

Compartment models to study human impact on climate change

Modelos compartimentales para estudiar el impacto humano en el cambio climático

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Abstract

Climate change is subject to lively public discussions. Especially the question of whether climate change is man-made is a central point of contention. Since climatic processes are interleaved and complex, it is difficult to evaluate public, political or scientific statements. In this work, we look at mathematical models behind climate processes on a simplified level. We present a simple model of the Earth's energy budget developed by high school students themselves during a project week. The model allows the students to compute the global Earth's surface temperature and effects of varying solar activity and land surface changes. More precisely, the model describing the energy budget of the Earth forms a systems of partial differential equations and can be linked to compartment models. We show how this mathematically challenging question can be didactically reduced in such a way that students can be enabled to develop, solve and extend compartment models independently without having been taught the theoretical background. We implemented this course as an interactive online workshop and present our experiences with gifted student groups. We believe that our material presents an opportunity to demonstrate the power of mathematical modeling, to understand natural phenomena, and to critically reflect on discussions.

El cambio climático es objeto de animados debates públicos. Especialmente la cuestión de si el cambio climático está provocado por el hombre es un punto central de controversia. Dado que los procesos climáticos están entrelazados y son complejos, es difícil evaluar las declaraciones públicas, políticas o científicas. En este trabajo, examinamos los modelos matemáticos que subyacen a los procesos climáticos a un nivel simplificado. Presentamos un modelo sencillo del brote energético de la Tierra desarrollado por los propios estudiantes de secundaria durante una semana de proyectos. El modelo permite calcular la temperatura global de la superficie de la Tierra y los efectos de la variación de la actividad solar y los cambios en la superficie terrestre. Más concretamente, el modelo que describe el balance energético de la Tierra forma un sistema de ecuaciones diferenciales parciales y puede vincularse a modelos de compartimentos. Mostramos cómo esta cuestión matemáticamente difícil puede reducirse didácticamente de manera que los estudiantes puedan desarrollar, resolver y ampliar los modelos de compartimentos de forma independiente sin que se les haya enseñado la base teórica. Implementamos este curso como un taller interactivo en línea y presentamos nuestras experiencias con grupos de estudiantes superdotados. Creemos que nuestro material presenta una oportunidad para demostrar el poder de la modelización matemática, para comprender los fenómenos naturales y reflexionar críticamente sobre los debates.

Keywords: Compartment models, human impact, climate change, high school students, project week
Palabras clave: Modelos compartimentales, impacto humano, cambio climático, estudiantes de secundaria, semana de proyectos

1. Introduction

The analysis of natural and anthropogenic impacts on the surface temperature of the Earth is a highly relevant and authentic research topic. Students protest against the lack of action on the climate crises, e.g. “Fridays for future”. Therefore, the relevance is given not only by its importance for the students, who experience the effects of climate change and will experience them even more in future, but also for the general world population, which has to deal with the effects of climate change (cf. Paris Agreement).

The major results, which summarize the current state of science with regard to the characteristics and (future) development of the climate, are published approximately every seven years in a report by the Intergovernmental Panel on Climate Change (IPCC). The predictions made in these reports are based on simulation models that build on past experience to make predictions about the future climate development and its consequences. For this purpose, climate driving causes are analyzed and their influence is determined. Since it is in practice extremely difficult to measure and simulate the effect of individual factors, metrics are introduced to intermediate between cause and effect. One of the most popular metrics quantifying the deviation between model and reality is radiative forcing, which describes the net change of the energy balance of the Earth system due to posed perturbations (cf. IPCC, 2013). We use the metric of radiative forcing to map the natural and anthropogenic radiative influences on the Earth’s temperature budget, and investigate whether we can validate the hypothesis that the currently visible climate change is primarily based on natural effects. More precisely, we use radiation balance equations to model and analyze the influences of various natural and anthropogenic radiative forcings on the climate.

This work also demonstrates that students can independently develop scientific methods that form the mathematical basis of university mathematics already at high school level.

A simple climate model therefore considers the energy budget of the Earth. The transport of energy between two bodies, in this case between the Earth and the Sun as its main energy supplier, takes place exclusively by directed radiation fluxes. In principle, this can be described by (stationary) *radiative transfer equations*. For details see section 1.1. Not only is the mathematics necessary for modeling the energy budget a challenge. Also solving partial differential equations is usually not covered in school mathematics. Additionally, describing the Earth’s climate is very difficult. Processes on the Earth and in the atmosphere are linked to and interact with each other. Still, our experience shows that with appropriate guidance according to ¶. Stender (2016) and Hattebuhr (2014), upper level students are able to create a discretized model (compartmental model) on their own. This involves dividing the Earth’s atmosphere into concentric spherical shells, so-called compartments, where the radiative flux density in each spherical shell is assumed to be constant. This leads to radiative flux balance equations similar to finite volume models. The components of these equations can be given to students as a matter of fact. They therewith construct – without knowing it – discretizations of a partial differential equation. Thus, they derive a simple model which allows a realistic calculation of the average surface temperature of the Earth based on solar radiation. The influence of natural and anthropogenic radiative forcings can also be estimated quantitatively.

1.1. Physical and mathematical background information

The following section serves as a summary of deeper physical and mathematical knowledge for the interested reader.

The interplay between matter and particles is described by the (stationary) radiative transfer equations

$$\Omega \frac{\partial \psi(x, \Omega, \lambda)}{\partial x} = \sigma_a(x) (B_\lambda(x, T(x)) - \psi(x, \Omega, \lambda)), \quad (1a)$$

$$0 = \int_0^\infty \int_{S^2} \sigma_a(x) (\psi(x, \Omega, \lambda) - B_\lambda(x, T(x))) d\Omega d\lambda, \quad (1b)$$

where ψ is the “radiative flux density at spacial position” $x \in \mathbb{R}$ in direction $\Omega \in \{-1, 1\}$ and at wavelength $\lambda \in \mathbb{R}$.

To understand these, one uses the wave-particle duality. Imagine that radiation is transmitted of particles. Then, radiative transfer equations describe the propagation of these particles. Each of these particles is at a place x , moves in direction Ω , and has a wavelength λ . The radiation density ψ now tells how many particles of the wavelength λ are moving at a location x in the direction Ω . If the density packets interact with matter, they are either absorbed or scattered. The scattering process is complicated as it changes the moving direction of the particles. Therefore, one may neglect this process for simplification as in our model only two directions $\Omega \in \{-1, 1\}$ are allowed. The opacity σ_a plays the role of an absorption coefficient and controls the interaction strength between particles and the background materials. Obviously, these particles are after absorption no longer part of the incoming density packets. This is described by the part $\psi_a(x, \Omega, \lambda)$. However, their energy is not lost. Matter, that can interact with light at all possible wavelengths, is called a *blackbody*. The emission of particles by a material is described by the Stefan-Boltzmann equation $B_\lambda(x, T(x)) = \epsilon \sigma_{SB} T^4$ with the Stefan-Boltzmann constant σ_{SB} . Over all, the Earth is a very good blackbody and interacts with light of all wavelengths in contrast to the Earth’s atmosphere: It interacts only with certain wavelengths of light. The emissivity quantifies how good a blackbody an object is. A perfect blackbody, which is later assumed for the Earth, has the emissivity $\epsilon = 1$. The lower bound is $\epsilon = 0$. Therefore, the emissivity of the atmosphere is in between. The wavelengths of light emitted by an object depend on the temperature T of the body. For example, the light emitted by the Sun has its highest intensity in the range of visible light. Whereas the Earth emits light in the infrared range, because it is much cooler than the Sun. Equation 1b says that over all directions and all wavelengths no energy is lost and is either in the form of radiative flux density or thermal radiation.

2. Implementation of a workshop for high school students

In the following sections the formulated problem description is introduced. It has already been implemented in a modeling week for (upper level) students¹ and during an internship in the mathematics department at the Karlsruhe Institute of Technology (KIT, Germany). Some general settings of the modeling week are described in section 2.1. Here, also recommendations for an introductory presentation are given within a description of feasible application areas including possible (school) settings. One possible way of solving the given task is presented in section 2.3 and is based on the students’ work and reflects the students’ approach.

¹From here on, the term “students” refers to upper level students aged 16 years and above. Our experiences are based on the implementation with high school students, although the research question is also interesting and appropriate for a learning clientele having completed high school. In Maren Hattebuhr (2014), a requirements profile was developed for the students. This provides information on what previous knowledge the students should ideally have in order to participate successfully in a modeling week and offer them good support. Students are selected on this basis.

2.1. Modeling weeks as a possible application area

During a modeling week², selected students with special interest in mathematics work in small teams of 5–6 students on real problems from economy or research. The problems are usually unsolved, and no solution is given. Each group is supervised by a scientific staff member of the university and backed up by a representative of the company or institute responsible for the research topic. The supervision is based on the principle of minimal help (cf. F. Zech (1998)). This is supported by student-centered and cooperative group work, in which learners actively explore mathematical methods based on student-centered discussions. In this way, the students can develop their ideas and test their resilience with like-minded people. Errors or improvements are found in the exchange with each other. This can be supported by the supervisor by temporarily withdrawing and letting the group discuss alone, or by specifically asking for an explanation of the ideas and their implementation. Experience shows that uncertainties of the students can be resolved by formulating their thoughts (cf. Hattebuhr 2014).

At the beginning of the week, the supervisors give a short presentation of the problem. In doing so, the topic is motivated and the necessity of working on the problem, but also the complexity and difficulty of it is emphasized. In our case, it means reference can be made to current, special weather events like heat records, cold waves, floods, etc., or to political statements of climate conferences, politicians, scientists or well-known persons. Students can also be asked which role climate change plays in their every-day life. Then, an easy compartment model of the radiation flux should be presented and technical terms should be introduced:

- The Earth's main supplier of energy is the Sun.
- The Sun radiates at different wavelengths. It has its highest intensity in the range of visible light.
- The solar radiation is partly reflected and partly absorbed by the Earth. The coefficient which represents the proportion of incoming sun radiation being reflected is called albedo. The absorptivity tells how good energy is absorbed by a body (here the Earth).
- The Earth itself emits radiation in the form of heat energy in all directions. The emissivity tells how good energy is emitted by a body (here the Earth).
- The impact of the atmosphere is unclear and has to be analyzed.
- We are looking for a quantitative description of natural and anthropogenic radiative forces to answer the question of whether climate change is mainly caused by natural impacts.
- We recommend discussing the natural and anthropogenic radiative forces during group work. The students themselves can give suggestions of possible impacts on climate (change).
- If the question arises: As each compartment layer is assumed to be homogenous and constant over time optical phenomena in the atmosphere, such as Rayleigh and Mie scattering, light refraction, diffraction are not considered. Likewise, thermodynamics, clouds, and vertically occurring processes, which lead to a more accurate but also much more complex model, are left out. The models presented in this article are not intended for making predictions. They can rather be used for understanding and examine different impacts on the Earth's temperature.

²For further information: <https://www.scc.kit.edu/forschung/CAMMPweek.php>

The problem description for the students contains further background information, as well as references to available data (see sec. 2.2). It is quite intentional that the students gather further information themselves, for example by searching the internet. It should always be clear: The supervisor is not omniscient! The project draws on knowledge from a wide variety of fields. Therefore, the supervisor may admit “I don’t know the answer to your question, but we will certainly find an answer together.” The supervisor is a mentor and is meant to find help – not to solve the problem. The problem is intentionally open-ended. Many approaches are possible and lead to a (good) solution. It surely helps to tell all students right at the beginning that there are many possible ways of solving and there is no one true solution. If a supervisor is open minded and motivates the students to follow their own ideas, she or he will be surprised and overwhelmed of what they can achieve by themselves.

The mathematical models were implemented with the tool MATLAB³ in order to hand over complex calculations to the computer. It is also easier to improve and extend the models. The students do not necessarily need programming knowledge for the application. The handling of MATLAB comes close to use a graphic-capable pocket calculator. In the past, there were several groups that coded all together via a projector. Nevertheless, it is strongly recommended that supervisors know basics of programming and can give assistance, for example, how to create graphics. In addition, a (digital) whiteboard or flipchart proved itself extremely helpful and became a central element for collecting and processing ideas, as well as for explanations. At the end of the week, student groups present their results to a broad audience like problem advisors as well as interested scientists, teachers, family members, and friends. In doing so, they realize that within one week they themselves have become experts within their topic. Overall, this week replicates the approach to project assignments in industry and business and is therefore authentic in terms of working methods.

With our experience, we can well imagine the use of this problem in further modeling weeks. We can also recommend it for online events, as the project was successfully completed by students in an interactive online workshop. Used settings are introduced in section 3. Digital tools, that enable collaborative mathematical modeling online in general are discussed in Schönbrodt et al. (2021). In addition, programs at school or university that run for a longer period of time work as well, i.e. a week with full-time work or half a year with weekly meetings, since the students need to familiarize themselves with the topic and then need to develop and implement (mathematical) models on their own. The topic is appropriate for an interdisciplinary project-oriented work as it combines the subjects mathematics, physics, geography, and computer science.

Note: The author herself has already been responsible for the supervision of nearly ten student groups and the organization of seven modeling weeks. For the latter, she contributed to the selection and formulation of the problems and was responsible for the selection of the students, as well as the introduction and the accompaniment of the group supervisors during the week.

2.2. Problem description for introducing the task to the students

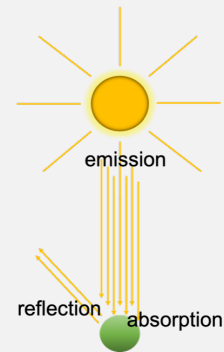
The students discuss the task on the basis of the following problem description.

³The student program CAMMP, which organizes the modeling week presented here twice a year, usually uses MATLAB. On the one hand, MATLAB is convincing due to its easy handling and syntax and can thus be quickly grasped even by people with little or no programming knowledge. It has also very good online documentation. On the other hand, it is used by many developers in industry and thus represents an authentic tool. MATLAB can also be used online, which allows access via a browser window without prior installation of the program. In principle, any other programming language such as Python or Julia could also be used.

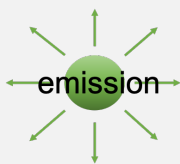
Politicians and the media discuss whether climate change is caused by natural impacts and occurs, for example, due to solar fluctuations, changes in land-use, and volcanic eruptions, or whether greenhouse gases cause climate change.

The decisive energy on Earth is supplied by the Sun. It provides the Earth with a certain amount of radiation power. This is called solar constant. The energy flux arriving on the Earth is given by $I_{\text{solar}} = 1368 \text{ W/m}^2 \pi r_{\text{terra}}^2$. The factor πr_{terra}^2 stems from the fact that the Earth is a disk as seen from the Sun.

Part of the incoming solar radiation is reflected by the Earth's surface. The reflectivity of the Earth is called albedo and is denoted by α . The remaining fraction $(1 - \alpha)I_{\text{solar}}$ is absorbed. The ratio between absorption and reflection is determined by the composition of the Earth's surface: The brighter the surface, the more sunlight is reflected and the greater the α .

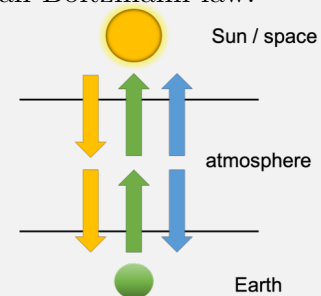


The energy absorbed by the Earth is converted into thermal energy and re-emitted. This thermal radiation can be described by the Stefan-Boltzmann law: The radiation power of the Earth is given by $I_{\text{terra}} = 4\pi r_{\text{terra}}^2 \sigma \epsilon_{\text{terra}} T_{\text{terra}}^4$. The factor of $4\pi r_{\text{terra}}^2$ comes from the fact that the Earth is a sphere and emits thermal energy over its entire surface.



The energy radiated by the Earth is partly absorbed by the atmosphere (denoted by ϵ_{atm}) and the rest permeates into outer space. The absorbed energy is subsequently re-emitted by the atmosphere. The radiation power of the atmosphere is again determined by the Stefan-Boltzmann law.

Problem: With the help of an atmospheric (multi-layer) model, we try to answer the following questions: What is the Earth's temperature? What impact do the various natural processes, such as solar fluctuations or the Earth's surface composition, have? How do volcanic eruptions affect the Earth's temperature? What is the role of greenhouse gases?



Important values and their explanations:

- Albedo: The Earth's surface (including the atmosphere) has an average albedo of about $\alpha \approx 30 \%$. If the Earth's surface changes, this leads to a change of the albedo. The albedo is always between 0 and 1.
- $\sigma \approx 5.670374419 \cdot 10^{-8} \text{ W/m}^2 \text{ K}^4$: Stefan-Boltzmann constant
- ϵ_{terra} : Emissivity of the Earth. The emissivity describes how well a body radiates energy. For the Earth it is about 94 %, for simplification it can be assumed as 100 %.
- ϵ_{atm} : Emissivity of the atmosphere. It has a value of 39 – 70 % depending on the composition and respective proportions of gases, aerosols, ... in the atmosphere.
- T_{terra} : Temperature of the Earth
- T_{atm} : Temperature of the atmosphere

Good luck!

2.3. Presentation of the model and its extensions including the solutions

The task presented above is intentionally formulated as open question to allow for various solution possibilities. In this article, we present one solution approach fulfilling the requirements that was developed by an independent student group. It was successfully completed by the students, which means that the groups autonomously set up a compartment model and improved it with several model extensions. They analyzed different impacts on the Earth's temperature using these models and interpreted the mathematical solutions. In the end, they quantified which process has the most significant impact on the temperature and evaluated, on this basis, whether the climate change is mainly based on natural forces or not.

All models developed by the students make the following assumptions:

1. There exists a radiation equilibrium state. That is, the incoming amount of radiation is equal to the outgoing amount of radiation. In other words, the absorbed energy is equal to the emitted energy in each compartment. If this was not the case, one of the compartments would heat up or cool down by itself over time, and external impacts could not be traced. Furthermore, there is no time dependency to consider.
2. Radiation has a direction. Only the two directions *Sun–Earth* and *Earth–Sun* are allowed.
3. The exchange of energy between bodies takes place exclusively by directed radiation fluxes. Radiation is not bound to matter. It can only be absorbed or emitted by a body. Scattering is neglected for simplicity.
4. The Earth is a perfect sphere with a radius of $r_{\text{terra}} = 6371$ km.

The students derived assumptions 1–3 from the problem description and the illustrations on it in a discussion with the supervisor. Assumption 4 was added completely independently.

In this article, I always denotes the radiation power per square meter. The source of the radiation is defined by the index. Thereby, *solar* stands for the solar, *terra* for the terrestrial (emanating from Earth) and *atm* as short term of atmospheric radiation.

Understanding the principle of (radiative) flux and compartment models: The Bare Rock model

In the simplest model, a system of Earth and space (incl. the Sun) is considered. This model is called the Bare Rock model (see Fig. 1, and cf. Archer, 2012; Kraus, 2004). This model ignores the existence of an atmosphere.

The Sun represents the only source of radiation. The solar radiation power is denoted by I_{solar} and defines the radiation power per square meter related to a surface orthogonal to the incoming radiation. On its way from Sun to Earth, the radiation power attenuates of about 50,000 times. The solar radiation power arriving Earth is about $I_{\text{solar}} = 1368 \frac{\text{W}}{\text{m}^2}$ (cf. Deutscher Wetterdienst). This is called the solar constant. Part of the radiation is reflected by the Earth's surface, denoted by $I_{\text{solar, reflected}}$. The reflectivity of the Earth is called albedo and is denoted by α . It holds

$$I_{\text{solar, reflected}} = \alpha \cdot I_{\text{solar}}.$$

Note, that the Earth receives this radiation power with its cross-section. This is a circle of the Earth's radius: πr_{terra}^2 . As the Earth rotates, the radiation power is distributed all over the Earth's surface, which is a sphere, and from which the radiation is then re-emitted. Therefore,

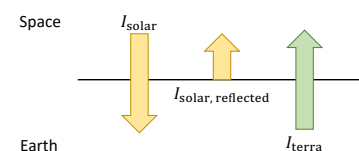


Figure 1 – Illustration of the incoming and outgoing radiation fluxes in the Bare Rock model

the ratio of the cross-sectional area of the Earth to the Earth’s surface area, which is given by $4\pi r_{\text{terra}}^2$, needs to be determined. It is 1 to 4. Thus, on average, only one quarter of the solar radiation power is available per square meter of the Earth’s surface. In opposite direction to the incoming solar radiation flux shown by a yellow arrow in Figure 1 are the reflected solar radiation (also marked in yellow) and the terrestrial radiation, denoted by I_{terra} (green arrow). While the solar radiation has its intensity maximum in the visible light range, the terrestrial radiation has its intensity maximum at bigger wavelengths. The Earth radiates thermal. Therefore, its emission is described by the Stefan-Boltzmann law.

According to the assumptions given above, the absorbed and the emitted radiation fluxes are in an equilibrium. This means that the sum off all fluxes have to add to zero.

Ejemplo 6.1 Possible solution

The radiation equilibrium described above is now translated into the language of mathematics and the Stefan-Boltzmann law for the emitted Earth’s radiation is considered. Here, the incoming radiation is equal to the outgoing radiation.

$$\begin{aligned} \frac{1 - \alpha}{4} \cdot I_{\text{solar}} &= I_{\text{terra}} \\ &= \epsilon_{\text{terra}} \cdot \sigma \cdot T_{\text{terra}}^4 \end{aligned} \tag{2}$$

The Earth’s temperature T_{terra} is now determined by rearranging equation 2 and inserting the following literature values. With the

albedo (reflection percentage)	$\alpha = 0.3,$
solar constant	$I_{\text{solar}} = 1368 \frac{\text{W}}{\text{m}^2},$
emissivity of the Earth	$\epsilon_{\text{terra}} = 1$
and the Stefan-Boltzmann constant	$\sigma = 5.670374419 \cdot 10^{-8} \frac{\text{W}}{\text{m}^2\text{K}}$

it is determined by about $255 \text{ K} \approx -18 \text{ }^\circ\text{C}$. Note, that equation 2 gives the temperature in Kelvin. This temperature unit is common in meteorology. More familiar from everyday life (in Germany) is the temperature unit $^\circ\text{C}$. From now on, only this unit will be used for more intuitive evaluation of the results for the students.

Understanding the use of the atmosphere: The One Layer model

The effects of the atmosphere to the radiation budget are completely neglected in the Bare Rock model. The One Layer model includes the effects of an atmosphere (cf. Archer, 2012). For this model, the students make the following additional model assumptions:

- The atmosphere consists of a single, infinitely-thin layer with glass properties. Solar radiation passes through it unhindered and is not absorbed.
- The albedo acts exclusively and completely at the Earth’s surface. That is, the reflection of solar radiation occurs at

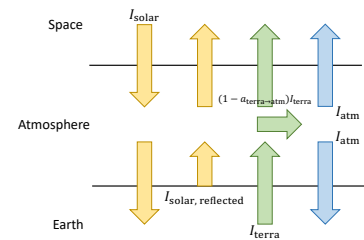


Figure 2 – Illustration of the incoming and outgoing radiation fluxes in the One Layer model

the Earth's surface. Reflections within the atmosphere by, for example, clouds, are not part of this model.

A new compartment has been added by the supplemented atmosphere. The radiation equilibrium still applies to the entire system. In Figure 2, the solar radiation is indicated by the yellow arrows, the terrestrial radiation by the green arrows, and the atmospheric radiation by blue arrows.

The solar constant I_{solar} first hits the atmospheric compartment. It is neither reflected nor absorbed. But it acts at the next compartment: the Earth. Like in the Bare Rock model before, part of the solar radiation is reflected at the Earth's surface and the major part is absorbed. The reflected solar radiation passes freely the atmosphere and vanishes into space. The Earth emits its energy I_{terra} as thermal radiation to the atmosphere, where it is partly absorbed with the absorptivity $0.39 < a_{\text{terra} \rightarrow \text{atm}} < 0.70$. In this process, especially water vapor, carbon dioxide, and ozone, which are known as the greenhouse gases, play a significant role. Radiation in the wavelength ranges of $3.4 - 4.1 \mu\text{m}$ and $8 - 13 \mu\text{m}$ – which is both thermal radiation – can pass the atmosphere almost freely. These intervals are called the atmospheric windows (cf. Armstrong et al., 2021). The atmosphere itself emits thermal radiation I_{atm} toward space and Earth, since in general a body emits heat in all directions equally. This is described analogously to the terrestrial thermal radiation by the Stefan-Boltzmann law.

There are two equivalent approaches for the mathematical modeling of this situation: Either the fluxes are set up at the two boundary layers and equilibrated, or the energy absorbed and emitted of each compartment are set equal. It is up to the students which way they choose. For conciseness, only the first one will be presented here as it already has been carried out independently by students.

Ejemplo 6.2 *Possible solution*

For the fluxes between space and atmosphere applies:

$$I_{\text{atm}} = \frac{1 - \alpha}{4} I_{\text{solar}} - (1 - a_{\text{terra} \rightarrow \text{atm}}) \cdot I_{\text{terra}} \quad (3)$$

Both the Earth radiation absorbed by the atmosphere and the radiation emitted by it are thermal radiations. Therefore, the absorptivity $a_{\text{terra} \rightarrow \text{atm}}$ is equal to the emissivity ϵ_{atm} . It varies between 39 – 70 % depending on the composition of the respective proportions of gases and aerosols.

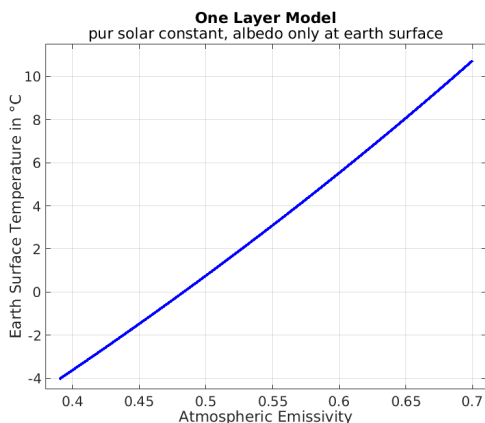
We can model the fluxes between atmosphere and Earth as:

$$I_{\text{terra}} = \frac{1 - \alpha}{4} \cdot I_{\text{solar}} + I_{\text{atm}} \quad (4)$$

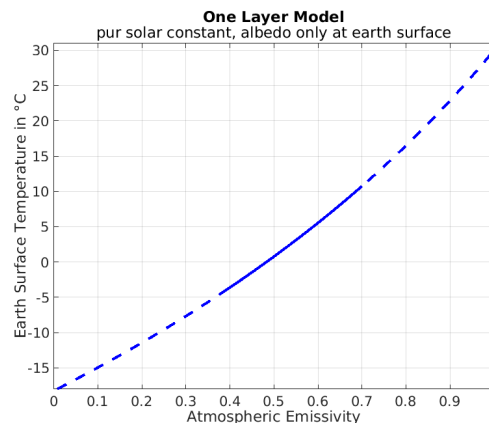
As in the Bare Rock model, the albedo is given as $\alpha = 0.3$ and the solar constant as $I_{\text{solar}} = 1368 \text{ W/m}^2$.

In the equations 3 and 4, the temperatures of the atmosphere and the Earth are unknown parameters. Solving the system of equations yields an Earth's surface temperature between -4.04°C to 10.74°C depending of the atmospheric emissivity (see Fig. 3a).

In Figure 3b, the temperature curve is extended for smaller and larger emissivities of the atmosphere. This shows consistency of the One Layer model with the Bare Rock model at an emissivity of 0 %: The One Layer model correctly yields a temperature of the Earth of about -18°C . This is plausible since an atmospheric emissivity of 0 % means that no (terrestrial) radiation is absorbed and thus reverts to the special case of the Bare Rock model. It can also be seen in Figure 3b that the current real (global) Earth's surface temperature of about 14°C would be reached at an atmospheric emissivity of about 76 %.



(a) Atmospheric emissivity of 39 – 70 % given by literature



(b) Extended atmospheric emissivity (0 – 100 %)

Figure 3 – One Layer model: Surface temperature of Earth depending on the atmospheric emissivity

Furthermore, in both Figures 3, the large variation in Earth temperature becomes immediately apparent: Temperature differences of up to nearly 15 °C occur due to changes in atmospheric composition alone. Accordingly, it appears to have a large impact on the change of the global temperature on Earth.

Overall, the solution of the One Layer model is closer to the real temperature on Earth than in the Bare Rock model. Therefore, the model could be improved.

One Layer model including terrestrial and atmospheric albedo

The students improved their model even further. The albedo is no longer exclusively reflected at the Earth’s surface, but additionally already in the atmosphere: In reality, about $\alpha_{atm} := 22.5\%$ of the incoming solar radiation is reflected in the atmosphere by clouds and aerosols and only $\alpha_{terra} := 8.8\%$ is reflected by the Earth’s surface.

Analogous to the first One Layer model (see sec. 2.3), the radiation flux equations can be set up on the basis of Figure 4. Again, yellow arrows mark solar radiation, green arrows represent terrestrial radiation and blue arrows describe atmospheric radiation.

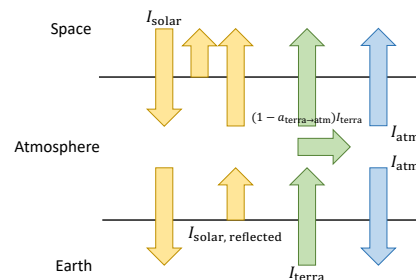


Figure 4 – Illustration of the incoming and outgoing radiation fluxes in the One Layer model with a terrestrial and atmospheric albedo

Ejemplo 6.3 Possible solution

For the fluxes between space and atmosphere applies:

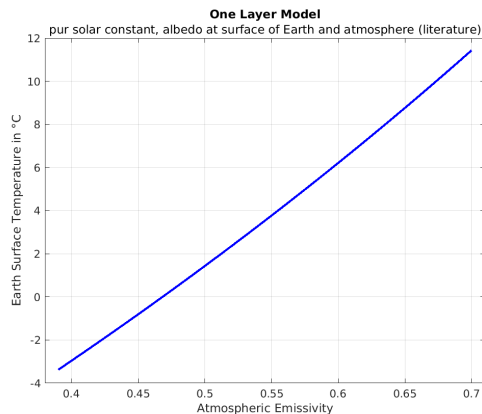
$$I_{atm} = \frac{(1 - \alpha_{atm})(1 - \alpha_{terra})}{4} I_{solar} - (1 - a_{terra \rightarrow atm}) \cdot I_{terra} \tag{5}$$

For the fluxes between atmosphere and Earth applies:

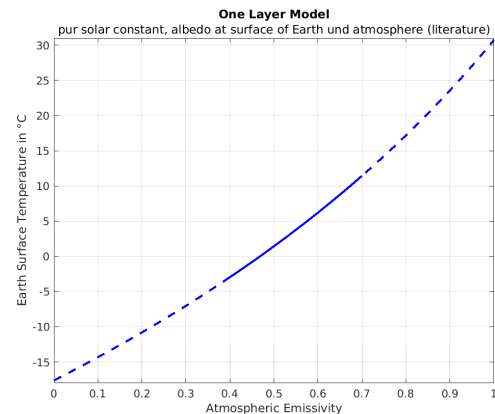
$$I_{terra} = \frac{(1 - \alpha_{atm})(1 - \alpha_{terra})}{4} \cdot I_{solar} + I_{atm} \tag{6}$$

The equations 5 and 6 form the new system of equations to be solved with the two unknown temperatures of the atmosphere and the Earth.

Furthermore, the absorptivity and emissivity of the atmosphere is the same and varies between 39 – 70 %. As a solution, the Earth's surface temperature varies from -3.38°C to 11.44°C (see Fig. 5). According to Figure 5b, the real temperature of the Earth – about 14°C – would be reached at an atmospheric emissivity of about 75 %.



(a) Atmospheric emissivity of 39 – 70 %



(b) Extended atmospheric emissivity (0 – 100 %)

Figure 5 – One Layer model including terrestrial and atmospheric albedo: Surface temperature of Earth depending on the atmospheric emissivity

Including the atmospheric albedo into the model made it more realistic because more atmospheric processes are now included. This approximately provides an upward shift in the solution curve of the simple One Layer model by nearly 0.7°C . As the change in the result is relatively small the students asked themselves how complex the model would have to get in order to reflect the real Earth's situation and if it would be possible with this approach at all. Still, the impact of the atmospheric emissivity is as big as in the previous model.

With this in mind, various possibilities for further action are now given. The students, who have already worked on this task, split into small groups of 2–3 persons and proceeded in parallel work, where each person could focus on her or his own preferences. This is very motivating for each group member. It is important that the supervisor assures that the small groups continue to exchange ideas and question each other's approaches and results. Critical questions are welcome. It is important to create a non-judgmental discussion culture. All group members should be able to explain the group's approach and the current status (in broad outline) at any time.

Following further actions are suggested:

1. The existing One Layer model including the atmospheric albedo can be made more realistic by including absorption of solar radiation by the atmosphere. The absorption coefficient $a_{\text{solar} \rightarrow \text{atm}}$ is now (real) greater than zero. Overall, this should result in a lower Earth's temperature, since less solar radiation is absorbed by the Earth. Nevertheless, it better reflects natural processes. This point is implemented in the following passage.
2. Using the One Layer model, the impacts of natural and anthropogenic radiative forcings can be estimated.
 - a) For this purpose, the phenomenon of solar fluctuations can be added to the model.

These lead to a (natural) variation of the solar constant. This is explained in more detail subsequently.

- b) Furthermore, the melting of snow and ice changes the albedo of the Earth. Bright, well-reflecting surfaces are replaced by darker, poorly reflecting surfaces. Therefore, a model will be established for the melting process of the polar ice shields. This model extension is described later on. Other changes in land-use also cause variations in the terrestrial albedo, but these will not be addressed further here. Overall, these effects are less amenable to assessment. In part, they are man-made, for example, by deforestation. But bushfires, such as those that broke out in Australia and parts of the United States in recent years, also affect the albedo. Often these changes have further effects, which in turn are reflected in atmospheric chemistry. The IPCC reports a “robust evidence that anthropogenic land use change has increased the land surface albedo, which leads to an radiative forcing of $-0.15 \pm 0.1 \text{ W/m}^2$.” (cf. IPCC 2013, 2016).
 - c) One can also study the impact of volcanic eruptions. “Only explosive volcanic eruptions have a significant impact on the global climate system, because then large amounts of particles (aerosols) can be hurled up to the stratosphere.” (cf. Kasang, 2014). The larger amount of particles in the atmosphere increases the atmospheric reflectivity: The Earth cools down. However, this effect lasts only for a few years, until the aerosols return to the troposphere, from where they are washed out by rain. For analysis, therefore, it is possible to look at the last 120 years if and when such explosive volcanic eruptions occurred and whether their effects are visible in time series of the global Earth’s surface temperature.
 - d) To examine the impact of humans on the climate system, (anthropogenic) carbon dioxide emissions can be studied. Here, literature research is appropriate. Some notes are found at the end of this section.
3. The last point to be mentioned is the extension of the One Layer model by further atmospheric compartments, each of which has different properties with respect to absorption, reflection, and emission of radiation. This was not implemented by the learning group due to the limit of time. It is up to the reader to pursue this independently.

One Layer model including absorption of solar radiation in the atmosphere

As an extension to the previous model, the atmosphere is no longer transparent for solar radiation. Gases of the atmosphere absorb 20 % of solar radiation. Another 5 % is absorbed by clouds. These values are summarized in the absorption coefficient $a_{\text{solar} \rightarrow \text{atm}} = 25 \%$. The reader can find a detailed explanation in S. Burger (2018). Again, the goal is to establish the radiative equilibrium. Figure 6 can assist in this.

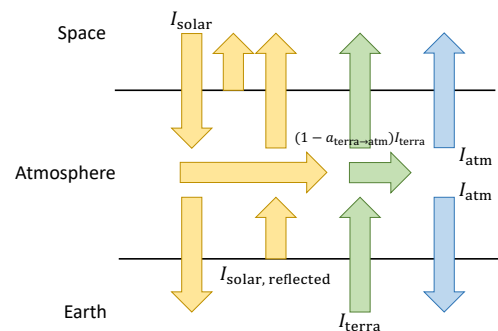


Figure 6 – Illustration of the incoming and outgoing radiation fluxes in the One Layer model with a terrestrial and atmospheric albedo including the absorption of solar radiation in the atmosphere

Ejemplo 6.4 Possible solution

For the radiation fluxes between space and atmosphere applies:

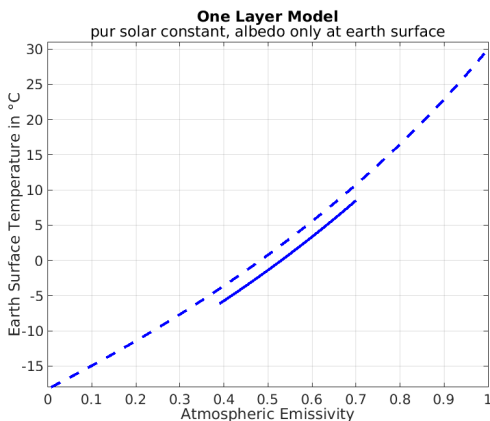
$$\frac{1 - \alpha_{\text{atm}}}{4} I_{\text{solar}} = \frac{(1 - \alpha_{\text{atm}})\alpha_{\text{terra}}(1 - a_{\text{solar} \rightarrow \text{atm}})^2}{4} I_{\text{solar}} + (1 - a_{\text{terra} \rightarrow \text{atm}})I_{\text{terra}} + I_{\text{atm}} \quad (7)$$

For the radiation fluxes between atmosphere and Earth applies:

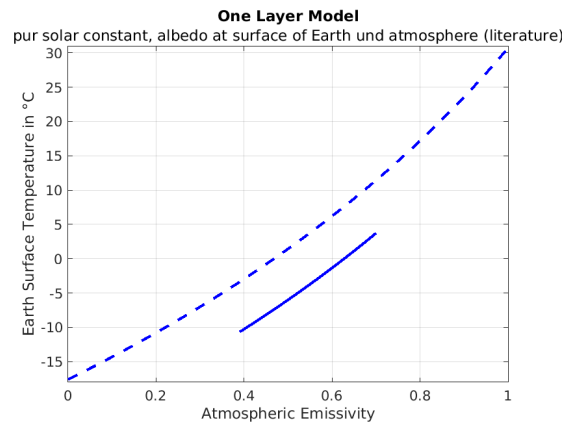
$$\frac{(1 - \alpha_{\text{atm}})(1 - a_{\text{solar} \rightarrow \text{atm}})}{4} I_{\text{solar}} + I_{\text{atm}} = \frac{(1 - \alpha_{\text{atm}})\alpha_{\text{terra}}(1 - a_{\text{solar} \rightarrow \text{atm}})}{4} I_{\text{solar}} + I_{\text{terra}} \quad (8)$$

Solving this system of equations yields the following results:

- a) If only the terrestrial albedo is considered as $\alpha = 0.3$, the temperature of the Earth as a function of the atmospheric emissivity for terrestrial radiation ranges between -6.17°C and 8.50°C . The blue continuous line in the Figure 7a shows exactly this result. The dashed line shows the prior solution (without the atmospheric emission of solar radiation, (see passage “One layer model including terrestrial and atmospheric albedo”).
- b) If, in addition to the terrestrial albedo $\alpha_{\text{terra}} = 0.088$, the reflection from the atmosphere with $\alpha_{\text{atm}} = 0.225$ is also taken into account, the temperature of the Earth as a function of the atmospheric emissivity for terrestrial radiation ranges between -10.68°C and 3.20°C . Again, the blue continuous line in Figure 7b shows exactly this result. The dashed line shows again the prior solution.



(a) One Layer model including only terrestrial albedo



(b) One Layer model including terrestrial and atmospheric albedo

Figure 7 – Surface temperature of Earth depending on the atmospheric emissivity. Dashed: Solution without a solar radiation absorbing atmosphere, solid: including a solar radiation absorbing atmosphere

Both results agree with our previous conjecture that the introduction of an absorption coefficient for solar radiation in the atmosphere leads to a lower temperature. The difference especially in case b) is very large with a difference of more than 5°C . Accordingly, this model also shows a large influence by the atmosphere on the Earth’s temperature.

Effect of variations of the solar constant in the One Layer model

The solar constant I_{solar} changes over time. First of all, solar activity has increased by about 30 – 35 % since the beginning of the Earth. This change of radiation intensity is not examined in detail here, because this development extends over such a long period of time (4.5 billion years) that it cannot cause a sudden temperature increase in the last 120 years. In addition, solar radiation is subject to variations due to changes in the Earth’s orbital parameters. These variations also relate to long-term time scales of tens of thousands of years (cf. D. Kasang, 2019). Therefore, this aspect is also neglected in this analysis. Instead, cyclically occurring fluctuations in solar activity are considered. Beside the often known Schwabe cycle, which repeats about every 11 years, there is also the Gleissberg cycle with a periodicity of about 80 years. The Schwabe cycle has an amplitude of about 0.1 % of the solar constant, the Gleissberg cycle of about 0.24 – 0.3 % (cf. Cubasch, U. (2002)).

Ejemplo 6.5 Possible solution

To investigate the effect of the two named sun cycles, the solar constant needs to be modified according to the above data. For the analysis only the maximum change in the solar radiation is considered which is equal to the amplitude of the cycles. For the Schwabe cycle, the solar constant is multiplied by 1.001 for an activity maximum and by 0.999 for an activity minimum. Analogously, the impact of the Gleissberg cycle is studied.

Note: In the following calculations the One Layer model with both atmospheric and terrestrial albedo was used. The values of the albedos correspond to their literature values of $\alpha_{\text{atm}} = 22.5\%$ and $\alpha_{\text{terra}} = 8.8\%$. The emissivity of the atmosphere for terrestrial radiation is still kept variable as in the previous studies.

The resulting fluctuations in the solar radiation are illustrated in Figure 8.

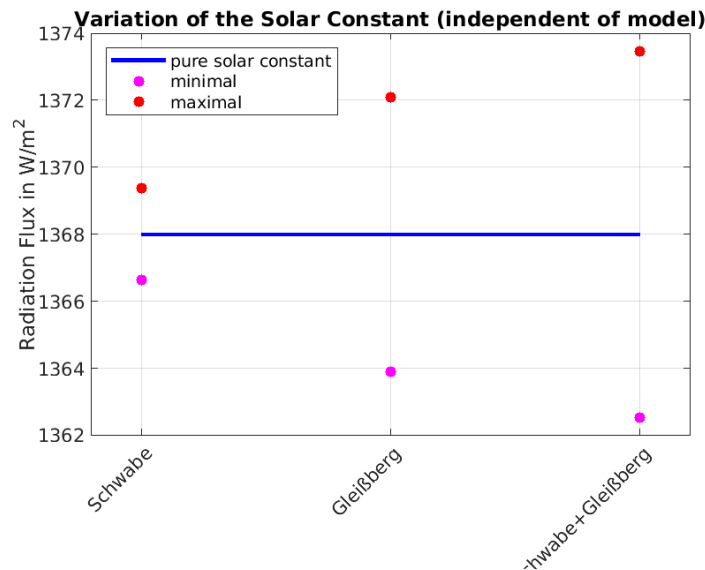
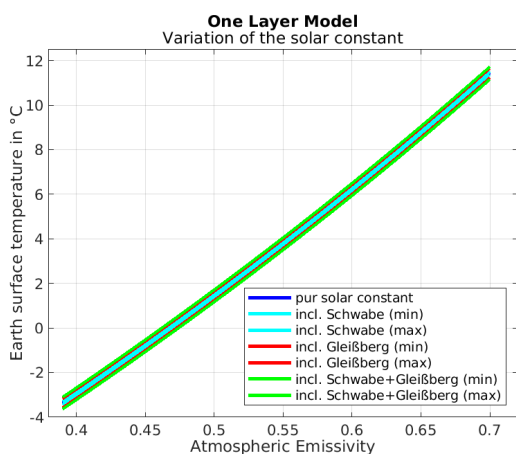


Figure 8 – Variations in the solar constant due to the Schwabe cycle, Gleissberg cycle and both cycles

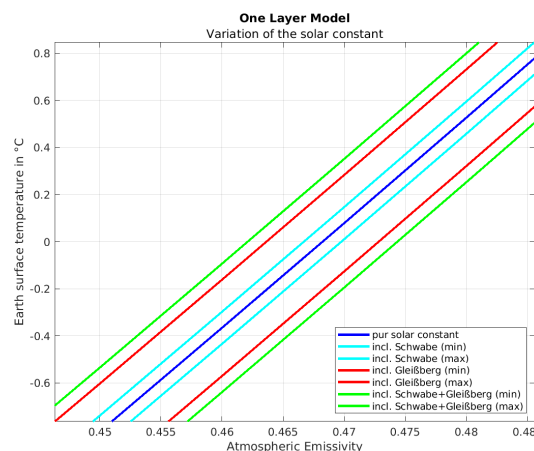
pur solar constant:	$I_{\text{solar}} = 1368 \frac{W}{m^2}$	
incl. Schwabe cycle:	$I_{\text{solar, min}} = I_{\text{solar}} \cdot 0.999$	$\Delta T_{\text{terra}} \approx -0.06 \text{ } ^\circ\text{C}$
	$I_{\text{solar, max}} = I_{\text{solar}} \cdot 1.001$	$\Delta T_{\text{terra}} \approx 0.06 \text{ } ^\circ\text{C}$
incl. Gleissberg cycle:	$I_{\text{solar, min}} = I_{\text{solar}} \cdot 0.997$	$\Delta T_{\text{terra}} \approx -0.19 \text{ } ^\circ\text{C}$
	$I_{\text{solar, max}} = I_{\text{solar}} \cdot 1.003$	$\Delta T_{\text{terra}} \approx 0.19 \text{ } ^\circ\text{C}$
incl. both cycles:	$I_{\text{solar, min}} = I_{\text{solar}} \cdot 0.996$	$\Delta T_{\text{terra}} \approx -0.25 \text{ } ^\circ\text{C}$
	$I_{\text{solar, max}} = I_{\text{solar}} \cdot 1.004$	$\Delta T_{\text{terra}} \approx 0.25 \text{ } ^\circ\text{C}$

If only the Schwabe cycle were active, the surface temperature of the Earth would change periodically by a maximum of $2 \cdot 0.06 \text{ } ^\circ\text{C}$. The Gleissberg cycle considered alone leads to a periodic temperature change of maximal $2 \cdot 0.19 \text{ } ^\circ\text{C}$. Even both cycles superimposed lead to a maximal temperature increase of $2 \cdot 0.25 \text{ } ^\circ\text{C}$. These results are illustrated in Figure 9a. To distinguish the graphs Figure 9b shows a cutout of Figure 9a. In it, the temperature differences between the individual models can be read more easily.

These calculations show the temperature fluctuations due to solar cycles are not big enough to explain a temperature increase of about $1 \text{ } ^\circ\text{C}$ during the last century. In addition, if solar cycles would cause such large temperature fluctuations, they would have to show up regularly in the temperature data. The IPCC report of 2001 confirms that solar radiation variability accounts for only about one-third of the radiative forcing caused by the increase in anthropogenic greenhouse gases from 1880 to the present (cf. IPCC, 2001).



(a) Atmospheric emissivity of 39 – 70 %



(b) Cutout of Figure 9a

Figure 9 – Effects on the surface temperature of the Earth due to different solar cycles

Note: The Gleissberg cycle was not considered by the students. It has been included here for completion. Instead, variations in the Earth’s orbital parameters were first investigated. As indicated here in the introductory text of this passage, the students neglected this forcing due to its long impact period.

Effect of melting ice caps on the terrestrial albedo in the One Layer model

The terrestrial albedo is determined by the Earth’s surface properties. Light surfaces have a greater reflectivity than darker ones. Snow reflects sunlight very well, whereas asphalt absorbs

it strongly. If the Earth were completely covered with ice, it would have an albedo of about 84 %. Most sunlight would be reflected. If the Earth were completely covered with forest, the albedo would be about 14 %. Most sunlight would be absorbed. Thus, it is clear that changes in the Earth's surface, such as ice cover, forest cover, desert areas, or urban areas, will result in a change of its albedo and as a consequence its surface temperature.

The melting process of polar ice surfaces will be modeled. The modeled terrestrial albedo is then integrated into the radiation balance model. In a first step, the One Layer model with only the terrestrial albedo is considered. In the second step, the reflectivity of the atmosphere is added again as given in literature.

Figure 10 shows the terrestrial albedo as a function of latitude. The albedo shown is averaged over a year to exclude seasonal variations. The graph shows that oceans and tropical regions have a low albedo, whereas polar regions, which are covered with snow and ice most of the year, have a much higher albedo. This Figure serves as the basis for the own albedo model.

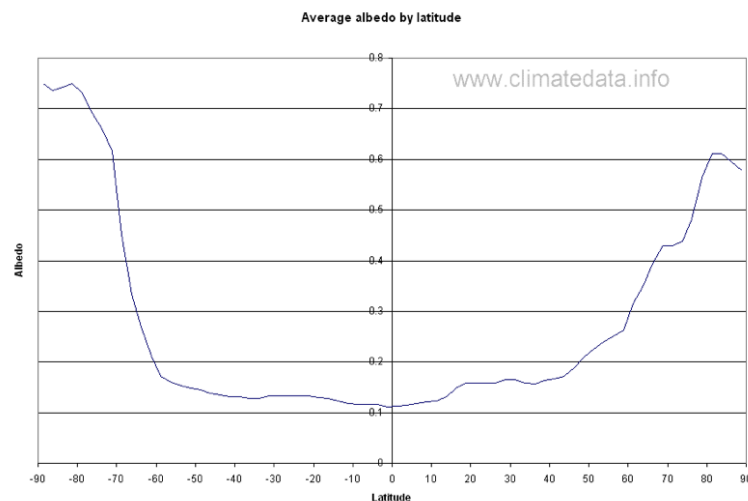


Figure 10 – Average albedo as a function of latitude, Source: <http://www.climatedata.info/forcing/albedo/> (Cited March 07, 2021)

The latitudes are discretized into blocks over 10° . Each block has a uniform albedo that is given by the mean value estimated from the graph. It is assumed here that the polar regions are approximately bounded by the Northern and Southern Arctic Circles. The model reflecting snow and ice melting is therefore limited solely to the regions north of 60°N and south of 60°S , respectively.

The present state of the ice and snow cover on Earth is set as the reference state for the melting process. Furthermore, the melting process is described by a linear progression. Ice and snow are replaced by water, which has a significantly lower albedo. For the albedo of water, an average value of 11 % is assumed.

The inclination of the Earth axis is not considered. In a first approximation – model m1 – also the curvature of the Earth is neglected. It is assumed that all Earth's surfaces are oriented perpendicular to the Sun. As a next step to improve the model the cosine effect is introduced, noting that the more northern or southern Earth surfaces make an effectively smaller contribution to the albedo. The solar radiation on a surface is the highest, if the surface faces the Sun directly, or in other words, if the surface is orthogonal to the incoming rays of the Sun. Since the earth is a sphere, this is the case only at the equator. The further one approaches the poles coming from the equator, the stronger is the deviation from the normal (see Fig. 11). The solar radiation is reduced in proportion to the cosine of the angle. This model is named m2 to make

referencing easier in the following discussion.

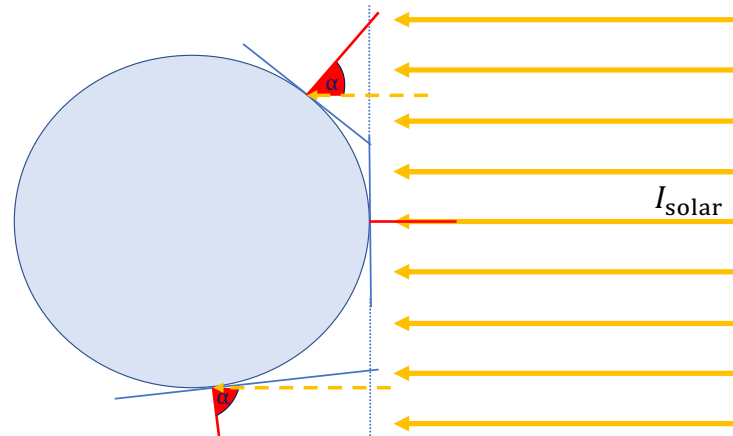


Figure 11 – Reduction of the incoming solar radiation due to the cosine effect. In yellow: incoming solar radiation; on the left: the Earth; blue lines: tangent on the Earth’s surface; in red: normals on the Earth’s surface and the angle to the incoming solar radiation

The process of deglaciation is shown in Figure 12 for both models. Both cases reflect a decrease in the albedo. The decrease is much larger in the model m1 than in model m2. This was to be expected: The polar surfaces are responsible for the change and have a relatively strong impact due to the model. The albedo in model m2 decreases from about 9.76 % to 8.07 %.

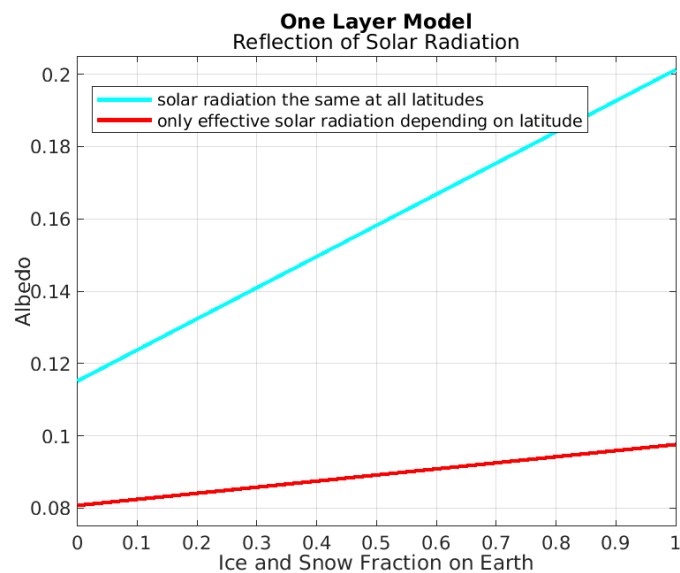
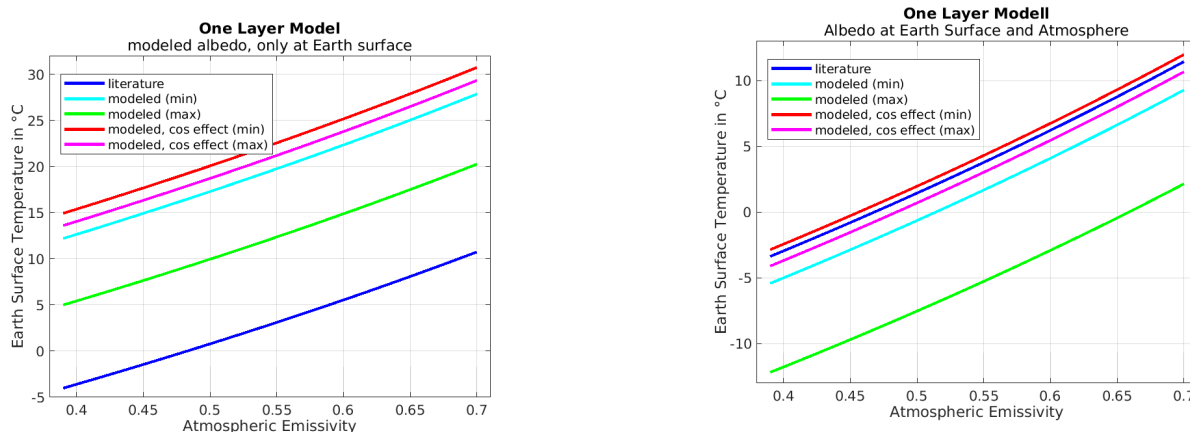


Figure 12 – The process of deglaciation. In cyan: model m1; in red: model m2.

In the two Figures 13 the effect on the temperature as a function of atmospheric emissivity is shown. The maximum and minimum albedo of each of the two models m1 and m2 were used. The maximum albedo corresponds to the present state of the ice surfaces (green and pink graphs), whereas the minimal albedo corresponds to completely melted polar surfaces (cyan and red graphs). The blue graph refers to the terrestrial albedo given by literature $\alpha_{\text{terra}} = 30\%$. Figure 13a shows the effects of polar ice melting on the Earth’s temperature without any atmospheric

reflection. The melting process in model m1 would lead at all atmospheric emissivities to a temperature increase of about 7°C . In contrast, the temperature in model m2 would rise of about 1.3°C if all ice was melted. In Figure 13b the atmospheric albedo is added again. Still, model m1 shows a great increase in temperature for the melting process. There the modeled albedo with consideration of the cosine effect leads approximately to an albedo of the current real situation.



(a) Only terrestrial albedos

(b) Terrestrial (modeled) and atmospherical (given by literature) albedos

Figure 13 – Effect on the Earth’s surface temperature by varying the terrestrial albedo

Human impact

As already mentioned before human being has an impact on the terrestrial albedo. This change of the land surface albedo goes along with a radiative forcing of $-0.15 \pm 0.1 \text{ W/m}^2$, which corresponds to $0.01 \pm 0.007 \%$ of the solar constant.

“Mankind is increasingly influencing the climate and temperature on Earth by using fossil fuels, cutting down rain forests, and raising livestock. Thus, the amount of greenhouse gases in the atmosphere increases enormously, which intensifies the greenhouse effect and global warming.” (cf. European Commission).

A change in the chemical composition of the atmosphere primarily affects its emissivity. All presented models and their extensions show large temperature differences of nearly 15°C caused by varying the atmospheric emissivity.

Referring to literature, mankind has increased the CO_2 content in the air by about 40–48 % since 1750. About half of this is absorbed by the oceans, plants, and soils. The other half enters the atmosphere (cf. European Commission, Kasang, 2020; Spiegel Wissenschaft, 2017). The importance of carbon dioxide becomes clearer from a physical point of view: Because of its chemical structure, it primarily absorbs and emits thermal radiation. The more carbon dioxide is present in the atmosphere, the more thermal radiation is absorbed. This results in an increase in the atmospheric emissivity and thereby, in an increase of the Earth’s surface temperature.

The IPCC (2013) states: “It is unequivocal that anthropogenic increases in the well-mixed greenhouse gases have substantially enhanced the greenhouse effect, and the resulting forcing continues to increase. The total anthropogenic effective radiative forcing over the Industrial Era⁴ is $2.3 \pm 1.0 \text{ W/m}^2$.” This radiative forcing would correspond to an increase of the solar constant of $0.17 \pm 0.07 \%$.

⁴since 1750

3. Conclusion and outlook

The outline of a workshop presented in this article allows students to explore the highly relevant subject of climate change using scientific tools. With simple models, the Earth's energy budget can be well represented and its surface temperature can be calculated.

In a first step, a very simplified model – the so-called Bare Rock model – was developed. This model is mainly used to get into the topic and to understand the principle of (radiative) flux and compartment models. Subsequently, the model has been extended by the atmosphere. Such a model, which considers one atmospheric layer (compartment), is called the One Layer model. The comparison between the Bare Rock model and the One Layer model provides an understanding of the atmosphere and its general effects: If there was no atmosphere around the Earth, it would have a surface temperature of about -18°C . Human life is hardly conceivable at these temperatures. Extensions to the model representing natural and anthropogenic perturbations have been incorporated: First, the atmospheric albedo was added to the One Layer model. In another modeling step, the absorption of solar radiation in the atmosphere was added. Both extensions are a better representation of the real situation.

The models allowed investigations of different impacts on the Earth's surface temperature. These included variations of the solar constant due to natural processes which lead to a maximum temperature increase of 0.25°C . Subsequently, the melting of large snow and ice areas in polar regions was modeled and integrated into the compartment models. A realistic model, which considers the curvature of the Earth by using the cosine effect, results in a temperature increase of about 1.3°C . Changes in the albedo due to anthropogenic land use change were neglected. Still, the influence of volcanic eruptions needs to be quantified. The influence of mankind was discussed on the basis of impacts due to changes of the atmospheric emissivity. Thus, even with a very simple model, the effects of natural and anthropogenic impacts can be compared quantitatively. Furthermore, insights into a mathematically challenging topic was provided.

The implementation of the research topic in the context of an online modeling week worked very well. The support and the exchange within the group took place via the tool Mattermost⁵ combined with jitsi⁶. Thus, discussions could be held in the whole team, but also outsourced in breakout rooms. In addition, the chat function could be used and data, such as images or diagrams, could be exchanged. Ideas were actively collected on an online whiteboard⁷, model extensions were schematically sketched and mathematical descriptions were developed. These tools were available to every team member at any time, so that everyone could keep track of things relatively easily. The use of screensharing was mainly used for programming, since MATLAB online⁸ does not allow simultaneous editing of the code of several users. This circumstance encouraged “public” programming as a team in particular, so that each team member could find their way around the code. All in all, the supervisor only had to provide a few aids. These were mainly in the area of motivational aids and rarely in the area of indirect general-strategic aids (cf. Zech,1998) and in the organization of the exchange (e.g. convening discussion groups, stimulating task sharing, reminding about the preparation of a presentation and a report). Students were particularly frustrated by the wide definitional range of atmospheric emissivity, which could not be constrained by any literature. Even though this makes the predominant influence on the Earth's temperature very clear, it led to disillusionment because

⁵For further information: <https://mattermost.com/>

⁶For further information: <https://meet.jit.si/>

⁷for further information: <https://miro.com/>

⁸For further information: <https://de.mathworks.com/products/matlab-online.html>




the students wished for a better approximation to reality (which, however, is not possible with this model). Ideas about the chemical balance of the atmosphere and the absorption abilities of individual chemical elements to model the emissivity itself⁹, could not be mathematized in the short time available and thus could not be included in the existing model. Therefore, a possibility to further improve the models would be given by extending the compartment model to several atmospheric layers. In such a model, the processes of emission and absorption could be addressed in more detail, and the influence of clouds could be better represented.

Although climate driving forcings could be quantified, predictions of climate remain very complex and difficult. They do not only consider natural influences, but also the behavior of human beings in the future (e.g. emissions, resettlement, drying of land areas, ...).

To get started with the topic of climate change, it is also possible to address the question of whether it exists at all and whether it can be significantly measured. This issue has already been addressed in a one-day modeling workshop: The examination of a temperature time series from 1900 to 2018 shows with a very high significance that the global average temperature is increasing (cf. Hattebuhr et al., 2018); Hattebuhr and Frank, 2019; Hattebuhr, accepted).

In conclusion, we think a workshop as presented in this contribution offers an excellent opportunity for students to engage in a highly relevant sociological and scientific topic. Investigation of the energy budget of the Earth allows to study natural and anthropogenic contributions to the surface temperature. As a consequence, this topic does not only introduce the students to the concept of mathematical modeling. They also learn about physical and chemical processes in the atmosphere, but most importantly come to a position where they are able to contribute to a lively public discussion on scientifically well-founded grounds. From a pedagogical point of view, working on this topic brings a lot of benefits for the students which is why we are convinced of the high value of this workshop.

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⁹The impetus for this idea came from the supervisor.

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