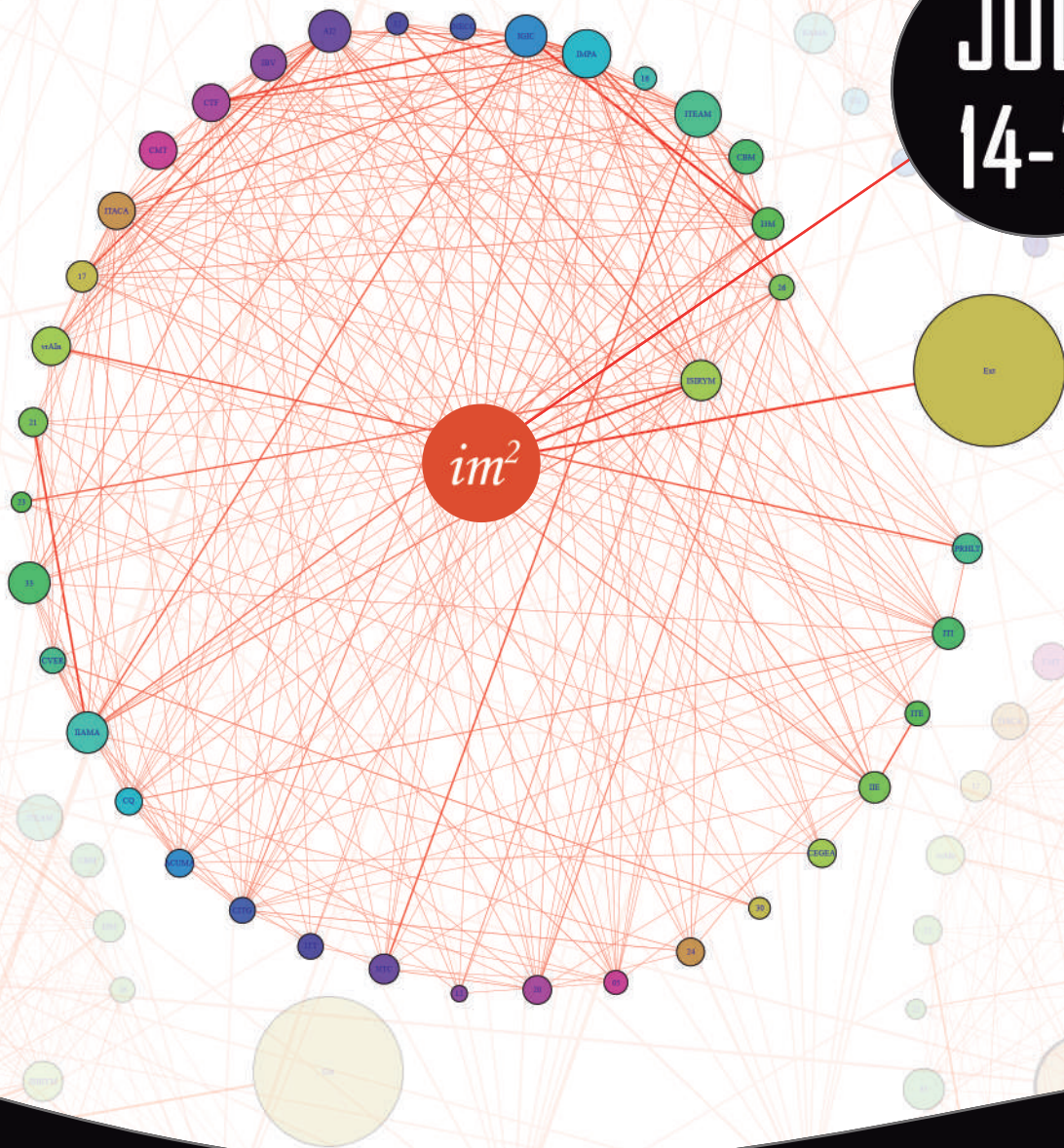


MODELLING FOR ENGINEERING & HUMAN BEHAVIOUR

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Edited by

Juan Ramón Torregrosa

Juan Carlos Cortés

Antonio Hervás

Antoni Vidal

Elena López-Navarro



UNIVERSITAT
POLITÈCNICA
DE VALÈNCIA

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Instituto Universitario
de Matemática Multidisciplinar

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Edited by: I.U. de Matemàtica Multidisciplinar, Universitat Politècnica de València.
J.R. Torregrosa, J-C. Cortés, J. A. Hervás, A. Vidal-Ferràndiz and E. López-Navarro

im²

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Contents

Density-based uncertainty quantification in a generalized Logistic-type model	1
Combined and updated H -matrices	7
Solving random fractional second-order linear equations via the mean square Laplace transform	13
Conformable fractional iterative methods for solving nonlinear problems	19
Construction of totally nonpositive matrices associated with a triple negatively realizable	24
Modeling excess weight in Spain by using deterministic and random differential equations	31
A new family for solving nonlinear systems based on weight functions Kalitkin-Ermankov type	36
Solving random free boundary problems of Stefan type	42
Modeling one species population growth with delay	48
On a Ermakov–Kalitkin scheme based family of fourth order	54
A new mathematical structure with applications to computational linguistics and specialized text translation	60
Accurate approximation of the Hyperbolic matrix cosine using Bernoulli matrix polynomials	67
Full probabilistic analysis of random first-order linear differential equations with Dirac delta impulses appearing in control	74
Some advances in Relativistic Positioning Systems	79
A Graph–Based Algorithm for the Inference of Boolean Networks	84
Stability comparison of self-accelerating parameter approximation on one-step iterative methods	90
Mathematical modelling of kidney disease stages in patients diagnosed with diabetes mellitus II	96
The effect of the memory on the spread of a disease through the environment	101
Improved pairwise comparison transitivity using strategically selected reduced information	106
Contingency plan selection under interdependent risks	111
Some techniques for solving the random Burgers’ equation	117
Probabilistic analysis of a class of impulsive linear random differential equations via density functions	122

Probabilistic evolution of the bladder cancer growth considering transurethral resection	127
Study of a symmetric family of anomalies to approach the elliptical two body problem with special emphasis in the semifocal case.....	132
Advances in the physical approach to personality dynamics	136
A Laplacian approach to the Greedy Rank-One Algorithm for a class of linear systems	143
Using STRESS to compute the agreement between computed image quality measures and observer scores: advantages and open issues	149
Probabilistic analysis of the random logistic differential equation with stochastic jumps	156
Introducing a new parametric family for solving nonlinear systems of equations	162
Optimization of the cognitive processes involved in the learning of university students in a virtual classroom	167
Parametric family of root-finding iterative methods	175
Subdirect sums of matrices. Definitions, methodology and known results.	180
On the dynamics of a predator-prey metapopulation on two patches.....	186
Prognostic Model of Cost / Effectiveness in the therapeutic Pharmacy Treatment of Lung Cancer in a University Hospital of Spain: Discriminant Analysis and Logit.....	192
Stability, bifurcations, and recovery from perturbations in a mean-field semiarid vegetation model with delay	197
The random variable transformation method to solve some randomized first-order linear control difference equations.....	202
Acoustic modelling of large aftertreatment devices with multimodal incident sound fields	208
Solving non homogeneous linear second order difference equations with random initial values: Theory and simulations.....	216
A realistic proposal to considerably improve the energy footprint and energy efficiency of a standard house of social interest in Chile	224
Multiobjective Optimization of Impulsive Orbital Trajectories.....	230
Mathematical Modeling about Emigration/Immigration in Spain: Causes, magnitude, consequences	236
New scheme with memory for solving nonlinear problems	241
SP_N Neutron Noise Calculations	246
Analysis of a reinterpretation of grey models applied to measuring laboratory equipment uncertainties	252
An Optimal Eighth Order Derivative-Free Scheme for Multiple Roots of Non-linear Equations	257
A population-based study of COVID-19 patient's survival prediction and the potential biases in machine learning.....	262
A procedure for detection of border communities using convolution techniques.....	267

Conformable fractional iterative methods for solving nonlinear problems

Giro Candelario^{b,1}, Alicia Cordero[‡], Juan R. Torregrosa[‡] and María P. Vassilev^b

(b) Área de Ciencias Básicas y Ambientales, Instituto Tecnológico de Santo Domingo
Avenida de Los Próceres #49, Los Jardines del Norte 10602,
Santo Domingo, Dominican Republic.

(‡) Departamento de Matemática Aplicada, Universitat Politècnica de València
Camino de Vera, s/n 46022 Valencia, Spain.

1 Introduction

Fractional derivatives have attracted increasing interest from the first steps of fractional calculus in the XVII-th century up to now, when they have become an excellent tool to model physical phenomena in porous media, for example.

However, fractional derivatives have been not been widely used, in general, to develop iterative schemes for solving nonlinear problems. Recently, in [5], two fractional Newton-type methods were proposed,

$$x_{k+1} = x_k - \left(\Gamma(\alpha + 1) \frac{f(x_k)}{cD_{a+}^{\alpha} f(x_k)} \right)^{1/\alpha}, \quad k = 0, 1, 2, \dots \quad (1)$$

and

$$x_{k+1} = x_k - \left(\Gamma(\alpha + 1) \frac{f(x_k)}{D_{a+}^{\alpha} f(x_k)} \right)^{1/\alpha}, \quad k = 0, 1, 2, \dots \quad (2)$$

with employ Caputo and Riemann-Liouville derivatives respectively. The order of convergence of these methods is $\alpha + 1$. Let us denote them as CFN and R-LFN, respectively. Let us remark that, when $\alpha = 1$, we obtain the classical Newton's method for each case above.

The use of Caputo and Riemann-Liouville fractional derivatives require the evaluation of special functions as Gamma and Mittag-Leffler functions, which both involve a high computational cost to compute these fractional derivatives. Theoretically, the order of convergence of these methods tends to be quadratic when $\alpha \approx 1$, but in the practice, the approximated computational order of convergence (ACOC, see [5]) is linear if α is different from 1.

In order to design a new iterative scheme able to avoid these problems, we introduce the conformable fractional derivative. It can be seen in [1,2] that the left conformable fractional derivative starting from a of a function $f : [a, \infty) \rightarrow \mathbb{R}$ of order $\alpha \in (0, 1]$, being $\alpha, a, x \in \mathbb{R}$, is defined as

$$(T_{\alpha}^a f)(x) = \lim_{\varepsilon \rightarrow 0} \frac{f(x + \varepsilon(x - a)^{1-\alpha}) - f(x)}{\varepsilon}. \quad (3)$$

¹giro.candelario@intec.edu.do

Let us remark that, if f is differentiable, then $(T_\alpha^a f)(x) = (x-a)^{1-\alpha} f'(x)$ and if f is α -differentiable in (a, b) , for some $b \in \mathbb{R}$, then $(T_\alpha^a f)(a) = \lim_{x \rightarrow a^+} (T_\alpha^a f)(x)$.

The left conformable fractional derivative holds the property of integer derivatives, $T_\alpha^a C = 0$, being C a constant. Conformable derivative is the most natural definition of fractional derivative, also, it does not require the evaluation of special functions, which involves a low computational cost compared with existing fractional derivatives.

In [6], a Taylor power series of $f(x)$ is provided, where the conformal derivative is defined at a different point from where it is evaluated, which is a key fact in the convergence analysis of our proposed scheme.

Theorem 1 (Theorem 4.1, [6]). *Let $f(x)$ be an infinitely α -differentiable function for $\alpha \in (0, 1]$, at the neighborhood of a_1 with conformable derivative starting from a . The fractional power series for $f(x)$ is:*

$$f(x) = f(a_1) + \frac{(T_\alpha^a f)(a_1)\delta_1}{\alpha} + \frac{(T_\alpha^a f)^{(2)}(a_1)\delta_2}{2\alpha^2} + R_2(x, a_1, a), \quad (4)$$

being $\delta_1 = H^\alpha - L^\alpha$, $\delta_2 = H^{2\alpha} - L^{2\alpha} - 2L^\alpha\delta_1$, \dots , and $H = x - a$, $L = a_1 - a$.

It is easy to prove that $\delta_2 = \delta_1^2$, $\delta_3 = \delta_1^3$, etc.

In next Section, the conformable fractional Newton-type method is obtained from Taylor power expansion (4).

2 Methods

To obtain a fractional Newton-type method from (4), let us regard the approximation of this Taylor power series to order one evaluated at the solution \bar{x} ,

$$f(x) \approx f(\bar{x}) + \frac{(T_\alpha^a f)(\bar{x})\delta_1}{\alpha}. \quad (5)$$

As it is known that $f(\bar{x}) = 0$, and $\delta_1 = H^\alpha - L^\alpha$, being $H = x - a$ and $L = a_1 - a$ ($a_1 = \bar{x}$), expression (5) can be rewritten as

$$f(x) \approx \frac{(T_\alpha^a f)(\bar{x})}{\alpha} [(x - a)^\alpha - (\bar{x} - a)^\alpha]. \quad (6)$$

Now, $(\bar{x} - a)^\alpha$ can be isolated as

$$(\bar{x} - a)^\alpha \approx (x - a)^\alpha - \alpha \frac{f(x)}{(T_\alpha^a f)(\bar{x})}. \quad (7)$$

So, from $(\bar{x} - a)^\alpha$, \bar{x} can be estimated as

$$\bar{x} \approx a + \left((x - a)^\alpha - \alpha \frac{f(x)}{(T_\alpha^a f)(\bar{x})} \right)^{1/\alpha}. \quad (8)$$

Regarding the iterates x_k and x_{k+1} as approximations of the solution \bar{x} , we obtain the Conformable fractional Newton-type method as

$$x_{k+1} = a + \left((x_k - a)^\alpha - \alpha \frac{f(x_k)}{(T_\alpha^a f)(x_k)} \right)^{1/\alpha}, \quad k = 0, 1, 2, \dots \quad (9)$$

Let us call this method TFN.

In the next result, the order of convergence of this method is stated. This is the first optimal fractional method according to Kung and Traub's conjecture (see [6]).

Theorem 2. Let $f : D \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function in the interval D containing the zero \bar{x} of $f(x)$. Let $(T_\alpha^a f)(x)$ be the conformable fractional derivative of $f(x)$ starting from a , with order $\alpha \in (0, 1]$. Let us suppose $(T_\alpha^a f)(x)$ is continuous and not null at \bar{x} . If an initial approximation x_0 is sufficiently close to \bar{x} , then the local order of convergence of the conformable fractional Newton-type method

$$x_{k+1} = a + \left((x_k - a)^\alpha - \alpha \frac{f(x_k)}{(T_\alpha^a f)(x_k)} \right)^{1/\alpha}, \quad k = 0, 1, 2, \dots$$

is at least 2, being $0 < \alpha \leq 1$, and the error equation is

$$e_{k+1} = \alpha(\bar{x} - a)^{\alpha-1} C_2 e_k^2 + O(e_k^3),$$

being $C_j = \frac{1}{j! \alpha^j} \frac{(T_\alpha^a f)^{(j)}(\bar{x})}{(T_\alpha^a f)(\bar{x})}$ for $j = 2, 3, 4, \dots$

Once the method has been designed and its convergence analyzed, it is necessary to test its performance on some nonlinear problems.

3 Results

In this section, we make some numerical tests on nonlinear equations in order to check its efficiency and reliability. We compare methods CFN and TFN with the classical Newton-Raphson scheme is made (when $\alpha = 1$).

For our test, we use Matlab R2019b with double precision arithmetics, $|f(x_{k+1})| < 10^{-8}$ or $|x_{k+1} - x_k| < 10^{-8}$ as stopping criterium, and a maximum of 500 iterations. For CFN method we use $a = 0$, the program made in [8] for computing the Gamma function, and the code provided by Igor Podlubny in Mathworks for the calculation of Mittag-Leffler function. For TFN method we consider $a = -10$. The initial estimation used is the same in each procedure.

Our test function is $f_1(x) = -12.84x^6 - 25.6x^5 + 16.55x^4 - 2.21x^3 + 26.71x^2 - 4.29x - 15.21$ with roots $\bar{x}_1 = 0.82366 + 0.24769i$, $\bar{x}_2 = 0.82366 - 0.24769i$, $\bar{x}_3 = -2.62297$, $\bar{x}_4 = -0.584$, $\bar{x}_5 = -0.21705 + 0.99911i$ and $\bar{x}_6 = -0.21705 - 0.99911i$. In Table 1, we can see that TFN method requires less iterations than CFN method for the same values of α , even less than classical Newton's scheme when $\alpha \leq 0.4$. It can also be observed that ACOC is 1 if $\alpha \neq 1$ in CFN method, whereas ACOC keeps being 2 or even greater than 2 if $\alpha \neq 1$ in TFN method.

α	CFN method					TFN method				
	\bar{x}	$ f(x_{k+1}) $	$ x_{k+1} - x_k $	iter	ACOC	\bar{x}	$ f(x_{k+1}) $	$ x_{k+1} - x_k $	iter	ACOC
1	\bar{x}_3	$4.16 \cdot 10^{-12}$	$3.47 \cdot 10^{-8}$	11	2.00	\bar{x}_3	$4.16 \cdot 10^{-12}$	$3.47 \cdot 10^{-8}$	11	2.00
0.9	\bar{x}_3	$7.96 \cdot 10^{-5}$	$8.11 \cdot 10^{-9}$	68	0.98	\bar{x}_3	$6.18 \cdot 10^{-13}$	$7.17 \cdot 10^{-9}$	11	2.00
0.8	\bar{x}_1	$1.94 \cdot 10^{-5}$	$9.99 \cdot 10^{-9}$	123	0.99	\bar{x}_3	$4.18 \cdot 10^{-12}$	$1.41 \cdot 10^{-9}$	11	2.00
0.7	\bar{x}_2	$1.1 \cdot 10^{-14}$	$9.94 \cdot 10^{-9}$	389	1.00	\bar{x}_3	$1.6 \cdot 10^{-12}$	$2.67 \cdot 10^{-10}$	11	2.00
0.6	-	-	-	500	-	\bar{x}_3	$1.6 \cdot 10^{-12}$	$4.81 \cdot 10^{-11}$	11	2.00
0.5	-	-	-	500	-	\bar{x}_3	$2.26 \cdot 10^{-12}$	$8.3 \cdot 10^{-12}$	11	2.00
0.4	-	-	-	500	-	\bar{x}_3	$2.91 \cdot 10^{-9}$	$8.89 \cdot 10^{-7}$	10	2.01
0.3	-	-	-	500	-	\bar{x}_3	$4.62 \cdot 10^{-10}$	$3.54 \cdot 10^{-7}$	10	2.01
0.2	-	-	-	500	-	\bar{x}_3	$7.36 \cdot 10^{-11}$	$1.38 \cdot 10^{-7}$	10	2.00
0.1	-	-	-	500	-	\bar{x}_3	$2.26 \cdot 10^{-12}$	$5.27 \cdot 10^{-8}$	10	2.00

Table 1: CFN and TFN results for $f_1(x)$ with initial estimation $x_0 = -2.2$

In order to check also the stability of fractional Newton-type methods, we analyze the dependence on initial estimates by using convergence planes as defined in [9] and used in [3] and [5].

To construct the convergence planes, we regard the initial estimates in horizontal axis and values of $\alpha \in (0, 1]$ in vertical axis. Each color represents a different solution found, and it is painted in black when no solution was found in 500 iterations. Each plane is made with a 400×400 grid, and tolerance of 0.001.

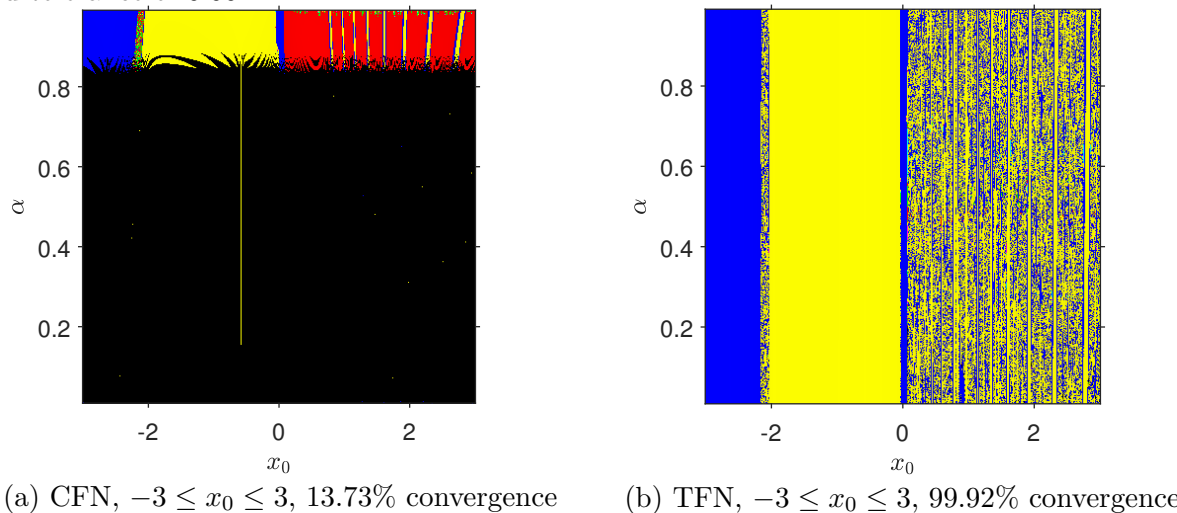


Figure 1: Convergence planes of CFN and TFN on $f_1(x)$

In Figure 1 we see that TFN method has a much higher percentage of convergence than CFN method, and the convergence is guaranteed even for lower values of α for a wide range of initial estimations.

4 Conclusions

As far as we know, we have proposed the first optimal fractional Newton-type method, by using Conformable derivatives. The fractional derivative used has the most natural definition, so, in this method is not required the evaluation of special functions, which involves a low computational cost compared with the existing fractional Newton-type methods. Also, the order of convergence of this method is quadratic, unlike the existing ones. Numerical tests were made, and the dependence on initial estimates was analyzed, confirming the theory. It can be concluded that this method shows a better numerical behavior than fractional Newton-type methods previously proposed, even than classical Newton-Raphson method in some cases. It was also observed that is possible to obtain both, real and complex roots, with real initial estimates, and that it is possible to get different roots not only by choosing a different initial estimate, but also by choosing a different value of α .

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