Advances in the physical approach to personality dynamics

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1 Introduction

The objective of this paper is to advance in the mathematical formalism that states a bridge between Physics and Psychology presented in [1]. The short-term personality dynamics can be modelled by a stimulus-response model: an integro-differential equation. The bridge between Physics and Psychology is provided when the stimulus-response model can be formulated as a Newtonian equation with its corresponding minimum action principle. This principle provides the current Lagrangian and Hamiltonian functions. This Hamiltonian function is a non-conserved energy because it depends explicitly on time. Then, some changes provided by the physical scientific literature of the last decades [2, 3] can derive into an approach where a Hamiltonian function is conserved: the Ermakov-Lewis energy. A theoretical application case is presented for the case of an individual that consumes 10 mg of methylphenidate. The stimulus dynamics, the Ermakov-Lewis energy with its kinetic and potential energies, and the GFP dynamical response are presented and discussed for this case.

2 Precedents

The stimulus-response model is given by the following integro-differential equation:

$$\left.\begin{array}{c}\dot{q}(t) = a(b - q(t)) + \delta \cdot s(t) \cdot q(t) - \sigma \cdot \int_{t_0}^{t} e^{\frac{r-t}{\tau}} \cdot s(r) \cdot q(r)dr \\ q(t_0) = q_0\end{array}\right\} \tag{1}$$

In Eq. 1 $q(t)$ is the General Factor of Personality (GFP) dynamics measured in activation units (au) and $s(t)$ an arbitrary stimulus. The conserved Ermakov-Lewis energy obtained in [1] is:

$$E = T_e + V_e = \frac{1}{2}\left(\sqrt{u(t)} \cdot C(t) \cdot \dot{q} + C^2(t) \cdot \dot{A}(t) + \frac{1}{2}\left(C(t)\frac{\dot{u}(t)}{\sqrt{u(t)}} - \sqrt{u(t)} \cdot C(t)\right) q\right)^2 +$$

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$$+\frac{k}{2}\left(\frac{\sqrt{u\left(t\right)}}{C\left(t\right)}q+A\left(t\right)\right)^{2} \quad (2)$$

In Eq. 2 k is an arbitrary constant and $u(t)$ is the generalized mass, given by $u\left(t\right)=u_0 e^{\left(a+\frac{\tau}{t}\right)(t-t_0)-\delta\int_{t_0}^{t}s(r)dr}$, where u_0 is an arbitrary constant, and the $C(t)$ and $A(t)$ variables hold the equations:

$$\ddot{C}(t)+\Omega(t)\cdot C(t)=\frac{k}{C^3(t)} \quad (3)$$

$$\ddot{A}(t)+2\frac{\dot{C}(t)}{C(t)}\dot{A}(t)+kA(t)C^4(t)+\frac{a\cdot b}{\tau}\frac{\sqrt{u(t)}}{C(t)}=0 \quad (4)$$

The analytical solution of $q(t)$ can be written in function of these two variables as:

$$q\left(t\right)=\begin{cases}\frac{C(t)}{\sqrt{u(t)}}\left(-A(t)+k_1\frac{dr}{C^2(r)}+k_2\right): & k=0\\ \frac{C(t)}{\sqrt{u(t)}}\left(-A(t)+k_1\cdot\sin\left(\sqrt{k}\int_{t0}^{t}\frac{dr}{C^2(r)}\right)+k_2\cdot\cos\left(\sqrt{k}\int_{t0}^{t}\frac{dr}{C^2(r)}\right)\right): & k>0\\ \frac{C(t)}{\sqrt{u(t)}}\left(-A(t)+k_1\cdot\exp\left(-\sqrt{-k}\int_{t0}^{t}\frac{dr}{C^2(r)}\right)+k_2\cdot\exp\left(-\sqrt{-k}\int_{t0}^{t}\frac{dr}{C^2(r)}\right)\right): & k<0\end{cases} \quad (5)$$

3 Advances

The advances presented in the present work are specified in the following. First of all, in order to choose the initial conditions for $A(t)$ and $C(t)$, the following assumptions in $t=t_0$ in Eq. 2 are done: $u_0=1$, $C_0=1$, $A_0=0$ au, $\dot{A}_0=0$ au $\cdot t^{-1}$ and $\frac{1}{2}C_0\frac{\dot{u}_0}{\sqrt{u_0)}}-\sqrt{u_0}\cdot\dot{C}_0=0$, which provide $\dot{C}_0=1/2\dot{u}_0 t^{-1}=1/2(a+\frac{1}{\tau}-\delta\cdot s_0)t^{-1}$, and also provide the initial value of the Ermakov-Lewis energy:

$$E=E_0=\frac{1}{2}\dot{q}_0^2+\frac{k}{2}q_0^2\ \mathrm{au}^2\cdot\mathrm{t}^{-2}=\frac{1}{2}(a(b-q_0)+\delta\cdot s_0\cdot q_0)^2+\frac{k}{2}q_0^2\ \mathrm{au}^2\cdot\mathrm{t}^{-2} \quad (6)$$

Note that Eq. 2 is a classical addition of kinetic and potential energy, whose value is conserved for all the GFP evolution period as a consequence of a stimulus, which is equal to the value of Eq. 6. In addition the choice of q(t) in Eq. 5 is clear: the $k>0$ case. The case k=0 has the unstable term $k_1\int_{t_0}^{t}\frac{dr}{C^2(r)}$, and the $k<0$ case has the unstable term $k_1\cdot exp\left(\sqrt{-k}\int_{t_0}^{t}\frac{dr}{C^2(r)}\right)$. Once the case $k>0$ has been chosen as the stable one, the comparison of Eq. 5 in $t=t_0$ with the initial values in Eq. 1 provides that $k_1=\frac{\dot{q}_0}{\sqrt{k}}$ and $k_2=q_0$ with $\dot{q}_0=a(b-q_0)+\delta\cdot s_0\cdot q_0$. Observe that finally one parameter is non-fixed. The preferred option is taking k_1 as the free parameter due to the k parameter (with dimensions T^{-2}) can be considered in future studies as a measure of the resistance of the individual to change its personality (as compared with a harmonic oscillator in Physics).

Then, the conclusion is that the Ermakov-Lewis energy of Eq. 2 can be written as:

$$E=\frac{1}{2}\dot{q}_0^2+\frac{1}{2}\frac{\dot{q}_0^2}{k_1^2}q_0^2=T_e+V_e=$$

$$=\frac{1}{2}\left(\sqrt{u(t)}\cdot C(t)\cdot\dot{q}+C^2(t)\cdot\dot{A}(t)+\left(\frac{1}{2}C(t)\frac{\dot{u}(t)}{\sqrt{u(t)}}-\sqrt{u(t)}\cdot\dot{C}(t)\right)q\right)^2+$$

$$+\frac{1}{2}\frac{\dot{q}_0^2}{k_1^2}\left(\frac{\sqrt{u(t)}}{C(t)}q+A(t)\right)^2 \quad (7)$$

Moreover, the $q(t)$ dynamics is written as:

$$q(t) = \frac{C\,(t)}{\sqrt{u\,(t)}}\left(-A(t) + k_1 \cdot \sin\left(\frac{\dot{q}_0^2}{k_1^2}\int_{t_0}^{t}\frac{dr}{C^2 r}\right) + q_0 \cos\left(\frac{\dot{q}_0^2}{k_1^2}\int_{t_0}^{t}\frac{dr}{C^2 r}\right)\right) \tag{8}$$

Note in Eqs. 7 and 8 that $\dot{q}_0 = a(b - q_0) + \delta \cdot s_0 \cdot q_0$, and that k_1 is a free but positive-valued parameter.

4 Application case

An application of the theoretical approach here developed is presented now in order to study the personality dynamics as a consequence of one methylphenidate dose consumption. To obtain the simplest mathematical structure of methylphenidate dynamics $s(t)$ a two level pharmacokinetics model [4] is considered:

$$\left.\begin{array}{l} \frac{dm(t)}{dt} = -\alpha \cdot m(t) \\ m(t_0) = M \end{array}\right\} \tag{9}$$

$$\left.\begin{array}{l} \frac{ds(t)}{dt} = \alpha \cdot m(t) - \beta \cdot s(t) \\ s(t_0) = s_0 \end{array}\right\} \tag{10}$$

In Eq. 9 $m(t)$ represents the evolution of methylphenidate before entering in the organism's plasma and metabolizing system, being M the methylphenidate initial amount and being α the methylphenidate assimilation rate. In Eq. 10 the $s(t)$ variable represents the methylphenidate amount in organism, assuming that its initial value is s_0, i.e., the neither metabolized nor excreted methylphenidate of a possible previous consumption, and β is the methylphenidate elimination rate. This coupled differential equations system can be integrated:

$$s\,(t) = s_0 e^{-\beta \cdot t} + \begin{cases} \alpha \cdot M\beta - \alpha e - \alpha \cdot t - e^{-\beta \cdot t}: & \alpha \neq \beta, \\ \alpha \cdot M \cdot t \cdot e - \alpha \cdot t: & \alpha = \beta. \end{cases} \tag{11}$$

The application case is a theoretical case in which one subject consumes 10 mg of methylphenidate, and his GFP is measured every 7.5 minutes during 180 minutes (3 hours), with the 5 adjectives scale of the GFP-FAS in the hedonic scale [5] in units called as activation units (au), inside the interval [0,50] au, i.e., each adjective is scored inside the interval [0,10] in the hedonic scale. A real ABC experimental design can be seen in [6]. The initial condition q_0 is also measured before consumption, and its value is $q_0 = 20.0$ au. The model parameters are chosen to reproduce a U-inverted GFP response with a recovering period under the tonic level with an asymptotic convergence to this value as $t \to +\infty$. Besides, the assimilation and elimination rates values for methylphenidate vary inside the following confidence intervals: $\alpha \in [0.00617, 0.02173]$ min^{-1} and $\beta \in [0.00566, 0.01451]$ min^{-1} (95% confidence) [7]. The concrete parameter values proposed for this application case but inspired in [6] are presented in Table 1.
Observe in Table 1 that the initial value of methylphenidate is $s_0 = 0$ mg, i.e., the individual has not consumed methylphenidate from very long before. In addition, all the computations and figures here presented have been done with MATHEMATICA in a period three times the period of the experimental design of [7], i.e., in a period of $3 \cdot 180$ min $= 540$ min $= 9$ hours, in order to appreciate its asymptotic dynamical behaviours. Note that Figure 1 presents the methylphenidate dynamics given by Eq. 11 and its asymptotic trend to zero.
Take into account to compute the evolution of the Ermakov-Lewis energy and the kinetic and potential energies given by Eq. 7, as well as the $q(t)$ or GFP dynamics by Eq. 8, that: (a) the

Table 1: Parameter values of the application case

Parameter name	Symbol	Values with units
Initial GFP	q_0	20 au
Initial stimulus	s_0	0 mg
Methylphenidate initial amount	M	10 mg
Methylphenidate assimilation rate	α	0.01746 min^{-1}
Methylphenidate elimination rate	β	0.01385 min^{-1}
Homeostatic control power	a	0.00536 min^{-1}
Tonic level	b	27.49 au
Excitation effect power	δ	0.0061267 $\text{mg}^{-1} \cdot \text{min}^{-1}$
Inhibitor effect power	σ	0.000161 $\text{mg}^{-1} \cdot \text{min}^{-2}$
Inhibitor effect delay	τ	40.64 min

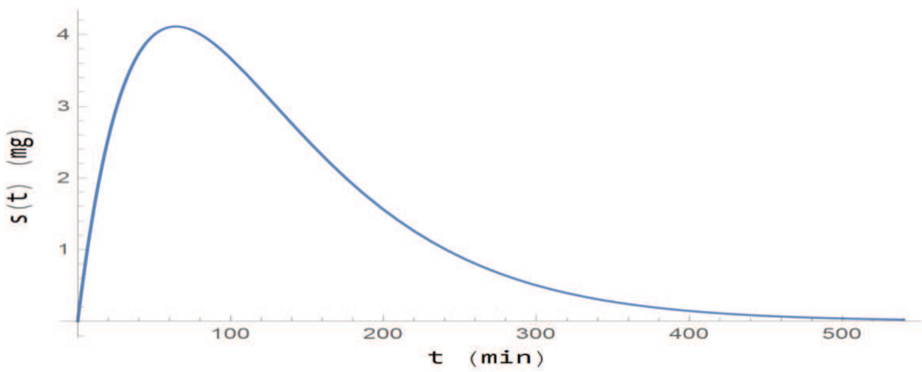

Figure 1: Stimulus dynamics (methylphenidate's amount evolution inside organism) given by Eq. 11 versus time.

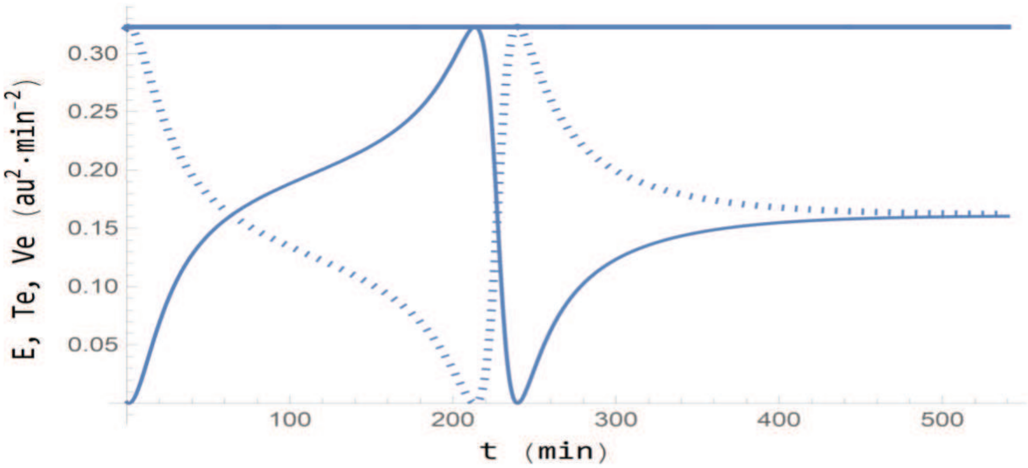

Figure 2: Ermakov-Lewis energy (E, upper straight line), kinetic energy (Te, first increasing line), and potential energy (Ve, first decreasing dotted line), versus time.

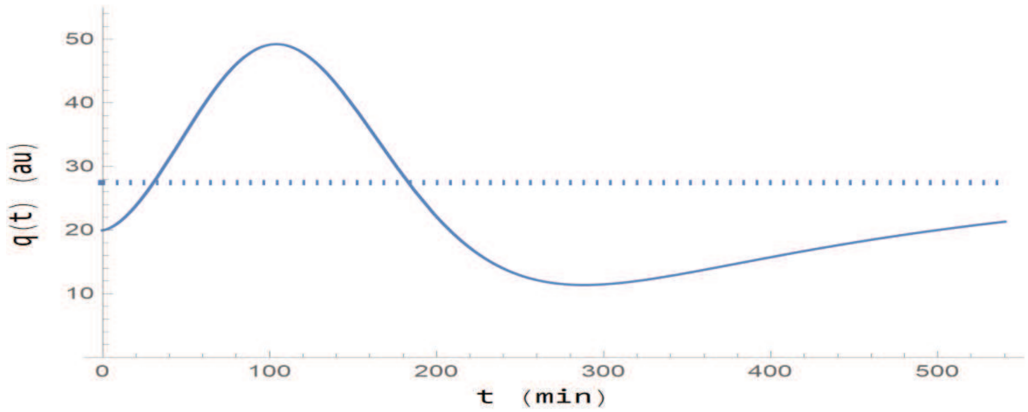

Figure 3: GFP or $q(t)$ dynamics (curve line) and the tonic level $b = 27.49$ au (dotted straight line), versus time.

$A(t)$ and $C(t)$ auxiliary variables dynamics have been solved numerically by Eqs. 3 and 4 with the initial values provided in above; (b) the free parameter value k_1 has been chosen as $k_1 = 1$ au, thus $k = \frac{\dot{q}_0^2}{k_1^2} = \frac{(a(b-q_0)+\delta \cdot s_0 \cdot q_0)^2}{1} = 0.0016$ min^{-2} (note that $s_0 = 0$ mg).
In addition, Figure 2 presents the evolution of the Ermakov-Lewis energy with value $E = E_0 = \frac{1}{2}\dot{q}_0^2 + \frac{1}{2}\frac{\dot{q}_0^2}{k_1^2}q_0^2 = 0.3228$ au (constant), jointly with its kinetic and potential partial energies. Note that, at the beginning, almost all energy is potential but it transforms continuously into kinetic energy, until an instant after 200 minutes in which suddenly the evolution is inverted to converge towards a common value, approximately to half the total Ermakov-Lewis energy.
Moreover, Figure 3 presents the GFP evolution or $q(t)$ dynamics together with the tonic level ($b = 27.49$ au). Note the U-inverted shape GFP response with a recovering period under the tonic level ($b = 27.49$ au) and an asymptotic convergence to this value as $t \to +\infty$. Thus, this hypothetical individual clearly reproduces the response pattern pointed out by the literature as a consequence of a stimulant drug.

5 Conclusions and Future work

The here presented finding about a bridge or "isomorphism" between Physics and Psychology, concretely between analytical dynamics and personality theory, must be emphasized. The conclusion is that we can apply the energy conservation principle of Physics to obtain the GFP dynamics produced by some environmental stimuli; in fact we can consider some psychological mechanisms as analogous to those of Physics. An application of the theoretical concept of energy is to consider the invariant Ermakov-Lewis energy as an amount of personality energy involved in a dynamic response to a stimulus, i.e., having a characteristic number that represents this complex dynamics, since energy is a scalar magnitude. This characteristic energy amount could be used in inferential statistics, with the sense that a dynamics could be reduced to a representative scalar. In fact, it has been done already in the context of an application case where 28 individuals consumed alcohol [8]. On the other hand, the inspiration obtained from the application cases of the Ermakov-Lewis energy in Physics should be considered. One of the most important application for the authors is the one related with the quantum approach, similar for instance to that considered in the work [9]. In this approach, the tonic level as asymptotic state in Eq. 1 is not considered, and the quantization rules are applied on the Hamiltonian. Then, a time-dependent Schrödinger equation arises, from which the wave function can be solved analytically in a similar way that it has been provided for the Ermakov-Lewis energy. The wave function provides the quantum version of Hamilton equations deduced by Bohm & Hiley [10] from the Schrödinger equation, which are stochastic, and from which quantized trajectories and bifurcations can be studied. Thus, multiple GFP dynamical response patterns with their corresponding asymptotic states can arise. Therefore, the authors' hypothesis is that this approach could provide: (a) a way to study the normal and the disorder dynamical patterns of personality; (b) how a bifurcation can steer, as a consequence of a stimulus, from a normal pattern of personality to a disordered one. Then, those sudden changes that many times are observed in personality theory could have a mathematical explanation.

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