A proposal for quantum short time personality dynamics

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1 Introduction

The present work is an attempt to obtain a quantum formulation for the short-term dynamics of personality as a consequence of an arbitrary stimulus. This daring goal lies on the hypotheses that the quantum approach can provide some new richness to the dynamical solutions and individual personality changes observed in behavioural sciences that a classical differential equation approach cannot provide.

First of all, a minimum action principle must be stated to describe this dynamics, i.e., a classical mechanics approach to personality, through the Lagrangian and the Hamiltonian functions [1]. The Hamiltonian permits to state the quantum approach through the corresponding Schrödinger equation.

In addition, these classical and quantum formulations allow postulating a bridge between physics and psychology. In fact, the current problem in physics consists in getting the dynamics (by a set of coupled second order differential equations) from a known Lagrangian. Besides, the inverse Lagrange problem [2] consists in finding the Lagrangian from the known dynamics. In the context of this paper, the inverse Lagrange problem, solved in [3] for the short term dynamics of personality as a consequence of an arbitrary stimulus, is taken: the Lagrangian and the Hamiltonian are presented and, as a consequence, the Schrödinger equation is got by applying the quantization rules on the Hamiltonian.

Personality is here measured by the Five-Adjective Scale of the General Factor of Personality (GFP-FAS) [4], which measures dynamically the General Factor of Personality (GFP), i.e., it is a way to measure the overall human personality [5]. The so-called response model is the mathematical tool used to model the personality dynamics [6]. However, the response model here presented has a slight different mathematical structure, which produces a more realistic dynamics [7]. The response model here presented is an integro-differential equation where the

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stimulus is an arbitrary time function. It is transformed into a second order differential equation for which a Lagrangian and a Hamiltonian are found, solving like this the corresponding inverse Lagrange problem.

Subsequently, the Schrödinger equation is found by applying the known quantization rules, and it is solved as a time-dependent equation, whose exact solution is found by following Ciftja's method in [8]. The corresponding quantum Hamilton equations [9] show its richness in dynamical solutions, in contrast to the classical approach solution.

2 The response model and its Lagrangian-Hamiltonian approach

The response model [7] is given by the integro-differential equation:

$$\dot{q}(t) = a(b - q(t)) + \delta \cdot s(t) \cdot q(t) - \gamma \int_{t_0}^{t} exp((x-t)/\tau) \cdot s(x) \cdot q(x)\, dx \tag{1}$$

$$q(t_0) = q_0 \tag{2}$$

In (1), $q(t)$ represents the GFP dynamics; and b and q_0 are respectively its tonic level and its initial value. Its dynamics is a balance of three terms, which provide the time derivative of the GFP: the homeostatic control ($a(b - q(t))$), i.e., the cause of the fast recovering of the tonic level b, the excitation effect ($\delta \cdot s(t) \cdot q(t)$), which tends to increase the GFP, and the inhibitor effect ($\int_{t_0}^{t} exp((x-t)/\tau) \cdot s(x) \cdot q(x)\, dx$), which tends to decrease the GFP and is the cause of a continuous delayed recovering. Parameters a, δ, γ and τ are named respectively the homeostatic control power, the excitation effect power, the inhibitor effect power and the inhibitor effect delay. In addition, the $s(t)$ time function represents the dynamics of an arbitrary stimulus. For more details about the interpretation of (1) its variables and parameters see [6] and [7]. Note that the tonic level (the b parameter) is the asymptotically stable GFP state that personality would take when the stimulus vanishes. However, in the quantum approach presented below, this parameter is avoided because, from this approach, a family of stable states represents the richness of dynamical states and personality changes found.

Taking the time derivative in (1) and subsequently substituting the integral term in this equation, the second order differential equation and the initial conditions arise:

$$\ddot{q}(t) = (-a - 1/\tau + \delta \cdot s(t))\, \dot{q}(t) + (-a/\tau + (\delta/\tau - \gamma)s(t) + \delta \cdot \dot{s}(t))\, q(t) + a \cdot b/\tau \tag{3}$$

$$q(t_0) = q_0 \tag{4}$$

$$\dot{q}(t_0) = a(b - q_0) + \delta \cdot s_0 \cdot q_0 \tag{5}$$

Equation (3) is an equivalent version of (1). In it, s_0 is the amount of stimulus in the initial time $t = t_0$. From now onwards (3) is the version of the response model to be used.

The corresponding Lagrangian, (L), momentum (p) and Hamiltonian (H) to (3) are [3]:

$$L(t, q, \dot{q}) = \frac{1}{2}u(t) \cdot \dot{q}^2 - \frac{1}{2}u(t) \cdot v(t) \cdot q^2 + u(t)\frac{a \cdot b}{\tau} \tag{6}$$

$$p = \frac{\partial L}{\partial \dot{q}} = u(t) \cdot \dot{q}(t) \tag{7}$$

$$H(t, q, p) = \frac{\partial L}{\partial \dot{q}} \dot{q} = \frac{1}{2u(t)} p^2 + \frac{1}{2} u(t) \cdot v(t) \cdot q^2 - u(t) \frac{a \cdot b}{\tau} \tag{8}$$

Where in (6), (7) and (8):

$$u(t) = exp((a + 1/\tau)(t - t_0) - \delta \int_{t_0}^{t} s(x)\, dx) \tag{9}$$

$$v(t) = a/\tau + (\gamma - \delta/\tau) s(t - t_0) - \delta \cdot \dot{s}(t - t_0) \tag{10}$$

And the corresponding Hamilton equations are:

$$\dot{q} = \frac{\partial H}{\partial p} = \frac{p}{u(t)} \tag{11}$$

$$\dot{p} = -\frac{\partial H}{\partial p} = -u(t) \cdot v(t) \cdot q + u(t) \cdot a \cdot b/\tau \tag{12}$$

Note that (12) provides only one asymptotic stable state through the term $u(t) \cdot a \cdot b/\tau$.

3 The Schrödinger equation and its exact solution

As above commented, consider that the b parameter is considered as zero from now onwards. Then, to get the quantum formalism, let $\Psi(t, q)$ be the wave function of the Schrödinger equation. To get this equation, the quantization rules are applied to the Hamiltonian given by (8) (with $b = 0$):

$$i\sigma \frac{\Psi(t, q)}{\partial t} = H(t, q, p \to -i \frac{\partial}{\partial q}) \Psi(t, q) \tag{13}$$

That is:

$$i\sigma \frac{\partial \Psi(t, q)}{\partial t} = -\frac{\sigma^2}{2u(t)} \frac{\partial^2 \Psi(t, q)}{\partial q^2} + \frac{1}{2} u(t) \cdot v(t) \cdot q^2 \cdot \Psi(t, q) \tag{14}$$

In (13) and (14) σ represents the Planck constant. But why not $\sigma = \hbar$, i.e., the true Planck constant? Because the q variable (GFP) is not spatial-type, it is rather an abstract, although measurable, personality variable. If the presented formalism were able to be experimentally contrasted, then the hypothesis about whether the equality $\sigma = \hbar$ holds or not should be clarified.

To solve the time-dependent Schrödinger equation given by (14), the method developed in [8] by Ciftja has been followed. First of all the following change of the wave function is made in (14):

$$\psi(t, q) = exp(-A(t) \cdot q^2 - B(t))\, \phi(t, q) \tag{15}$$

In (15), $A(t)$ and $B(t)$ are undetermined time-functions by the moment. This change provides (16) plus (17) and (18):

$$i\sigma\frac{\partial\phi}{\partial t} = \phi1(t,q) + \phi2(t,q) \tag{16}$$

$$\phi1(t,q) = -\frac{\sigma^2}{2u(t)\cdot}\frac{\partial^2\phi(t,q)}{\partial q^2}(i\sigma\cdot\dot{A}(t) - 2\sigma^2\frac{A^2(t)}{u(t)} + \frac{1}{2}u(t)\cdot v(t))\cdot q^2\cdot\phi(t,q) \tag{17}$$

$$\phi2(t,q) = (i\sigma\cdot\dot{B}(t) + \sigma^2\frac{A(t)}{u(t)})\phi(t,q) + 2\sigma^2\frac{A(t)}{u(t)}q\frac{\partial\phi(t,q)}{\partial q} \tag{18}$$

Now, in (16), the following change of the x independent variable is made:

$$x = \frac{q}{\rho(t)} \tag{19}$$

In (19), $\rho(t)$ is, as well by the moment, an undetermined time-function. This change provides (20) plus (21) and (22):

$$i\sigma\frac{\partial\Gamma(t,x)}{\partial t} = \Gamma1(t,x) + \Gamma2(t,x) \tag{20}$$

$$\Gamma1(t,x) = -\frac{\sigma^2}{2u(t)\cdot\rho^2(t)}\frac{\partial^2\Gamma(t,x)}{\partial x^2} + (i\sigma\cdot\dot{A}(t) - 2\sigma^2\frac{A^2(t)}{u(t)} + \frac{1}{2}u(t)\cdot v(t))\rho^2(t)\cdot x^2\cdot\Gamma(t,x) \tag{21}$$

$$\Gamma2(t,x) = (i\sigma\cdot\dot{B}(t) + \sigma^2\frac{A(t)}{u(t)})\Gamma(t,x) + (i\sigma\frac{\dot{\rho}(t)}{\rho(t)} + 2\sigma^2\frac{A(t)}{u(t)})x\frac{\partial\Gamma(t,x)}{\partial x} \tag{22}$$

In (20): $\Gamma(t,x) = \phi(t, q = \rho(t)\cdot x)$. Now, the following equations are forced to hold in this equation:

$$i\sigma\dot{B}(t) + \sigma^2\frac{A(t)}{u(t)}) = 0 \tag{23}$$

$$i\sigma\frac{\dot{\rho}(t)}{\rho(t)} + 2\sigma^2\frac{A(t)}{u(t)} = 0 \tag{24}$$

$$(i\sigma\cdot\dot{A}(t) - 2\sigma^2\frac{A^2(t)}{u(t)} + \frac{1}{2}u(t)\cdot v(t))\rho^2(t) = \frac{\sigma^2}{2u(t)\cdot\rho^2(t)} \tag{25}$$

From (23), (24) and (25), (20) becomes:

$$i\sigma\frac{\partial\Gamma(t,x)}{\partial t} = -\frac{\sigma^2}{2u(t)\cdot\rho^2(t)}\frac{\partial^2\Gamma(t,x)}{\partial x^2} + \frac{x^2}{2u(t)\cdot\rho^2(t)}\Gamma(t,x) \tag{26}$$

A final change is made in the time independent variable of (26):

$$T = \int_{t_0}^{t}\frac{dr}{u(r)\cdot\rho^2(r)} \tag{27}$$

That provides:

$$i\sigma\frac{\partial\Omega(T,x)}{\partial T}=-\frac{\sigma^2}{2}\frac{\partial^2\Omega(T,x)}{\partial x^2}+\frac{x^2}{2}\Omega(T,x) \tag{28}$$

In (28) $\Omega(T,x)=\Gamma(T=\int_{t_0}^{t}\frac{dr}{u(r)\cdot\rho^2(r)},x)$. Note that (28) is the Schrödinger equation corresponding to a harmonic oscillator. From the known boundary conditions of stability in (28) as $x\to\pm\infty$, its energies (eigenvalues) and exact eigenfunctions are [8]:

$$E_n=(n+\frac{1}{2})\sigma\ ;\ n=0,1,2,... \tag{29}$$

$$\Omega_n(T,x)=exp(-i\frac{E_n}{\sigma})(\frac{1}{\sigma\pi})^{1/4}\frac{2^{-n/2}}{\sqrt{n!}}exp(-\frac{x^2}{2\sigma})H_n(\frac{x}{\sqrt{\sigma}}) \tag{30}$$

In (30) H_n are the Hermite polynomials. By unmaking the changes proposed above, the eigenfunctions of the Schrödinger equation (14) are:

$$\Psi_n(t,q)=\frac{1}{\sqrt{2^n n!}}(\frac{1}{\sigma\pi\rho^2(t)})^{1/4}exp(-i\frac{E_n}{\sigma}\int_{t_0}^{t}\frac{dr}{u(r)\rho^2(r)}+i\frac{u(t)}{\sigma}\frac{\dot\rho(t)}{\rho(t)}q^2)exp(-\frac{q^2}{2\sigma\rho^2(t)})H_n(\frac{1}{\sqrt{\sigma}}\frac{q}{\rho(t)}) \tag{31}$$

Note in (31) that it depends only on the undetermined $\rho(t)$ time-function, which implies that it will depend on the $\rho(t)$ solution. In fact, handling appropriately the system provided by (23), (24) and (25), the system can be reduced to one only second order differential equation for $\rho(t)$:

$$\ddot\rho(t)+\frac{\dot u(t)}{u(t)}\dot\rho(t)+v(t)\cdot\rho(t)=\frac{1}{u^2(t)\cdot\rho^3(t)} \tag{32}$$

4 The quantum Hamilton equations

An alternative interpretation of the Quantum Mechanics was proposed by Bohm and Hiley [9], putting the emphasis on the quantum Hamiltonian that can be derived from the exact solution of the Schrödinger equation (31) and (32). This Hamiltonian is obtained by splitting the wave function into the amplitude $\Delta(t,q)$ and the phase $S(t,q)$. The amplitude square provides the probability conservation, while the phase is a correction of the Hamilton-Jacobi equation. It permits to write the quantum Hamiltonian H_q [9]:

$$H_q(t,q,p)=\frac{1}{2u(t)}p^2+\frac{1}{2}u(t)\cdot v(t)\cdot q^2-\frac{u(t)}{\sigma^2}\frac{1}{\Delta_n(t,q)}\frac{\partial^2\Delta_n(t,q)}{\partial q^2} \tag{33}$$

Where, in (33), under the hypothesis that the $\rho(t)$ solution is real in (32):

$$\Delta_n(t,q)=|\Psi_n(t,q)|=\frac{1}{\sqrt{2^n n!}}(\frac{1}{\sigma\pi\rho^2(t)})^{1/4}exp(-\frac{q^2}{2\sigma\rho^2(t)})H_n(\frac{1}{\sqrt{\sigma}}\frac{q}{\rho(t)}) \tag{34}$$

Thus, the corresponding quantum Hamilton equations to (33) are:

$$\dot q=\frac{\partial H_q}{\partial p}=\frac{p}{u(t)} \tag{35}$$

$$\dot{p} = -\frac{\partial H_q}{\partial p} = -u(t) \cdot v(t) \cdot q + \frac{\sigma^2}{2u(t)} \frac{\partial}{\partial q} \left(\frac{1}{\Delta_n(t,q)} \frac{\partial^2 \Delta_n(t,q)}{\partial q^2} \right) \tag{36}$$

The second order formulation can be recovered from (35) and (36):

$$\ddot{q}(t) + \frac{\dot{u}(t)}{u(t)} \dot{q}(t) + v(t) \cdot q(t) = \frac{\sigma^2}{2u^2(t)} \frac{\partial}{\partial q} \left(\frac{1}{\Delta_n(t,q)} \frac{\partial^2 \Delta_n(t,q)}{\partial q^2} \right) \tag{37}$$

which can be compared with the initial second order formulation of (3) by using the $u(t)$ and $v(t)$ functions:

$$\ddot{q}(t) + \frac{\dot{u}(t)}{u(t)} \dot{q}(t) + v(t) \cdot q(t) = a \cdot b/\tau \tag{38}$$

Note that (37) provides an infinite family of dynamical evolutions and their corresponding asymptotic stable states through the term $\frac{\sigma^2}{2u^2(t)} \frac{\partial}{\partial q} \left(\frac{1}{\Delta_n(t,q)} \frac{\partial^2 \Delta_n(t,q)}{\partial q^2} \right)$, in contrast to the simple asymptotic stable state given by the term $a \cdot b/\tau$ in (38).

5 Conclusions and future work

Note that the richness in dynamical evolutions and their corresponding asymptotic stable states (37), must be investigated in a future time, moreover if it is compared with the simplicity of (38). This investigation could contribute to discover how an individual personality can change, by a bifurcation [9], to another one that could be radically different. In other words, it could provide the answer to the following question: why an individual can develop a disordered personality dynamics after a given stimulus and the same stimulus produces a non-disordered personality dynamics in a different individual?

However, the mathematical work here developed must be specified much more. On a hand, the initial conditions for $\rho(t)$ in (32) that provide real-valued solutions must be found in coherence with the theory units with which the GFP is measured.

On the other hand, note that the general solution of (14) should be expressed as $\Psi(t,q) = \sum_{n=0}^{+\infty} C_n \Psi_n(t,q)$ where C_n are complex numbers. However, the conditions under which the work with the pure states $\Delta_n(t,q) = |\Psi_n(t,q)|$ in (37) is right must be investigated: is a determined stimulus related with an only pure state? Is this approach the right one to describe the arising of the personality bifurcation and the consequent personality change? These questions and other similar must be answered.

Finally, the quantitative and qualitative solutions of (37) must be related with the results of different experimental designs, such as, for instance, the one presented in [3] with methylphenidate, or with other similar stimuli. This should be a definitive point to understand the personality change by the help of the quantum formalism here developed.

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