Second harmonic propagation in Coupled Oscillators

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Abstract
In this work, we studied numerically, analytically and experimentally the nonlinear dynamics for a chain of magnates. Propagation the second harmonic depends on the medium parameters and the excitation signal (amplitude and frequency). From the experimental results which have a good agreement with the theoretical results, several phenomenons in nonlinear behavior can be study. The experiment also shows the generation of subharmonics.

Keywords: Discrete System, FPU, Second harmonic.

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1 Introduction

Nonlinear waves propagation in one-dimension chain has been extensively studied in a continues system [1] and discrete system in the form of granular chain [4]. The nonlinearity of the system depends on the medium, the excitation amplitude and the driven frequency. This model can explain the evolution of the harmonic in space. The balance between the nonlinearity and the dispersion allow to the solitary wave to propagate [5]. Studying soliton waves in artificial dispersion medium have been presented in [2].

2 Theoretical model

We consider a finite chain of identical magnets with mass m aligned along the x-axis. The equation of motion of the chain is described by coulomb potential. The force between the nearest neighbor is the magnetic dipole interaction.

$$m\ddot{u_n} = \epsilon\left(\frac{1}{(a-u_{n+1}+u_n)^4}\right) - \left(\frac{1}{(a-u_n+u_{n+1})^4}\right) - \frac{g}{L}u_n$$

where u_n represent the displacement of magnet nth and measured with respect to equilibrium position, ϵ is coupling constant, a is the distance between magnets and g is the gravity. In the experiment setup each magnet is attached to T-shaped rod with length L. This equation is difficult to solve. We assumed that the amplitude $u_{n+1} + u_n$ is very small comparing to the lattice constant a. For that, we transferred this equation to well-known problem α-FPU [4].

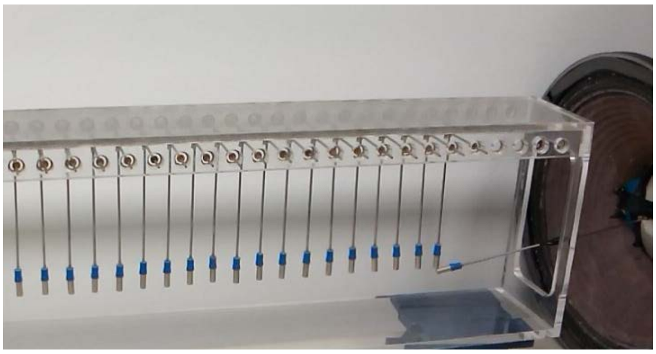

Fig. (1) The experimental setup shows that the pendulums are totally "hanging" by the repulsion force between two disc magnets.

3 Experimental Setup

The chain consists of 53 magnets arranged to be in one-dimension. All magnets are identical with material NdFeB (Supermagnete S-05-14-N); magnetization grade is N45, the diameter is 5 mm and height is 13.96 mm. The magnets were placed at equally space a = 20 mm. Each magnet is attached to T-shaped rod with length L = 100mm which hanging on air by repulsion force between two disc magnets and there is no friction, as shown in Fig. (1). We record the motion by using camera GoPro.

4 Results

In this work we investigated the behavior of the chain under different frequencies and different amplitudes. We chose three cases to study; 1st case when driven harmonic and second harmonic lies into non-dispersive part of Pass-band. In this case, the wave behaves as propagating in homogenous medium. It's the zone of non-dispersive regime which all the harmonics propagate with the same velocity. The amplitude of the second harmonic starts to increase with decreasing first harmonic, as shown in Figure (1-a).

2nd case, when the driven frequency is in pass band but the second harmonic lies into the Gap. We observed constant value for 2nd harmonic along space, as shown in Figure (1-b). Finally, 3rd case, when the second harmonic generated in the dispersive curve. The second harmonic will propagate with "beating" due to phase-mismatch between forced wave $k(2\omega)$ and a propagative wave $2k(\omega)$, as shown in Figure (1-c).

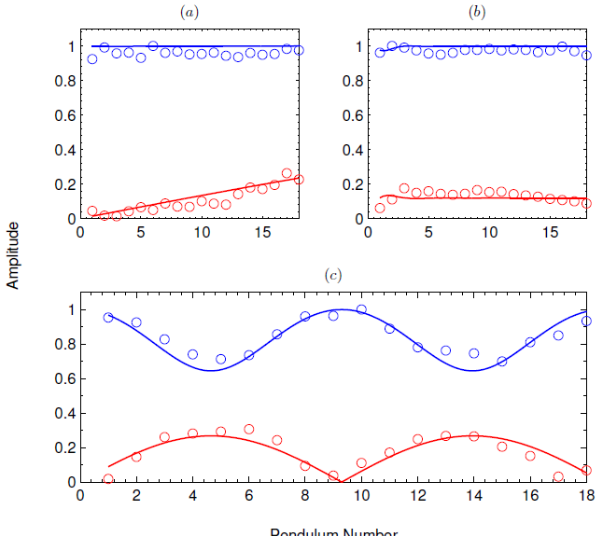

Fig. (1) (Line) Analytically, (Circles) Experimentally, (Blue) First harmonic at generates the second harmonic (Red); n = 18 pendulums (a) f_d = 5 Hz, (b) f_d = 10 Hz, (c) f_d = 8.7 Hz

5 Conclusions

We have devolped a model of magnets that provides the studying the nonlinear acoustic wave. This model greatly smplifies the complex bihaviuor of the nonlinear dynamic. Generation of the second harmonic depend on the position of the deriven frequancy. We introduced three different way for second harmonic behavior, in cases of propagating in dispersive and non-dispersive regime. Finally, the case the second harmonic in Gap. The proposed model can be applicable to intense study for solitary wave (Soliton, Kink...).

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