MODELLING FOR ENGINEERING & HUMAN BEHAVIOUR 2019

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Instituto Universitario de Matemática Multidisciplinar
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Edited by
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Exponential time differencing schemes for pricing American option under the Heston model

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1 Introduction

The classic Black-Scholes model makes assumptions that are not empirically valid. The model is widely employed as a useful approximation to reality, but proper application requires understanding its limitations and constant volatility of the stock returns is one of them. In fact, this assumption is one of the biggest source of weakness, because the variance has been observed to be non-constant leading to models, such as GARCH, to model volatility changes. There are other approaches to model the asset volatility, as consider that follows a random process or, in other words, consider the volatility as a stochastic process. This point of view lead us to a Partial Differential Equation (PDE) different from the classic Black-Scholes, now there are involved two different variables, apart of the time: asset level S and variance ν. Deal with this PDE and the presence of cross-derivatives is a challenging task. It is even more difficult to deal with American options which allows to exercise the option at any time before the expiration date. But the solution to this problem is of great interest to the financial markets.

2 The pricing problem

To the pricing of American options we use the Heston model [5]:

$$\begin{gathered} dS(t) = \mu S(t)dt + \sqrt{\nu(t)}S(t)dW_1, \\ d\nu(t) = \kappa(\theta - \nu(t))dt + \sigma\sqrt{\nu(t)}dW_2, \\ dW_1 dW_2 = \rho dt, \end{gathered} \tag{1}$$

and a penalty method similar as in [3]. With this assumptions, applying Itô's lemma and standard arbitrage arguments we achieve the following PDE:

$$\frac{\partial U}{\partial t} + \frac{1}{2}\nu S^2 \frac{\partial^2 U}{\partial S^2} + \rho\sigma\nu S \frac{\partial^2 U}{\partial S \partial \nu} + \frac{1}{2}\sigma^2 \nu \frac{\partial^2 U}{\partial \nu^2} + rS\frac{\partial U}{\partial S} + \bar{\kappa}(\bar{\theta} - \nu)\frac{\partial U}{\partial \nu} - rU + f(E, S, U) = 0, \tag{2}$$

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at which we will remove the cross-derivatives with the classical technique for the reduction of second order linear PDE to canonical form [4, chapter 3]. It is well known that, using finite differences, cross-derivatives involves negative coefficients. So, like we are talking about prices we must guarantee the solution's positivity. This fact motivates the transformation of the problem.

The following step of the semi-discretization. We apply centered finite difference to the spatial derivatives, letting alone the temporal-derivatives, achieving a system of ODEs:

$$\frac{dP}{dt} = A(\xi)P(t) + f\left(\xi, P\right). \tag{3}$$

Now we apply the ETD method [2] and the temporal discretization. Finally, making some assumptions to provide solutions, we achieve a numerical scheme to the PDE (2):

$$P^{n+1} = e^{Ak}P^n + k\ \varphi(A,k)\ f(\xi, P^n). \tag{4}$$

3 Positivity and stability

Like we are computing prices, we must assure the positivity an stability of the provided solutions. And in the case that we were interested in computing put prices, we also must assure that our numerical scheme provides bounded profits.

We can assure the positivity of our numerical scheme bounding the numerical derivative's step-size of the spatial variables. Specifically:

$$h \leq \frac{\alpha}{\delta}, \tag{5}$$

where α is the minimum main diagonal coefficient of matrix $A(\xi)$ and δ the maximum of non-diagonal elements.

The stability condition is fulfilled if the temporal step-size verify the following:

$$k \leq \frac{h^2}{(\lambda + r)h^2 + 2\alpha_m\left(\frac{1+m^2}{m^2}\right)}, \tag{6}$$

where α_m is the maximum main diagonal coefficient of matrix $A(\xi)$, r the risk-free rate, m the relationship between the spatial step-sizes and λ a constant dependent of the penalty term.

It can be verified for put options, using the induction principle, that at any time step:

$$\parallel P^n \parallel_\infty \leq E. \tag{7}$$

4 Numerical experiments

Fig. 1 shows the numerical solution for American put options under the set of parameters: $S_1 = 0.25,\ S_2 = 40,\ \nu_1 = 0.002,\ \nu_2 = 1.2,\ r = 0.1,\ \rho = 0.1,\ E = 10,\ T = 0.25,\ \lambda = 200,\ \kappa =$

5, $\theta = 0.16$, $\sigma = 0.9$ for k and h verifying the stability condition.

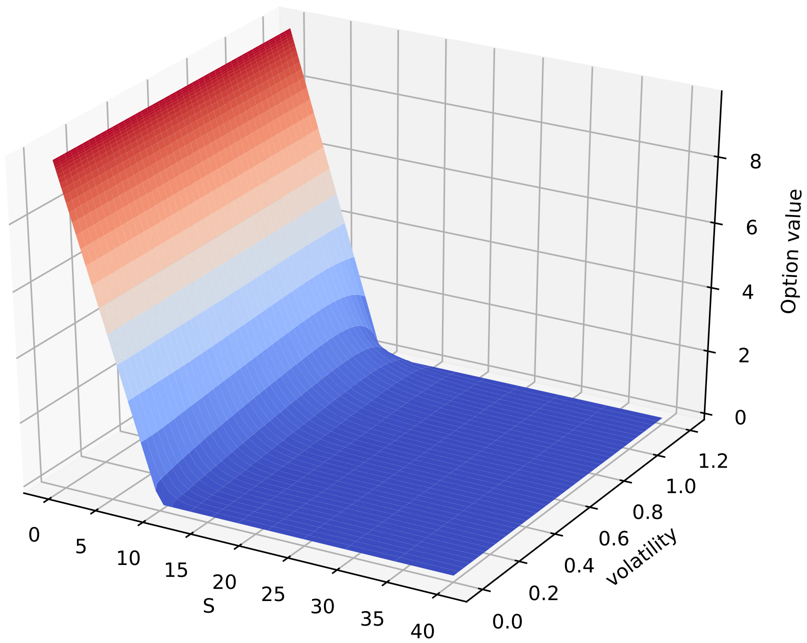

Figure 1: Numerical solution for $\tau = T$, $h = 0.07$ and $k = 5 \cdot 10^{-5}$.

We can see that for a big values of the underlying asset, the option values tends to zero. On the other hand, when the asset tends to zero the option value tends to the strike price E, as we expect because of (7). Other relevant issue that our numerical solution catches is that for a big values of the volatility the option value is bigger than for low values, but this is only relevant when the asset is near to the strike price. Proposed numerical solution are competitive with other approaches in the literature [1, 6–10].

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References

[1] Clarke, N. and Parrott, K. The multigrid solution of two-factor American put options. *Oxford Computing Laboratory, Research Report*, 96-16, 1996.

[2] Cox, S.M. and Matthews, P.C. Exponential Time Differencing for Stiff Systems. *Journal of Computational Physics*, 176(2):430-455, 2002.

[3] Forsyth, P. A. and Vetzal, K. R. Quadratic Convergence for Valuing American Options Using a Penalty Method. *SIAM Journal on Scientific Computing*, 23(6):2095-2122, 2002.

[4] Garabedian, P. R. *Partial Differential Equations*. Springer Berlin Heidelberg, 1998.

[5] Heston, S.L. A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *Review of Financial Studies*, 6(2):327-343, 1993.

[6] Ikonen, S. and Toivanen, J. Efficient numerical methods for pricing American options under stochastic volatility. *Numerical Methods for Partial Differential Equations*, 24(1):104-126, 2007.

[7] Oosterlee, C.W. On multigrid for linear complementarity problems with application to American-style options. *ETNA. Electronic Transactions on Numerical Analysis [electronic only]*, 15:165-185, 2003.

[8] Yousuf, M. and Khaliq, A.Q.M. An efficient ETD method for pricing American options under stochastic volatility with nonsmooth payoffs. *Numerical Methods for Partial Differential Equations*, 29(6):1864-1880, 2013.

[9] Zhu, S.-P and Chen, W.-T. A predictor–corrector scheme based on the ADI method for pricing American puts with stochastic volatility. *Computers & Mathematics with Applications*, 62(1):1-26, 2011.

[10] Zvan, R., Forsyth, P. and Vetzal, K. Penalty methods for American options with stochastic volatility. *Journal of Computational and Applied Mathematics*, 91(2):199-218, 1998.