases have been analysed within this project in order to illustrate the performance of the process. This was meant to prove that the algorithm may work for any application, not only those that have been defined in this project. For each case, the final solution that assures the best performance within their field of application has been included. These cases can be found on following sections (check "4. Results" in page 36 for further information).

Finally, a set of findings and future lines were extracted out of the results of this project, which intended to illustrate the advantages and applications that this process may have, not only for the studied cases used as examples, but for any acoustic application, which is in turn the main aim of this project (check Section "5. Findings and future developments" in page 51).

1.2. Scope

This project develops an evolutionary optimization process for acoustic applications where size and geometry matters. Nevertheless, its scope is slightly wider since additional tasks must be approached. These tasks required collecting information on several fields, such as:

- Design of sound diffusers (principles of specular and diffuse reflections, Schroeder diffusers, design frequencies, etc).
- Design of sonic crystals for propagation of sound beams (properties, Insertion Losses calculations, band gaps, etc).
- Genetic algorithms for evolutionary optimization processes (definition of parameters, crossovers and mutation processes, etc).

1.3. Backgrounds

The development of an optimization process for any acoustic applications answers to one of the their most critical conditions: **their physical dimensions**. (i.e. **their size**). As in any engineering project, size goes along with expensiveness: the bigger the product is, the more expensive it shall become. **It is an everyday aim to minimize costs by all means when developing an engineering project**. On the other hand, acoustic applications have the property of being heavily dependant on geometry and size. This is basically because its frequency components have an inverse relation to length: **the lower the frequency of a wave, the higher its wave-length**.

This is a well-known issue when designing acoustic implementations, and it is easily represented by expression (1):

$$f = \frac{c}{\lambda} \tag{1}$$

Where:

- $\boldsymbol{f}$ is the frequency of the wave we are analysing.
- $\boldsymbol{c}$ represents the propagation speed of sound in the medium where the application is intended to work in. The most common medium is air, which is the one used in the examples of this project, and its value for environmental conditions (i.e., T = 20º C; p = 1 atm) is 343 m/s.
- $\boldsymbol{\lambda}$ is the associated wave-length of the analysed frequency.

Therefore, it is vital to keep **acoustic devices as small as possible**: not only because the smaller an application is, the easier it is to implement, but also, and most importantly in engineering problems, the cheaper it is to produce.

Nevertheless, engineering projects, and more specifically, acoustic projects, are often (if not always) affected by not only one parameter (in this case, size), but by several. Levels of attenuation, frequency limits, performance stability and so on are also factors that must be taken into account when analysing the quality of an acoustic implementation.

And, on top of that, if we are trying to optimize applications that are affected by several dimensions (such as acoustic diffusers, sonic crystals for acoustic barriers, etc) **they may present fairly different behaviours due to slightly variations of their geometry.** This produces a high amount of possibilities that need to be analysed in order to find the best solution. This factor makes this kind of problems a perfect example for a computerized calculation.

By implementing an optimization algorithm, i.e. taking advantage of the calculation potential of computers, this process can be automatized and shortened substantially. Moreover, it can be analysed how suitable and different an algorithm is when making it more complex, and in the end, determining until which point it is worth it, based on its time of operation.

Several scientific papers have started already to develop these methods in fields such as sonic crystals [5], which may be applied in acoustic implementations, or in devices like sound diffusers [1] [3], or noise barriers [10].

1.4. Work plan

In order to carry out this project, a **work plan** had to be developed, aiming to structure and organize each of the required steps. Thus, this plan has been divided into several activities that, all together, sum up the total amount of **300 hours** that were required to invest in this project. All these activities, and their respective amount of hours, have been represented in Table 1.

Activity	**Time (hours)**
Bibliographic review	75
Implementation of the evolutionary algorithm in a MATLAB code	100
Export of results and graphs. Development of figures	5
Writing of the memory	75
Review of the writing in English	5
Development of the presentation and audiovisual material	20
Meetings with the directors of the thesis	10
Fulfilment of documents to be delivered	10
TOTAL	**300**

Table 1: Work plan

A brief explanation of each of the activities explained in Table 1 can be found in the following list:

- **Bibliographic review**: Research and documentation based on the titles presented in the section "6. Bibliography" in page 53.

- **Implementation of the evolutionary algorithm in a MATLAB code**: Development and design of all functions, scripts and codes needed for the optimization algorithm, as well as the definition of those cases used as examples within this project.

- **Export of results and graphs. Development of figures**: Once the results are obtained thanks to the codes, they were represented in both analytical and graphical ways.

- **Writing of the memory**: Development of this document, as well as its structure, sections, design, etc.

- **Review of the writing in English**: Since this document has been entirely written in a language different from the author's native one, it requires a slightly greater amount of time to grammatically revise its content, making sure no errors or ambiguities were committed.

- **Development of the presentation and audiovisual material**: Along with this document, several files must be designed in order to show and explain this project during its presentation (PowerPoint files, animation videos, graphs, etc).

- **Meetings with the directors of the thesis**: Several verifications with the directors of this project have been carried out, paying attention on how it developed throughout time, and checking that the requirements and landmarks were reached on time and in an appropriate manner.

- **Fulfilment of documents to be delivered**: Once the project was finished, it was required to present it to the competent bodies of the Universitat Politècnica de València, being necessary to apply it before the established deadlines and making sure it fulfils all the requirements.

1.5. Software and design tools

The following software and design tools have been used in order to fulfil some of the tasks required during the project:

- **MATLAB R2020a**: Code development and creation of graphs.

- **Adobe InDesign CS6**: Creation and writing of this document, including its structure, style and layout.

- **Adobe Photoshop & Illustrator CS6**: Figure editing and processing.

- **Microsoft PowerPoint**: Creation of the presentation file used during its exposition.

- **MathMagic Pro 8.7**: Formulas and expressions designer.

2. State of the Art

2.1. Downhill simplex optimization method

An optimization problem requires finding a set of parameters with such characteristics that they fulfil a certain quality criteria that must be optimized. In other words, maximizing or minimizing a certain function $\boldsymbol{f}(\mathbf{x})$.

Before defining the optimization processes that were followed in this project, we considered appropriate to present first one of the basic approaches to this discipline. This way, we may understand more deeply the different enhancements that the method underwent through, and consequently, we will be able to gradually add different steps to it.

One of the basic approaches in any optimization problem is the **downhill simplex method**, also known as **Gradient descent** or **ascent method,** depending on the approach we are following (i.e. minimizing or maximizing). Hereafter, we will define the process based on a minimizing approach, being the other one developed in the opposite way.

This method is based on an iterative algorithm that aims to find a local minimum in a differentiable function $\boldsymbol{f}(\mathbf{x})$, by repeatedly taking steps in the opposite direction of the gradient. This is because this direction represents the **steepest descent**. Thus, if the function is defined and is differentiable for an specific range or neighbourhood, then $\boldsymbol{f}(\mathbf{x})$ at a point **A** decreases at a greater rate by going in the direction defined by the following expression:

$$-\nabla f(\mathbf{A}) \tag{2}$$

i.e, the negative gradient. This way, and through a previously set amount of iterations, the method can get closer to the minimum, as represented in Figure 1. The amount of iterations will have an effect on the results that the method could come up with.

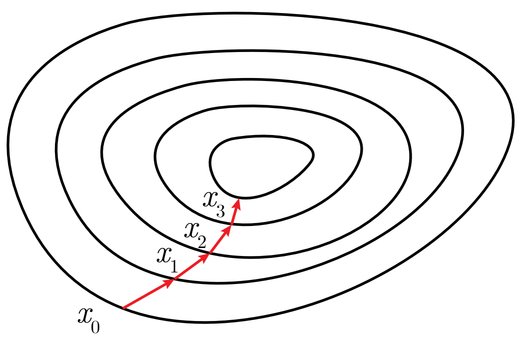

Figure 1: Gradient descent for a series of level sets. Red arrows represent the direction of steepest descent for each iteration.

2.2. Evolutionary optimization

Nevertheless, many of the optimization problems that show up in engineering fields are too difficult and complex to solve through traditional and analytical processes. This is the case of our studied applications of sound diffusers or acoustic barriers made up of sonic crystals: a subtle change in their configuration may produce a completely different response.

A common alternative to regular optimization is **evolutionary optimization**. This new approach is strongly based on the way evolution works in nature, **generating new individuals or samples based on the features of previous ones**. This task can be easily implemented within a computer, in order to make it find new solutions iteratively.

Evolutionary algorithms lie on a group of individuals that represents a set of solutions for a specific problem. This group undertakes then several **transformations** (which includes **crossovers and mutations processes**), and then go through a selection step which favours the best individuals of the updated group.

In order to always keep the population constant, some of the individuals are removed, making sure they are not part of the best samples so far. Once the condition of the optimization is reached, the algorithm offers a selection of the best individuals.

For every cycle of transformation and selection, the group of individuals reaches a new **generation**. This way, after a certain amount of them, the groups is expected to contain individuals that are closer to the ideal solution. To better understand this process, the schematic process of this evolutionary optimization algorithm has been represented in Figure 2.

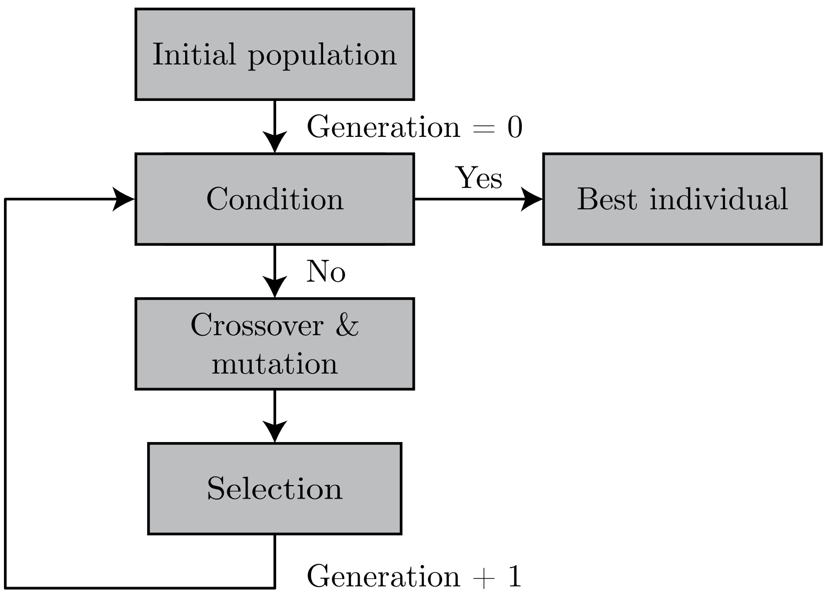

Figure 2: Basic scheme of an evolutionary optimization algorithm.

Therefore, the main components for an evolutionary algorithm are:

- **Population**: Initial samples that, throughout an specific amount of generations, gradually become a representation of potential solutions for our optimization problem.

- **Transformation process**: In order to generate new samples, several changes are applied within the group of individuals. They are based on:

    - **Crossovers**: A new individual is generated by mixing features of different samples, each of them in a certain proportion.

    - **Mutations**: Some of the features are randomly generated in order to add some more genetic variety into the process.

- **Selection process**: Once changes are applied, a group of individuals that fulfils the best with the requirements is chosen based on the qualities of its samples to solve the problem. Random individuals are also removed in order to keep the population constant.

These iterations may be run as many times as required. Depending on our needs, we might choose a certain amount or another. Besides, we must take into account that, due to the numerical approach of the method, there will be a point where finding better individuals would take enormous amounts of time. Therefore, to find the figure that fits the best with our requirements, several implementations should be analysed.

2.2.1. Local minimums

During the optimization process, it is possible to reach a result that might seem like a minimum or maximum in its neighbourhood, but that in turn does not represent the real minimum/maximum of the global study, like the case that has been represented in Figure 3.

These results are called **local minimums/maximums** and they have to be avoided by all means in order to achieve the best possible solution. In fact, the evolutionary implementation of our optimization process aims to avoid this local minimums, looking for the global one, which is also reflected in the developed codes that have been employed within this project.

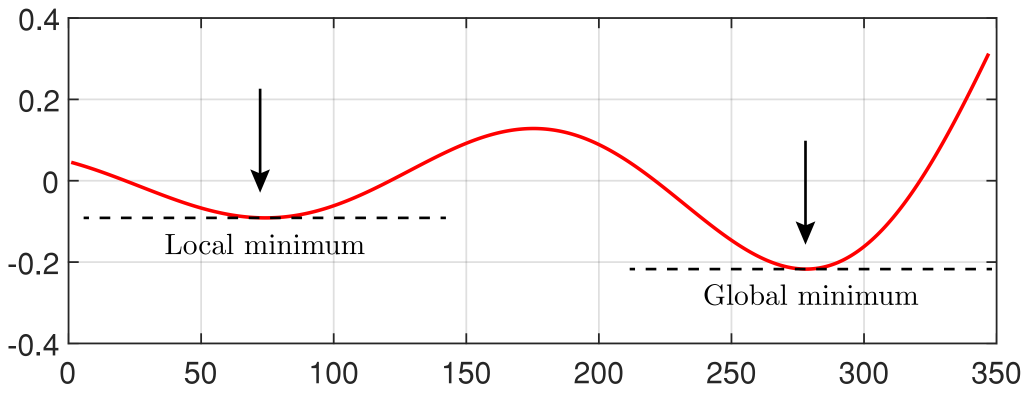

Figure 3: Difference between local and global minimums.

2.3. Genetic algorithms

As mentioned in Section "2.2. Evolutionary optimization", an evolutionary optimization problem shall require an **initial group of individuals**. This groups would be generated **randomly**, in order to provide enough **genetic diversity** into the process, which is the easiest and most popular initialization method [15]. This enhances the algorithm and makes it easier to find better solutions. This way, by introducing a certain number of samples, subjects with better acoustic results will be iteratively obtained.

For each iteration, a specific set of individuals were removed from the population in order to keep it constant, being replaced by the new samples. The removed ones were not necessarily those with the most unfavourable results but actually any individual apart from the best ones. This way, we assure that **the genetic diversity is kept high**, which is key for this process. These new individuals were generated by combining pairs of optimal ones, **randomly adapting geometric characteristics of both of them**, similar to a process of genetic evolution. As stated before, this implementation is intended to work with **any acoustic application that could be defined by geometrical factors**, and not only with those cases analysed in this project.

This way, an important factor to stablish is the **cost function** or **cost parameter**, i.e. the variable that determines the quality of the optimization [1]. This factor will be extensively described in following sections, but in most cases, there are not one, but two cost parameters involved (such as size, frequency homogeneity, costs, etc), and usually, they are in opposition to each other.

Finally, two different applications were described and developed as examples for the optimization process described in this document. Nevertheless, and as stated before, this method may as well be applicable for any other application.

The two cases that have been developed within this document are:

- Sound diffusers.
- Sonic crystals for acoustic barriers.

2.4. Multi-objective optimization

When applying optimization process to real situations, we find that they usually are multi-objective, i.e. multiple goals must be achieved in order to obtain the best results. And typically, these goals are in conflict with each other [15]. Using the cases previously defined as illustrative examples, we can find within them the following goals that need to be accomplished:

- When designing a sound diffuser, we might want to minimize its thickness and maximize its frequency response.
- When designing an acoustic barrier with sonic crystals, we might want to minimize its weight (and consequently, its size) and at the same time, maximize its performance.

Multi-objective optimization focuses then on minimizing a vector $\boldsymbol{f(x)}$ of fuctions, being $\boldsymbol{x}$ the independent variable compound of $\boldsymbol{n}$ dimension, as stated in the following expression [15]:

$$\min_x f(x) = \min_x [f_1(x), f_2(x), \cdots, f_k(x)] \tag{3}$$

2.5. Pareto front

When analysing the group of individuals that takes part into our multi-objective optimization problem, we must find those samples that are considered the best. Due to the multi-objective approach, we would not find only one a individual, but **several**. This way, we must identify those samples for which **none of the others are better for all the objectives at the same time**. In order to clearly fulfil this conditions, we must define at this point several concepts [15]:

- **Domination**: A point $\boldsymbol{x^*}$ is said to dominate $\boldsymbol{x}$ if $\boldsymbol{x^*}$ is at least as good as $\boldsymbol{x}$ for all objective function values, and it is better than $\boldsymbol{x}$ for at least one objective function value.
- **Weak domination**: A point $\boldsymbol{x^*}$ is said to weakly dominate $\boldsymbol{x}$ if $\boldsymbol{x^*}$ is at least as good as $\boldsymbol{x}$ for all objective function values. Note that if $\boldsymbol{x^*}$ dominates $\boldsymbol{x}$, then it also weakly dominates $\boldsymbol{x}$.
- **Non-domination**: A point $\boldsymbol{x^*}$ is said to be non-dominated if there is no $\boldsymbol{x}$ that dominates it.

Therefore, we can define a certain type of samples as **Pareto optimal points**:

A Pareto optimal point $\boldsymbol{x^*}$, also called a Pareto point, is one that is not dominated by any other $\boldsymbol{x}$ in the search space.

In Figure 4 it has been represented three Pareto points as red dots named **A**, **B** and **C**. It can be observed that there is no other point in the population that dominates them:

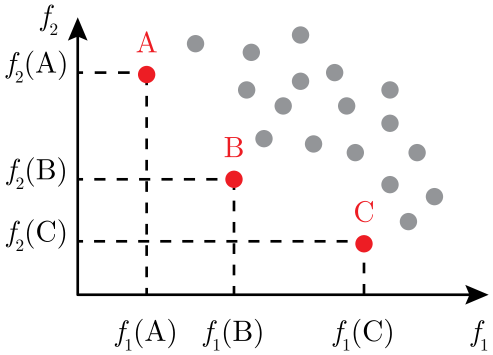

Figure 4: Three Pareto points within a group. None of them are dominated for any other individual in the population.

Thus, in any multi-objective optimization problem, there will be a group of individuals that fulfil this requirement and that could be therefore considered as Pareto points. This group conform then the so called **Pareto Front**, represented in Figure 5 by the red dots and blue line.

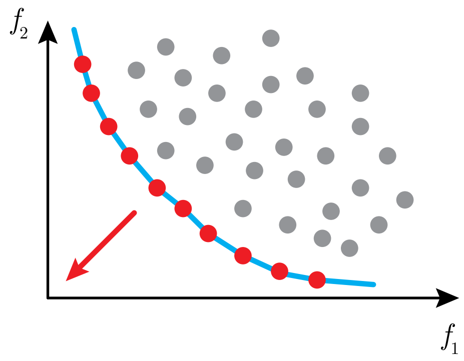

Figure 5: Pareto front. The red arrow represents the direction that the optimization process should follow and aim to, and for which the algorithm will define new individuals, and consequently, find better results in new generations of groups.

2.6. Cost parameters

As stated in Section 2.3 “Genetic algorithms” (page 16), cost parameters are the variables that determine the **quality of the individuals** [1], and in multi-objective optimization processes, there are always at least two of them. They are the functions that must be minimized or maximized, and usually, doing this to one of them means doing the opposite to the other one. This way, it is usually said that the optimization of the cost parameters **are in conflict with each other**.

In the bibliography, cost parameters can also be named as *fitness function* when referring to maximizing problems, while *cost function* is used for minimizing purposes [15].

Each of the cases that were studied in this project (i.e. sound diffusers and sonic crystals sound barriers) had two different cost parameters, which are going to be described in Section 3.1 “Cost parameters for analysed cases” (page 27).

Therefore, in order to achieve a proper result in any of these acoustic applications, the optimization process must aim at both cost parameters, minimizing or maximizing them at the same time. It is by doing this that the Pareto front could be updated in every new iteration, and with it, the group of individuals that represent the set of solutions for our application.

In order to clearly define and understand the cost parameters that are going to be defined for every proposed case, we will present a brief theoretical explanation of this two implementations. This way, we will have a better understanding of the phenomenons that take place during their performances.

2.7. Sound diffusers

2.7.1. Introduction to sound diffusers

The sound that individuals can hear is not only based on the direct sound coming from the source, but **the combination of it with the reflections caused by all the surfaces** around the source.

In regards to these reflections, when the sound is not specularly but spatially and temporally dispersed, it is then considered as **diffuse reflection**, and the surface which produces it is known as a **diffuser** (check Figure 6).

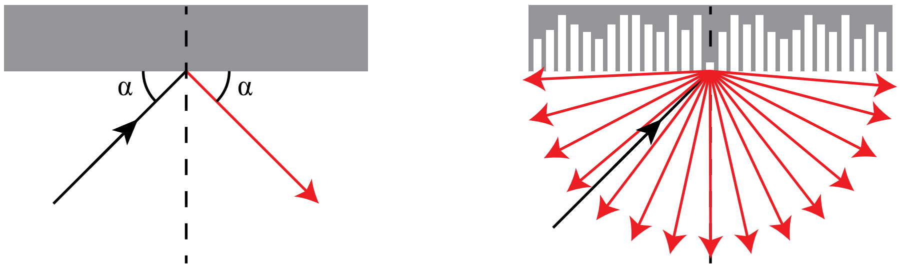

Figure 6: Specular and diffused reflections represented (red arrows) after an incident wave (black arrow) reaches different surfaces.

The first diffusers that were developed did not disperse sound in a predictable manner. This changed when Manfred R. Schroeder invented the phase grating diffuser, also known as Schroeder diffusers, which only required a small number of simple design equations [14].

2.7.2. Schroeder diffusers

Schroeder diffusers consist basically in a surface made of a series of wells, each of them having the same width but different depths. These depths are not randomly set, but determined by a **mathematical number sequence**.

Figure 7: Two typical Schroeder diffusers

If we consider a plane wave facing onto the diffuser, plane waves will be also reflected from the bottom of the wells, re-radiating again into the space. If we take a point external to the diffuser, its pressure value will result from the interference between the radiating waves from each well. All of them will have the same magnitude but different phase due to the time it takes the wave to enter and leave each of the wells independently.

The polar distribution of the reflected waves and their resultant pressure is then determined by the choice of **well depths**. Schroeder demonstrated that, by choosing a **quadratic residue sequence**, the energy reflected into each diffraction lobe direction is the same. This allowed to obtain an homogeneous polar distribution.

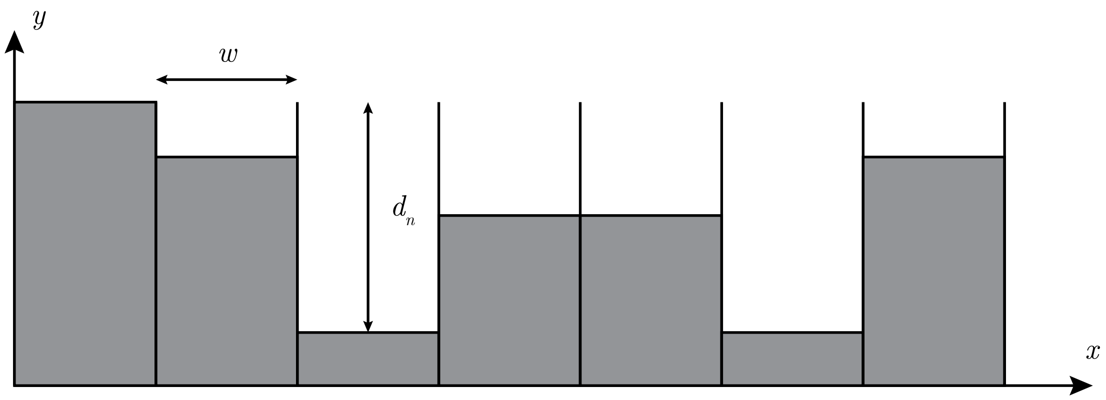

*Figure 8: Cross-section of a Quadratic Residue Diffuser with **N** = 7. Each well **n** will be defined by its depth $\boldsymbol{d_n}$, while all of them shall have the same width **w**.*

Therefore, using a quadratic residue sequence to come up with the set of well depths, the sequence number for the $\boldsymbol{n}^{\text{th}}$ well, $\boldsymbol{s_n}$, could simply be calculated making use of expression (4):

$$s_n = n^2 \cdot \bmod N \tag{4}$$

Where:

- **mod** indicates the least non-negative remainder.
- ***N*** is a prime number.

Therefore, if we take as an example $\boldsymbol{N} = 7$, the diffuser generated by it shall follow the sequence represented in Table 2.

$\boldsymbol{n}$	0	**1**	**2**	**3**	**4**	**5**	**6**
$\boldsymbol{n^2}$	0	**1**	**4**	**9**	**16**	**25**	**36**
$\boldsymbol{n^2 \cdot \bmod N}$	0	**1**	**4**	**2**	**2**	**4**	**1**

Table 2: Sequence for a Schroeder diffuser with N = 7.

2.7.3. Design frequency

At this point, it is important to remark that Schroeder diffusers are intended to work at **integer multiples of a design frequency**, f_0. Therefore, this design frequency is normally set as the **lower frequency limit**.

However, for design purposes, it is usually better to stablish formulations in terms of the corresponding **design wavelength**, λ_0. Thus, in order to calculate the depth d_n of the n^{th} well, we must use the elements from the previously defined sequence through the following expression [14]:

$$d_n = \frac{s_n \lambda_0}{2N} \tag{5}$$

It can be deduced out of expression (5) that well depths will then vary between 0 and $\lambda_0/2$, approximately. Anyhow, it is important to take into account that the design frequency is just the first frequency at which **even energy diffraction lobes can be achieved**.

2.8. Sonic crystals for acoustic barriers

2.8.1. Introduction to acoustic barriers

An acoustic barrier is a constructive element which size is fairly bigger than the wave-lengths that it is intended to affect on. They work as an obstacle in between the straight line that connects a sound source and a receptor [6], as represented in Figure 9.

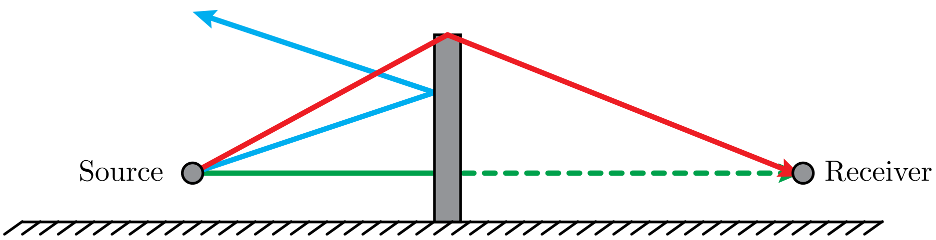

Figure 9: Effect produced by a barrier over acoustic waves between a source and a receiver.

Where:

- The **green line** is the path that the direct sound would have followed if there was no barrier. When the barrier is present, the green line represents the transmitted wave that goes through the barrier.
- The **red line** is the shortest path that the acoustic wave finds to reach the receiver without being affected by the barrier. It is important to remark that, when the front wave reaches the barrier, it works as a new source, producing new front waves within its limits.

- The **blue line** represents the sound waves reflected by the barrier, which do not reach, and therefore affect, the receiver.

Acoustic barriers are extensively employed in roads and highways with great amount of traffic, or close to train railways that go through urban settlements. They enhance the living conditions around these places by significantly reducing the acoustic contamination that are produced by these sound sources.

2.8.2. Performance of noise barriers

In order to determine the influence of the acoustic barrier, a parameter known as **Insertion Loss** (***IL***) can be calculated [7]. It is based on comparing the behaviour of sound before and after placing the acoustic barrier, measuring in both cases the sound pressure produced by the source (i.e. $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$).

With these two values, the Insertion Loss can be calculated. It represents the level of attenuation produced by the barrier in the sound pressure field, and it can be obtained by making use of expression (6).

$$IL = 10\log\left(\frac{p_1^2}{p_2^2}\right) \tag{6}$$

Similarly, the Insertion Loss that is produced in each frequency band can also be combined in order to produce a single figure of merit, known as **Global Insertion Loss** ($\boldsymbol{DL_{IL}}$). This can be calculated making use of expression (7).

$$DL_{IL} = -10\log\left|\frac{\sum_{i=1}^{18} 10^{0.1L_i}\,10^{-0.1IL_i}}{\sum_{i=1}^{18} 10^{0.1L_i}}\right| (\text{dBA}) \tag{7}$$

Being:

- $\boldsymbol{L_i}$ the ponderated value of the A-weightening factor at the $\boldsymbol{i}^{\mathbf{th}}$ one third octave band (there would be 18 in total, from 100 Hz to 5000 Hz).

- $\boldsymbol{IL_i}$ the value of the Insertion Loss at the $\boldsymbol{i}^{\mathbf{th}}$ one third octave band.

At this point, it is important to remark that the final Insertion Loss value generated by the acoustic barrier would not be calculated by only taking into account the inherent features of this element. Thus, the sound that is the diffracted by the upper edges of the barrier will also have an important effect in the isolation that it produces.

Figure 10: Acoustic barriers in between residential areas of the city of San Fernando (Spain).

As shown in Figure 10, acoustic barriers are usually rigid surfaces with a such a height that they can be problematic for locations were **rain and wind cause floods and drain problems**. These factors may even lead to the impossibility of using these barriers. Due to this, some studies based on acoustic barriers made of sonic crystals have been developing lately. This barriers would be made of periodic crystal structures that allow air and water to pass through them, and simultaneously, act over specific frequency bands. Sonic crystal acoustic barriers are usually made by 3 or 4 rows of this structures [9] [10].

2.8.3. Acoustic barriers made of sonic crystals

A crystal is any solid whose fundamental elements are **distributed spatially in a periodic way**. When studying how waves behave through those materials, it can be observed that there are frequency bands for which the medium is virtually transparent and others that are significantly diminished by the barrier. By using elements whose size is comparable to the wave-lengths (usually, cylinders such as the ones represented in Figure 11), barriers that affect on specific frequency ranges can be designed. These ranges are called **band gaps**, and their characteristics determine the quality of an acoustic barrier.

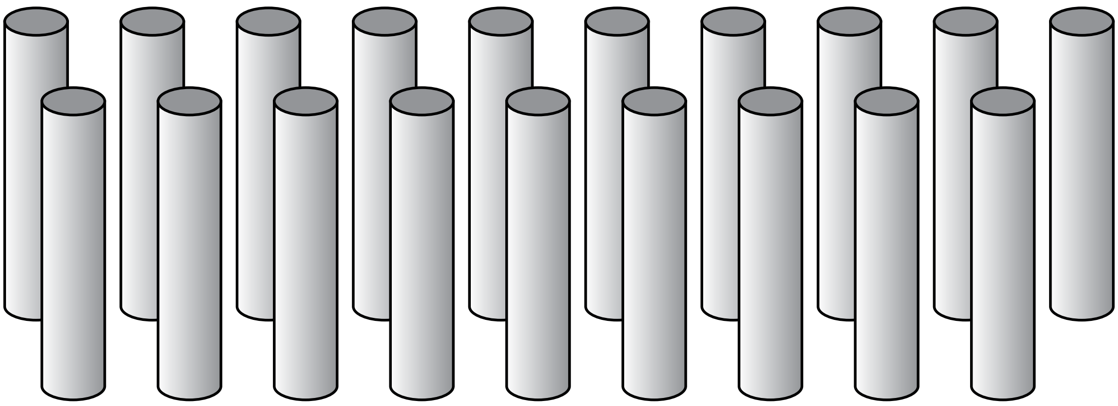

Figure 11: Acoustic barrier following a sonic crystal structure.

When an acoustic wave goes through a barrier made of this kind of cylinders, it will reflect on every diffuser element. These reflections may result in a destructive interaction, which can be replicated as many times as required to make the wave not to propagate at the other side of the barrier. This effect may be achieved by placing each element at a certain distance, depending on the desired frequency. This distance is defined by **Bragg's Law**, which is represented in expression (8):

$$2d\sin\theta = n\lambda \tag{8}$$

Being:

- $\boldsymbol{d}$ the distance between elements.
- $\boldsymbol{\theta}$ the angle between the incident and reflect waves.
- $\boldsymbol{n}$ positive integers (1, 2, 3, ...).
- $\boldsymbol{\lambda}$ the chosen wave-length.

Sonic crystals (SCs), also known as photonic crystals (PCs) in optics, are **spatially modulated materials** that have **temporal dispersion properties**, that also modify the **spatial dispersion** (also called **diffraction**). These properties allow to manage the diffractive broadening of acoustics beams.

2.8.4. Prediction of the performance of acoustic barriers

In order to characterize acoustically the different individuals of the population that were obtained throughout the optimization process, a simulation model was developed within MATLAB. It was based on the numerical technique known as **FDTD** (**Finite Difference Time Domain**).

Although this method was originally intended for simulations of the electromagnetic field, its discretization processes may be also implemented for acoustic waves, allowing us to simulate the behaviour of sound in the time domain. The fact of working in this time domain allows us to **run only one simulation and obtain information within a wide frequency range**, in contrast with other methods that need to calculate one frequency at a time.

This way, the wave equations that are going to be used in the simulation can be defined as follows:

$$\nabla p + \rho_0 \frac{\delta \vec{u}}{\delta t} = 0 \tag{9}$$

$$\frac{\delta p}{\delta t} + \rho_0 c^2 (\vec{\nabla} \cdot \vec{u}) = 0 \tag{10}$$

Where:

- $p(r,z,t)$ is the sound pressure.
- $\vec{u}(r,z,t)$ is the particle's speed.
- ρ_0 is the density of the medium.
- c is the propagation speed of sound in that medium.

For the purposes of this project, we are going to implement this simulations in a two dimensional layout. This way, we develop expression (9) and expression (10) in both $\boldsymbol{x}$ and $\boldsymbol{y}$ directions, obtaining the following:

$$\frac{\delta p}{\delta x} + \rho_0 \frac{\delta u_x}{\delta t} = 0 \tag{11}$$

$$\frac{\delta p}{\delta y} + \rho_0 \frac{\delta u_y}{\delta t} = 0 \tag{12}$$

$$\frac{\delta p}{\delta t} + \rho_0 c^2 (\frac{\delta u_x}{\delta t} + \frac{\delta u_y}{\delta t}) = 0 \tag{13}$$

These expressions must be discretised following the definition of a derivative, as the slope between two points that are separated by a differential space (both in the spacial and the time domain). Since the equations are discretised, so must be the space domain that is analysed. To do so, the space is divided into an specific amount of "**cells**" which a length **Δx**, both in the directions **x** and **y**. This length is known as **spacial increment**.

With this parameter, and knowing the characteristics of the medium that we are analysing, we may also obtain the **temporal increment**. With it, we can define the sample rate and the maximum frequency we can work at. This way, once we know the dimensions of the domain we are going to simulate, and its properties, three matrixes are generated: one for the calculation of sound pressure, and two for the calculation of speeds in each point of the domain, both in the directions **x** and **y**. Thus, we can finally obtain the discretised expressions for the computational implementation of this method:

$$v_x^{n+\frac{1}{2}}\left(i+\frac{1}{2},j\right) = v_x^{n-\frac{1}{2}}\left(i+\frac{1}{2},j\right) - \frac{\Delta t}{\rho \Delta x}[p^n(i+1,j) - p^n(i,j)] \tag{14}$$

$$v_y^{n+\frac{1}{2}}\left(i,j+\frac{1}{2}\right) = v_y^{n-\frac{1}{2}}\left(i,j+\frac{1}{2}\right) - \frac{\Delta t}{\rho \Delta y}[p^n(i,j+1) - p^n(i,j)] \tag{15}$$

$$\begin{aligned} p^{n+1}(i,j) = p^n(i,j) - k\frac{\Delta t}{\Delta x}\left[v_x^{n+\frac{1}{2}}\left(i+\frac{1}{2},j\right) - v_x^{n-\frac{1}{2}}\left(i-\frac{1}{2},j\right)\right] \\ -k\frac{\Delta t}{\Delta y}\left[v_y^{n+\frac{1}{2}}\left(i,j+\frac{1}{2}\right) - v_y^{n-\frac{1}{2}}\left(i,j-\frac{1}{2}\right)\right] \end{aligned} \tag{16}$$

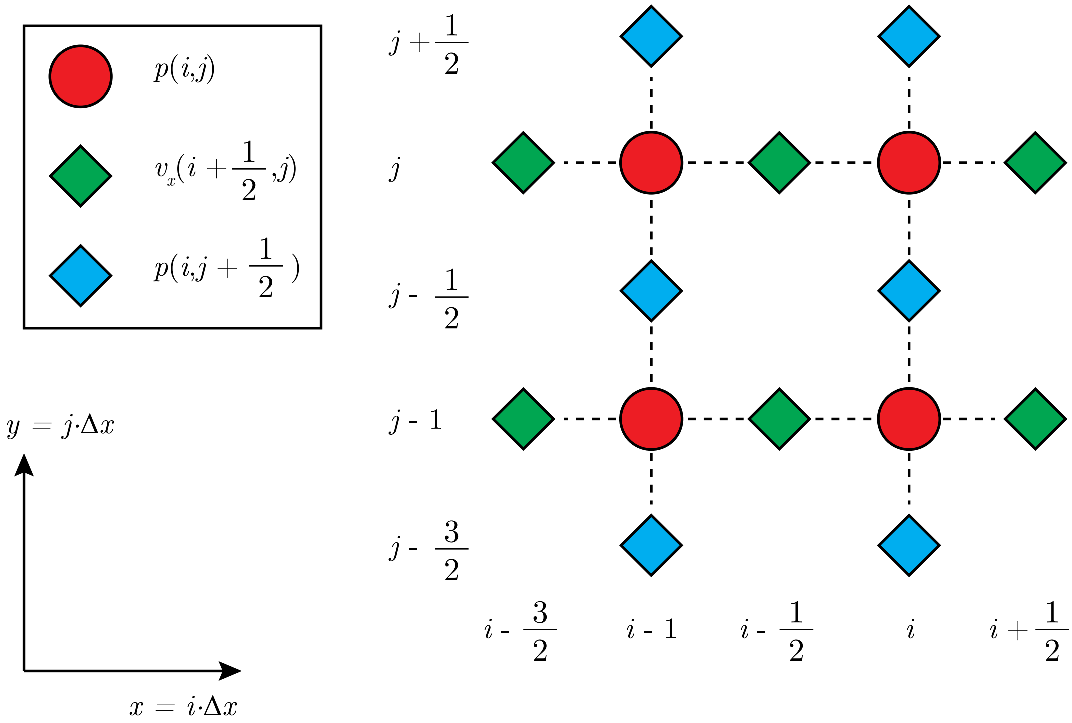

Figure 12: Pressure (red circles) and speed matrixes (blue and green squares) in a 2D simulation following the FDTD method.

This FDTD simulation worked simultaneously with the multi-objective optimization process (check Figure 13), and allowed us to calculate the required parameters for the analysis of sound barriers.

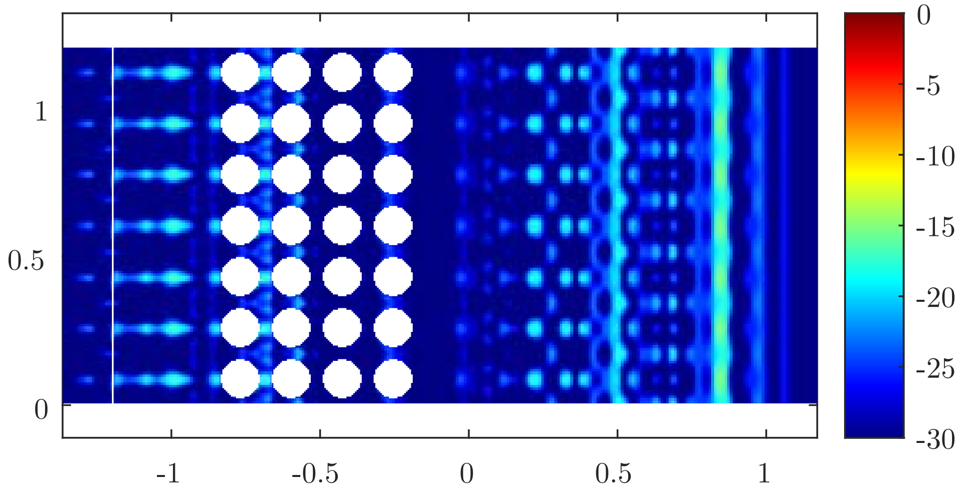

Figure 13: Snapshot from FDTD simulation implemented within the software MATLAB. In this case, for a 4 × 7 sonic crystal sound barrier.

3. Project development

3.1. Cost parameters for analysed cases

As stated in Section 2.3 "Genetic algorithms" (page 16), cost parameters are the variables that determine the **quality of the optimization** [1], and in multi-objective optimization processes, there are always at least two of them.

For each of the cases that were described in this project, two different cost parameters had to be set. This way, the optimization process could be defined and therefore, implemented. The following sections gather these parameters for each of the cases.

3.1.1. Sound diffusers cost parameters

As defined in Section 2.7 "Sound diffusers" (page 19), the polar distribution of the reflected waves and their resultant pressure is determined by the choice of **well depths**. This feature is what determines the physical size of the object, and it is vital to keep them **as thin as possible**.

Therefore, the **depth of the diffuser** shall be one of its cost parameters, and it will be necessary to **minimize its value**.

On the other hand, due to their own physical properties, Schroeder diffusers are only useful for a certain range of frequencies. Since in room acoustics, the most problematic range for absorption and distribution is the low frequency band, then it is key to know until which lower limit it is applicable. Beyond this limit, the diffuser will act as a flat surface in regards to the wave.

Then, this **lower limit** shall be the second cost parameter, and it will be necessary to **minimize its value**.

This means that, in regards to sound diffusers, there must be a compromise between **its greatest depth and the lowest frequency it works at**. It will not make sense if a diffuser is extremely thin, but its lowest applicable frequency is too high, and vice-versa.

3.1.2. Sonic crystal sound barriers cost parameters

As defined in Section 2.8 "Sonic crystals for acoustic barriers" (page 21), these implementations are made of elements whose size is comparable to the wave-lengths of the frequency we want to affect on. Usually, these elements are cylinders such as the ones represented in Figure 11, and their weights are the features that set their physical sizes. By increasing or reducing their radius, we will affect their weights accordingly. Therefore it is vital to keep them **as light as possible**, being their **weight** the first cost parameter.

However, we cannot forget that this weight depends on the density of the chosen material, and that why we are going to plot all the results using **arbitrary units.**, which may be then scalable whenever a specific material is selected.

On the other hand, the influence of an acoustic barrier is measured by the **Insertion Loss (*IL*)**, which was defined by expression (6) in page 22. As can be observed, this parameter compares the behaviour of sound before and after placing the acoustic barrier, measuring in both cases the sound pressure produced by the source (i.e., $\boldsymbol{p_1}$ and $\boldsymbol{p_2}$). Therefore, the Insertion Loss shall be the second cost parameter, and it will be required to obtain **the highest amount** of it.

This means that, in regards to sonic crystal sound barriers, there must be a compromise between **its weight and the greatest amount of losses it is capable to provide**. In other words, it will not make sense if a barrier is significantly light but its losses are negligible, and vice-versa.

Finally, it is important to remark the fact that, for the purpose of this project, the sound barriers that were designed are considered to have an **infinite height**. This is because the FDTD simulations that were carried out had been designed for a **two dimensional field**.

3.2. Genetic algorithm based on concepts

3.2.1. Definition

As explained before, multi-objective optimization problems can be approached by taking advantage of the calculation power of computers, by making them follow an evolutionary algorithm that generates new individuals from a group of existing ones. Concepts are an additional feature that can be implemented in genetic algorithms. When producing new individuals by randomly mixing features of two existing ones, we might come up with a sample that fits in the so called **Pareto Front** (check Section "2.5. Pareto front" for further information).

> A new interesting approach would be to see what happens when these individuals are changed proportionally according to one of their features. For the purpose of this project, which deals with acoustic applications, the features that are more suitable to be changed are their geometrical characteristics, i.e. changing their size-related cost parameters defined in previous sections.

The individuals that are obtained applying this method are known as **concepts**. As schematically represented in Figure 14, after generating them, it must be checked how many individuals each concept has in the Pareto front. In that respect, the worst ones shall be eliminated, Also, the best ones will be combined to generate new samples that will substitute those that were removed, keeping the population constant. The amount of concepts that are removed and substituted must be defined in the process.

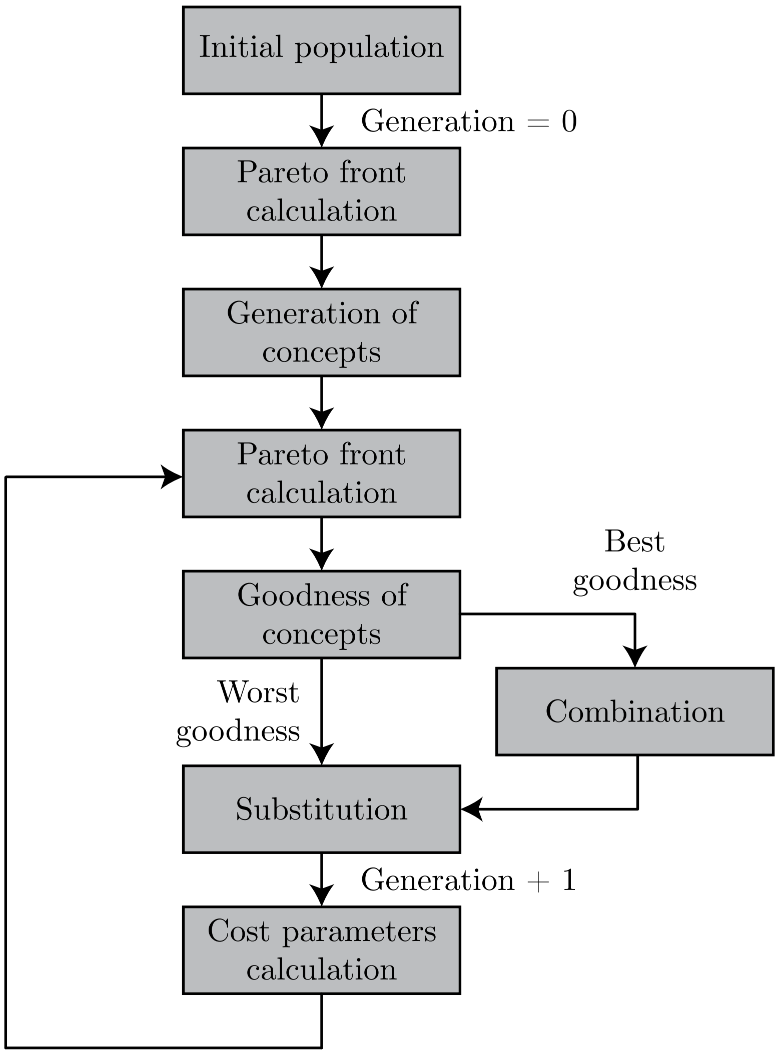

Figure 14: Basic scheme of the concepts implementation.

Nevertheless, it is also important to determine **the amount of concepts that are worth to be applied**. This is due to the fact that, the greater the amount of concepts, the longer it will take the algorithm (and therefore, the computer) to come up with a solution. This project aims to find the best amount of concepts that may be used for the purposes of the optimization.

In order to show an illustrative example of this implementation, we made use of the cases proposed previously. This way, the concepts used for each of the studied implementations that were studied in this project have been described in the following sections.

3.2.2. Concepts applied to sound diffusers

As explained in Section "3.1.1. Sound diffusers cost parameters", both the depth and the minimum frequency for which the diffuser can properly work at are the features that were analysed for their optimization.

When optimizing a set of diffusers (i.e. the **individuals**), it is interesting to check what happens when we generate new samples (the **concepts**) by changing their size accordingly to a **scale factor**.

This scale factor is approached in such a way that the deepest well of this group of individuals is normalized to 1, making the rest of the wells lie on an specific value between 1 and 0, as it was represented in Figure 8.

Thus, once the diffusers' depths are normalized, new individuals will be generated by scaling its deepest well to an interval between 0 and 1. The scale factor shall be divided in the same amount of steps as the number of individuals per concept, without applying the same scale to more than one individual.

In Figure 15 a), we can observe an original diffuser with a maximum depth of $\boldsymbol{d_n}$, that in terms of our implementation will have a value of 1, and in Figure 15 b), the same diffuser reduced by a factor of 0.5 (i.e. maximum depth of $\boldsymbol{d_n/2}$).

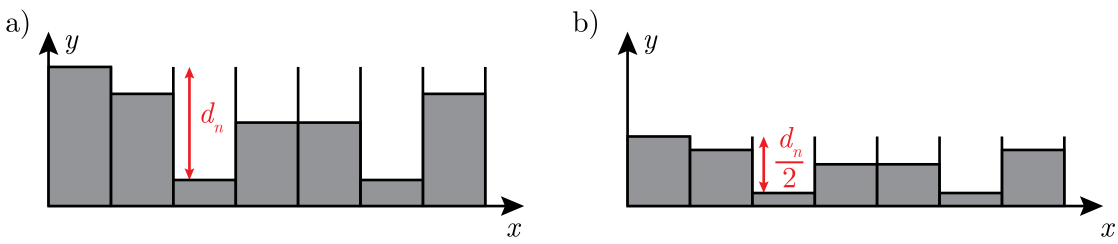

Figure 15: Generating a new sample from an original sound diffuser (concept). It can be observed how the depth of every well has been reduced to half its length.

The fact that a diffuser follows the same pattern as a bigger one that works in a proper manner does not mean that the first one will work too. This is due to the multi-objective approach of our optimization problems, and the fact that acoustic waves are affected by the size of the objects they interact with. Therefore, and as represented in Figure 16, each of the individuals within the concept shall have a different behaviour, with some enhancing the original diffuser, and others worsening its performance.

It is important to remark the fact that, having good individuals within a concept is not the only feature that we should pay attention to. Different individuals of the same concept may have **very erratic performances**, such as the one presented in Figure 16. This particular case shows extreme changes in the minimum frequency they affect on between very close scale factors.

> Thus, and if we follow a practical approach, for a concept to be considered totally appropriate not only should it present several good individuals, but also **they must be consecutive to each other**.

This way, we will assure that slightly changes or inaccuracies in their production do not produce extreme changes in their behaviour. In other words, we must take into account only those cases that show **enough stability** (or looseness) between their individuals.

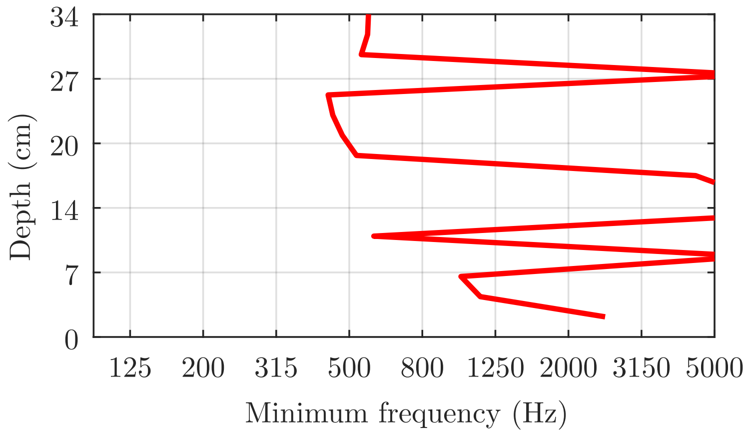

Figure 16: Example of individuals for a sound diffuser concept. The red line joins each of the samples (vertexes of the line) that conform the whole concept.

Finally, the **amount of concepts** used for each iteration of the optimization process should have been defined. This was approach in such a way that several cases were tested.

For this comparison to work, **the same total amount of samples should remain constant**. This was set as a total of $\mathbf{2^{10} = 1024}$ **individuals**. Moreover, the amount of concepts should be such that it allowed to equally divide the individuals among them.

This requirement was met by using the first six powers of 2, resulting on the six different cases that are defined in Table 3. We had to **compare** the performance of each of this cases in order to figure out which implementation would be more suitable for our optimization process.

Individuals per concept	Total individuals	Amount of concepts
$2^0 = \mathbf{1}$	1024	$1024/1 = \mathbf{1024}$
$2^1 = \mathbf{2}$	1024	$1024/2 = \mathbf{512}$
$2^2 = \mathbf{4}$	1024	$1024/4 = \mathbf{256}$
$2^3 = \mathbf{8}$	1024	$1024/8 = \mathbf{128}$
$2^4 = \mathbf{16}$	1024	$1024/16 = \mathbf{64}$
$2^5 = \mathbf{32}$	1024	$1024/32 = \mathbf{32}$

Table 3: Amount of concepts and individuals for sound diffusers

Each of this cases would result then in a different amount of concepts and **individuals per concept** (hereafter, **ipc**). For illustration purposes, two examples have been represented in Figure 17 (512 concepts, 2 ipc) and Figure 18 (128 concepts, 8 ipc), where individuals were plotted as black dots (with the Parento front highlighted in red), and with clearly distinguishable amount of concepts.

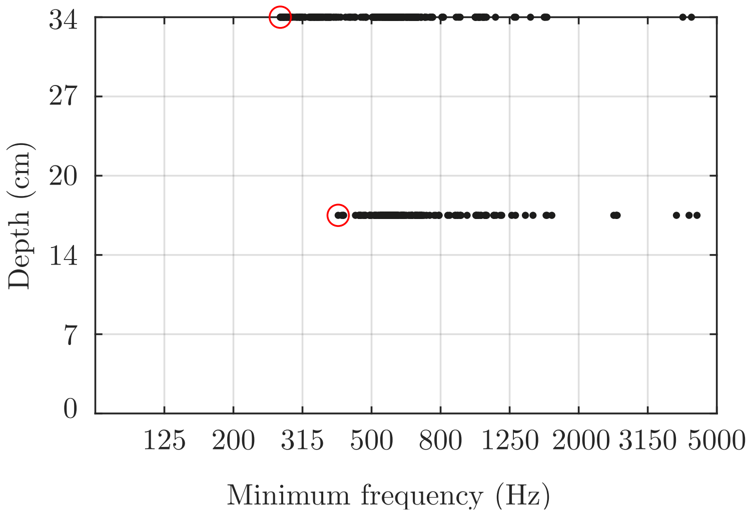

Figure 17: Diffusers for 2 ipc (512 concepts). Red dots represent the individuals within the Pareto front. Black ones represent the entire population. Based on the way the population is represented, the amount of individuals per concept is easily deduced.

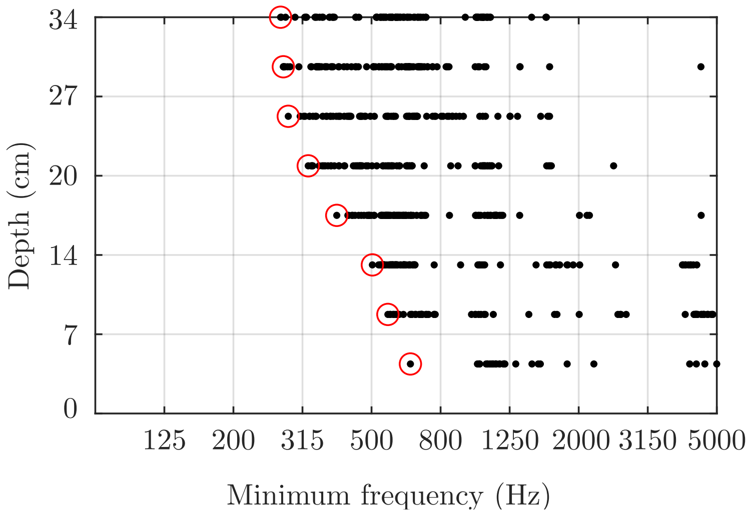

Figure 18: Diffusers for 8 ipc (128 concepts).

3.2.3. Concepts applied to sonic crystal sound barriers

As explained in Section 3.1.2 "Sonic crystal sound barriers cost parameters" (page 27), the weight of the barrier (measured by its radius and expressed in arbitrary units) and the Insertion Loss that it causes are both the features that were meant to be optimized. Thus, once a proper barrier with a suitable weight is found, we might generate new versions of this original one by adjusting its radius (and therefore, its weight) by making use of a **scale factor** that went from 0.6 to 2. In Figure 19 a), an optimal case of a sound barrier is represented. In Figure 19 b), the same structure is represented, but with a reduced scale, generating a concept from an original individual.

It is important to remark that in this case, and as stated in Section 3.1.2 "Sonic crystal sound barriers cost parameters" (page 27), **the height of the barrier was not taken into account**, and thus, not changed for any of the concepts.

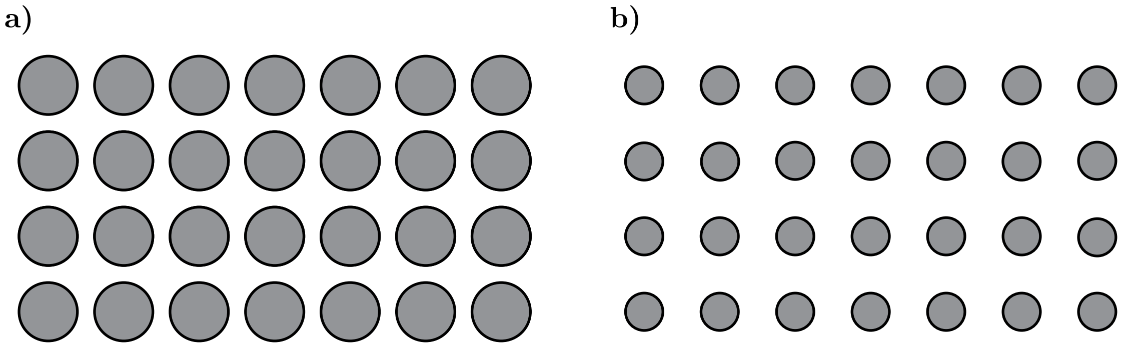

Figure 19: Generating a concept from a sonic crystal sound barrier.

Besides, the scale factor applied to the sound barriers might make them smaller, but also bigger. It may be counterproductive for the weight cost parameter, but at the same time it may generate individuals with better Insertion Loss properties. This might be interesting for some applications.

Nevertheless, this scale changes will not always generate good individuals for the purpose of the optimization. As happened with the sound diffusers, the multi-objective approach of our optimization problems and the fact that acoustic waves are affected by the size of the objects they interact with, may cause that only the original individual fits within the requirements of our problem,

Therefore, and as represented in Figure 20, each of the concepts shall have a different behaviour, with individuals enhancing the already existing barrier, and others worsening its performance. As can been observed, **sometimes barriers with slightly greater weight have significantly better acoustic performances** (weight is expressed through arbitrary units that may be scalable once a specific material is chosen).

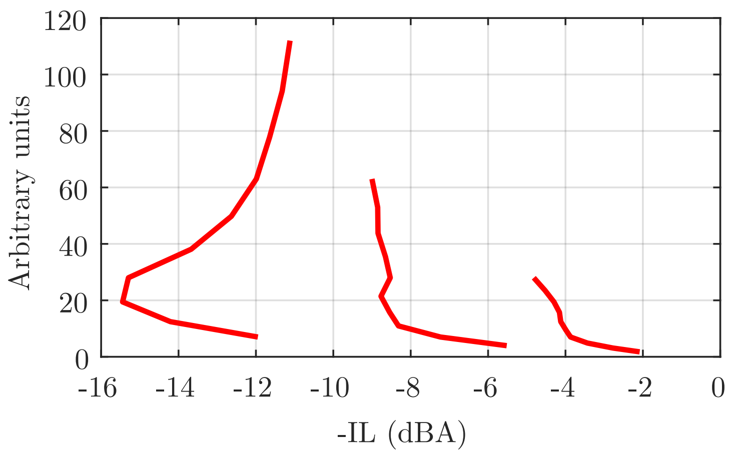

Figure 20: Example of concepts for a sonic crystal sound barrier.

Unlike what happened with the sound diffusers, where the behaviour of each new concept should have been calculated since it produced a totally different result from the others, the sonic crystal sound barriers present a **way easier approach**. Every new concept shall have the same frequency response, but shifted in position. This means that, if the original had a band gap at 1 kHz, a barrier with half the weight shall have the same band gap at 500 Hz, and so on.

This made their calculation way easier and faster, being only needed to invest time in **the calculation of the response of the original individual**.

Finally, and as with sound diffusers, **the different amounts of concepts** should have been defined. This way, the process could be evaluated in order to find its best possible implementation, and at the same time, compare the advantages and disadvantages of each approach. For this comparison to work, **the same total amount of samples should again remain constant**. Once more, this was set as a total of **2^{10} = 1024 individuals**. Moreover, the amount of concepts should be such that it allowed to equally divide the individuals among them. Thus, 5 different cases were defined, as presented in Table 4. For the case of sound barriers, and on the contrary of sound diffusers, we ran less cases due to the greater amount of time required to calculate the behaviour of each individual. This has to be taken into account when comparing results and on the general evaluation of the method.

Individuals per concept	Total individuals	Amount of concepts
$2^0 =$ **1**	1024	1024/1 = **1024**
$2^2 =$ **4**	1024	1024/4 = **256**
$2^3 =$ **8**	1024	1024/8 = **128**
$2^4 =$ **16**	1024	1024/16 = **64**
$2^5 =$ **32**	1024	1024/32 = **32**

Table 4: Amount of concepts and individuals for acoustic barriers

For illustration purposes, three examples have been represented in Figure 21, Figure 22 and Figure 23, where individuals were plotted as black dots (with the Parento front highlighted in red circles), and in this case, with not-so-clearly distinguishable amount of concepts. It can be observed that a limit of -6 dB of Insertion Loss was established. This was set this way in order to prevent the algorithm from generating individuals whose weight is so light that the Insertion Loss produced is almost negligible. This way, every time an individual with more than -6 dB of Insertion Loss was generated, its value was set to 0 dB, making its removal from the population more likely.

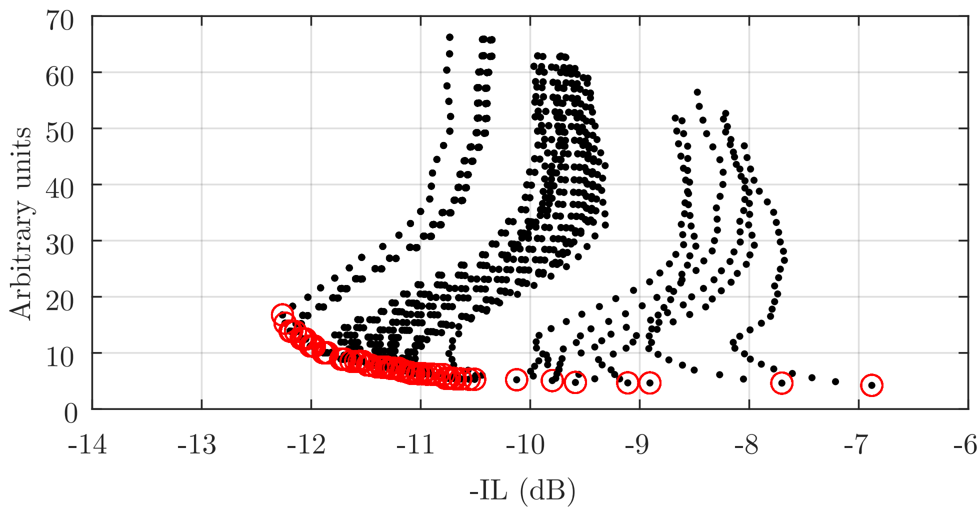

Figure 21: Acoustic barriers for 32 ipc (32 concepts). Red dots represent the individuals within the Pareto front. Black ones represent the entire population.

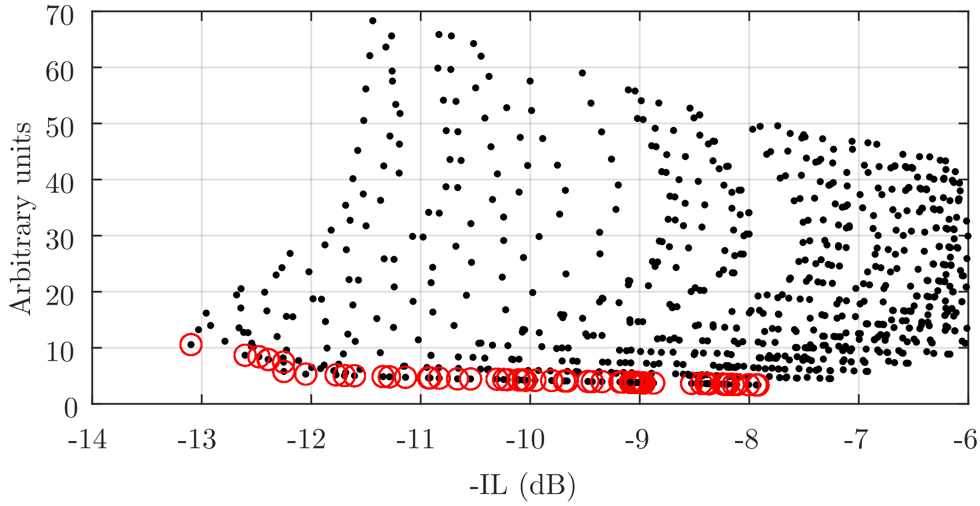

Figure 22: Acoustic barriers for 16 ipc (64 concepts).

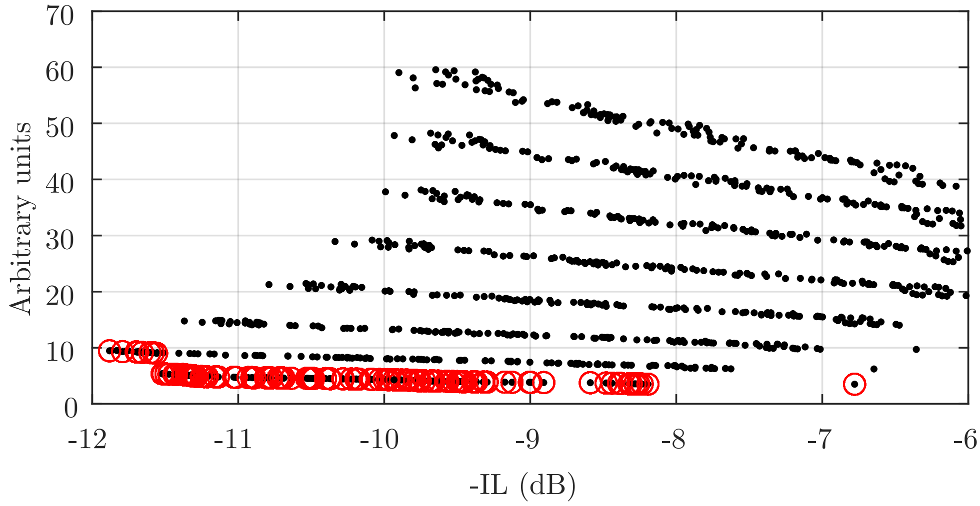

Figure 23: Acoustic barriers for 8 ipc (128 concepts).

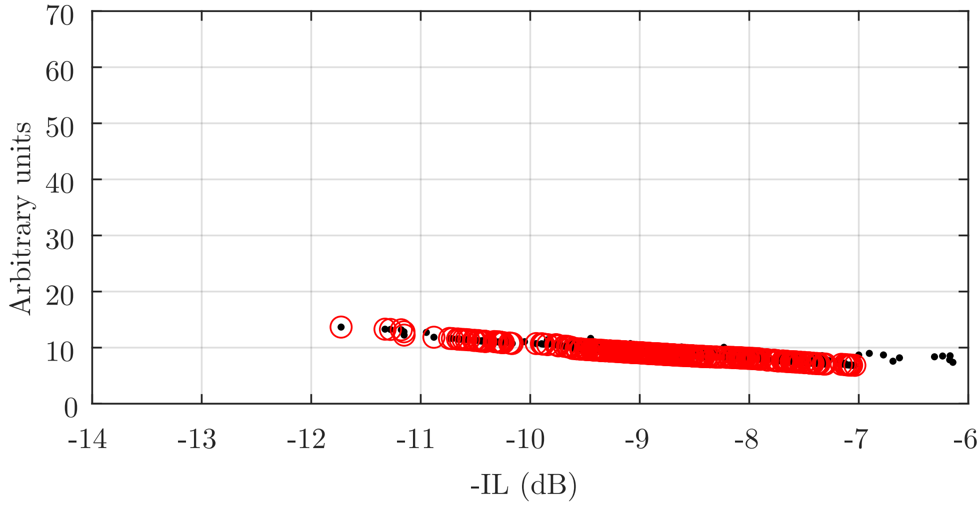

Figure 24: Acoustic barriers for 1 ipc (1024 concepts).

4. Results

In this section, we will present the results that were obtained in this project after the application of the optimization process. Thus, we will carry out a general interpretation of the performance of the multi-objective optimization by concepts, which may be relevant for any acoustic application we want to enhance.

This way, we will conclude if the method gives better results than other approaches or implementations, and in that case, which are its strongest features. Also, when needed, we will stablish in which cases or situations the method is not recommended to be applied, or could be problematic.

But before this general approach, we will also present the results obtained for each particular studied case (sound diffusers and sonic crystal acoustic barriers), that may be more helpful and illustrative for a practical interpretation and understanding of the method. They will be at the same time an evidence that this implementation is either productive or not. For each of the cases, we will present the amount of time spent for their optimization, which is indeed another factor that must be taken into account to determine the suitability of the method.

4.1. Results for sound diffusers

As stated in Section 3.1.1 “Sound diffusers cost parameters” (page 27), both the depth of the diffuser and its minimum frequency at which it works in a proper manner were the cost parameters chosen to optimize their acoustic behaviour. As expected, their behaviours are in opposition to each other.

Also in Section 3.2.2 “Concepts applied to sound diffusers” (page 29), it was established that the individuals that conformed each concept had a set of normalized depths that went from 0 to 1, being this value for the thickest diffuser.

Besides, in the same section was defined that several approaches were going to be followed by changing the amount of concepts, with specific quantities expressed in Table 3. In total, six different amount of concepts were implemented in the process. Moreover, due to the randomness of the method, **two different calculations** were ran for each implementation. This allowed the algorithm to provide more reliable results.

Therefore, we came up with **12 different set of solutions** (2 versions of 6 different individual per concept implementations). Taking into account that every situation had the same amount of individuals (a population of 1024 cases), it took approximately the same time to run each of the calculations.

A suitable amount of generations was established as $2^{12} - 2^{10} = 3073$ generations, which gave us an approximate time of **30 minutes for each case**. This way, **it took approximately 6 hours to complete the entire process**, which is a more that fair amount of time for an optimization process like this. The obtained results for each case are described in the following section.

4.1.1. General comparison of sound diffusers

In order to have a basic understanding on the results provided by our algorithm, the Pareto front (i.e. the best individuals within the group) of each situation was represented together in Figure 25, using different colours for each case. This means that, for every amount of concepts, **two different Pareto front were represented** (this is due to the implicit randomness of the process previously explained).

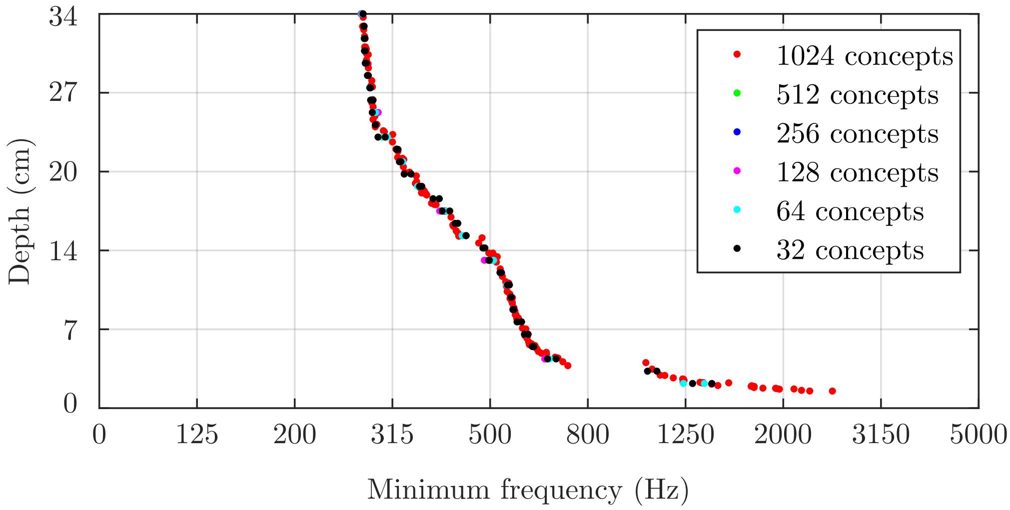

Figure 25: Diffusers' Pareto front of all cases represented together. Each colour represents the individuals obtained after applying different amount of concepts.

First of all, we are going to check the amount of individuals that compounded each of the Pareto fronts. This has been presented in Table 5. It could be interpreted as **there is a correlation between the amount of concepts employed and the amount of individuals generated within the front**. Nevertheless, it is interesting to remark the results for 1024 and 32 concepts, whose performances are slightly different as expected.

Individuals per concept	Amount of concepts	Individuals in Pareto	
		Case 1	Case 2
1	1024	**68**	**76**
2	512	**2**	**2**
4	256	**4**	**4**
8	128	**8**	**8**
16	64	**16**	**16**
32	32	**30**	**30**

Table 5: Amount of individuals in the Pareto front for sound diffusers

As can be observed, the cases with 1024 and 32 concepts are the ones more visible (red and black dots). The other cases (rest of colours) are not noticeable since they produce nearly the same results as these two.

This means that there are **barely any difference among cases**: we can not determine which amount of concepts provides the best set of solutions for the purpose of this optimization process.

It is important to remark the fact that the approach of 1024 concepts (i.e. 1 individual per concept) is **equivalent to a classic multi-objective optimization**. Again, the results obtained with concepts are not substantially better.

This was somehow expected, since optimizing sound diffusers by concepts does not provide a significantly better result on the Pareto front. Nevertheless, in order to carry out a deeper analysis, and really come up with justified conclusions, we are going to analyse the behaviour of **each of the concepts of the individuals in the Pareto front**, i.e. checking how its minimum frequency varies throughout the established set of depths of its case.

This means that, for each of its points, we represented all the individuals of its corresponding concept, and then compared them between each other. This way, we would be able to see how different they are in regards to their stability (and in turn, its quality), and also if there are some patterns that might be interesting according to their implementation.

4.1.2. Sound diffusers comparison by concepts

At this point, it is important to define the term **stability** when referring to sound diffusers:

A concept will be considered stable when several of its individuals are not only in or in the neighbourhood of the Pareto front, **but they also appear in a consecutive way** (two or more individuals in correlative positions).

This way, we assure that small modifications on the depth of the diffuser does not produce extreme modifications on its frequency response.

After running all the optimizations, we noticed that the results provided by the designed MATLAB code showed, in general terms, **a very erratic response in regards to their minimum frequency**, independently of the amount of concepts employed. Several examples have been represented in Figure 26, Figure 27 and Figure 28. In these figures, the concept is represented by a black line, while the Pareto front is illustrated as the red circles.

This meant that, even though a set of individuals were found to be appropriate under specific circumstances, they were extremely unstable and could be hardly applicable for real cases. This is because slightly changes or errors in their depth may lead to a completely different frequency response.

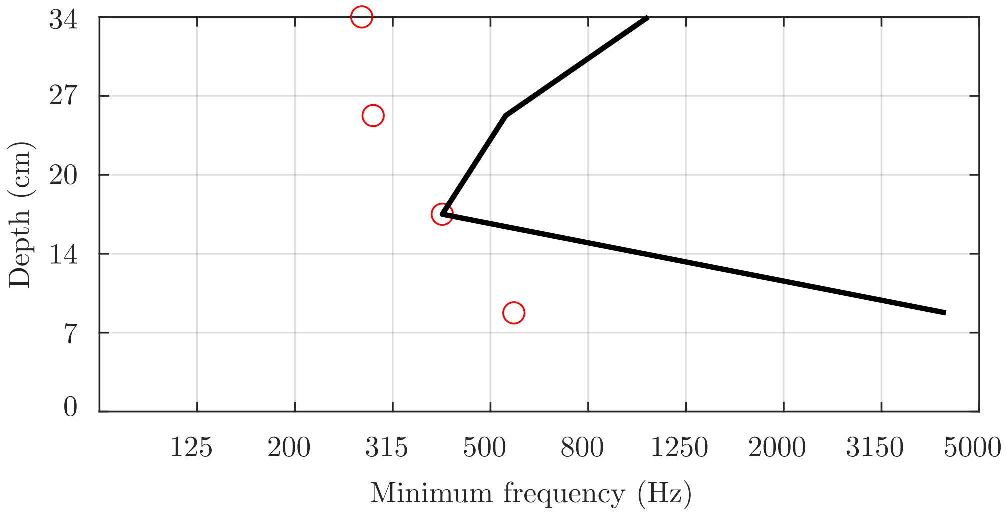

Figure 26: Case for an 4 ipc implementation (256 concepts). Red dots represent the individuals within the Pareto front. Black solid line represents an specific concept.

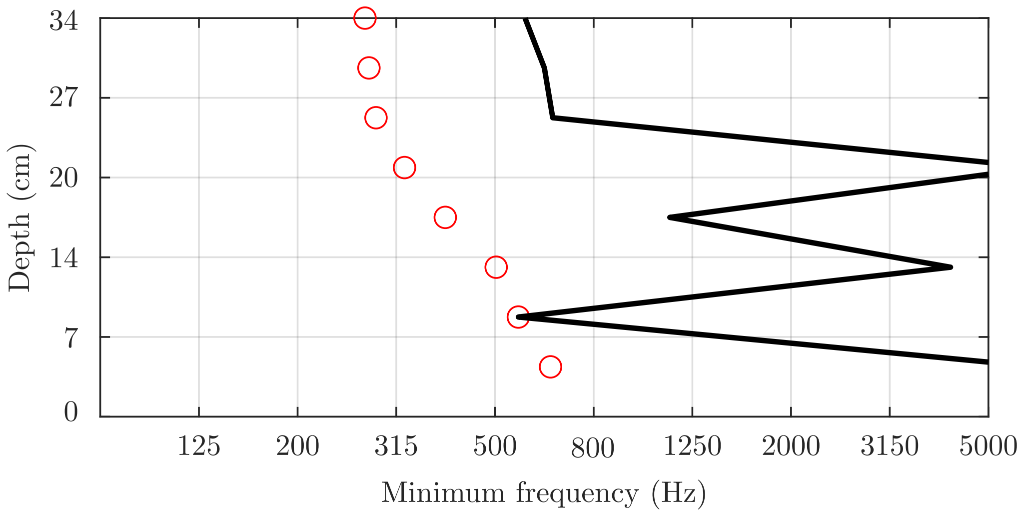

Figure 27: Case for an 8 ipc implementation (128 concepts).

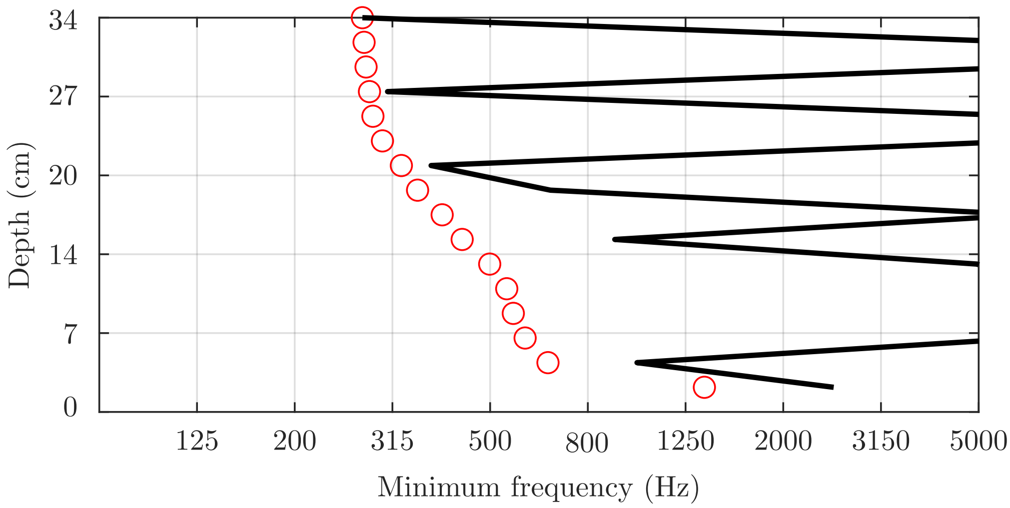

Figure 28: Case for a 16 ipc implementation (64 concepts).

Nevertheless, there were cases that showed a fairly good response in terms of stability. Despite of the fact of not being actually in the Pareto front, they were close enough to be considered good individuals. This led us to set three groups of individuals that could be found within the implementation of this algorithm:

- **Case 1 - Poor stability**: The dominant case, and the one explained previously. Its minimum frequency has a very erratic behaviour depending on the depth (i.e., its stability is highly fragile). An example is represented in Figure 29.

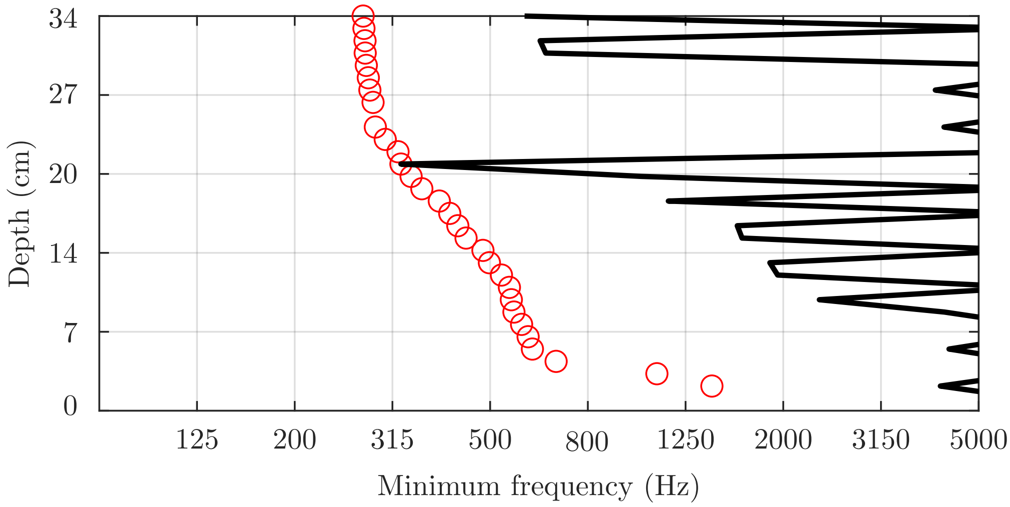

Figure 29: Case 1: Poor stability.

- **Case 2 - Moderate stability**: Its minimum frequency is relatively stable for some individuals. Not all of them are necessarily in the Pareto front, but are close enough to it to be considered good samples. An example is represented in Figure 30.

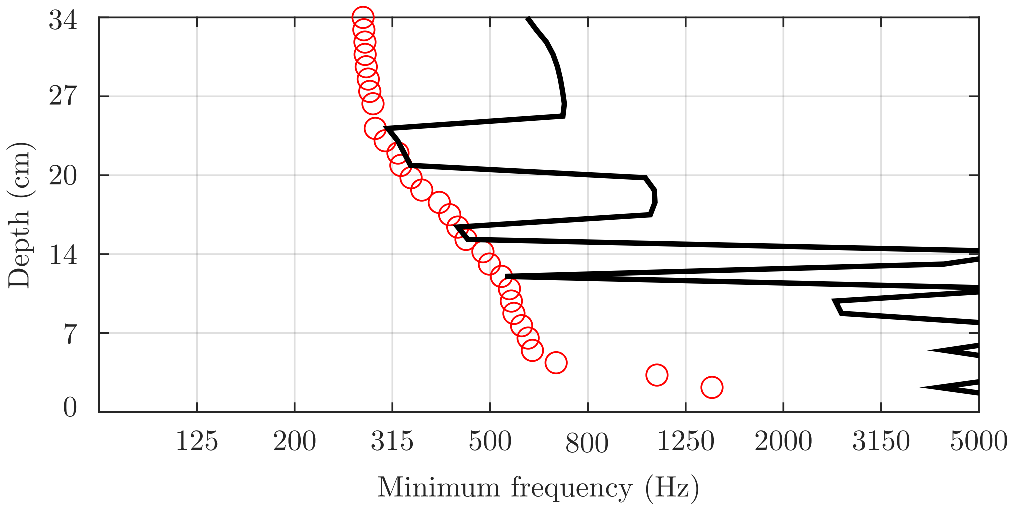

Figure 30: Case 2: Moderate stability.

- **Case 3 - Good stability**: Their minimum frequency is fairly stable for a high amount of individuals. Most of the times they do not lie within the Pareto Front, but they are close enough to be considered good. An example is represented in Figure 31.

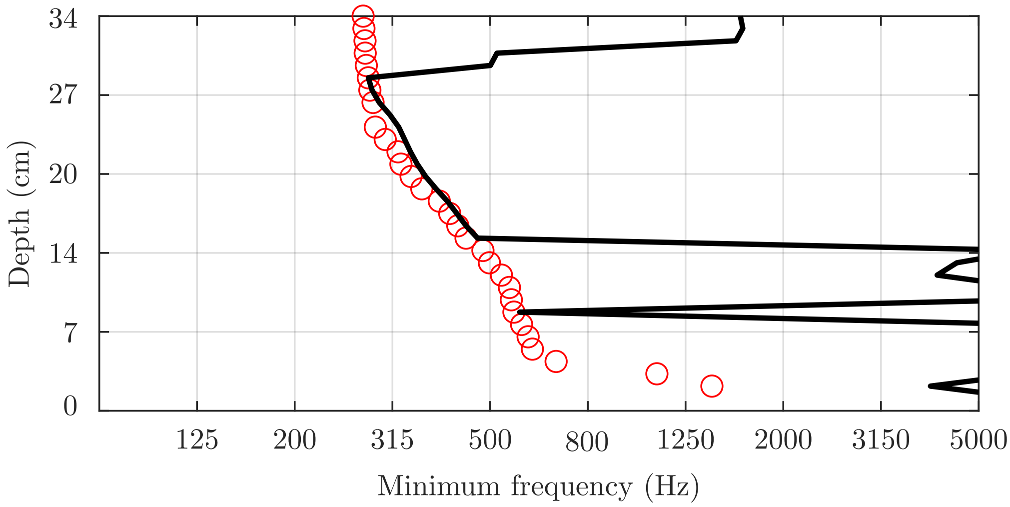

Figure 31: Case 3: Good stability

> Nevertheless, we can not forget the fact that the majority of results had concepts with very few individuals within the Pareto Front, with two samples in a row at best in the most favourable cases.

4.1.3. Comparison with randomly generated sound diffusers

In order to check the virtues of the method, we are going to compare the results obtained after the optimization process with individuals that were **randomly generated**. To do a proper comparison, we must use the same amount of individuals in both approaches, and also, compare them with the same scale factor. This means two conditions:

- Since in each iteration we generated 32 new individuals, and in total we ran 10^{12} - 10^{10} = 3072 generations, we must then generate **3072 × 32 = 98304** new individuals randomly.
- Since the individuals generated randomly were unique (i.e. no concepts were applied), we must compare them with the individuals that had a scale factor of 1 within our optimization process.

This way, we obtained the results represented in Figure 32. As can be observed, the algorithm produces nearly the same edge-individuals in the Pareto front. However, it is visible that the implemented method produced slightly better results. Besides, the randomly generated individuals are seem to appear almost everywhere in the graph, while the method reduced those samples substantially.

Anyhow, these results can be interpreted once again as a sign of the complexity of the problem and the lack of significant improvement obtained with the method, which may be a consequence of the nature of the application itself and not necessarily of the optimization method.

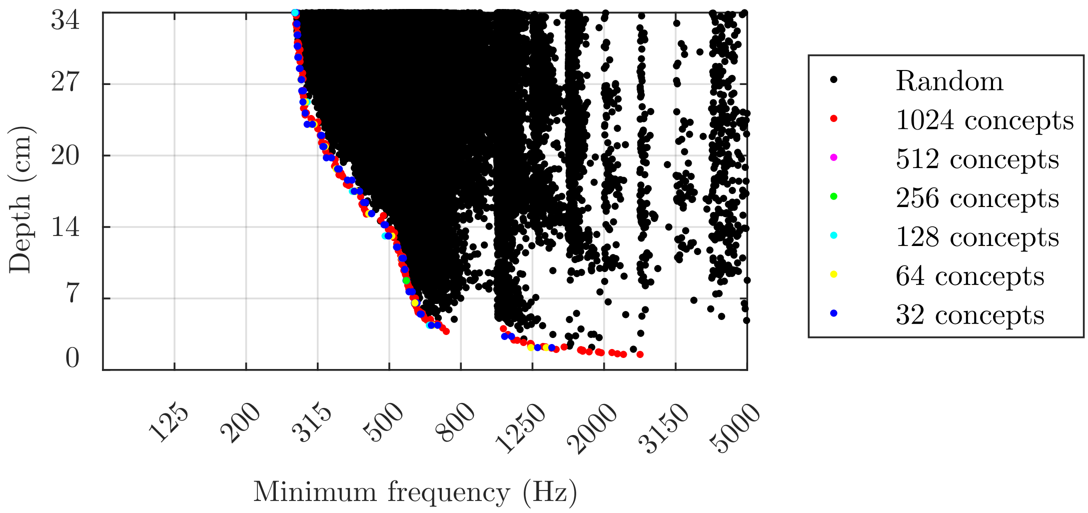

Figure 32: Comparison with randomly generated sound diffusers.

4.1.4. General results for sound diffusers

As could be observed in the results, we can conclude that **the amount of concepts offers little or no improvement to the development of the Pareto front**. Therefore, the enhancement of this feature does not depend on the amount of concepts. However, it has been illustrated that, **the higher the amount of individuals per concepts, the more reliable were the results in terms of stability**. Since we had more accuracy throughout the depths, we could check if the individual remained stable for all these values.

Nevertheless, we could see that the stability of the diffusers has proven to be quite fragile among the different cases analysed. Even though some good and stable individuals were obtained, it is important to bare in mind that **not all the samples that lie within the Pareto front are potentially good implementations in real situations**, due to their lack of looseness or error margin. However, those individuals that presented good stability are a good example of the virtues and potentials of this methodology.

Finally, it has been observed that the conditions of the problem are too tight to come up with really significant improvements. The method can not produce significant better solutions than those analysed in other experiments.

4.2. Results for acoustic barriers made of sonic crystals

As stated in Section 3.1.2 "Sonic crystal sound barriers cost parameters" (page 27), **the Insertion Loss produced by the barrier, along with its weight** (i.e., its radius) were the cost parameters chosen to optimize their acoustic behaviour. As in the case of sound diffusers, both parameters response are in opposition to each other. Also in Section 3.2.3 "Concepts applied to sonic crystal sound barriers" (page 32), the scale factor applied to the concepts was established, going from 0.6 to 2. At this point, we should also remind that **the height of the barriers were not taken into account** for the optimization process.

Moreover, it was defined that several implementations were going to be carried out by adjusting the amount of concepts, with specific quantities expressed in Table 4. In total, 5 different amount of concepts were implemented in the process. Unlike what was done with the sound diffusers, in this case only one calculation per amount of concept was ran. This was because of the time each of implementation spent calculating, duplicating its length for every different amount of concepts. Therefore, the results are not as reliable as with the diffusers, and it is something that might be enhanced in future calculations.

Therefore, we came up with 5 different set of solutions. However, even though every situation had again the same amount of individuals (a population of 1024 samples), not every case required the same amount of time to be calculated. In fact, the greater the amount of concepts, the longer it took to carry out the calculations. This is because, for each generation, 32 new individuals were generated. Depending on the amount of individuals per concept, this meant a different amount of simulations, going from 1 simulation (32 concepts with 32 ipc, i.e. only one barrier) up to 32 simulations per generation (1024 concepts with 1 ipc, i.e. 32 new barriers). This is something that we should take into account once we evaluate the performance of the algorithm.

Additionally, an amount of $2^{12} - 2^{10} = 3073$ generations was again considered appropriate. As stated before, the amount of time that was required to calculate each implementation **duplicated its length for every amount of concepts**. This way, **it took approximately 6 hours for 32 concepts, 12 hours for 64 concepts, and so on**.

Finally, the results obtained for each of the cases are described in the following section.

4.2.1. General comparison of acoustic barriers

Similarly to the approach we took with the sound diffusers, we are going to represent together the Pareto front of all the implementations. This helped us to have a quick understanding of the results provided by our algorithm, and has been represented in Figure 33, using different colours for each case.

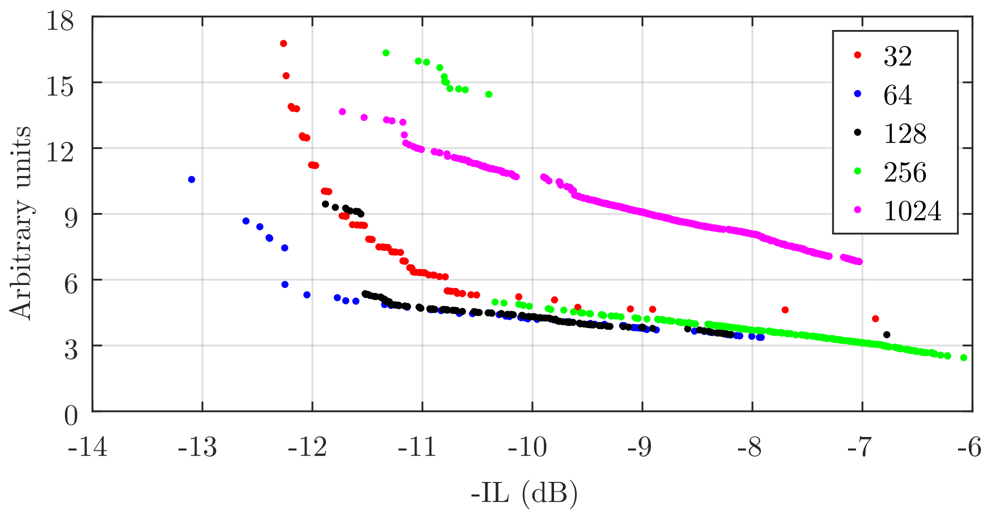

Figure 33: Acoustic barriers' Pareto front of every amount of concepts.

First of all, the amount of individuals that compounded each of the Pareto fronts has been gathered in Table 6. It could be interpreted as **there is a correlation between the amount of concepts employed and the amount of individuals generated within the front**. Nevertheless, for the case of 32 concepts, we can see there are more individuals than expected, and the opposite for 1024 concepts. This might be interpreted as **a lack of correlation between these two parameters**.

Individuals per concept	Amount of concepts	Individuals in Pareto
1	1024	**350**
4	256	**242**
8	128	**122**
16	64	**62**
32	32	**80**

Table 6: Amount of individuals in the Pareto front for acoustic barriers

Also, it can be seen that each case generated a rather different solution in most areas of the Pareto front, with some similar behaviours between pairs (check Figure 33). At first sight, it is clear that the approach with **64 concepts provides the best set of solutions** (blue dots), with individuals going further in the Insertion Loss values, and also, being substantially lighter from -11,5 dB and below. Moreover, the cases of 32 and 128 concepts show mixed behaviours, with generally heavier individuals (red dots) or less wide isolation properties (black dots). Nevertheless, it is also visible that **all the implementations show a fairly linear response from around -11 dB and above**, with interesting and different results below this area. In fact, is that section of the graph, where solutions diverge, that shows the greatest advantage of the 64 concepts implementation. Additionally, if we assume the case for 1024 concepts (1 ipc) as a **classic multi-objective approach**, we can conclude that optimizing by making use of concepts generally produces better results.

This means that there are clear differences among cases: **we can easily determine which amount of concepts provides the best set of solutions** for the purpose of this optimization process.

This is also visible when we represent these Pareto fronts along with the behaviour of barriers that are used as references. If we think of a barrier as a matrix of cylinders, these references represent different cases of thickness for their cylinders. Thus, a barrier with reference value of 1 is compound of cylinders whose diameters match the length of the quadrants of the matrix, as represented in Figure 34 a). A reference of 0.5 would have half the length, as in Figure 34 b), and so on.

The performances of some of these reference barriers have been represented in Figure 35, along with the different Pareto fronts obtained after the optimization process. It can be observed that **virtually all the reference barriers have been surpassed at some point by at least one of the Pareto fronts generated**. This results on lighter individuals for the same Insertion Loss level or vice-versa. This is something that shows the advantages of this method.

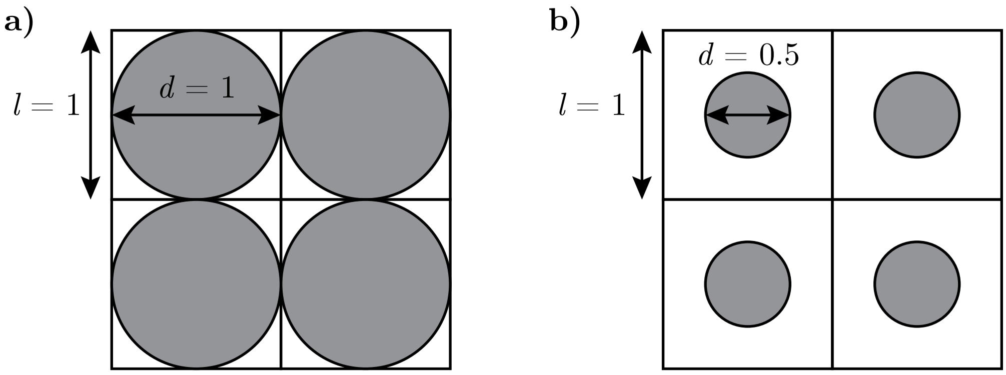

Figure 34: Reference barriers for different values of thickness.

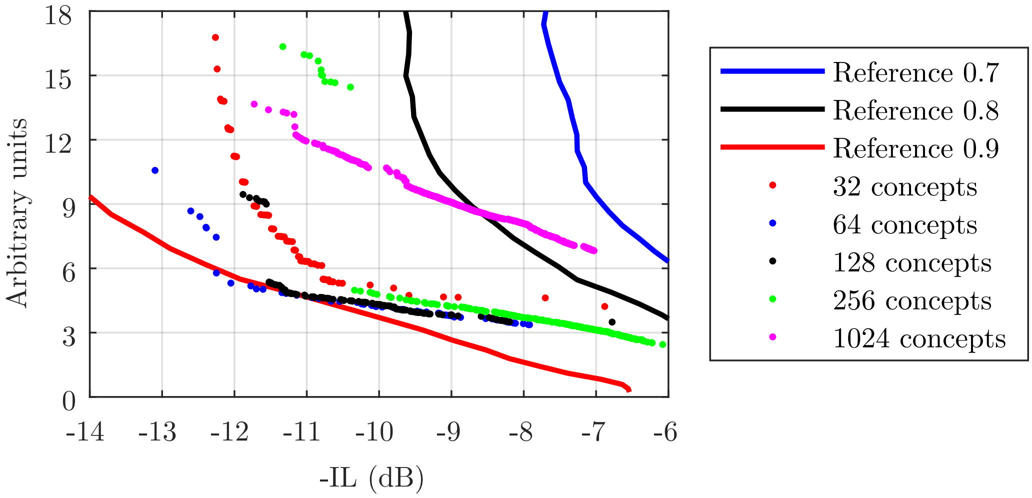

Figure 35: Reference values (solid lines) for acoustic barriers compared to the different Pareto fronts (dotd) obtained after the implementation of the algorithm.

However, and as implemented with the sound diffusers, we are going to carry out a deeper analysis by analysing the behaviour of **each of the concepts of the individuals in the Pareto front**, i.e. checking how its weight (expressed through arbitrary units that may be scaled afterwards)varies throughout the established set of Insertion Losses. This way, we would be able to really come up with justified conclusions.

Thus, for every sample in the Pareto front, we represented all the individuals of its corresponding concept, and then compared them between each other. This way we would be able to see if there are some patterns that might be interesting in regards to their implementation.

4.2.2. Acoustic barriers comparison by concepts

In this section, we are going to present the concepts of some of the individuals that reached the Pareto front. This way, we tried to look for interesting behaviours along the results of every implementation. This has been represented in Figure 36, Figure 37 and Figure 38.

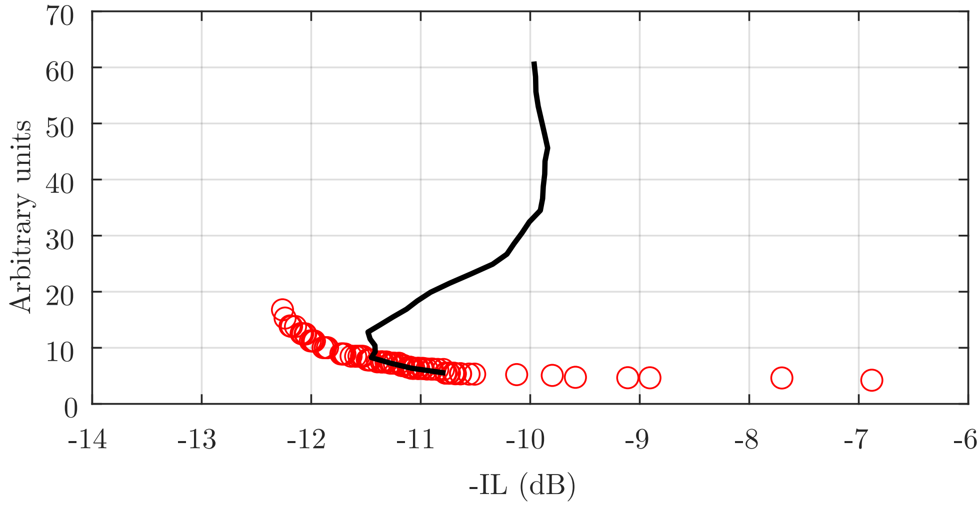

Figure 36: Most common performance for 32 concepts barriers. Red dots represent the individuals within the Pareto front. Black solid line represents an specific concept.

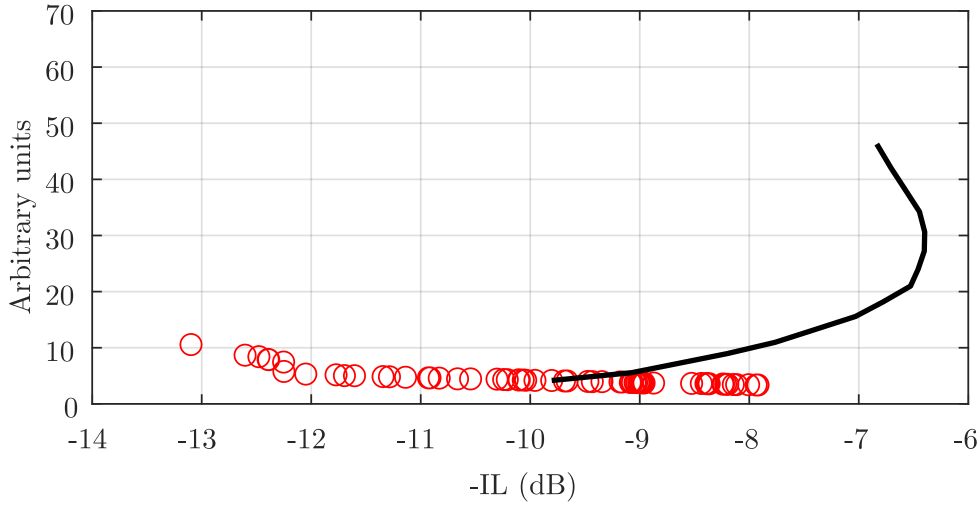

Figure 37: Most common performance for 64 concepts barriers.

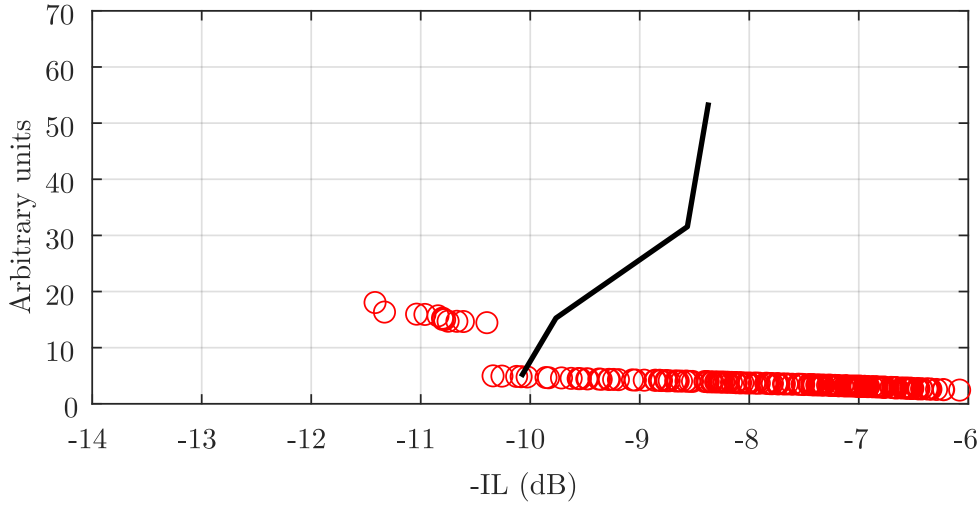

Figure 38: Most common performance for 128 concepts barriers.

First of all, it can be observed that for every single concept, the individuals that reached the Pareto front were always **the lightest ones**, specially in the area where the ratio weight/losses seems to be linear. These scale factors could be adapted in further implementations in order to avoid **unnecessary calculations**. Even though it was explained that calculating the response of all the individuals within a concept was just a matter of shifting their frequency response, the sum of all of them might require enough time to make it important to reduce them. Also, it might be a better way to find individuals closer to the Pareto front.

Additionally, it can be observed that, despite the fact that the 64 concepts implementation provided the most interesting results, it is the 32 concepts case that generally produces more stable ones. As happened with the sound diffusers, we considered a concept as stable when several of its individuals are not only in or in the neighbourhood of the Pareto front, **but they also appear in a consecutive way** (two or more individuals in correlative positions). This situation could be found in every implementation, but it was way more common for 32 concepts, which additionally produced a fair amount of individuals in the Pareto front, as presented in Table 6.

In order to evaluate the behaviour of a stable concept in each implementation, we represented two examples in Figure 39, Figure 40 and Figure 41. They were more common around the area where the performances diverge, i.e. around -11.5 dB in Figure 33. It can be observed that, in the case of 32 concepts, the resulting barriers are way more stable according to the Pareto front than the cases of 64 or 256 concepts. Even though the individuals are not specifically part of the front, they are close enough to it to be considered good samples.

Moreover, for 32 concepts, virtually every concept has more than 2 individuals in the front, with enough cases gathering 3 or more samples. On the contrary, none of the others has concepts with more than one individual in the Pareto front, making all of them extremely unstable. This reinforces the idea of this case being the most stable of all of them.

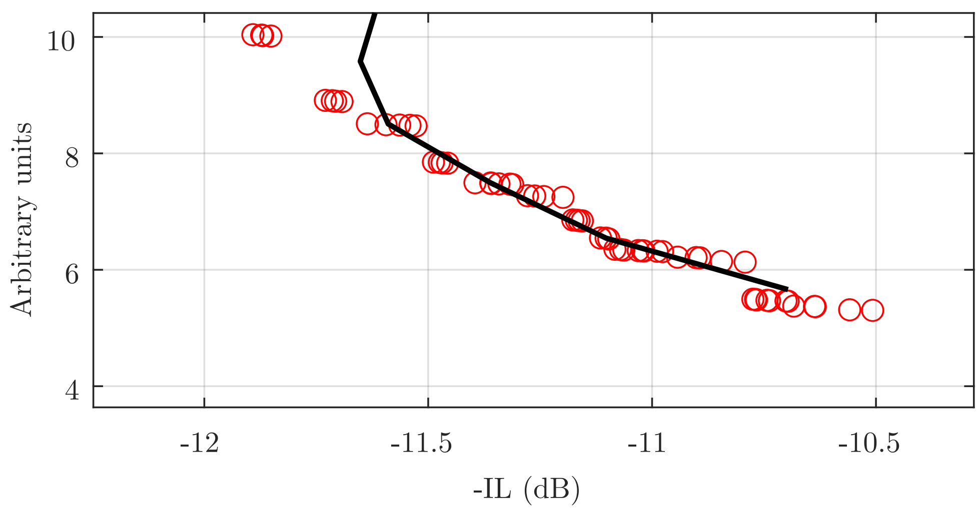

Figure 39: Detailed view of a 32 concepts stable result. It can be observed how this specific concept follows a rather stable behaviour, with individuals either in the Pareto front, or sufficiently close to it.

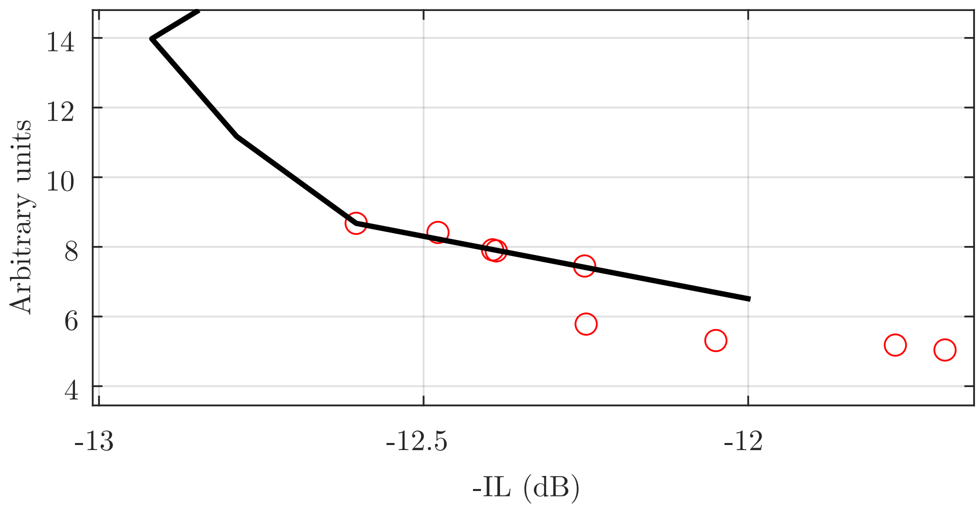

Figure 40: Detailed view of a 64 concepts stable result. In this case, although the stability of the results seem to be good enough, the lack of accuracy due to the amount of concepts prevent us from making reliable conclusions.

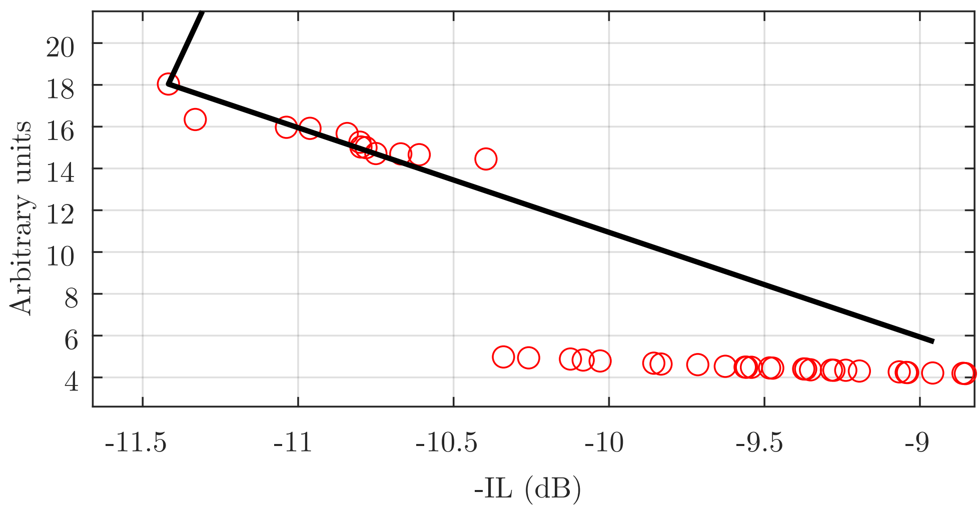

Figure 41: Detailed view of a 256 concepts stable result. Because of the extremely low amount of individuals per concept, the accuracy of the results does not ensure that an apparent good individual behaves the same throughout its concept.

It may be interesting as well to represent all the concepts together as lines in order to see the different patterns followed by them. This way, we would be able to check if there are some kind of groups of concepts or "families" that lie within the same area and that follow a similar behaviour. This was represented in Figure 42 with the implementation of 256 concepts as an example.

As can be observed, there are several groups with similar performances. Using different colours, we grouped them in 5 families, with virtually all of them presenting a rather weak stability. It is on the critic area (where the linear behaviour diverges) where we find the concepts with more individuals in the Pareto front. Nevertheless, and as represented in the detailed view of Figure 41, even they are not as stable as desired.

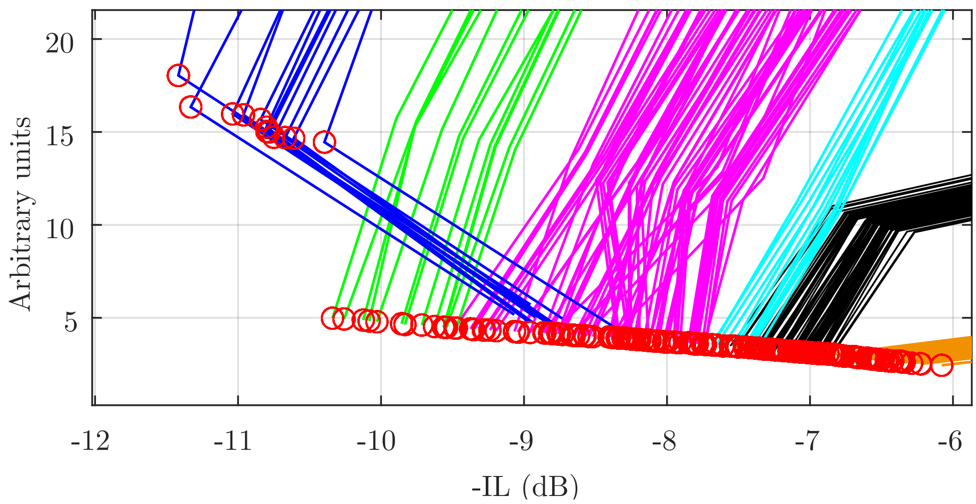

Figure 42: Different families of stability among barriers. Each visible pattern has been represented with a different colour.

4.2.3. Comparison with randomly generated barriers

As with the sound diffusers, we checked the virtues of the method by compareing the results obtained after the optimization process with individuals that were **randomly generated**. To carry out a proper comparison, we must use the same two conditions defined in Section 4.1.3 "Comparison with randomly generated sound diffusers" (page 41). However, in this case, the generation of these random individuals **took much longer** than with the sound diffusers, since it was required to run the FDTD simulation of each of them.

This way, we obtained the results represented in Figure 43. As can be observed, the algorithm produces significant better results than those generated randomly. It is visible that the random method tends to produced samples in a very specific area, while the different implementations of our optimization process are capable of define new limits.

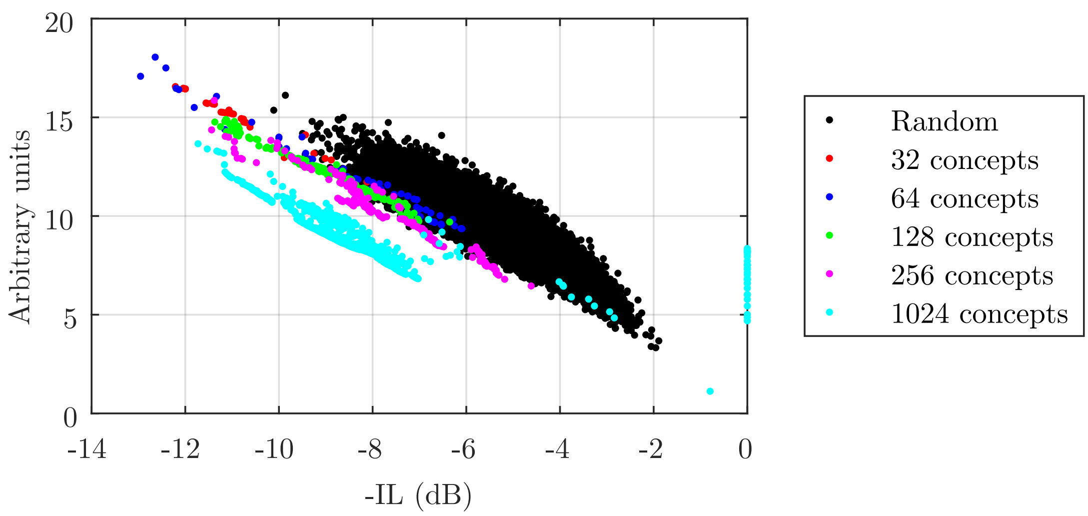

Figure 43: Comparison with randomly generated acoustic barriers. It can be observed the tendency of this method when generating new individuals.

In this case, where the individuals are only those that followed the original scale factor of 1, we can also observe that the best samples came from the 1024 concepts implementation. This case stands out substantially, while the rest produced nearly the same “fronts”. However, all of them generated better individuals that those we obtained randomly.

These results can be seen as a representation of the virtues of our method. Oppositely to the sound diffusers, whose nature is too complex to come up with substantially better results, this implementation allowed us to obtain a **significant enhancement**.

4.2.4. General results for acoustic barriers

After analysing the results presented in previous sections, we can conclude that **the amount of concepts does affect the process**, with better performances in the Pareto front depending on its value. This is even more evident in the area where the ratio weight/Insertion Loss loses its linear character. However, we could see that **the stability of the barriers is quite fragile** among the different cases analysed.

As with the diffusers, we must take into account that not all the samples in the Pareto front are potentially good implementations in real situations, due to their lack of error margin. However, those individuals that presented good stability are a good example of the virtues and potentials of this methodology.

Additionally, **the stability was also affected by the amount of concepts**, although this parameter had its best performance in a different amount than the best Pareto front. This may lead to a different evaluation of the results **depending on our preference**.

> Whether we look for the **best performance or the most stable behaviour**, we must use the results originated from a specific amount of concepts. Usually these amounts do not match.

5. Findings and future developments

The main aim of this project was to develop a genetic algorithm for a multi-objective optimization process for acoustic applications, in order to make them as small as possible. Also, it was intended to prove that these multi-objective processes could be enhanced by making use of **concepts**, i.e. generating new individuals by adjusting one of the physical features of the samples according to a scale factor. To illustrate this process, two different acoustic applications were analysed.

In the case of sound diffusers, it could be seen that **not big differences were found in the Pareto front when changing the amount of concepts**. This may be interpreted as a independence between this parameter and the efficiency of the process. Additionally, it has been shown that the results obtained for sound diffusers were extremely **unstable**. Values that lied on the Pareto front were only adequate for a highly accurate measure: few cases were found where the depth of the wells could have a fairly error range or tolerance.

This can also be interpreted as a sign that **the efficiency of the algorithm has prevailed over the stability of the results**, due to the intrinsic complexity of the application. However, it could be observed that the stability of the individuals was indeed affected by the amount of concepts employed. **The higher the amount of concepts, the more stable the optimal individuals were**.

In regards to the sound barriers made up of sonic crystal structures, **we did find a relation between the amount of concepts and the results in the Pareto front**, and on top of that, again **a relation between this parameter and the stability of the individuals**. Nevertheless, these two criteria did not match. This meant that we must make a choice in order to produce either the most efficient or the most stable results.

Additionally, we observed that the computational time spent in each application was quite different. Thus, depending on the complexity of the case, the amount of time to generate sufficiently reliable results may differ. Requirements like simulations may substantially increase the required calculation time.

The difference of complexity between applications was also reflected when comparing the optimization with concepts with a classical multi-objective approach (equivalent to our 1024 concepts, or 1 individual per concept). In the case of sound diffusers, the improvement was very subtle, while in the acoustic barriers the difference was very clear.

Also, for both cases, we observed that randomly generated individuals were always worse than those obtained with concept based optimization. Nevertheless, the degree of improvement depended again on the complexity of the application that was being analysed.

Thus, after all these conclusions, we may enunciate some properties that can be deduced out of these results:

- Although the performance of the Pareto front (i.e. the best individuals of the optimization process) may be directly influenced by the amount of concepts, **this does not necessarily occur in every possible application**.

- The stability of the concepts, and consequently the practical applicability of the results, **is directly influenced by the amount of concepts**. This is because, the higher the amount of concepts, the more accurate the results can be. However, it has not to be necessarily a direct relation between them.

- The different amount of concepts provides enough sets of results to, once the algorithm is finished, decide which approach fits the best with our application. Thus, depending on our needs, we might choose the best results according to our cost parameters, or those individuals with the most stable performance.

- The complexity of the application directly influences the results of the algorithm. Some of them may produce better results than other when applying concept based optimizations.

- The method always generates better results than classic multi-objective methods and random approaches.

Therefore, after all the implementations and analysis of the obtained results, we can come up with the conclusion that this process has proven to be **useful for reducing the size of the acoustic applications**. As previously stated, it might not always be suitable to look for the smallest results, but in the end, it is up to the developer to choose one criteria or another.

Also, in the future this algorithm may be implemented in any other acoustic applications such as loudspeakers, room design, etc. It would be interesting to analyse how the algorithm behaves for such different applications. This would also be helpful to determine the limitations and weaknesses of the algorithm.

Thus, a good way to enhance the method would be to define with total precision the cost parameters that need to be optimize, as well as the applicable limits and the optimal scale factors, which can sometimes be problematic or misleading. Nevertheless, this is a step that will always be required in order to produce proper results. Thus, once this is achieved, the optimization process may be easily applied, generating sufficiently good results.

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