

MODELLING FOR ENGINEERING & HUMAN BEHAVIOUR 2019

im²

Instituto Universitario de Matemática Multidisciplinar
Polytechnic City of Innovation

Edited by

R. Company, J.C. Cortés,
L. Jódar and E. López-Navarro

July 10th - 12th 2019



UNIVERSITAT
POLITÈCNICA
DE VALÈNCIA



CIUDAD POLITÈCNICA
DE LA INNOVACIÓN



Modelling for Engineering & Human Behaviour 2019

València, 10 – 12 July 2019

This book includes the extended abstracts of papers presented at XXIst Edition of the Mathematical Modelling Conference Series at the Institute for Multidisciplinary Mathematics “Mathematical Modelling in Engineering & Human Behaviour”.

I.S.B.N.: 978-84-09-16428-8

Version: 18/11/19

Report any problems with this document to ellona1@upvnet.upv.es.

Edited by: R. Company, J. C. Cortés, L. Jódar and E. López-Navarro.

Credits: The cover has been designed using images from [kjpargetter/freepik](https://www.kjpargetter.com/).

im²

Instituto Universitario de Matemática
Multidisciplinar

This book has been supported by the European Union through the Operational Program of the [European Regional Development Fund (ERDF) / European Social Fund (ESF)] of the Valencian Community 2014-2020. [Record: GJIDI/2018/A/010].



**GENERALITAT
VALENCIANA**
Conselleria d'Hisenda
i Model Econòmic



UNIÓ EUROPEA

Fons Europeu de
Desenvolupament Regional

Una manera de fer Europa

Contents

A personality mathematical model of placebo with or without deception: an application of the Self-Regulation Therapy	1
The role of police deterrence in urban burglary prevention: a new mathematical approach	9
A Heuristic optimization approach to solve berth allocation problem	14
Improving the efficiency of orbit determination processes	18
A new three-steps iterative method for solving nonlinear systems	22
Adaptive modal methods to integrate the neutron diffusion equation	26
Numerical integral transform methods for random hyperbolic models	32
Nonstandard finite difference schemes for coupled delay differential models	37
Semilocal convergence for new Chebyshev-type iterative methods	42
Mathematical modeling of Myocardial Infarction	46
Symmetry relations between dynamical planes	51
Econometric methodology applied to financial systems	56
New matrix series expansions for the matrix cosine approximation	64
Modeling the political corruption in Spain	70
Exponential time differencing schemes for pricing American option under the Heston model	75
Chromium layer thickness forecast in hard chromium plating process using gradient boosted regression trees: a case study	79
Design and convergence of new iterative methods with memory for solving nonlinear problems	83
Study of the influence falling friction on the wheel/rail contact in railway dynamics ..	88
Extension of the modal superposition method for general damping applied in railway dynamics	94
Predicting healthcare cost of diabetes using machine learning models	99

Sampling of pairwise comparisons in decision-making	105
A multi-objective and multi-criteria approach for district metered area design: water operation and quality analysis	110
Updating the OSPF routing protocol for communication networks by optimal decision-making over the k-shortest path algorithm	118
Optimal placement of quality sensors in water distribution systems	124
Mapping musical notes to socio-political events	131
Comparison between DKGa optimization algorithm and Grammar Swarm surrogated model applied to CEC2005 optimization benchmark	136
The quantum brain model	142
Probabilistic solution of a randomized first order differential equation with discrete delay	151
A predictive method for bridge health monitoring under operational conditions	155
Comparison of a new maximum power point tracking based on neural network with conventional methodologies	160
Influence of different pathologies on the dynamic behaviour and against fatigue of railway steel bridges	166
Statistical-vibratory analysis of wind turbine multipliers under different working conditions	171
Analysis of finite dimensional linear control systems subject to uncertainties via probabilistic densities	176
Topographic representation of cancer data using Boolean Networks	180
Trying to stabilize the population and mean temperature of the World	185
Optimizing the demographic rates to control the dependency ratio in Spain	193
An integer linear programming approach to check the embodied CO_2 emissions of the opaque part of a façade	199
Acoustics on the Poincaré Disk	206
Network computational model to estimate the effectiveness of the influenza vaccine <i>a posteriori</i>	211
The key role of Liouville-Gibbs equation for solving random differential equations: Some insights and applications	217

Exponential time differencing schemes for pricing American option under the Heston model

R. Company ^b, F. Fuster^{†1} and L. Jódar ^b

(b) Institut Universitari de Matemàtica Multidisciplinar,
Universitat Politècnica de València,

(†) Banco Santander,
Av. de Cantabria, s/n, 28660 Boadilla del Monte, Madrid.

1 Introduction

The classic Black-Scholes model makes assumptions that are not empirically valid. The model is widely employed as a useful approximation to reality, but proper application requires understanding its limitations and constant volatility of the stock returns is one of them. In fact, this assumption is one of the biggest source of weakness, because the variance has been observed to be non-constant leading to models, such as GARCH, to model volatility changes. There are other approaches to model the asset volatility, as consider that follows a random process or, in other words, consider the volatility as a stochastic process. This point of view lead us to a Partial Differential Equation (PDE) different from the classic Black-Scholes, now there are involved two different variables, apart of the time: asset level S and variance ν . Deal with this PDE and the presence of cross-derivatives is a challenging task. It is even more difficult to deal with American options which allows to exercise the option at any time before the expiration date. But the solution to this problem is of great interest to the financial markets.

2 The pricing problem

To the pricing of American options we use the Heston model [5]:

$$\begin{aligned} dS(t) &= \mu S(t)dt + \sqrt{\nu(t)}S(t)dW_1, \\ d\nu(t) &= \kappa(\theta - \nu(t))dt + \sigma\sqrt{\nu(t)}dW_2, \\ dW_1dW_2 &= \rho dt, \end{aligned} \tag{1}$$

and a penalty method similar as in [3]. With this assumptions, applying Itô's lemma and standard arbitrage arguments we achieve the following PDE:

$$\frac{\partial U}{\partial t} + \frac{1}{2}\nu S^2 \frac{\partial^2 U}{\partial S^2} + \rho\sigma\nu S \frac{\partial^2 U}{\partial S \partial \nu} + \frac{1}{2}\sigma^2\nu \frac{\partial^2 U}{\partial \nu^2} + rS \frac{\partial U}{\partial S} + \bar{\kappa}(\bar{\theta} - \nu) \frac{\partial U}{\partial \nu} - rU + f(E, S, U) = 0, \tag{2}$$

¹e-mail: ferran.ffv@gmail.com

at which we will remove the cross-derivatives with the classical technique for the reduction of second order linear PDE to canonical form [4, chapter 3]. It is well known that, using finite differences, cross-derivatives involves negative coefficients. So, like we are talking about prices we must guarantee the solution's positivity. This fact motivates the transformation of the problem.

The following step of the semi-discretization. We apply centered finite difference to the spatial derivatives, letting alone the temporal-derivatives, achieving a system of ODEs:

$$\frac{dP}{dt} = A(\xi)P(t) + f(\xi, P). \quad (3)$$

Now we apply the ETD method [2] and the temporal discretization. Finally, making some assumptions to provide solutions, we achieve a numerical scheme to the PDE (2):

$$P^{n+1} = e^{Ak}P^n + k \varphi(A, k) f(\xi, P^n). \quad (4)$$

3 Positivity and stability

Like we are computing prices, we must assure the positivity and stability of the provided solutions. And in the case that we were interested in computing put prices, we also must assure that our numerical scheme provides bounded profits.

We can assure the positivity of our numerical scheme bounding the numerical derivative's step-size of the spatial variables. Specifically:

$$h \leq \frac{\alpha}{\delta}, \quad (5)$$

where α is the minimum main diagonal coefficient of matrix $A(\xi)$ and δ the maximum of non-diagonal elements.

The stability condition is fulfilled if the temporal step-size verify the following:

$$k \leq \frac{h^2}{(\lambda + r)h^2 + 2\alpha_m \left(\frac{1+m^2}{m^2}\right)}, \quad (6)$$

where α_m is the maximum main diagonal coefficient of matrix $A(\xi)$, r the risk-free rate, m the relationship between the spatial step-sizes and λ a constant dependent of the penalty term.

It can be verified for put options, using the induction principle, that at any time step:

$$\|P^n\|_{\infty} \leq E. \quad (7)$$

4 Numerical experiments

Fig. 1 shows the numerical solution for American put options under the set of parameters: $S_1 = 0.25$, $S_2 = 40$, $\nu_1 = 0.002$, $\nu_2 = 1.2$, $r = 0.1$, $\rho = 0.1$, $E = 10$, $T = 0.25$, $\lambda = 200$, $\kappa =$

5, $\theta = 0.16$, $\sigma = 0.9$ for k and h verifying the stability condition.

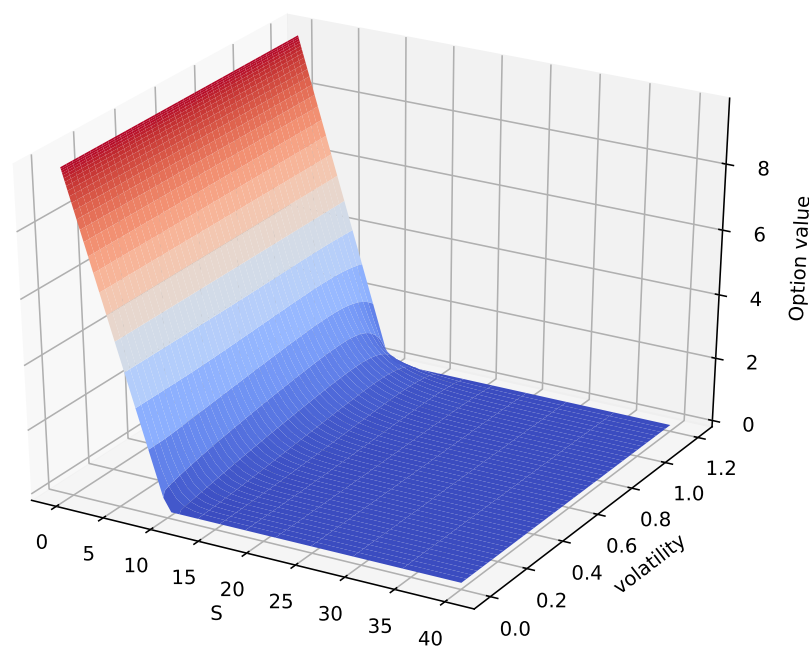


Figure 1: Numerical solution for $\tau = T$, $h = 0.07$ and $k = 5 \cdot 10^{-5}$.

We can see that for a big values of the underlying asset, the option values tends to zero. On the other hand, when the asset tends to zero the option value tends to the strike price E , as we expect because of (7). Other relevant issue that our numerical solution catches is that for a big values of the volatility the option value is bigger than for low values, but this is only relevant when the asset is near to the strike price. Proposed numerical solution are competitive with other approaches in the literature [1,6–10].

Acknowledgements

This work has been partially supported by the Ministerio de Ciencia, Innovación y Universidades Spanish grant MTM2017-89664-P.

References

- [1] Clarke, N. and Parrott, K. The multigrid solution of two-factor American put options. *Oxford Computing Laboratory, Research Report*, 96-16, 1996.
- [2] Cox, S.M. and Matthews, P.C. Exponential Time Differencing for Stiff Systems. *Journal of Computational Physics*, 176(2):430-455, 2002.
- [3] Forsyth, P. A. and Vetzal, K. R. Quadratic Convergence for Valuing American Options Using a Penalty Method. *SIAM Journal on Scientific Computing*, 23(6):2095-2122, 2002.

- [4] Garabedian, P. R. *Partial Differential Equations*. Springer Berlin Heidelberg, 1998.
- [5] Heston, S.L. A Closed-Form Solution for Options with Stochastic Volatility with Applications to Bond and Currency Options. *Review of Financial Studies*, 6(2):327-343, 1993.
- [6] Ikonen, S. and Toivanen, J. Efficient numerical methods for pricing American options under stochastic volatility. *Numerical Methods for Partial Differential Equations*, 24(1):104-126, 2007.
- [7] Oosterlee, C.W. On multigrid for linear complementarity problems with application to American-style options. *ETNA. Electronic Transactions on Numerical Analysis [electronic only]*, 15:165-185, 2003.
- [8] Yousuf, M. and Khaliq, A.Q.M. An efficient ETD method for pricing American options under stochastic volatility with nonsmooth payoffs. *Numerical Methods for Partial Differential Equations*, 29(6):1864-1880, 2013.
- [9] Zhu, S.-P and Chen, W.-T. A predictor–corrector scheme based on the ADI method for pricing American puts with stochastic volatility. *Computers & Mathematics with Applications*, 62(1):1-26, 2011.
- [10] Zvan, R., Forsyth, P. and Vetzal, K. Penalty methods for American options with stochastic volatility. *Journal of Computational and Applied Mathematics*, 91(2):199-218, 1998.