A FINITE ELEMENT METHOD FOR NEUTRON NOISE ANALYSIS IN HEXAGONAL REACTORS

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ABSTRACT

The early detection of anomalies through the analysis of the neutron noise recorded by in-core and ex-core instrumentation gives the possibility to take proper actions before such problems lead to safety concerns or impact plant availability. The study of the neutron fluctuations permits to detect and differentiate anomalies depending on their type and possibly to characterize and localize such anomalies. This method is non-intrusive and does not require any external perturbation of the system. To effectively use the neutron noise for reactor diagnostics it is essential to accurately model the effects of the anomalies on the neutron field. This paper deals with the development and validation of a neutron noise simulator for reactors with different geometries. The neutron noise is obtained by solving the frequency-domain two-group neutron diffusion equation in the first order approximation. In order to solve this partial differential equation a code based on a high order finite element method is developed. The novelty of this simulator resides on the possibility of dealing with rectangular meshes in any kind of geometry, thus allowing for complex domains and any location of the perturbation. The finite element method also permits automatic refinements in the cell size (h-adaptability) and in its polynomial degree (p-adaptability) that lead to a fast convergence. In order to show the possibilities of the neutron noise simulator developed a perturbation in a hexagonal two-dimensional reactor is investigated in this paper.

KEYWORDS: Neutron noise; Finite element method; Hexagonal geometry

1. INTRODUCTION

Being able to monitor the state of nuclear reactors while they are running at nominal conditions is a safety requirement. The early detection of anomalies gives the possibility to take proper actions before such problems lead to safety concerns or impact plant availability. The CORTEX project [1], funded by the European Commission in the Euratom 20162017 work program, aims at developing an innovative core monitoring technique that allows detecting anomalies in nuclear reactors, such as excessive vibrations of core internals, flow blockage, coolant inlet perturbations, etc. The technique is based on using the inherent fluctuations in neutron flux recorded by in-core and ex-core instrumentation, referred to as neutron noise, from which the anomalies will be detected and differentiated depending on their type, location and characteristics. The method is non-intrusive and does not require any external perturbation of the system.

To be able to detect, localise and quantify a perturbation in real-time, an automatic algorithm based on machine learning has to be provided with a large set of simulation data [2]. As the number of experiments to effectively train the machine learning algorithms is huge, these experiments must be carried out in a time efficient manner, i.e. fast running techniques are required to carry out the simulations. One useful technique to solve the effect of a perturbation in the neutron noise is to resolve the frequency-domain first-order neutron noise equation in the diffusion approximation. This is a partial differential equation with complex numbers.

This work presents a neutron noise simulator developed with the finite element method, called *FEMFFUSION*. It can deal with any kind of geometry allowing complex domains and any location of the perturbation. In other words, it computes the same quantities as the frequency domain code *CORE SIM* [3] but allowing any kind of geometry and a more adaptable structure. Also, the finite element method also permits automatic refinements in the cell size (h-adaptability) and in its polynomial degree (p-adaptability) that leads to an exponentially fast convergence.

2. THE NEUTRON DIFFUSION EQUATION

In the two energy group approximation, the time-dependent neutron diffusion equation with one group of delayed neutrons, where the matrices are denoted by [ ], are defined as [4]

$$[\mathrm{v}^{-1}]\frac{\partial \phi}{\partial t} - \vec{\nabla}\cdot\left([D]\vec{\nabla}\phi\right) + [\Sigma_T]\phi = (1-\beta_{\text{eff}})\chi(\nu\Sigma_f)^T\phi + \lambda_{\text{eff}}\chi\mathcal{C}, \tag{1}$$

$$\frac{\partial \mathcal{C}}{\partial t} = \beta_{\text{eff}}(\nu\Sigma_f)^T\Phi - \lambda_{\text{eff}}\,\mathcal{C}, \tag{2}$$

where the cross sections matrices are defined as

$$[\mathrm{v}^{-1}] = \begin{bmatrix} \frac{1}{\mathrm{v}_1} & 0 \\ 0 & \frac{1}{\mathrm{v}_2} \end{bmatrix}, [\Sigma_T] = \begin{bmatrix} \Sigma_{a1}+\Sigma_{12} & 0 \\ -\Sigma_{12} & \Sigma_{a2} \end{bmatrix},$$

$$[D] = \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix}, \quad \nu\Sigma_f = \begin{bmatrix} \nu\Sigma_{f1} \\ \nu\Sigma_{f2} \end{bmatrix}, \quad \chi = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The main unknown of the neutron diffusion equation is the space- and time dependent neutron flux, in its usual separation in the fast and thermal energy groups $\phi = \left[\phi_1(\vec{r},\,t),\quad \phi_2(\vec{r},\,t)\right]^T$, and the neutron precursor concentration $\mathcal{C}(\vec{r},\,t)$. All other quantities have their usual meaning [4].

2.1. Static problem

For a given transient analysis in a core reactor, usually, a static configuration of the reactor is considered as initial condition. Associated with the time dependent neutron diffusion equation, (1) and (2), the static solution takes the form

$$-\vec{\nabla}\cdot\left([D]\vec{\nabla}\phi\right) + [\Sigma_T]\phi = \frac{1}{k_{\text{eff}}}\chi(\nu\Sigma_f)^T\phi. \tag{3}$$

This problem is known as the *Lambda Modes* problem for a given configuration of the reactor core. To solve the problems (3), a spatial discretization of the equations has to be selected. In this work, a high order continuous Galerkin finite element method is used leading to an algebraic eigenvalue problem associated with the discretization of equation (3) with the following block structure,

$$\begin{bmatrix} L_{11} & 0 \\ -L_{21} & L_{22} \end{bmatrix} \tilde{\Phi} = \frac{1}{\lambda} \begin{bmatrix} M_{11} & M_{12} \\ 0 & 0 \end{bmatrix} \tilde{\Phi} \,, \tag{4}$$

where $\tilde{\Phi} = \left[\tilde{\phi}_1,\ \tilde{\phi}_2\right]^T$ are the algebraic vectors of weights associated with the fast and thermal neutron fluxes. More details on the spatial discretization used can be found in [5].

The resulting algebraic eigenvalue problem is solved using the Block Inverse-Free Preconditioned Arnoldi Method (BIFPAM) [6] or Newton iteration solver [7].

3. FIRST-ORDER NEUTRON NOISE THEORY

The first-order neutron noise theory is based on splitting every time dependent term, expressed as $X(\vec{r},t)$, into their mean value, X_0,which is considered as the steady-state solution, and their fluctuation around the mean value, δX as,

$$X(\vec{r},\, t) = X_0(\vec{r}) + \delta X(\vec{r},\, t). \tag{5}$$

The fluctuations are assumed to be small compared to the mean values. This allows to neglect second-order terms $(\delta X(\vec{r},\, t) \times \delta X(\vec{r},\, t)) \approx 0$. Also, the fluctuations of the diffusion coefficients are neglected and $\delta D_g \approx 0$ is assumed. This approximation was demonstrated to be valid for light water reactor applications [8]. Thus, the first-order neutron noise equation can be written as [9].

$$-\vec{\nabla} \cdot \Big([D] \vec{\nabla} \delta\phi(\vec{r},\, \omega) \Big) + [\Sigma_{\text{dyn}}]\ \delta\phi(\vec{r},\, \omega) = \delta S(\vec{r},\, \omega), \tag{6}$$

The perturbation source term $\delta S(\vec{r},\, \omega)$ is given by the frequency-domain as changes in the cross sections :

$$\delta S(\vec{r},\, \omega) = \begin{bmatrix} \delta S_1(\vec{r},\, \omega) \\ \delta S_2(\vec{r},\, \omega) \end{bmatrix} = \phi_s\, \delta\Sigma_{12} + \phi_a \begin{bmatrix} \delta\Sigma_{a1} \\ \delta\Sigma_{a2} \end{bmatrix} + \frac{1}{k_{\text{eff}}} [\phi_f] \begin{bmatrix} \delta\nu\Sigma_{f1} \\ \delta\nu\Sigma_{f2} \end{bmatrix}, \tag{7}$$

where

$$[\Sigma_{\text{dyn}}] = \begin{bmatrix} \Sigma_1 & -\nu\Sigma_{f2}\left(1 - \frac{j\omega\beta_{\text{eff}}}{j\omega + \lambda_{\text{eff}}}\right) \\ -\Sigma_{12} & -\Sigma_{a2} + \frac{j\omega}{\text{v}_2} \end{bmatrix}, \qquad \phi_s = \begin{bmatrix} -\phi_1 \\ \phi_1 \end{bmatrix},$$

$$[\phi_a] = \begin{bmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{bmatrix}, \qquad [\phi_f] = \left(1 - \frac{j\omega\beta_{\text{eff}}}{j\omega + \lambda_{\text{eff}}}\right) \begin{bmatrix} \phi_1 & \phi_2 \\ 0 & 0 \end{bmatrix},$$

$$\Sigma_1 = \Sigma_a + \frac{j\omega}{\text{v}_1} + \Sigma_{12} - \nu\Sigma_{f1}\left(1 - \frac{j\omega\beta_{\text{eff}}}{j\omega + \lambda_{\text{eff}}}\right).$$

By comparing with Eqs. (3), it can be seen that the neutron noise equation is an in-homogeneous equation with complex quantities that has to be solved after the steady-state solution is obtained because ϕ_1 and ϕ_2 represent the steady state fast and thermal neutron fluxes, respectively. The related static eigenvalue problem must be solved with the same spatial discretization as the frequency domain neutron noise equation to get coherent results.

Applying the continuous Galerkin finite element discretization to Eq. (6) leads to an algebraic linear system of equation with the following block structure

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \delta\tilde{\Phi} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} , \tag{8}$$

where $\delta\tilde{\Phi} = \left[\delta\tilde{\phi}_1,\ \delta\tilde{\phi}_2\right]^T$ are the algebraic vectors of weights associated with the fast and thermal neutron noise fluxes.

4. NUMERICAL RESULTS

To study the possibilities of the *FEMFFUSION* code, a two-dimensional hexagonal reactor (2D IAEA benchmark) was considered. This benchmark has a 1/12 reflective symmetry but as the inserted perturbation is not symmetrical, the whole reactor is solved. The fuel assembly pitch is 20.0 cm. Table 1 shows the cross section data for the this reactor. Figure 1 shows the materials layout.

A perturbation in the fuel assembly marked with material 5 of 10% of its cross sections reference values needs to be solved inserted to verify the developed noise simulator:

$$\begin{aligned} &\delta\Sigma_{a1} = 0.00095042, &&\delta\Sigma_{a2} = 0.00750058, &&\delta\Sigma_{s12} = 0.00177540, \\ &\delta\Sigma_{f1} = 0.00058708, &&\delta\Sigma_{f2} = 0.00960670. \end{aligned}$$

Table 2 shows the convergence of the solution depending on the polynomial degree used in the FEM shape functions (FED). We have defined the following error indicators

$$\begin{aligned} \Delta k_{\text{eff}} &= k_{\text{eff}} - k^*_{\text{eff}}, \\ \varepsilon_g &= 100 \times \frac{1}{N_A} \sum_{a=1}^{N_A} \frac{\phi_{a,g} - \phi^*_{a,g}}{\phi^*_{a,g}}\ \%, \qquad g = 1,\ 2, \\ \eta_g &= 100 \times \frac{1}{N_A} \sum_{a=1}^{N_A} \frac{\delta\phi_{a,g} - \delta\phi^*_{a,g}}{\delta\phi^*_{a,g}}\ \%, \qquad g = 1,\ 2. \end{aligned}$$

where values with * represent reference results extracted from [10] for the steady-state results and a very fine FEM computation with a finite element polynomial degree equals to 7 and each cell refined into 16 cells for the neutron noise results. $\phi^*_{a,g}$ and $\delta\phi^*_{a,g}$ are the mean flux and the average noise flux, respectively, at assembly a. N_A is the number of assemblies in the reactor. Figure 2 represents the mean assembly flux values for the steady state solution using FED $= 3$. Figure 3 presents the neutron noise magnitude and Figure 4 displays the neutron noise phase. The results shows that the fast neutron noise has ab influence over a wider region. On the other hand, the thermal neutron noise is mostly localized. Also, for this perturbation, the phase of the neutron noise is similar throughout the entire reactor.

Table 1: Cross section data for 2D IAEA reactor.

Material	Group	D_g	Σ_{ag}	$\nu\Sigma_{fg}$	Σ_{12}
1	1	1.5	0.010	0.000	0.020
	2	0.4	0.080	0.135	
2	1	1.5	0.010	0.000	0.020
	2	0.4	0.085	0.135	
3	1	1.5	0.010	0.000	0.020
	2	0.4	0.013	0.135	
4	1	1.5	0.010	0.000	0.040
	2	0.4	0.013	0.000	
5	1	1.5	0.010	0.000	0.020
	2	0.4	0.013	0.135	

Code	FED	DoFs	Δk_{eff} (pcm)	ε_1 (%)	ε_2 (%)	η_1 (%)	η_2 (%)
FEMFFUSION	1	553	680	23.31	19.34	21.53	15.81
FEMFFUSION	2	2119	95	2.51	1.97	2.40	1.79
FEMFFUSION	3	4699	6	0.25	0.17	0.23	0.17
FEMFFUSION	5	12 901	0	0.00	0.00	0.04	0.04

Table 2: Convergence table for 2D IAEA reactor.

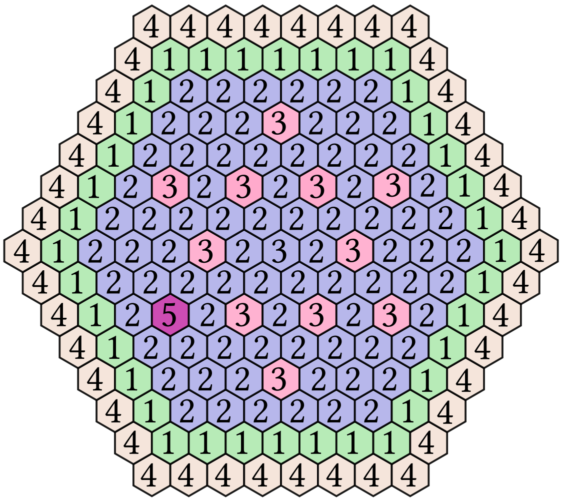
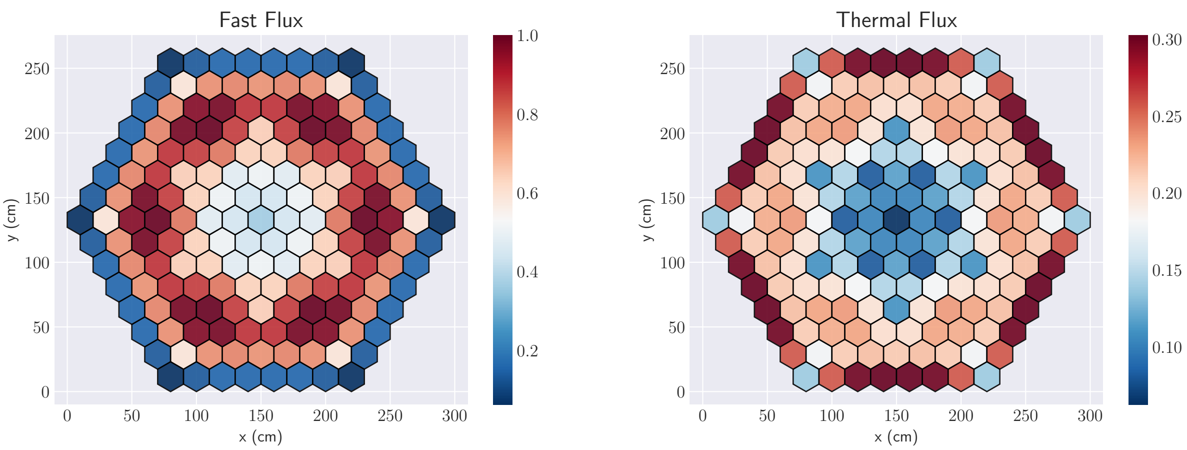


Figure 1: Materials layout in the 2D-IAEA benchmark.



(a) Fast Flux **(b) Thermal Flux**

Figure 2: Static neutron fluxes in the IAEA reactor.

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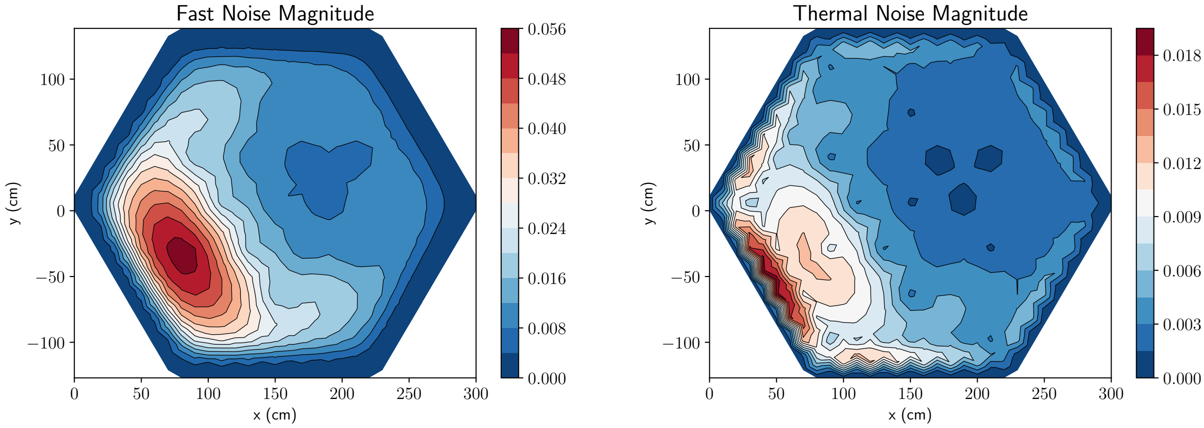
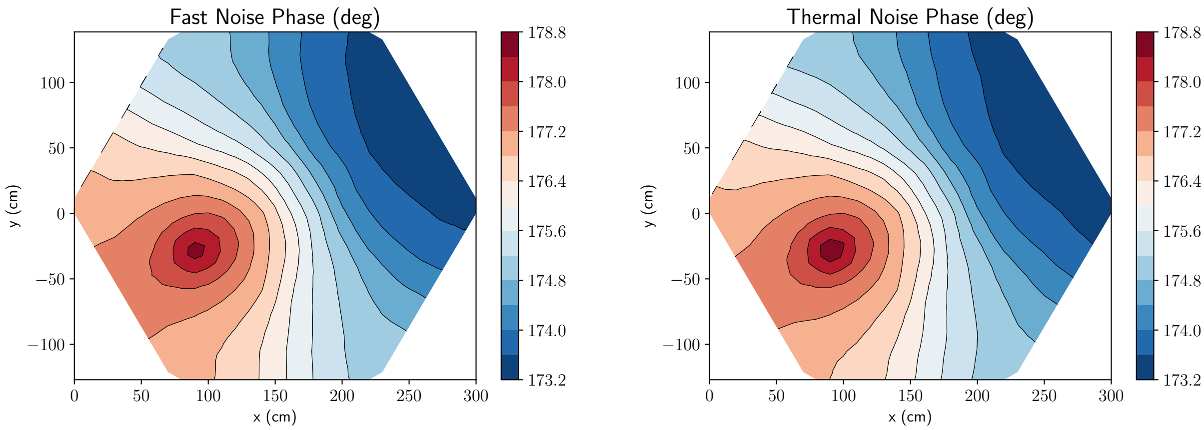


(a) Fast Flux Noise **(b) Thermal Flux Noise**

Figure 3: Magnitude of the noise in the 2D IAEA reactor.



(a) Fast Flux Noise **(b) Thermal Flux Noise**

Figure 4: Phase of the noise in the 2D IAEA reactor

5. CONCLUSIONS

This work presents a neutron noise simulator developed with the finite element method. It can deal with different kinds of geometry allowing complex domains as hexagonal reactors and any location of the perturbation. Also, the finite element method permits automatic refinements in the cell size and in its polynomial degree that leads to fast spatial convergence. This code will permit to train machine learning algorithms to detect perturbations in real-time in operating nuclear reactors.

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REFERENCES

[1] C. Demazière, P. Vinai, M. Hursin, S. Kollias, and J. Herb. "Noise-Based Core Monitoring and Diagnostics - Overview of the project." In *Advances in Reactor Physics (ARP-2017)*, pp. 1–4. Mumbai, India (2017).

[2] F. D. S. Ribeiro, F. Caliva, D. Chionis, A. Dokhane, A. Mylonakis, C. Demaziere, G. Leontidis, and S. Kollias. "Towards a Deep Unified Framework for Nuclear Reactor Perturbation Analysis." In *2018 IEEE Symposium Series on Computational Intelligence (SSCI)*. IEEE (2018). URL https://doi.org/10.1109/ssci.2018.8628637.

[3] C. Demazière. "CORE SIM: A multi-purpose neutronic tool for research and education." *Ann Nucl Energy*, **volume 38**(12), pp. 2698–2718 (2011).

[4] W. M. Stacey. *Nuclear Reactor Physics*. Wiley, Weinheim, Germany (2007).

[5] A. Vidal-Ferràndiz, R. Fayez, D. Ginestar, and G. Verdú. "Solution of the Lambda modes problem of a nuclear power reactor using an h–p finite element method." *Annals of Nuclear Energy*, **volume 72**, pp. 338–349 (2014).

[6] A. Vidal-Ferràndiz, A. Carreño, D. Ginestar, and G. Verdú. "A Block Arnoldi Method for the SPN Equations." *International Journal of Computer Mathematics*, pp. 1–22 (2019).

[7] A. Carreño, L. Bergamaschi, A. Martinez, A. Vidal-Ferràndiz, D. Ginestar, and G. Verdú. "Block Preconditioning Matrices for the Newton Method to Compute the Dominant λ-Modes Associated with the Neutron Diffusion Equation." *Mathematical and Computational Applications*, **volume 24**(1), p. 9 (2019).

[8] V. Larsson and C. Demazière. "Comparative study of 2-group and diffusion theories for the calculation of the neutron noise in 1D 2-region systems." *Annals of Nuclear Energy*, **volume 36**(10), pp. 1574–1587 (2009).

[9] I. Pázsit and C. Demazière. "Noise Techniques in Nuclear Systems." In *Handbook of Nuclear Engineering*, pp. 1629–1737. Springer US (2010).

[10] Y. A. Chao and Y. A. Shatilla. "Conformal Mapping and Hexagonal Nodal Methods —II: Implementation in the ANC-H Code." *Nuclear Science and Engineering*, **volume 121**(2), pp. 210–225 (1995).