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Study of the influence falling friction on the wheel/rail contact in railway dynamics

J. Giner-Navarro[♭1], V. Andrés-Ruiz[♭], J. Martínez-Casas[♭] and F. D. Denia[♭]

(♭) Centro de Investigación en Ingeniería Mecánica,
Universitat Politècnica de València.

1 Introduction

The complexity of the railway interaction comes from the coupling between the train and the track introduced through the forces appearing in the wheel/rail contact area. These forces are governed by the friction coefficient through the Coulomb's law, characterised by the static and kinematic values, although most of the contact models in railway dynamics consider a constant friction along the simulations. Nevertheless, it is well known that the friction coefficient falls with the slip velocity [1, 2] from a maximum point determined by the static value to a point of saturation corresponding to the kinematic value. The question under debate is if the slope of this fall since recent test-rig experiments seem to reduce it ostensibly compared to friction curves generally estimated in the literature [3].

Rudd [4] proposed this negative slope as mechanism responsible for the generation of an instability phenomenon called railway curve squeal, which has received special attention from researchers [5–9]. The self-excited oscillations that characterise this phenomenon occur when the train is passing along a narrow curve, generating a strong tonal noise in the high-frequency domain. Although falling friction is the most widely accepted mechanism, other possibilities have been proposed to explain, getting more credit the mode-coupling mechanism [10, 11]. For this instability, the oscillation frequencies of two structural modes of an undamped system come closer and closer together until they merge and a pair of an unstable and a stable mode results [12, 13].

This work proposes a model based on a mass-spring-damper oscillator to evaluate its stability when submitted to a variable friction curve. Considering a single-dof (degrees of freedom) model, the paper studies the unstable conditions of the slip-dependent friction that can make the steady-state unstable. The study is extended to a two-dof case with two different geometric configurations in order to analyse the influence of the geometric coupling between the normal and tangential directions arisen from the contact and if it may instabilise the system even considering constant friction.

[1]e-mail: juanginer@upv.es

2 Overview of the mathematical approach

Fig. 1 shows a single degree of freedom oscillator excited by friction over a moving belt [2]. As mentioned in the previous section, there exists two means to get sustained oscillations with an oscillator: either by a decreasing slope of the creepage-creep force phenomenological law, or by a variation of the vertical force applied to the moving mass. In the former case, the motion of the belt is transformed into self-excited vibrations of the mass. In the latter case, the mass is subjected to a forced vibration imposed by the variation of the vertical force.

First, consider the case of a decreasing slope. The equation of motion reads

$$m\ddot{x} + c\dot{x} + kx = F_x, \tag{1}$$

where m is the mass of the oscillator, k the stiffness of the spring, c the damping coefficient and F_x the creep force. The dependency between the relative speed between the mass and the belt and the friction force is given by the Coulomb's law

$$F_x = (\mu_s - \delta_\mu v_x) N_0 sign(v_x), \tag{2}$$

where $v_x = \frac{(V - \dot{x})}{V}$, and N_0 is the static load, and δ_μ the decreasing slope of the friction curve. From the convenient variable transformation, Eq. (1) can be adimensionalised and expressed as

$$q'' + 2\zeta q' + q = (\mu_s - \delta_\mu \widetilde{v}_x) sign(\widetilde{v}_x), \tag{3}$$

where $q = \frac{kx}{N}$, $q' = \frac{dq}{d\tau} = \omega_n \frac{dq}{dt}$, $\widetilde{v}_x = \widetilde{V} - q'$, $\widetilde{V} = \frac{kV}{\omega N}$, $\omega_n = \sqrt{\frac{k}{m}}$ is the natural frequency and $\zeta = \frac{c}{(2m\omega_n)}$ is the damping rate.

For a given dimensionless sliding velocity $\widetilde{V}$, the equilibrium state is associated with a stationary slip where the conveyor belt moves at speed $\widetilde{V}$ but not the oscillator ($q'_0 = 0$). The equilibrium may be stable or unstable. As it is well known for this kind of friction-induced self-excited oscillator that the equilibrium state can undergo instability through a Hopf bifurcation leading to a cycle solution, i.e. a periodic vibration. This stability problem may be analysed by the first Lyapunov method reconsidering the problem in the phase space.

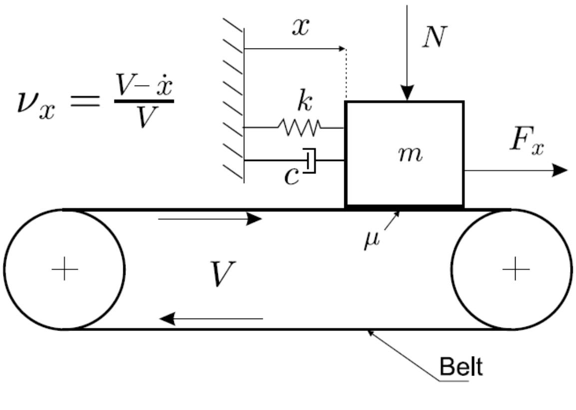


Figure 1: Single-dof oscillator excited by friction.

There is an increase of interest in the direction of mode-coupling phenomena in addressing curve squeal, which have been explained in a simplified form by Hoffmann et al. [10, 12] and Sinou and Jezequel [13], through frequency-domain models. This type of instability can occur even considering a constant coefficient of friction, arising from non-conservative displacement-dependent forces.

Fig. 2 shows the typical system adopted to illustrate this mechanism, in which the friction coefficient μ is constant. Here the mass has two dof and two springs. As the mass vibrates, variations in the normal load occur, leading to variations in the friction force. The modes of the wheel may have both vertical and lateral components and the contact angle of the wheel with the rail may vary. At least two modes are necessary to initiate this mechanism.

By considering small oscillations around the equilibrium of steady-state sliding, the system in Fig. 2 can be mathematically described as

$$\begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + \begin{pmatrix} k_{11} & k_{12} - \mu K_H \\ k_{21} & k_{22} \end{pmatrix} \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}, \tag{4}$$

where the terms $k_{ij}(\alpha_1, \alpha_2)$ in the stiffness matrix depend on the orientation and stiffness of the springs which in turn depend on angles α_1 and α_2 [13]. K_H represents the linearised Hertzian contact stiffness; x and y are the vibration displacements in tangential and normal directions, respectively, and F and N are the corresponding friction and normal forces. The most important feature of Eq. (4) is that the stiffness matrix is non-symmetric, making the system unstable if the upper diagonal term of the stiffness matrix $k_{12} - \mu K_H \leq 0$ due to the value of friction coefficient μ.

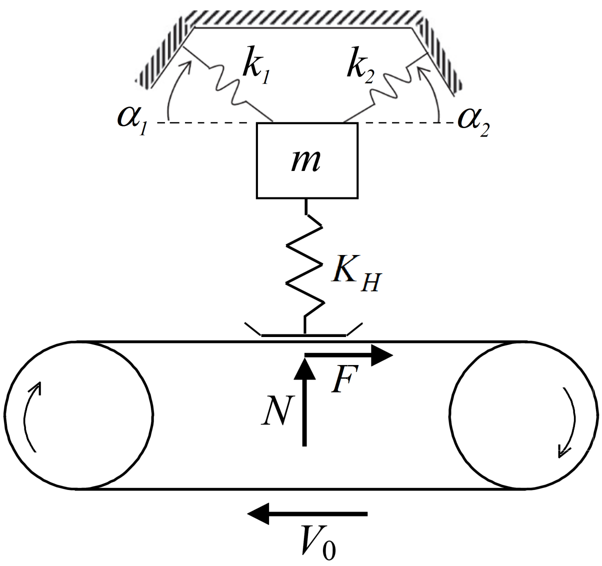


Figure 2: Two-dof system on moving belt.

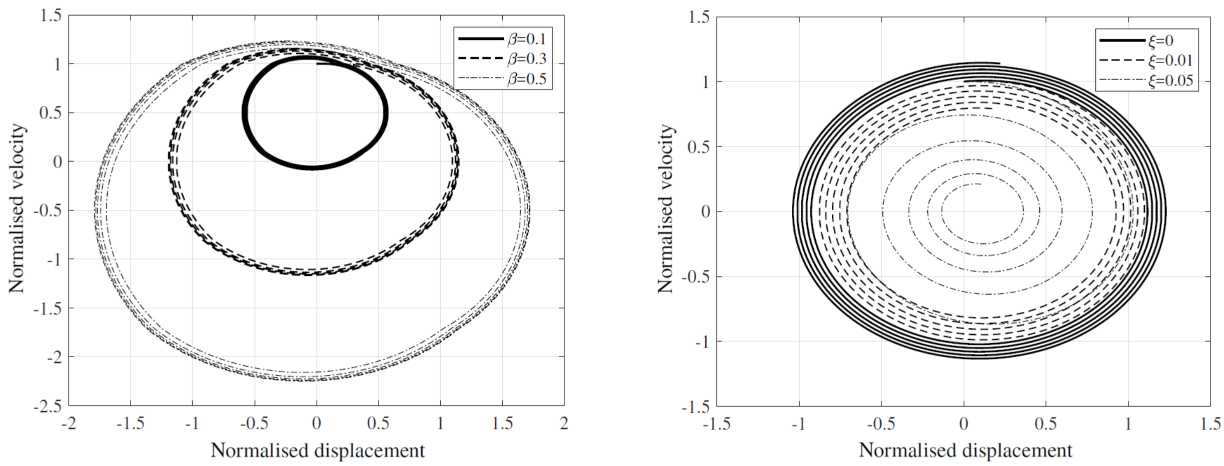
3 Results

As shown in [14], the following non-dimensional parameter indicates the relative importance of the stick and slip phases:

$$\beta = (\mu_s - \mu_k)\frac{N}{Vm\omega_n}. \quad (5)$$

This value is usually in the range 0.1–1 [14] for curve squeal situations. The parameter permits to evaluate the stick-slip motion of the single-dof oscillator, as seen in Fig. 3a for three values of β, in which the velocity (normalised by the belt velocity V_0) is plotted against the displacement (normalised by V_0/ω_0). It can be seen a 'limit cycle' as the formation of a stable periodic motion from different initial conditions. For small values of β, the slip phase predominates since the motion is close to elliptical on the phase plane and the oscillation frequency is close to the natural frequency. The stick phase predominates for large values of β and the oscillation frequency is lower than the natural frequency [14].

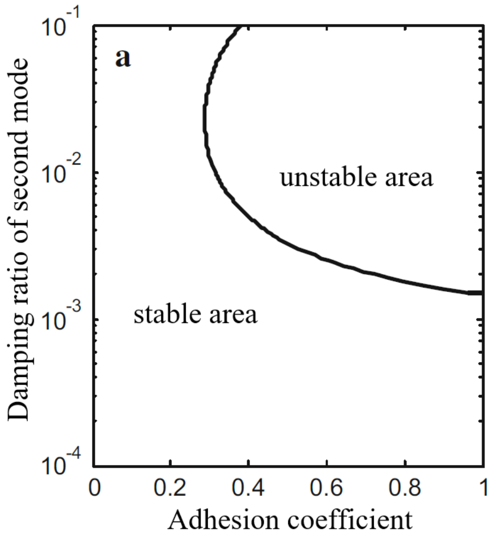
The effect of damping is also assessed in Fig. 3b for $\beta = 1$ and three values of damping ratio. It is observed a small effect on the amplitude of the limit cycle when the damping is increased, until the damping reaches a value where the oscillations are suppressed. For $\zeta = 0.05$ in this case, the damping exceeds the limiting value and the oscillations decay. The limiting value of damping ratio can be approximated as $\zeta > \beta^2/4\pi$ [14].



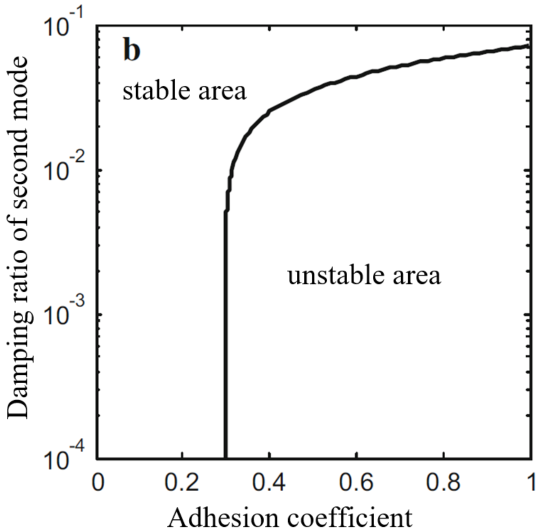
(a) (b)

Figure 3: Normalised displacement vs. non-dimensional velocity of a simplified stick-slip mechanism: $\mu_s = 0.4$, $\mu_k = 0.3$. (a) Without damping; (b) for different damping levels ($\beta = 1$).

Using now the two-dof oscillator model to assess the mode coupling, the effect of damping is evaluated. It can be observed from Fig. 4 that an increase in damping can favour instability in some situations or can improve stability in others. On the one hand, Fig. 4a shows the stability map for varying friction coefficient when the damping ratio of only the second mode of the system is varied, while the damping ratio of the first mode is kept at 10^{-4}. For low values of damping, the system remains stable. Nevertheless, it becomes more unstable when the damping of the second mode is between about 2×10^{-3} and 10^{-1}. On the other hand, increasing together the damping ratios of both modes while keeping their ratio fixed, Fig. 4b shows that damping has no effect on the stability up to about 10^{-2}, while the system is quickly stabilised above this value.



(a)



(b)

Figure 4: Stability maps for two-mode system for contact angle 3° and lateral contact position of 8 mm showing effect of damping ratio. (a) Damping ratio of second mode only is varied; (b) damping ratio of both modes is varied, keeping the ratio between them fixed.

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