Optimization of trajectories under the presence of uncertainties

Aerospace's engineering bachelor's final thesis

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Abstract

This academic work is about solving optimization problems that are applied in the aerospace field. In particular in this case a interplanetary transfer problem between Earth and Mars is going to be modelled and optimized by introducing and using a convex algorithm which is a very efficient type of algorithm. This kind of algorithms suppose a field under investigation for model engineering optimization problems. The paper explains qualitatively how this algorithm works, how the transformations of problems are made to apply this algorithm and it shows a practical example by modelling and solving the transfer already commented. Also, the importance of the mission is explained and some parametric studies of the mission from the solution obtained are performed. This parametric studies consist on studying how a design parameter of the spacecraft, the maximum thrust, and how the configuration of mesh of the computational domain affects the solution i.e. increasing the number of nodes of the mesh. And finally general conclusions about the use of the algorithm and from the parametric studies are exposed.

Key words: Transfer problem, convex algorithm, parametric study

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Introduction

In recent days, with the incipient entrance of modern and big private enterprises to the space sector the number of incoming missions is growing up very quick, so this is an interesting time to research for new methods and technologies that applies to the space missions since the financial situation of projects is reaching a prosperity point[1]. One of the main interests of some of the most important investors are the exploration and travel to Mars in order to try to reproduce the habitability conditions of the Earth in the future and be able to stablish colonies there for preventive reason in case the Earth become unavailable to keep sustaining the human race life or to exploit new natural resources. This is why the transfer problem from Earth to Mars is kept on eye for an important number of space missions.

Nowadays space missions count on high precision instrument devices as well as high-performance control systems that are continuously supervised remotely. Also the development of the computer systems provides an extremely high capacity of accurately predict the trajectory of interplanetary missions, so that the expected errors are at the minimum. Anyway, during the mission, shall environmental disturbances occur and

[1] Morgan Stanley. 2020. *Space: Investing in the Final Frontier | Morgan Stanley*. [online] Available at: <https://www.morganstanley.com/ideas/investing-in-space> [Accessed 14 August 2021].

some trajectory parameters need to be recalculated in order to take that into account. So it is necessary to develop a tool to manage these calculations as fast as possible.

Of course this recalculations have to be oriented into finding the optimal solution in order to minimize the mission costs. In addition, the tool has to be as efficient and computationally fast as possible, since this could open the possibility of computing the trajectory continuously in real-time, which would allow for an effective way to handle the presence of disturbances or uncertainties on the system model.

One of the possible solutions consists in using a convex algorithm to solve the problem involving the trajectory optimization as the speed and robustness of this solution approach is well known in the literature, especially with respect to traditional (nonlinear) algorithms [1]. The key point in using the convex approach is reformulating the transfer problem, which is non-convex, in order to convert it into a convex optimization problem by means of convexification techniques.

It should be noted that for the trajectories convex algorithms work very well, with a good precision, but maybe for other kind of missions where the conditions of the problem are more extreme or demand higher performances than this problems the applicability of this kind of algorithm should be carefully studied.

This work aims to introduce the application of convex algorithms for the optimization of trajectories for orbit transfers, with application to an Earth-Mars transfer, explaining in detail the theoretical background behind this method and performing some parametric studies related to the effects changing the thrust and the grid discretization, to the validity of the algorithm.

First, the convex optimization will be explain in detailed. After that, the application of the convex optimization to the transfer problem by its technics will be explained. Finally, the parametric results of studies on the transfer problem between Mars and Earth will be shown and discussed.

Convex optimization

A convex optimization problem is an optimization problem in which the objective function is convex and the feasible set is a convex set [2]. A function is said to be convex if the following relation holds

$$f(\theta x + (1-\theta)y) \leq \theta f(x) + (1-\theta) f(y) \quad \forall \theta \in [0,1] \qquad (1)$$

And a set denoted by S is said to be convex if for this set the following condition holds

$$\theta x + (1-\theta)y \in S \quad \forall (x, y \in S; \theta \in [0,1]) \qquad (2)$$

There are some subclasses of convex optimization depending on the type of objective function, the constraints of the problem and the method to find the solution. As an example, least-squares problems and the linear programming problems are two special cases of convex problems. Even though, there is no analytical formula for the solution of convex optimization problems, very effective methods exist for solving them, which are currently being investigated. One of these methods is the interior-point method, which already works well for some subclasses of the convex optimization problem [2].

The main advantage of the convex optimization problems is that they can be solved in a very efficient way. The difficult part is to recognise if the optimization problem that it is being studied is convex, or if it could be reformulated as a convex problem.

So, the key point for being able to solve most of the optimization problems that can be found (not only in the space field but also in engineering) is being able to formulate them in a convex way. In order to do so, some techniques can be applied that are called convexification methods.

For the non-convex problems also the convex optimization can play an important role since a convex approximation of the problem can represent a starting point for applying more complex and further methods to solve them.

The ease of solving convex optimization problems lies in the fundamental property of convex optimization problems for which any locally optimal point is also globally optimal, thus, if the objective function is strictly convex and defined over a convex feasible set that implies that there exist only one optimal point at most [2].

The convex optimization problem is formulated as follows:

$$\begin{aligned} & \textit{minimize } f_o(x) \\ & \textit{subject to } f_i(x) \leq 0, \qquad i = 1, \dots, m \\ & \qquad\qquad\quad h_i(x) = 0, \qquad i = 1, \dots, p \end{aligned} \tag{3}$$

where the inequalities of the form $f_i(x) \leq 0$ are called inequality constraints and the $f_i(x)$ are called inequality constraint functions, while the equalities $h_i(x) = 0$ are called equality constraints and the functions $h_i(x)$ are called equality constraint functions. In the case that there are no constraints the problem is said “unconstrained”. Also, it is important to remark that the problem is said to be expressed in standard form when the equality and inequality have the constraints right hand side equal to zero [2]. So, any problem can be expressed in standard form by rearranging the constraints in such a way that the right hand side of them becomes zero. A special subclass of the convex problem is the feasibility problem in which the objective function is identically zero and the optimal value for instance is either zero or infinite depending on if the feasible set is nonempty and empty respectively [2]. Solving this problem is useful to check that the constraints are consistent.

As it was previously commented most of the problems of interest are not directly presented in a convex form, but they can be more efficiently solved using a different form, thus they are needed to be reformulated. In these cases, the best way to solve them is to solve equivalent problem. Two problems are said to be equivalent if, from a solution of one, a solution of the other can be readily obtained and vice versa [2]. A simple example would be the maximization of the objective function instead of its minimization, the equivalent problem is to apply the convex standard form at the negative objective function, so use $-f_o$ instead of f_o. Some of these transformations that can be used to obtain equivalent convex problems are:

- Eliminating equality constraints: Given the equality constraints in the matrix form $Ax = b$; by finding a particular solution of the equality constraints x_0 and a matrix F whose range is the null space of A , the equality constraints can be eliminated and a new equivalent problem of variable z is found which has the following shape:
$$\begin{aligned}&minimize\ f_o(Fz + x_0)\\&subject\ to\ f_i(Fz + x_0) \leq 0, \qquad i = 1, \dots, m\end{aligned}$$
This equivalent problem has the advantage of not having to focus on the equality constraints but this is not always beneficial since sometimes the problem is more difficult to be understood or analysed or even worsen the efficiency of the solver when eliminating these equality constraints.
- Introducing equality constraints: Sometimes it could be useful to introduce new variables and constraints. Exploiting the linearity of the equality constraints it is possible to introduce a new variable y_i and replace a constraint function (or the objective function in case it is also linear) by $f_i(y_i)$ and add the equality constraint $y_i = A_i x + b_i$.
- Slack variables: Also it is possible to transform an inequality constraint into an equality constraint by introducing a new variable that is called slack variable. For example, $f_i(x) \leq 0 \rightarrow\ f_i(x) + y_i = 0\ \ with\ \ y_i \geq 0$.

These are just a few examples of how to obtain equivalent convex. More complex techniques and with different scopes have been devised according to the complexity of the convexification of the problem to be solved.

And said before, different subclasses of problem can be found according with the features of their objective function and the type of constraints and the constraint functions. For this work, it is interesting to introduce a special subclass which is close to the quadratic program problem called Second-Order Cone Programming(SOCP) problems. It has the shape,

$$\begin{aligned}&minimize\ f^T x\\&subject\ to\ \|A_i x + b_i\|_2 \leq c_i^T x + d_i, \qquad i = 1, \dots, m \qquad (4)\\&\qquad\qquad Fx = g,\end{aligned}$$

The inequality constraint is called a second-order cone constraint, since it is the same as requiring the affine function to lie in the second-order cone of the space $\mathbb{R}^{n_i+1}$, where n_i comes from the definition of the matrix A_i, $A_i \in \mathbb{R}^{n_i \times n}$ being n the dimension of the space where f is defined. SOCPs can be solved by using the aforementioned interior-point methods in a very efficient way. [2]

Earth to Mars transfer problem

The Earth to Mars transfer problem consists in the transfer of a spacecraft from Earth orbit to Mars orbit. The problem is modelled under the assumption of considering the spacecraft as a point mass, which is a more than acceptable assumption for interplanetary missions [3]. For describing the state vector of the spacecraft, an Earth-Centred Inertial (ECI) reference frame is adopted for the position, and a Local-Vertical-Local-Horizontal (abbreviated as LVLH) reference frame is adopted for the velocity [4]. So, the state vector has seven components, the three components of the spacecraft position, the three components of the spacecraft velocity and the mass of the spacecraft. The notation adopted can be seen in equation 5, where the state vector is represented by the letter x:

$$\bar{x} = [r \quad \theta \quad \varphi \quad v_r \quad v_t \quad v_n \quad m] \qquad (5)$$

Where r is the distance from the Earth center to the spacecraft, θ is the right ascension angle, φ is the latitude angle, v_r is the radial component of the velocity, v_t is the eastward component of the velocity, v_n is the northward component of the velocity and m is the spacecraft mass.

The dynamical model assumes that the forces acting on the spacecraft are the gravity force and the own thrust of the spacecraft. This thrust vector can be expressed also in the LVLH reference frame and it is the control vector of the problems since it is the parameter that can be changed. This control vector is

$$\bar{u} = \bar{T} = [T_r \quad T_t \quad T_n] \qquad (6)$$

Of course the thrust is limited by the actual technology incorporated in the spacecraft or by the remaining resources of it. So for the thrust it is necessary to set an upper bound, so that

$$||\bar{T}|| \leq T_{max} \qquad (7)$$

that is , $||\bar{T}|| \in [0; T_{max}]$.

The goal of the optimization algorithm is to determine the magnitude and the direction of the available spacecraft thrust to complete the transfer in an optimum way.

It is known the thrust is related with the mass flow rate and the effective exhaust velocity as

$$||\bar{T}|| = -\dot{m}c \qquad (8)$$

where c is the effective exhaust velocity that is calculated as the gravity acceleration at sea level and the specific impulse in vacuum of the engine:

$$c = g_0 I_{sp} \qquad (9)$$

where g_0 is the standard gravity constant and I_{sp} is the specific impulse in seconds. [3]

For the gravity force, it is considered only the gravitational effect of the Sun since for the interplanetary transfers gravity forces of the other bodies can be neglected without any loss of accuracy, thus converting the problem into a two bodies problem, which is far simpler than the 3 or 4 body problem, that would result from accounting for the presence of other planets such as Jupiter. The gravity acceleration is given by

$$\bar{g} = \frac{-\mu}{r^3}\bar{r} \qquad (10)$$

where the gravitational parameter of the Sun is $\mu = 132712440000\ km^3/s^2$.[2]

The dynamical model that governs the problem is given by the following set of differential equations of the form $\dot{\bar{x}} = \bar{f}(\bar{x}, \bar{u}, t)$:

$$\begin{cases} \dot{r} = v_r \\ \dot{\theta} = \dfrac{v_t}{r\cos\varphi} \\ \dot{\varphi} = \dfrac{v_n}{r} \\ \dot{v_r} = \dfrac{{v_t}^2 + {v_n}^2}{r} - \dfrac{\mu}{r^2} + \dfrac{T_r}{m} \\ \dot{v_t} = -\dfrac{v_r v_t}{r} + \dfrac{v_n v_t}{r}\tan\varphi + \dfrac{T_t}{m} \\ \dot{v_n} = -\dfrac{v_r v_n}{r} - \dfrac{{v_t}^2}{r}\tan\varphi + \dfrac{T_n}{m} \\ \dot{m} = -\dfrac{T}{c} \end{cases} \qquad (11)$$

The objective in this mission is to minimize the propellant used. Thus, the objective function would be to maximize the spacecraft mass at the end of the transfer, that is, when only the payload and structural mass remain and the propellant mass has been completely consumed. Therefore, the higher the final mass is, the less the propellant used in the transfer.

Besides the differential constraints, the problem has to consider also the boundary conditions. First, the spacecraft state at the initial instant it is known. For the terminal conditions, orbital parameters of Mars at the final state are used. By assuming a final circular orbit, the constraints for the problem are as follows

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[2] Ssd.jpl.nasa.gov. 2021. *Astrodynamic Parameters*. [online] Available at: <https://ssd.jpl.nasa.gov/astro_par.html> [Accessed 30 August 2021].

$$\begin{cases} \bar{x}(t_0) = \bar{x}_0 \\ r(t_f) = r_f \\ v_r(t_f) = 0 \\ v_t^2(t_f) + v_n^2(t_f) = \frac{\mu}{r_f} \\ \frac{v_t(t_f)\cos\varphi(t_f)}{\sqrt{v_t^2(t_f)+v_n^2(t_f)}} = \cos i_f \end{cases} \tag{12}$$

where i_f is the inclination of the final orbit.

So, as it can be seen the optimal problem to solve is the following:

$$\begin{aligned} &minimize \quad -m(t_f) \\ &subject\ to \quad (11)-(12) \end{aligned} \tag{13}$$

Parameter	Value	Units
r_0	14959787	km
θ_0	0	rad
φ_0	0	rad
v_{r0}	0	km/s
v_{t0}	0	km/s
v_{n0}	29.7846928993	km/s
m_0	659.3	kg
t_f	21859200	s
r_f	227936640	km
i_f	1.85	rad
v_f	24.12952351	km/s

Table 1: Data set for the optimization problem

The particular conditions considered for the problem are listed in Table 1.

It should be noted that Mars position is expressed in the ECI inertial reference frame, so despite a relative movement between Mars and Earth exists it is not necessary to consider it since it is already considered in the final position information provided to the algorithm. Also it should be remarked that the components of the eastward and northward velocity are put together in a unique velocity representing the tangential velocity of the final orbit called v_f.

Reformulation of the problem

With the introduction to the convex optimization problems it can be seen that the actual one is not accomplishing the convex conditions, so a series of transformations are going to be performed in order to convexify the problem into a SOCP problem, which was previously commented, and it also will improve the performance of the solver. The solver method consists in the aforementioned interior-point methods, which are high-efficient methods for this kind of problem.

First a change of variables is going to be carried out. New control variables are introduce to replace the initial ones by normalizing the thrust with respect the mass and adding one dimension to the control vector which is the normalized thrust magnitude:

$$\bar{u} = [u_r \quad u_t \quad u_n \quad u_N] = \left[\frac{T_r}{m} \quad \frac{T_t}{m} \quad \frac{T_n}{m} \quad \frac{T}{m}\right] \quad (14)$$

Also, defining a new change of variable of the mass of the form:

$$z = \ln m \qquad (15)$$

And by differentiating the new state variable:

$$\dot{z} = \frac{\dot{m}}{m} = -\frac{T}{mc} = -\frac{u_N}{c} \qquad (16)$$

So, the new state vector is defined:

$$\bar{x} = [r \quad \theta \quad \varphi \quad v_r \quad v_t \quad v_n \quad z] \qquad (17)$$

And rewriting the dynamical model with these new changes of variables [3]:

$$\begin{cases} \dot{r} = v_r \\ \dot{\theta} = \dfrac{v_t}{r\cos\varphi} \\ \dot{\varphi} = \dfrac{v_n}{r} \\ \dot{v_r} = \dfrac{{v_t}^2 + {v_n}^2}{r} - \dfrac{\mu}{r^2} + u_r \\ \dot{v_t} = -\dfrac{v_r v_t}{r} + \dfrac{v_n v_t}{r}\tan\varphi + u_t \\ \dot{v_n} = -\dfrac{v_r v_n}{r} - \dfrac{{v_t}^2}{r}\tan\varphi + u_n \\ \dot{z} = -\dfrac{u_N}{c} \end{cases} \qquad (18)$$

The new defined control variables are related to each other by the condition [3]:

$$ {u_r}^2 + {u_t}^2 + {u_n}^2 = {u_N}^2 \qquad (19)$$

But this is a non-convex equality constraint. So in order to convexify the constraint it is going to be performed a constraint relaxation which allows to convexify the non-convex equality constraint into a convex constraint preserving the original formulation. The constraint relaxation will lead to the following convex constraint [3]:

$$ {u_r}^2 + {u_t}^2 + {u_n}^2 \leq {u_N}^2 \qquad (20)$$

which corresponds to a second-order cone constraint.

Also it is necessary to define the limits of the new variable control u_N that should be bounded by the maximum thrust of the engine [3]:

$$0 \leq u_N \leq T_{max} e^{-z} \qquad (21)$$

But it is seen that the constraint is non-convex too, so in this case to convexify the constraint it is necessary to linearise the constraint around a reference solution (k). So, as a result it is obtained the constraint [3]:

$$0 \leq u_N \leq T_{max} e^{-z^{(k)}} \left(1 - \left(z - z^{(k)}\right)\right) \qquad (22)$$

Finally, the rest of the non-convexities that are located on the differential equations and the boundary conditions are convexified by means of linearisation around a reference solution (k) as it was did before. It should be noted that this reference solution is constantly being updated by the iterations of the algorithm.

The dynamical model linearization can be obtained easily by exploiting the control-affine dynamics that the change of variables performed before allows to it. [3]

$$\dot{\bar{x}} = \hat{\bar{f}}(\bar{x},t) + B(\bar{x},t)\bar{u} \approx \hat{\bar{f}}\left(\bar{x}^{(k)},t\right) + \hat{\bar{f}}_x\left(\bar{x}^{(k)},t\right)\left(x - x^{(k)}\right) + B\left(\bar{x}^{(k)},t\right)\bar{u} \qquad (23)$$

where $\hat{\bar{f}}$ is defined as $\hat{\bar{f}} = \bar{f}(\bar{u} = \bar{0})$ and B is the control coefficient matrix, which has a number of rows equal to the number of state variables and a number of columns equal to the number of control variables. So in this case, its dimensions are 7x4. And $\hat{\bar{f}}_x$ is the partial derivative matrix of the dynamic equations of the model with respect the state variables.

Finally, by adding the definition for simplicity of [3]:

$$A = \hat{\bar{f}}_x\left(\bar{x}^{(k)},t\right) \quad ; \bar{c} = \hat{\bar{f}}\left(\bar{x}^{(k)},t\right) - A\bar{x}^{(k)} \qquad (24)$$

The linearized simplified dynamics which governs the optimization problem arises into [3]:

$$\dot{\bar{x}} \approx A\left(\bar{x}^{(k)},t\right)\bar{x} + B\left(\bar{x}^{(k)},t\right)\bar{u} + \bar{c}\left(\bar{x}^{(k)},t\right) \qquad (25)$$

Once the problem has been convexified it is needed to discretize the domain since the state and control variables are continuous and the inputs that the systems is going to receive are not going to be continuous(despite the assumption of quasi-continuous signals can works for some system) and also the numerical methods used by the solver require a finite domain.

For achieving this purpose the independent variable, which is the time, is discretized in M points by dividing the mission duration (t_f) by M-1 intervals. It is important to remark that is not necessary that the intervals have the same length, this means that the nodes can be focus in the zones of the domain that require more attention for accuracy reasons. For example if in some interval of the domain it is found that there are inaccuracies, lower intervals of time and for instance more nodes can be used in that zone to improve the accuracy. Finally, to integrate the state and control variables that are discretized over this time grid the integration scheme used in this application is a simple trapezoidal integration scheme.

The quality of the mesh and the solution is after checked by using the Betts and Huffman approach that ensures that the discrete-time solutions meet the proper tolerances of error with respect the solution [3]. This refinement technique also makes the improve of the mesh and the solution by adding the minimum nodes, so that the grid size does not increase much and only in the required zones of the domain.

And with this, the reformulation of the problem ends and as a result a convex approximation has been obtained. But by iteratively applying the SOCP solver the approximated problem solution converges to the real one, by establishing a determined tolerance between two consecutive reference solutions in order to ensure the stop condition of the algorithm when it reaches the solution.

Nominal solution

The nominal solution, that is the starting point of the parametric study, obtained by using the convex algorithm is shown by means of the plot of the trajectory, both in 2D and 3D, the control variable evolution (which is the thrust), the mass evolution and the final mass of the spacecraft. The plots of the trajectory are shown in Figure 1 and 2, in 2D and 3D respectively.

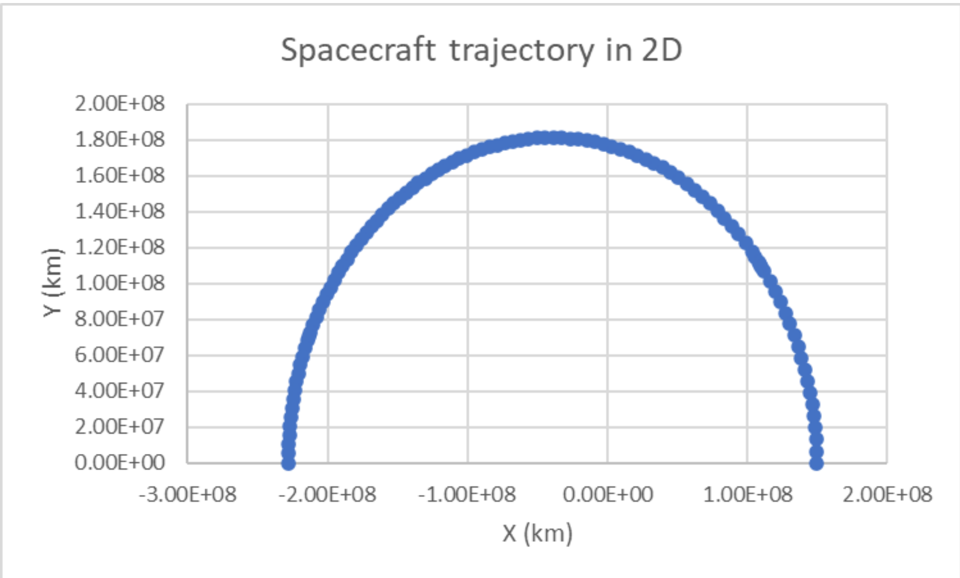

Figure 1: Trajectory of the spacecraft in 2D

Here it is seen how the spacecraft moves in the XY plane which is the equatorial Earth plane, between the initial and final position.

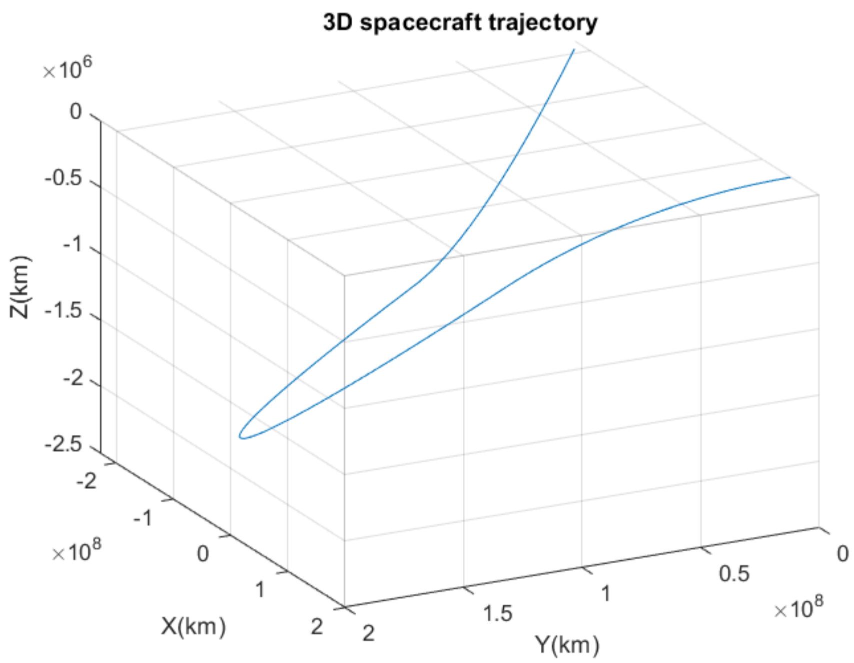

Figure 2: Trajectory of the spacecraft in 3D

In Figure 2 the trajectory is shown by adding the third dimension which corresponds to the Earth rotation axis with the North as the positive axis direction.

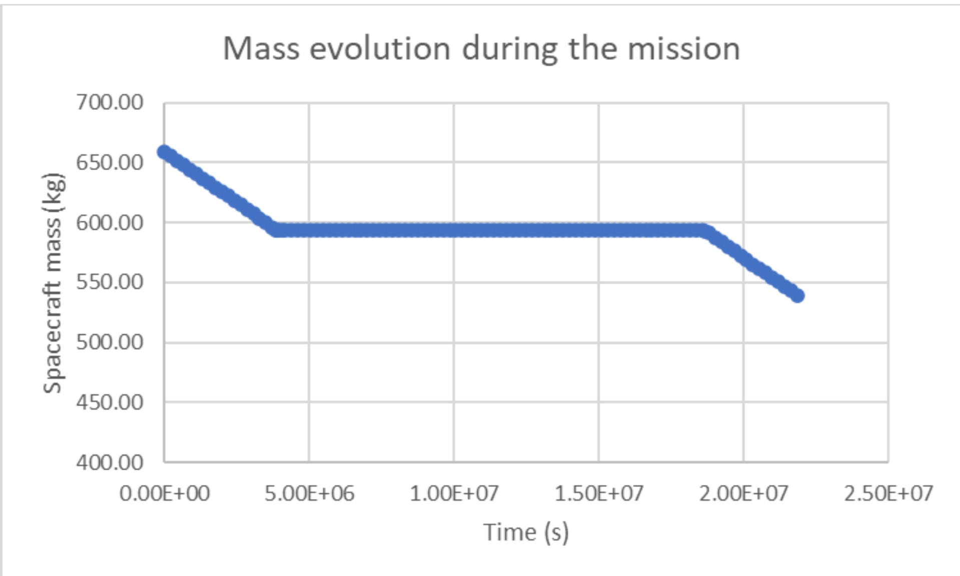

Figure 3: Spacecraft mass evolution during mission

The mass evolution can be seen in Figure 3. It is appreciated how the change during the intermediate phase of the transfer is practically null that means that in this phase no propellant is used because almost no control is applied. The propellant consumption is located at the beginning and at the end of the transfer, this is because these are the points where it is necessary to apply a ΔV to scape of the initial orbit and couple with the final orbit respectively.

This is complemented with Figure 4, which shows the control of the thrust evolution during the mission.

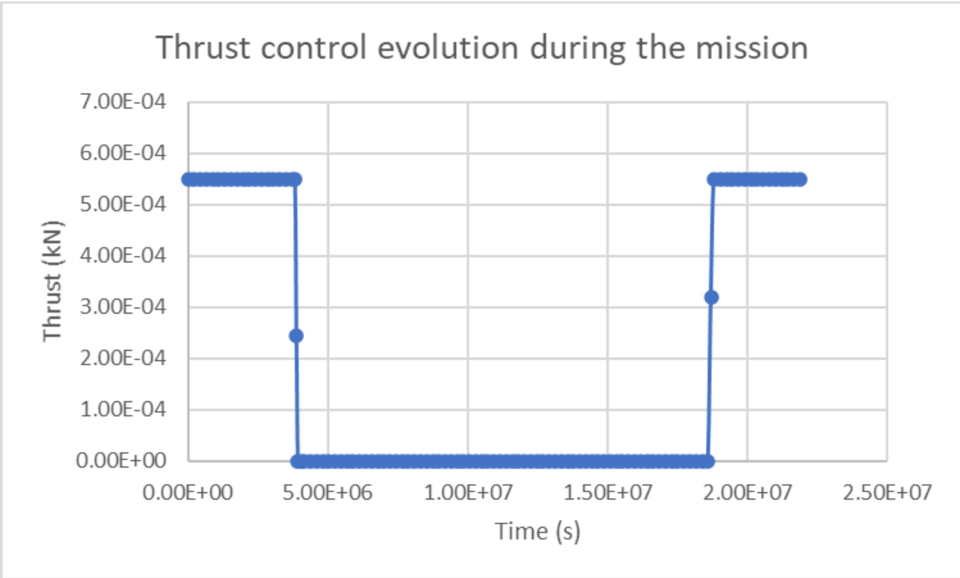

Figure 4: Thrust control during the mission

And as it was expected, since their relationship was already considered in the model, the thrust plot concords with the mass plot. The mass decreases when the thrust is applied since propellant is needed to produce this thrust.

The solution final mass is 539.3865197 kg. So, the propellant mass used during the mission is 119.9134803 kg.

Thrust variation

In this parametric study, an evaluation of how the maximum thrust of the spacecraft affects the propellant mass, and for instance the final mass of the spacecraft is going to be carried out. It is already known that final mass and propellant mass are related each other as $m_0 = m_f + m_p$. So, this is useful since the final mass will be given for a certain structural mass plus the payload mass. So, given a determined initial mass it is seen that the less propellant is used the more payload that can be carried by the spacecraft.

For this study, the initial thrust evaluated is the original value used in Reference [5]. That value has been changed progressively by decreasing 0.05 N until the mission is no longer possible to be completed with that engine specification. The obtained results are shown in Figure 5.

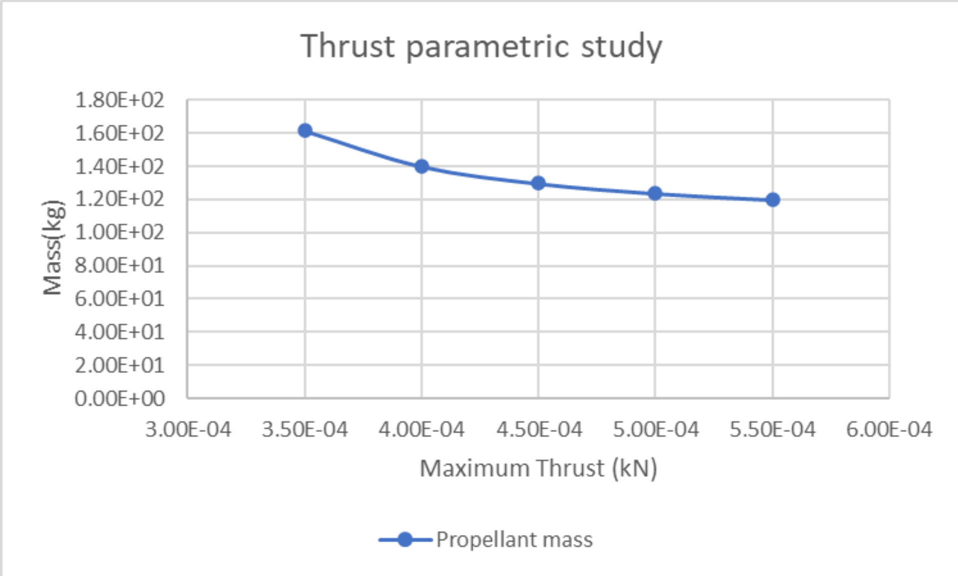

Figure 5:Propellant mass as a function of the maximum thrust

It is seen how the greater the maximum thrust available is the less propellant mass needed to perform the transfer and for instance more payload mass available to carry. The last value that allowed to complete the mission was 0.35 N of maximum thrust. For obtaining a better resolution to locate this limit of the maximum thrust that allows to perform the transfer, from 0.35 N a zoom is going to be used meaning that in this case a lower step to decrease the maximum thrust is going to be used, being this one of 0.01 N. This “zoom” can be seen in Figure 6.

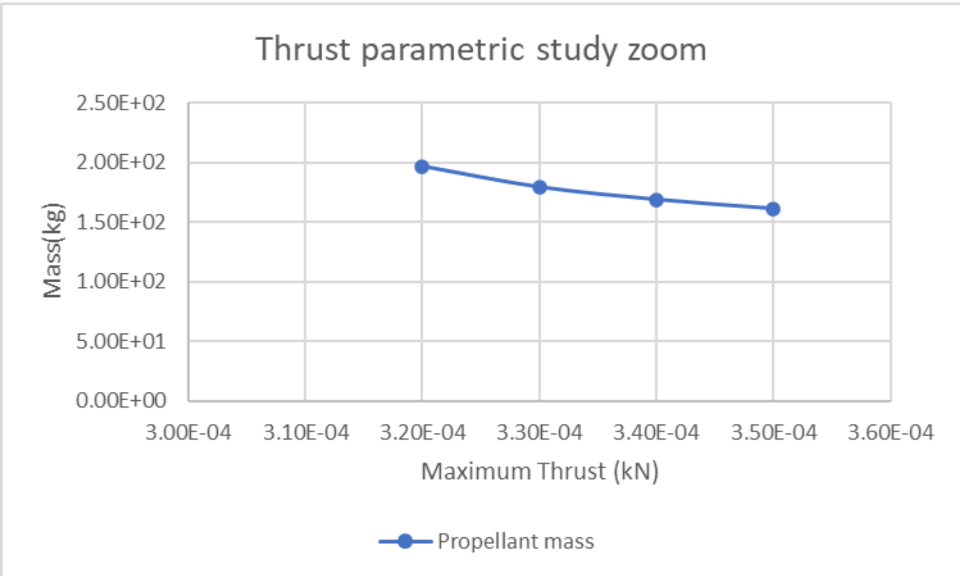

Figure 6: Propellant mass as a function of the maximum thrust zoomed

In this case the limit was found in 0.32 N, below this value using a 0.01 N step the mission cannot be carried out. The behavior in this region is the same as before and also it can be seen that the lower the maximum thrust of the spacecraft the more propellant that is needed for the mission and for instance, given an initial mass the less the payload that can be transferred.

Grid discretization variation

In this parametric study, the number of nodes of the discretization it is going to be increased to see how it affects the solution and to evaluate if this increase directly translates into a better accuracy and also if it is worthy or on the contrary it is not going to change significatively. The expected results are that the higher the nodes the accuracy is going to be better and it is going to tend to a stable value, this means that the differences of consecutive values are going to decrease. But this is going to happen infinitely since it is a numerical method, so this is was the reason of defining a tolerance as a stop condition. So, to analyze the variation of the solution by increasing the number of nodes, not only the values obtained are going to be shown but also the error between two consecutive errors and also the relative error in order to take perspective of the percentage of variation are going to be plot.

The formulas for the error, ε, and the relative error, ε_r, of a magnitude value m_k are:

$$\varepsilon = |m_k - m_{k-1}| \quad ; \qquad \varepsilon_r(\%) = \left|\frac{m_k - m_{k-1}}{m_{k-1}}\right| \cdot 100 \qquad (26)$$

So, for the final mass the results are shown in Figures 7, 8 and 9.

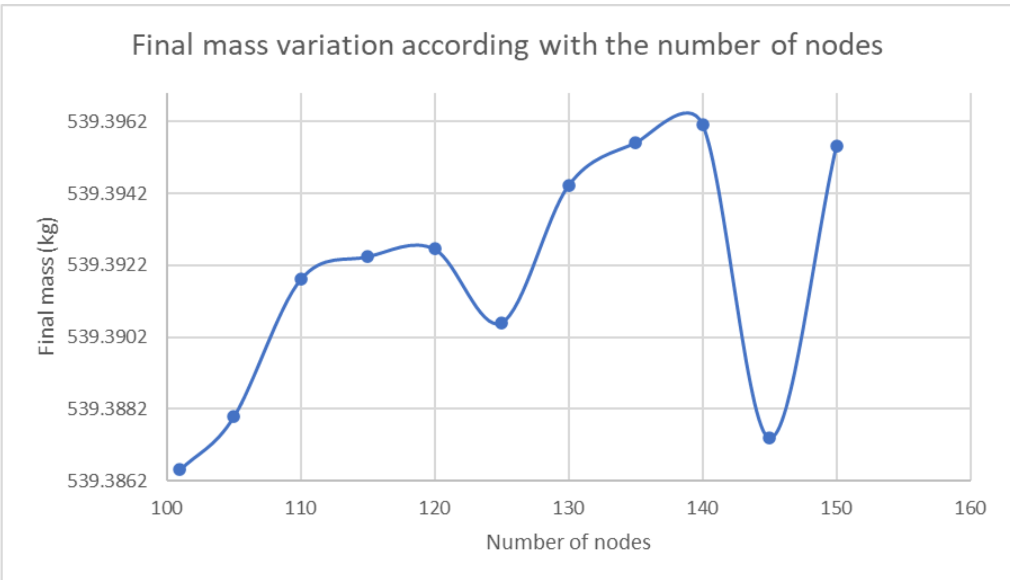

Figure 7: Final mass as a function of the number of nodes used

It can be seen that variations in the number of nodes do not produce any meaningful change in the final mass. So, from this graph it can be envisage that the increase of number of nodes beyond the original number is not worthy as the solution does not tend to stabilize and the differences found among the different values are negligible.

Therefore, it can be concluded that the grid was already sufficiently dense. For complementing this analysis also the error plots have to be carefully analyzed.

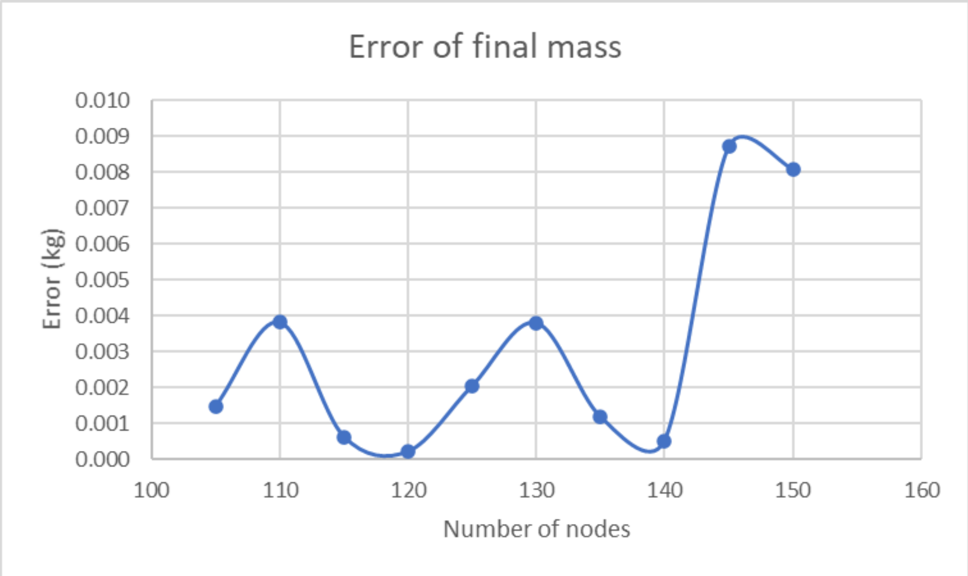

Figure 8: Final mass error as a function of the number of nodes

This graph encourages what it was stated in the previous paragraph. The error seems to be very low, so probably with the original number of nodes the accuracy is good enough and by increasing the number of nodes, the extra computational cost is not rewarded with an improve in the solution. To see this error with more perspective, the error in comparison of the magnitudes of the values of mass is useful to analyze the relative error.

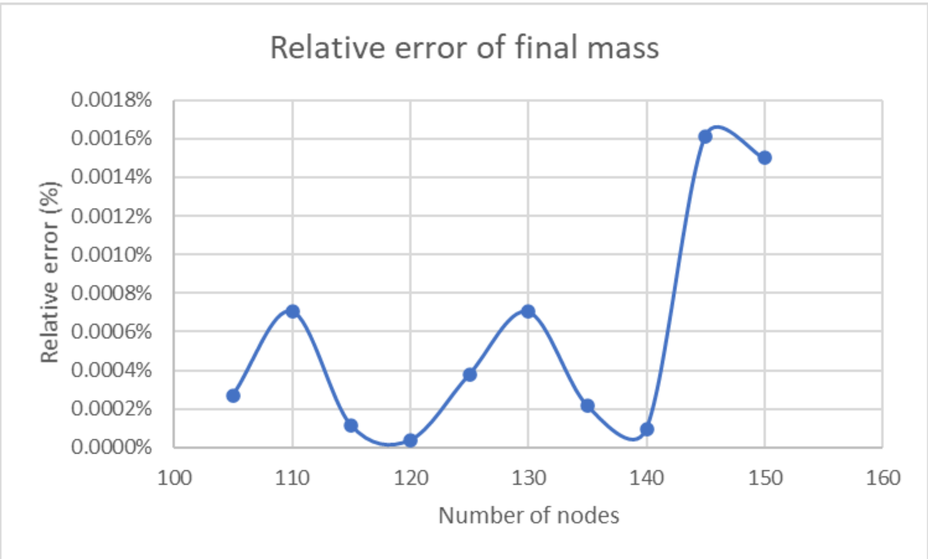

Figure 9: Relative error of the final mas as a function of the number of nodes

This graph of course presents the same behavior than before since the difference is that the result is multiply by a constant value (the inverse of the mass value of the reference solution) to nondimensionalize the error and see the percentage of error. In this figure it can be checked as the values of the error is very low.

As a conclusion, it can be said that the solution is already mesh-independent since changing the mesh i.e. the number of nodes, does not provide an stabilization of the variations and also this variations are very low and for instance the computational effort of increasing the number of nodes does not provide any benefit to the problem solution. Therefore the solution obtained with the original mesh is already stable and precise, and it is validated.

Conclusion

This academic work has introduced the convex optimization problems that are a powerful tool to solve optimization problems involving different fields, and it was applied for the one that corresponds to the aerospace engineering field. It was seen how a problem that initially is non-convex can be convexified through different techniques always being careful with the scope of the problem and the validity of the transformations performed. It has been interesting to analyze the importance of the transfer problem between the Earth and Mars, despite the algorithm can be applied to other transfer problem, and the velocity of the solution. All the calculations were done very quick by the algorithm, which is interesting when it is needed to perform fast recalculations of the trajectory or mass charges or consider potential scenarios of the mission.

And also the dynamical model that governs the planetary transfer problem has supposed an interesting review of the knowledge acquired during the degree, not only by the orbital equations themselves but also by the implementation of a dynamical model that also involves other disciplines as for example control systems subjects, math subjects and so on. The only possible change is to design a transformation from a more general reference frame, as for example an heliocentric one, to generalize it to different transfer and not only to Earth to Mars but anyway this reference frame also works well, since first all the spacecrafts can be relative related with the Earth's position and for instance have them in the ECI reference frame and in fact most of the missions of interest will have their starting point at Earth so it is also useful.

The parametric study was a good way to explore how the change, first in the properties of the spacecraft selected and second in the own mesh configuration affects to the solution obtained. The change in the maximum thrust showed how the diminution of the engine capacity despite making cheaper the construction of the spacecraft implies a larger amount of fuel needed to perform the transfer which suppose an increase of the costs. Also, it was possible to identify the minimum value of thrust capacity of the engine to carry out the mission.

With respect the second parametric study, related with the mesh configuration, it was shown that changing the number of nodes beyond the original number of the algorithm did not improve the solution since there were small random oscillations that did not attenuate proving for instance that the algorithm had already converged to the final solution. So, it was concluded that the increase in the grid and in consequence in the computational cost do not provide any benefit.

This second study, as a result also validated the solution obtained originally despite as it was explained the algorithm also counts with a Betts and Huffman approach to check the quality of mesh and solution.

Finally, it has been shown that convex algorithms are a powerful tool for optimization problems, so they will probably keep evolving and maybe will be key for some space missions in the coming years.

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