MODELLING IN SCIENCE EDUCATION AND LEARNING Volume 15(1), 2022 DOI: 10.4995/msel.2022.16550. Instituto Universitario de Matemática Pura y Aplicada Universitat Politècnica de València

Concept and structure of the Hamburg Modeling Days Concepto y estructura de las Jornadas de Modelización de Hamburgo

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Abstract

In Germany, there is quite a tradition of modeling projects, carried out by different universities all over Germany. Since 2001, different modeling projects has been conducted by the working group on Didactics of Mathematics of Department of Education of the University of Hamburg. In this paper, the current project, the Hamburg Modeling Days, is described in detail. One goal of this project is to enable students to gain experience in working on modeling problems and to acquire modeling competencies. Furthermore, prospective teachers are supported in their professionalization for teaching mathematical modeling, especially with respect to the supervision of students working on modeling problems. Finally, practicing teachers are in the implementation of modeling problems in mathematics classes.

En Alemania, existe una gran tradición de proyectos de modelización, llevados a cabo por diferentes universidades de todo el país. Desde 2001, el grupo de trabajo de Didáctica de las Matemáticas del Departamento de Educación de la Universidad de Hamburgo ha llevado a cabo diferentes proyectos de modelización. En este artículo se describe en detalle el proyecto actual, las Jornadas de Modelización de Hamburgo. Uno de los objetivos de este proyecto es posibilitar que los alumnos adquieran experiencia en resolver problemas de modelización y que adquieran competencias de modelización. Además, se apoya a los futuros profesores en su profesionalización de la enseñanza de la modelización matemática, especialmente en lo que respecta a la supervisión de los alumnos que tratan de resolver problemas de este tipo. Por último, los profesores de secundaria están implementando los problemas de modelización en las clases de matemáticas.

Keywords: modeling projects, modeling competencies, teaching competencies

Palabras clave: proyectos de modelización, competencias de modelización, competencias docentes

1. Introduction

The teaching of modeling competencies has been a mandatory part of the mathematics curriculum of all school levels in Germany for more than 10 years. The problems worked on in class are usually problems that can be worked on in a few school hours, often for 90 minutes. However, more complex problems that take much longer to solve are also addressed in schools in Germany. This is often done in the context of modeling projects, which have a long tradition and are often conducted by universities. The projects pursue different goals and take place under different circumstances.

In Hamburg, different modeling projects have been developed and carried out in the last 20 years. They differ in particular in the target group as well as the duration. What they have in common, however, is that both prospective teachers and practicing teachers are used to supervise the students in working through the modeling problems. This requires appropriate preparation. The core of this paper is therefore the presentation of the structure of the current modeling project in Hamburg, together with the preparatory activities for both prospective and practicing teachers.

2. Modelling projects in Germany

The implementation of university-supported modeling projects with students has a long tradition in Germany¹. As early as the late 1980s, modeling weeks for university students were organized at the University of Kaiserslautern by Helmut Neunzert's working group, and since the early 1990s also for older students at schools. The idea of modeling projects lasting several days has spread throughout Germany, with the framework conditions adapting to the respective locations and target groups. They differ, for example, in the duration (between two and five days), the place of implementation (in the school facilities or outside the school), the participants (individual interested students or whole classes or grades working on the modeling problems in known groups or with unknown persons) and the form of supervision (prospective teachers, teachers or university staff). Common features are that the modeling problems are usually given, but the participants can choose between several problems, that the work is done in small groups, that the work on the problems is organized as independently as possible by the small groups, and that, unlike in regular classes, the time available often allows for a revision of the models created or for several models to be compared (Bracke, Göttlich & Götz, 2013; Kaiser, Bracke, Göttlich & Kaland, 2013).

The goal of these modeling projects is different: While some projects aim to give an impression in which areas of daily life mathematics can and must be used as a tool, others aim to promote competencies for independent processing of modeling problems, where the necessary mathematical procedures may still have to be worked out or known procedures may have to be combined in a new way. In both approaches, students should be able to experience mathematics in a different way than in the classroom "and their mathematical world views or mathematical beliefs are broadened" (Kaiser et al., 2013).

A further goal of the thematization of modeling problems in the context of the mathematical education of students is to become a responsible citizen, i.e. students should experience that they themselves are able to question numbers and models propagated in the media or brought to them by advisors by working on the corresponding problems themselves with the methods available to them Kaiser (2017). This has an impact on the modeling problems that the students

 $^{^{1}}$ Some of them were even awarded in the past, for example https://www.profale.uni-hamburg.de/projekt/aktuelles/2017-06-12-hamburger-bildungspreis-stender.html and https://lehrportal.uni-kassel.de/moodle/course/view.php?id=12).

are asked to work on during the project. These often originate from industry or society and, if possible, have a relevance that is directly apparent to the students (examples can be found in Vorhölter & Haier (in this issue) or, for example, Kaiser et al. (2013). They are complex and authentic, have minimal, if any, simplifications, and are workable in a variety of ways, (Kaiser et al., 2013). In order for students to experience their competence in working on these problems, support from supervisors must be minimal and adaptive (e.g., Vorhölter, Kaiser & Borromeo Ferri, 2014; Stender & Kaiser, 2015). Therefore, the approach of these projects can be described as a holistic approach (Blomhoj & Jensen, 2003).

At the University of Hamburg, longer-term modeling projects with students were conducted for the first time in 2001. The Department of Mathematics and the working group of the Didactics of Mathematics of the Department of Education jointly conducted annual seminars for prospective mathematics teachers in cooperation for seven years, in the context of which the prospective mathematics teachers were to support students (16-18 years old) in working on a complex modeling problem. The modeling activities took place within regular mathematics classes and lasted one school semester. Due to organizational constraints, this concept was changed in 2009 in that the modeling activities now took place in a compressed form within one week and on the campus of the university. Until 2013, this form of modeling week was offered twice a year, with approximately 400 students participating annually. Here, it was not specified whether the schools participated with entire courses or whether only interested students signed up for the modeling weeks. The supervision of the students was initially carried out by prospective teachers, later in cooperation with staff of the Department of Mathematics, and in a further step together with teachers. The integration of the training of prospective mathematics teachers into the modeling projects was therefore an essential part of these projects in Hamburg from the beginning. The aim of these projects was not only to train the students, but also to broaden the image of mathematics on the part of the students: By working on authentic problems, they were to question the widespread attitude that in mathematics there are only right and wrong solutions, and to learn that decisions in society, politics and science are always based on models (that are also based on subjective assumptions and the use of different values). and must therefore be questioned and, if necessary, changed. (Kaiser & Schwarz, 2006, 2010; Kaiser et al., 2013).

With the aim of being able to convey this important knowledge to younger students, the first modeling days were held in 2010 at a large Hamburg high school for students in grade 9 (14-15) years old). The modeling days lasted three days and were held on the school's facilities. The students could choose between three different modeling problems and were divided into small groups. The small groups were supervised by a tandem of prospective teachers who had participated in a didactic seminar. In this seminar, the focus was on didactical content, but the modeling problems to be supervised were also worked on by the prospective teachers themselves. This was intended to provide the students with an initial protected field of practice in which they could test and develop their skills with the supervision of open complex problems - skills that are definitely useful (if not necessary for good teaching) for regular mathematics teaching in Germany. The participation of the entire grade was intended to ensure that the students' modeling experiences could also be built upon in further mathematics instruction a circumstance that is not guaranteed if only individual students participate in the modeling projects. By increasing the number of prospective teachers participating in the seminar to approximately 60 students (divided into two seminars), the modeling days were able to be held at a total of five schools in the years 2013 and 2014. In this context, training sessions were also held for the mathematics teachers of the participating classes, who were then also involved in supervising the students (Stender, 2016).

The different modeling projects in Hamburg, the associated seminars, the teacher training as

well as the structure of the modeling projects themselves have been regularly evaluated since the first implementation (Kaiser & Schwarz, 2010; Stender & Kaiser, 2015; Stender, 2016; Vorhölter 2018) and further developed according to the external framework conditions. An overview of the different modeling projects in Hamburg can be found in Table 1.

	Long lasting modeling project	Modeling weeks	Modeling days	Hamburg Modeling Days ²
Time period	2001-2008	2009-2013	2010-2014	2015-now
Target group	whole classes	Interested students and whole classes	whole grades	whole grades
Duration	1 double lesson per week for one semester	1 week	3 days	2 days
Numberofparticipatingstudentsperyear		400	150	300
Supervisor	Prospective teachers with help of practicing teachers	Prospective teachers, staff of the Department of Mathematics, practicing teachers	prospective teachers	prospective and practicing teachers

Table 1 – Overview of different modeling projects in Hamburg.

3. Hamburg Modeling days

The project Hamburg Modeling Days is aimed at schools that would like to participate in this project with their entire grade 9 or 10. The project was carried out for the first time in 2010, continuously developed further and has existed in its current form since 2015. The special feature of this project is that it is aimed at entire grades and not only at interested students, and that both the training of prospective mathematics teachers and the in-service training of practicing teachers are integrated³.

3.1. Procedure and structure of the project

The Hamburg Modeling Days take place every year in February, whereby the exact date is agreed upon with the participating schools and varies from school to school. During this time, students in the 9th or 10th grade have the opportunity to work on one of three modeling problems offered by the university's coordinator. Care is taken to ensure that the problems have different content and are different in terms of their subject context. The problems are briefly

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 $^{^{2}}$ Since the development of the modeling days was continuous, it is not possible to give an exact year of the beginning for the current form; the year mentioned refers to the the year in which the last major changes were implemented.

³Several groups of people are involved in the Hamburg Modeling Days: Students in grades 9 and 10 work on modeling problems and are supervised by student teachers, hereafter referred to as prospective teachers, and the students' mathematics teachers, hereafter referred to as practicing teachers.

presented together with the respective challenges before the modeling days start. In consultation with the schools, the work takes place either at the school itself or at the university's campus. It is important that there is enough space and that the students can work both days in the same room. Furthermore, there must be adequate technical equipment and access to the internet.

The students have two school days to work on the modeling problem, which is why they are



Figure 1 – Organization of the Hamburg Modeling Days

released from the rest of the lessons during this time. Basically, we try to give the students as much freedom as possible, but still support them especially in planning the working process, if necessary. After all, the goal of this project is not to achieve a specific result, but to acquire competencies. In addition to the acquisition of modeling competencies, this includes in particular competencies in independent work as well as competencies for effective cooperation in the group. Communication skills are also trained, since at the end of the project the students are expected to present their thoughts and results to the grade level and, if possible, to other members of the school community such as peers, parents and teachers.

Preparation for the modeling days, which take place in February, already starts at the beginning of the school year in late summer of the previous year (cf. Figure 1). At the beginning of the school year, the schools that participated in the modeling days in the previous year are asked whether they would like to participate again, how many students are expected and how many teachers will participate. Furthermore, teachers are asked whether they would like to have their own in-service training. In addition, consideration is given to which modeling problems will be offered this year. Among other things, attention is paid to the fact that

- if possible, the same problems are not used in consecutive years (unless as part of a revision of the newly designed problem),
- the contents and activities necessary to work on the problem differ as much as possible (for example, not all problems should require the use of a particular software), and
- the contexts differ and, if possible, have a regional or actual reference.

Attempts are made to have the choice of problems, as well as the days on which the modeling days will be held at each school, set at the beginning of the university semester in October. This allows prospective teachers to best adjust to the content and times of the modeling days. During the semester, the prospective teachers are prepared to supervise the students during the modeling days. Whereby care is taken, of course, to ensure that the content taught is not only aimed at the activity during the modeling days but contributes overall to the professionalization of the prospective teachers (cf. chapter 3.2). At the same time, the teachers are also prepared for the modeling days. Depending on the level of knowledge of the participating teachers (it is always the mathematics teachers teaching in the respective grade), either explicit further training with theoretical and practical content is offered (even for subgroups), or the teachers are simply integrated into the seminar work (cf. chapter 3.3). The students are also introduced to

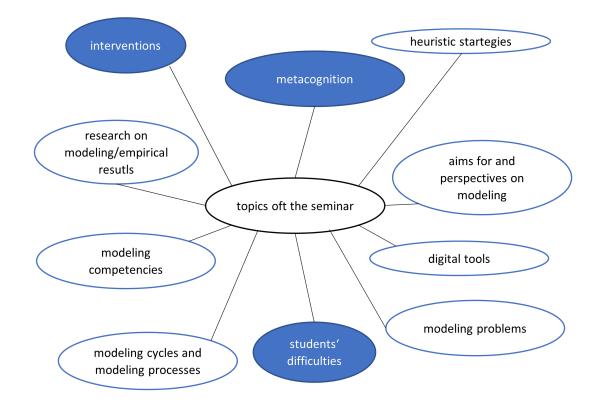


Figure 2 – Topics of the seminar. Topics were videos were used are marked by a blue background

modeling during this time, which in turn is used in the seminar to give the prospective teachers an impression of what they can expect from the students on the modeling days. For example, students are introduced into modeling cycles, and work on at least one modeling problem. If possible, written solutions will be followed up with feedback from prospective teachers. The described organizational conditions, in particular the number of participating students and the available space, have led to the fact that three high schools from Hamburg with a total of more than 300 students as well as approx.twelve mathematics teachers continuously participate in this project and are supported by approx.30 prospective teachers during the modeling days.

3.2. The preparatory seminar for the prospective teachers

The prospective teachers participating in the modeling seminar are in the first semester of their master studies. They intend to work as teachers in different kinds of schools (primary school, grammar school, comprehensive school). The modeling seminar is one out of three seminars, between which the preservice teachers can choose. The seminar lasts one semester, and there are 14 session of 135 minutes. At the beginning of the seminar, the prospective teachers usually only have basic knowledge of mathematical modeling. At the end of the seminar, prospective teachers get the opportunity to apply the knowledge they have gained during the seminar to support students during the modeling days and reflect on their experience in the seminar concluding exam. The aim of the seminar is to provide the prospective teachers with knowledge as well as skills for implementing modeling activities in their future lessons. According to Borromeo Ferri and Blum (2010), four dimensions of teacher competencies can be differentiated concerning modeling. The content of the seminar is structured along these four dimensions as following:

The first dimension (theoretical dimension) contains knowledge about mathematical modeling

as such. This includes knowledge about different modeling cycles as well as aims and perspectives of modeling (e.g., Kaiser, 2017), modeling competencies (e.g., Kaiser, 2007; Maaß, 2006), heuristics (e.g., Stender, 2018) and metacognitive strategies and their benefits (e.g., Vorhölter, 2019), digital tools for modeling (e.g., Greefrath & Siller, 2017) as well as modeling beliefs and types of modeling behaviour (e.g., Maaß, 2007). The prospective teachers are asked to read preparatory texts for each topic. During the seminar the respective aspects are discussed using various methods and applied to practical examples. Using this kind of procedure a deeper understanding of the respective contents shall be achieved.

The second dimension (*task dimension*) refers to the knowledge about modeling tasks. Thus, in a first step criteria for modeling problems (e.g., Blomhøj & Kjeldsen, 2006) are discussed. Later on, the prospective teachers have to work on multiple modeling problems of varying complexity - including the three problems of the Hamburg Modeling Days - and present their solutions during the seminar. Subsequently, the different solution processes are compared and discussed. The different problems are analysed in terms of their requirements and possible difficulties. Furthermore, the usage of digital tools for working on the modeling problems is discussed and prospective teachers are asked to use those tools on their own. Due to time restrictions, the prospective teachers are not asked to construct modeling problems on their own.

The third dimension includes independent planning and implementation of lessons (*instructio-nal dimension*). The prospective teachers are not asked to plan a whole lesson by themselves, but they are supposed to structure the activities during the Hamburg Modeling Days. Questions of how to introduce a modeling problem best, how to implement phases of reflection and discussion and how to intervene in a most helpful and adaptive way are addressed. In particular the latter aspect is raised several times: knowledge about possible and especially useful intervention strategies (e.g., Stender & Kaiser, 2015; Stender, 2016; Vorhölter et al., 2014), is provided as well as reflections about helpful interventions in relation to the concrete problem. The fourth dimension (*diagnostic dimension*) contains the competence to analyze students' difficulties and reconstruct phases of the modeling process in students' work. In preparation for the Hamburg Modeling Days, the prospective teachers have to analyze posters which were created during modeling days of previous years. On these posters, the previous students have presented not only their solution, but also their solving processes. In addition, further written problem solutions by small groups of students participating later on the modeling days are analyzed by the prospective teachers, and feedback is given directly back to the students.

According to Blömeke, Gustafsson and Shavelson (2015), not only dispositions (cognition and affective-motivational aspects) are of great importance for teaching modeling. In order for these to be effective in practical situations, additional situation-specific skills (i.e. perceiving a situation, interpreting it according to theoretical concepts and making decisions based on the interpretation) are required. Fostering those skills is crucial to implement knowledge into practice. Thus, within the seminar prospective teachers are guided to perceive and interpret situations and decide how to act on them. Not only written statements of students are used, but also different kinds of videos are used to foster prospective teachers' situation specific skills: Some of them were recorded during modeling lessons in schools, some of them are staged videos that comprise different aspects in a short time. The use of videos has the advantage that they can be selected in advance with regard to a certain theoretical content, so that the importance of theoretical background knowledge for the perception and interpretation of certain situations can become directly clear to the students. Therefore, these have proven useful for beginning the teaching of situation-specific skills. In addition to the videos, in some years prospective teachers have the opportunity to visit students in class prior to the start of the modeling days to experience regular instruction. This is then reflected on together within the seminar. Since prospective teachers are always expected to go to classes in small groups and describe their experiences there in this seminar, they become aware that different people perceive different things in the classroom. In addition, because it is a regular class, they are forced to use their theoretical knowledge flexibly to interpret the situations.

Overall, the work on the highly complex modeling tasks is spread throughout the semester and practical and theoretical elements alternate. The contents of the seminar are usually discussed on a theoretical level as well as practically applied and thus cannot be assigned to only one of the dimensions according to Borromeo Ferri and Blum (2010). In addition, situation-specific skills are addressed in connection with numerous contents (outlined in blue in Figure 2). The supervision of the students during the Hamburg Modeling Days takes place at the end of the seminar; a final meeting with the entire seminar after the modeling days has not taken place in the last years due to time restrictions.

The seminar is evaluated regularly (e.g., Vorhölter, 2018). In particular, a dissertation project is developing a video-based instrument that will be used to track the development of students' situation-specific skills over the course of the semester (Alwast & Vorhölter, 2021).

3.3. Interaction with the teachers

One of the goals of the Hamburg Modeling Days is to support teachers in teaching modeling competencies and to encourage them to implement appropriate tasks in the classroom. For this, it is important that they have appropriate knowledge. However, it is just as important that they experience that it is possible to work on modeling problems in the classroom and that they can trust their students to perform certain processes.

At the beginning of the project, the teachers, just like the prospective teachers, were assigned a small group to supervise during the modeling days. Therefore, in three preparatory training sessions, both theory about mathematical modeling was taught and the three modeling problems that were to be supervised during the modeling days were worked on. This was in order to be able to support students in working on these problems and the associated development of modeling competencies, the supervisors must have thought through the problems themselves. Therefore, similar to the seminar with the prospective teachers, the teachers were asked to develop possible ways of working on the individual problems. These were compared and discussed and it was reflected at which points problems presumably occur for students and how these can be dealt with in the best possible way in terms of adaptive help. In some cases, solutions developed by the university students were also integrated into the discussion.

In addition to imparting knowledge, the participating teachers always wanted to know what they could expect from students during the modeling days. Often, a certain skepticism towards the project was expressed. In order to counter this, student work from past modeling days was shown in the training and great importance was attached to the fact that teachers who had already participated (several times) shared their experiences.

With the change in the structure of the modeling days, the involvement of the teachers also changed: On the one hand, many prospective teachers felt overburdened with sometimes occurring motivation problems of students, and on the other hand, teachers were no longer released from teaching for the entire duration of the project. In addition, many teachers now had theoretical knowledge, but many were unsure due to a lack of practical experience and therefore expressed a desire to learn about the work of several small groups. Both aspects led to the fact that teachers are still offered explicit in-service training, but at the same time they are involved in the seminar work, although everyone can decide for themselves how intensively they do this:

• Teachers are asked if university students can participate in one of the mathematics lessons. If possible, a less complex modeling task should be worked on in this lesson, so that the

prospective teachers get first impressions of the modeling competencies of the students. Through the subsequent reflection carried out together with the teacher teaching the lesson, didactic decisions can be justified and questioned together.

- The teachers are given modeling problems for the students, on which the prospective teachers are supposed to give feedback. Due to time constraints, we have not yet been able to have prospective teachers and teachers reflect together on feedback.
- The teachers are invited to the sessions where the students present their own work on the complex problems. In this way, the teachers get an impression of the problems and can
 more authentically than the university lecturers make comments regarding a possible reduction or which is now more common encourage the prospective teachers to expect more from the students.

During the Hamburg Modeling Days, it is now no longer the teachers' task to supervise individual groups, but to support several university students in supervising their groups. This support refers to motivation problems as well as to hints which students should be challenged more and which should be supported more. As a result, the teachers get an overview of the work of different small groups and the handling of different problems and can thus better integrate the students' experiences into their later lessons.

3.4. Current developments

The current developments of the project and the seminar are less due to evaluation than to external circumstances. On the one hand, there is the Corona pandemic and the associated requirement that the seminar as well as the modeling days to be exclusively digital. The fact that the students are not allowed to work together in person during the Hamburg Modeling Days makes collaborative work, i.e. the appropriate distribution of tasks and consolidation of results, even more important. In particular, it was necessary here to work out with the prospective teachers how they could also instruct this digitally and what requirements they should make in order to be able to inform themselves about the students' work process at any time. For the work in the seminar, this meant a sensible restructuring of the seminar content with asynchronous work phases in individual work or group work and synchronous discussion phases in small groups or the overall group. Part of these restructurings and related innovations are to be maintained or even expanded:

- Communication between teachers and prospective teachers or prospective teachers and students is currently still handled by the lecturer. Here, a common work platform for direct exchange would be desirable.
- Flexible digital collaboration among university students enabled faster exchange and more intensive personal support. The development of collaborative documents, the uploading and sharing of these led to faster feedback loops and more transparency within the seminar as well as between lecturers and prospective teachers.

The second development concerns the cooperation with the Department of Mathematics at the university. As shown, the first modeling projects with students were carried out jointly by the Department of Mathematics and the working group of Math Didactics at the Faculty of Education. For several reasons, this cooperation did not take place for a long time, but in addition to the seminar presented in this paper, math seminars on mathematical modeling were also offered. However, both seminars were not linked. For the past two years, attempts have been made again to link the two seminars. In concrete terms, the aim is that as many students as possible first attend a math seminar on modeling, in which they themselves work on a complex modeling problem at university level. Afterwards, they should think about possible didactic reductions (without making the problems less authentic or realistic) as well as about adequate support in the context of the didactic seminar. This concern could not be implemented in the form so far due to structural problems despite good intentions. However, similar modeling problems were already used in both seminars, even if the university students were not the same and the didactic reduction was made by the lecturers.

4. Summary and Outlook

The implementation of modeling projects has a long tradition in Hamburg. The projects presented were always organized and accompanied by the university in different responsibilities. In the article it was explained how the Hamburg Modeling Days for students, which are currently carried out annually, are connected with the training of prospective teachers and the in-service training of teachers. In particular, the structure of the overall project, the preparation of the prospective teachers, and the cooperation with the teachers were presented. It became clear that one central aspect connects all measures: the problems worked on by the students on the modeling days. The supervising prospective and practicing teachers first work on the problem themselves and get to talk about different ways of solving it in order to be able to better support the students in the process of working on it.

The prospective teachers acquire necessary competencies in the university seminar to be able to supervise modeling activities. But also the practicing teachers should have theoretical knowledge of modeling processes, which they can get by offers of university teachers or by participation in the seminar for the prospective teachers.

In addition to the presentation of the current situation, further developments were also outlined, which result from the pandemic situation and changed structures both at the schools and in the training of prospective teachers. These regular adjustments are necessary to justify the continuation of the project, as this is the only way to ensure the greatest possible learning success for all involved.

In this paper, primarily the structure of the Hamburg Modeling Days was presented. Only briefly, the problems to be worked on by the students during the modeling days as well as organizational measures to be carried out by the university coordinators and practicing teachers have been discussed. These are presented, from the point of view of teachers and students in Vorhölter & Haier (this issue).

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