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Sustainability of Management Decisions in a Digital Logistics Network

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Abstract: Globalization has given a powerful impetus to the development of international commercial activity and logistics management systems taking full advantage of cross-border networking. The solution lies at the intersection of information technologies, technical means of machine-to-machine (M2M) interaction, mobile high-speed networks, geolocation, cloud services, and a number of international standards. The current trend towards creating digital logistics platforms has set a number of serious challenges for developers. The most important requirement is the condition of sustainability of the obtained solutions with respect to disturbances in the conditions of logistics activities caused not only by market uncertainty but also by a whole set of unfavorable factors accompanying the transportation process. Within the framework of the presented research, the problem of obtaining the conditions for the stability of solutions obtained on the basis of mathematical models is set. At the same time, the processes of transferring not only discrete but also continuous material flows through complex structured networks are taken into account. This study contains the results of the analysis of the stability of solutions of differential systems of various types that simulate the transfer processes in network media. Initial boundary value problems for evolutionary equations and differential-difference systems are relevant in logistics, both for the discrete transportation of a wide range of goods and for the quasi-continuous transportation of, for example, liquid hydrocarbons. The criterion for the work of a logistics operator is the integral functional. For the mathematical description of the transport process of continuous and discrete media, a wide class of integrable functions are used, which adequately describe the transport of media with a complex internal rheological structure.

Keywords: logistics; sustainability; digitalization; continuous mathematical model; discrete mathematical model; optimization

1. Introduction

Different factors could facilitate the transformation of economies to sustainable economies [1,2] as well as bioeconomies [3].

This study presents a thematic description of the process of transfer of continuous and discrete media by formalisms of differential systems with continuously varying time and differential-difference systems with discretely varying time. In 2010, the authors completed a qualitative analysis of evolutionary transport equations with a spatial variable changing on a network or a network-like domain (the existence and uniqueness of the solution, the correctness of the stated initial-boundary value problems). The latter led to the emergence of further research by the authors that in 2020 ended with scientific results in the direction of studying the properties of solutions (stability, stabilization, and controllability of system solutions) and the practical use of the obtained results, the logical conclusions of which were the results of this work. It should be noted that the use of the mentioned mathematical results is a distinctive feature of this work and represents an advantage over most studies of an economic orientation, in which the issues of the stability of mathematical models of an economic nature, and therefore the stability of processes, are extremely rarely and superficially studied or are not considered at all. The authors also note that the results obtained are new—the authors are not aware of similar publications in the studied direction.

The development of correct mathematical models reflecting the evolution of material flows in networks is a complex task and requires the involvement of a serious mathematical apparatus [4]. For the found theoretical solutions and algorithms, real-world business puts forward the additional condition of the study of their sustainability. It is necessary due to such factors as the instability of market conditions, the spread of parameters of mathematical models, and the inaccuracy of the initial data.

2. Materials and Methods

2.1. The Purpose of the Research and Methods

The goal of the study, due to its complexity, includes two tasks.

The first task is to find the stability of the solution of an evolutionary parabolic system with continuous time. The system has distributed parameters on the graph used as a prototype of the logistics transport network [5,6]. The mathematical model itself describes the process of transferring material flows through a spatial network. The parabolic system is studied using the class of integrable functions reflecting mass transfer with a complex internal rheology. A generalized solution or weak solution of the system in this case is an integrable function that determines the variational formulation of the initial boundary value problem.

The second problem involves the analysis of an analogue of the initial boundary value problem for a differential system of parabolic type with a discretely varying time variable: a differential-difference system of equations with a spatial variable changing on a graph (network) that defines a mathematical model of the transfer process in discrete time. To construct such an analogue, we use the E. Rote method of semi-sampling over a time variable. The conditions for the existence of a weak solution of a differential-difference system and the analysis of its stability are obtained. The results of the study can be used in problems, such as initial boundary value problems and differential-difference systems of equations, of the optimization of network-like transfer processes.

One of the most relevant problems is the discrepancy between the development of hardware and brainware. The problem is that although hardware for logistics platforms is represented by a full line of modern technical means for tracking the flow of goods and cargo, as well as the exchange of information between all participants in the logistics process, the brainware algorithms have only just begun to develop in the direction of network logistics. For practice, we need algorithms that, first of all, are optimal by economic criteria and acceptable for embedding in the software of servers of digital logistics platforms.

The following parameters are noted separately: the traffic level; the capacity of the routes; the spread of the waiting time and the process of loading and unloading operations; customs services when crossing state borders; the influence of weather conditions on the speed of movement; technical failures of carriers; and other factors known to logistics management. In addition, the costs are affected by fluctuations in the cost of stevedoring operations, mileage rates in multimodal logistics, a difficult-to-predict range of prices for the rental of port and station infrastructure, etc.

Their impact can be assessed using the theory of random functions and the theory of queuing (or queuing theory, QT). Traditionally, for these purposes, the methods of A. M. Lyapunov have been used to assess the sustainability of systems described by differential equations for small perturbations of the initial condition. The development of this approach lies in the area of logistics networks modeling taking into account the sustainability of material flows. This improvement considers the methods that use maps, graph theory, and systems of equations from the apparatus of mathematical physics and applies methods of the theory of optimal decision-making.

Within the framework of the presented research, the problem of obtaining the conditions for the stability of solutions obtained on the basis of mathematical models is set. At the same time, the processes of transferring not only discrete but also continuous material flows through complex structured networks are taken into account. This study contains the results of the analysis of the stability of solutions of differential systems of various types that simulate the transfer processes in network media. The problems under consideration are relevant to logistics, both for the discrete transportation of a wide range of goods and for the quasi-continuous transportation of, for example, liquid hydrocarbons.

A generalized solution is a function that defines the variational formulation of an initial boundary value problem or a differential-difference system. Going beyond the classical continuously differentiable solutions is dictated by the need to describe more precisely the physical nature of the processes of transport of continuous media (e.g., gas, oil, petroleum products) or discrete flows (commercial goods and cargo) through logistics networks.

The analysis of the influence of perturbations of the source data on the behavior of a real logistics system and an abstract mathematical model (the solutions obtained either in the class of initial boundary value problems or the system of differential-difference equations used) is critical primarily from the point of view of economic indicators of commercial activity. This makes it possible to assess the risks for investors as well as analyze the sustainability of the proposed business models.

Note that in the class of ordinary differential equations, the results of the mathematical theory of stability have been obtained and are widely applied. The new tasks, dictated by the need to manage economic processes, require optimization according to efficiency criteria containing at least two variables, one of which is the current time of the process. In such economic applications, due to the increasing complexity of algorithms for modeling evolutionary transfer processes, there is a natural need to use formalisms of equations and systems of partial differential equations that are more appropriate to the specifics of evolutionary processes.

In the present paper, the abovementioned formalisms are in practice taken to be the main ones in the mathematical description of evolutionary processes on networks, and their analysis is the main goal of the study. Namely, we will obtain the sustainability conditions for solutions for two classes of evolutionary differential systems—systems with a continuously varying time variable and systems with a discretely varying time variable.

2.2. Main Definitions

We use the classical notation and concepts introduced by the authors to describe differential systems with a spatial variable changing on a graph Γ (in applications on a logistics network). At the same time, the internal structure of the graph can be arbitrary (the graph can contain cycles) while remaining connected and bounded.

Due to the use of functions with a carrier on the graph (and, consequently, differential equations with a variable changing on the graph), at each edge of $\gamma \subset \Gamma$ a parametrization is introduced by the segment $[0,1]$.

The case of parametrization by different segments is not excluded. In accordance with the notation introduced in [3,4], $\partial\Gamma$, $J(\Gamma)$ are the sets of boundary ζ and internal ξ nodes of Γ, respectively.

By $\Gamma_0 \subset \Gamma$ we denote a graph that does not include boundary and end nodes: $\Gamma_0 = \Gamma \backslash (\partial\Gamma \cup J(\Gamma))$, and $\Gamma_t = \Gamma_0 \times (0,t)$ ($\gamma_t = \gamma_0 \times (0,t)$), $\partial\Gamma_t = \partial\Gamma \times (0,t)$ ($t \in (0,T]$,

$T < \infty$—constant). We use the concept of the Lebesgue integral on the graph Γ and on the domain Γ_t:

$\int\limits_{\Gamma} f(x)dx = \sum\limits_{\gamma} f(x)_\gamma dx$, $\int\limits_{\Gamma_t} f(x,t)dxdt = \sum\limits_{\gamma_t} f(x,t)_\gamma dxdt$, respectively;

$f(\cdot)_\gamma$ is the narrowing of the function $f(\cdot)$ to a fixed edge γ.

We use the classical space $C(\Gamma)$ of continuous functions on a graph and the Lebesgue space $L_p(\Gamma)$ $(p = 1,2)$ of pth-degree measurable and summable functions with norm $\| u \|_{2,\Gamma} = (\int\limits_{\Gamma} u^2(x)dx)^{1/2}$ and the space $L_p(\Gamma_T)$ with norm:

$$\| u \|_{2,\Gamma_T} = (\int\limits_{\Gamma_T} u^2(x,t)dxdt)^{1/2}$$

as well as a space $L_{2,1}(\Gamma_T)$ of functions from $L_1(\Gamma_T)$ with norm:

$$\| u \|_{L_{2,1}(\Gamma_T)} = \int\limits_0^T (\int\limits_{\Gamma} u^2(x,t)dx)^{1/2} dt$$

In the analysis of differential equations and systems of equations with a spatial variable changing on the graph Γ, we use the main Sobolev function spaces: $W_2^1(\Gamma)$ with elements from $L_2(\Gamma)$, for which the derivative of the first order (the generalized derivative) belongs to $L_2(\Gamma)$; the space of functions $W_2^{1,0}(\Gamma_T)$ with elements from $L_2(\Gamma_T)$, for which the derivative of the first order in the spatial variable x belongs to $L_2(\Gamma_T)$, and the space $W_2^1(\Gamma_T)$ is introduced similarly; and $V_2(\Gamma_T)$ is composed by the set of time-continuous t dependencies $u(x,t) \in W_2^{1,0}(\Gamma_T)$, which satisfy the finite norm

$$\| u \|_{2,\Gamma_T} \equiv \max_{0 \le t \le T} \|u(\cdot,t)\|_{L_2(\Gamma)} + \|u_x\|_{L_2(\Gamma)} \quad (1)$$

We further introduce the state space with auxiliary spaces. This is necessary not only for the decision-making process, but also for the development of the conditions for the existence of these decisions. To do this, in $W_2^1(\Gamma)$, consider the expression:

$$\ell(\mu,\nu) = \int\limits_{\Gamma} \left(a(x)\frac{d\mu(x)}{dx}\frac{d\nu(x)}{dx} + b(x)\mu(x)\nu(x) \right) dx \quad (2)$$

defining a bilinear form with fixed measurable and bounded on Γ_0 coefficients $a(x)$,$b(x)$ from the space $L_2(\Gamma)$:

$$0 < a_* \le a(x) \le a^*, |b(x)| \le \beta, x \in \Gamma_0$$

2.3. Theoretical Fundamentals

The authors consider a statement that is proved in the work [5].

Lemma 1. *Let the relation $\ell(u,v) - \int\limits_{\Gamma} f(x)\eta(x)dx = 0$ hold for the function $u(x) \in W_2^1(\Gamma)$, where $\eta(x) \in W_2^1(\Gamma)$ is an arbitrary function and $f(x) \in L_2(\Gamma)$ is fixed. Then, the narrowing of $a(x)_\gamma \frac{du(x)_\gamma}{dx}$ is continuous for the set of internal nodes of the graph under consideration for an arbitrary edge $\gamma \subset \Gamma$.*

We introduce a set of $\Omega_a(\Gamma)$ functions $u(x)$ that satisfy the statement of Lemma 1 and the following relations, called balance conditions in applications:

$$\sum_{\gamma \in R(\xi)} a(1)_\gamma \frac{du(1)_\gamma}{dx} = \sum_{\gamma \in r(\xi)} a(0)_\gamma \frac{du(0)_\gamma}{dx}$$

for any internal node $\xi \in J(\Gamma)$ of the graph (in applications, the nodes of accumulation and distribution of the transported medium of the logistics network); here, $R(\xi)$ and $r(\xi)$ denote the sets of edges γ oriented "to node ξ" and "from node ξ", respectively.

The operation of closing $\Omega_a(\Gamma)$ by the norm of the space $W_2^1(\Gamma)$ leads to the definition of the space $W^1(a, \Gamma)$. If we assume that for $u(x)$ from the set $\Omega_a(\Gamma)$ there is a boundary condition $u(x)|_{\partial\Gamma} = 0$, then another space $W_0^1(a, \Gamma)$ is defined.

We introduce a set of $\Omega_a(\Gamma_T)$ functions $u(x,t) \in V_2(\Gamma_T)$, whose traces exist on the sections of the domain Γ_T with the plane $t = t_0$ ($t_0 \in [0, T]$) and belong to the space $W_0^1(a, \Gamma)$. The following relations are also valid for them:

$$\sum_{\gamma \in R(\xi)} a(1)_\gamma \frac{\partial u(1,t)_\gamma}{\partial x} = \sum_{\gamma \in r(\xi)} a(0)_\gamma \frac{\partial u(0,t)_\gamma}{\partial x} \tag{3}$$

(here, $\xi \in J(\Gamma)$ is an arbitrary node of the logistics network). The operation of closing $\Omega_a(\Gamma_T)$ by norm (1) leads to the definition of the space $V^{1,0}(a, \Gamma_T)$; the following inclusion of $V^{1,0}(a, \Gamma_T) \subset W_2^{1,0}(\Gamma_T)$ is obvious.

We define another subspace for $W_2^{1,0}(\Gamma_T)$ by the closure in the norm $W_2^{1,0}(\Gamma_T)$ of the set of functions that are differentiable and meet the conditions (3) in $\xi \in J(\Gamma)$ and in $t \in [0, T]$.

We introduce the notation $W^{1,0}(a, \Gamma_T)$ for it and also define it similarly $W^1(a, \Gamma_T)$; in this case, $W^{1,0}(a, \Gamma_T) \subset W_2^{1,0}(\Gamma_T)$ is performed (in applications using this simulation, this corresponds to the absorption condition).

We have defined the $V^{1,0}(a, \Gamma_T)$ domain of states of the differential system as well as additionally $W^{1,0}(a, \Gamma_T)$ and $W^1(a, \Gamma_T)$. The main difference between $V^{1,0}(a, \Gamma_T)$ and the spaces $W^{1,0}(a, \Gamma_T)$, $W^1(a, \Gamma_T)$ is that the elements $W^{1,0}(a, \Gamma_T)$, $W^1(a, \Gamma_T)$ have no continuity over the time variable t.

The proven statement is of great practical importance in the formation of economic indicators for the evaluation of network logistics. Indeed, the integrated criterion that reflects the cost of moving goods depends on both the distance traveled and the time. Such dependencies have jumps caused by changes in tariffs for multimodal transportation, the approach to the expiration date of a wide range of consumer goods in the fast-moving consumer goods (FMCGs) segment, the terms of the transport lease, port infrastructure, demurrage, detention, or contractual penalties for short delivery or late shipment, and other factors related to logistics activities.

3. Results

3.1. Mathematical Model of Transfer with Continuously Changing Time

The use of the introduced formalisms for modeling the processes of moving commodities through the logistics network structure allows us to strictly mathematically describe the following algorithm.

On the elements of the spaces $W^{1,0}(a, \Gamma_T)$ and $V^{1,0}(a, \Gamma_T)$, we consider the differential equation

$$\frac{\partial y(x,t)}{\partial t} - \frac{\partial}{\partial x}\left(a(x)\frac{\partial y(x,t)}{\partial x}\right) + b(x)y(x,t) = f(x,t) \tag{4}$$

It is a system of parabolic differential equations with distributed parameters $a(x)$ and $b(x)$, which characterize the quantitative indicators of the flow state $y(x,t)$ (the transfer rate, transport efficiency, and other criteria) along the edges γ of the graph Γ as a portrait of the logistics network $f(x,t) \in L_{2,1}(\Gamma_T)$.

The state function $(x, t \in \overline{\Gamma}_T)$ $(x, t \in \overline{\Gamma}_T)$ for system (4) in the network domain $\overline{\Gamma}_T$ is defined by a generalized solution $y(x,t)$ of system (4) that satisfies the initial and boundary conditions of the form:

$$y\big|_{t=0} = \varphi(x), \quad x \in \Gamma, \quad y\big|_{x \in \partial\Gamma_T} = 0 \tag{5}$$

the $\varphi(x) \in L_2(\Gamma)$ is here.

Above, we have already defined the summability with a square for the established functions $a(x)$ and $b(x)$.

From the condition of belonging to $y(x,t) \in V^{1,0}(a, \Gamma_T)$ and summability with the square of the functions $a(x)$, $b(x)$, it follows that the state $y(x,t)$, as a map of the economic criterion on the graph, i.e., the map $y : [0, T] \to W_0^1(a, \Gamma) \subset L_2(\Gamma)$, is a continuous function on the graph, so that the first relation in (5) makes sense and is understood for almost all values of its variable.

For simplicity of further presentation, we use the Dirichlet boundary condition (the second relation in (5)), which in the applications means the absence of a flow (inflow) of a continuous medium in the boundary nodes of the network, i.e., we consider the first initial boundary value problem (4), (5).

We present the main statements and fragments of the proof of the validity of the obtained results in the part that is necessary for the study of the sustainability of solutions. This is primarily applied in the analysis of the adoption of management strategies by managers of logistics services and risk management of logistics business processes. To do this, consider the solution first in the auxiliary space $W^{1,0}(a, \Gamma_T)$, then in the main space $V^{1,0}(a, \Gamma_T)$.

Definition 1. *A generalized solution of the initial boundary value problem (4), (5) is understood as an element (function) $y(x,t) \in W^{1,0}(a, \Gamma_T)$ that satisfies the identity of the integral type:*

$$-\int_{\Gamma_T} y(x,t)\frac{\partial \eta(x,t)}{\partial t}dxdt + \ell_T(y, \eta) = \int_{\Gamma} \varphi(x)\eta(x,0)dx + \int_{\Gamma_T} f(x,t)\eta(x,t)dxdt \tag{6}$$

for an arbitrary element $\eta(x,t) \in W^1(a, \Gamma_T)$ equal to zero for $t = T$; $\ell_t(y, \eta)$ denotes the bilinear form with respect to the elements u, η:

$$\ell_t(y, \eta) = \int_{\Gamma_t} \left(a(x)\frac{\partial y(x,t)}{\partial x}\frac{\partial \eta(x,t)}{\partial x} + b(x)y(x,t)\eta(x,t) \right) dxdt, \quad t \in (0, T]$$

It should be noted that the solvability of problem (4), (5) in $W^{1,0}(a, \Gamma_T)$ (and $V^{1,0}(a, \Gamma_T)$) is established by representing the solution of $y(x,t) \in V^{1,0}(a, \Gamma_T)$ as a series using the system of generalized eigenfunctions of the boundary value problem (the problem for an elliptic equation):

$$-\frac{d}{dx}\left(a(x)\frac{du(x)}{dx} \right) + b(x)u(x) = \lambda u(x), u(x)|_{\partial\Gamma} = 0 \tag{7}$$

in the class of functions of the $W^1(a, \Gamma)$ space [6,7].

It follows that, under assumption (2), the spectral problem (7) has a set of eigenvalues $\{\lambda_i\}_{i \geq 1}$ forming a countable set. The following properties of the spectral characteristics of problem (7) are valid:

1. The eigenvalues are real and have finite multiplicity, they are numbered in ascending order of modules (taking into account multiplicities): $\{\lambda_k\}_{k \geq 1}$, each eigenvalue corresponds to its own real generalized eigenfunction, and the set of generalized eigenfunctions forms the sequence $\{\phi_k(x)\}_{k \geq 1}$;

2. The numbers λ_k are positive except for a finite number of the first ones; for $b(x) > 0$, all numbers λ_k are positive;
3. The set $\{\phi_k(x)\}_{k\geq 1}$ is an orthogonal basis in the spaces $W_0^1(a,\Gamma)$ and $L_2(\Gamma)$ (below $\parallel \phi_k \parallel_{L_2(\Gamma)} = 1$);
4. For the boundary value problem $\Lambda\phi = \lambda\phi + g$, $g \in L_2(\Gamma)$ generated by the differential expression:

$$\Lambda u = -\frac{d}{dx}\left(a(x)\frac{du(x)}{dx}\right) + b(x)u(x),$$

is an alternative to Fredholm in the $W_0^1(a,\Gamma)$ space.

Note 1. In applied problems of practical economics, the coefficient $b(x)$ is a positive function, which means that all eigenvalues of the boundary value problem described in the expression (7) are positive. To find the sustainability conditions of evolutionary systems with an elliptic part defined by the differential operator $\Lambda u = -\frac{d}{dx}\left(a(x)\frac{du(x)}{dx}\right) + b(x)u(x)$, the a priori inequality $b(x) > 0$ is the main one for establishing the stability conditions.

Theorem 1. *Let the following conditions be satisfied: $f(x,t) \in L_{2,1}(\Gamma_T)$, $\varphi(x) \in L_2(\Gamma)$, $b(x)$ is a non-negative function. The initial boundary value problem (4), (5) in the space $W_0^1(a,\Gamma)$ (and the space $V^{1,0}(a,\Gamma_T)$) has a unique generalized solution.*

To prove unambiguous weak solvability in the space $W_0^1(a,\Gamma)$ (and then in the space $V^{1,0}(a,\Gamma_T)$), we use the Faedo–Galerkin method with a special basis—the set of generalized eigenfunctions $\{\phi_k(x)\}_{k\geq 1}$ of the class $W_0^1(a,\Gamma)$ of the one-dimensional elliptic operator generated by the differential expression:

$$\Lambda u = -\frac{d}{dx}\left(a(x)\frac{du(x)}{dx}\right) + b(x)u(x)$$

The representation of the solution as a series in the system $\{\phi_k(x)\}_{k\geq 1}$ and the subsequent analysis of the convergence of this series together with its derivative in the time variable complete the proof. The uniqueness of the solution is established by the classical method using the linearity of the elliptic part of Equation (4).

3.2. Mathematical Model of Transport with Discretely Varying Time

Next, we consider a mathematical model of the transfer process in a discrete time change. The mathematical description is based on the Rote method of semi-discretization, which allows us to represent the mathematical model by formalisms of a differential-difference system of ordinary differential equations on a graph, i.e., a system of equations with a spatial variable changing on the graph. In this case, the theorem of the existence of a solution in the space $W_0^1(a,\Gamma)$ is established and the ways of analyzing the stability of this solution are indicated. In the space $W_0^{1,0}(a,\Gamma_T)$ of the states $u(x,t)$) of system (4), we consider the initial boundary value problem (4), (5).

We dissect the region Γ_T with the planes $t = k\tau$, $k = 0,1,2,\ldots,M$, $\tau = \frac{T}{M}$, while the sections Γ_T for any k are Γ. Equation (4) is replaced by the differential-difference equation:

$$\frac{1}{\tau}(u(k) - u(k-1)) - \frac{d}{dx}\left(a(x)\frac{du(k)}{dx}\right) + b(x)u(k) = f_\tau(k), \quad (8)$$

where $f_\tau(k) \equiv f_\tau(x,k) = \frac{1}{\tau}\int\limits_{(k-1)\tau}^{k\tau} f(x,t)dt$ and $k = 1,2,\ldots,M$.

It is clear that $f_\tau(k) \in L_2(\Gamma)$, $k = 1,2,\ldots,M$.

The functions $u(k)$ $k = 1, 2, \ldots, M$ are defined as weak solutions of the elliptic equation (8) satisfying the conditions

$$u(0) = \varphi(x), \quad u(k)|_{x \in \partial\Gamma} = 0. \tag{9}$$

Thus, for a fixed k relation (8), (9) is the boundary value problem for the elliptic equation (8) with respect to $u(k)$.

Note 2. The relations (8), (9) are analogous to the implicit first-order difference scheme of approximation with respect to the time variable t for the initial boundary value problem (4), (5), given in the space $W_0^{1,0}(a, \Gamma_T)$, with the elliptic operator:

$$-\frac{d}{dx}\left(a(x)\frac{d\omega}{dx}\right) + b(x)\omega, \ \omega \in W_0^1(a, \Gamma).$$

Definition 2. *Elements (functions) $u(k)$, $(k = 1, 2, \ldots, M)$ of the space $W_0^1(a, \Gamma)$ are called weak solutions of the differential-difference equation (8) with conditions (9) if they satisfy the integral identity:*

$$\int_\Gamma u(k)_t \nu(x)dx + \ell(u(k), \nu(x) = \int_\Gamma f_\tau(k)\nu(x)dx \tag{10}$$

for any element (function) $\nu(x) \in W_0^1(a, \Gamma)$; the equality $u(0) = \varphi(x)$, as equality in spaces e $L_2(\Gamma)$, is understood almost everywhere, $u(k)_t \equiv u(x, k)_t = \frac{1}{\tau}[u(k) - u(k-1)]$.

Theorem 2. *Under the conditions $f(x, t) \in L_{2,1}(\Gamma_T)$ and $\varphi(x) \in L_2(\Gamma)$, the functions $u(k)$ $(k = 1, 2, \ldots, M)$ for sufficiently small τ $(\tau < \tau_0)$ are uniquely defined as elements of the space $W_0^1(a, \Gamma)$.*

Proof. Note, first of all, that from the condition $f(x, t) \in L_{2,1}(\Gamma_T)$ and the representation $f_\tau(k)$, $f_\tau(k) \in L_2(\Gamma)$ follows by virtue of the very definition of the space $L_{2,1}(\Gamma)$. Let $k = 1$. Based on properties 3 and 4 of the boundary value problem $\Lambda\phi = \lambda\phi$, $\phi|_{\partial\Gamma} = 0$, and the relation:

$$\Lambda u(1) = -\frac{1}{\tau}u(1) + f_\tau(1) + \frac{1}{\tau}u(0), \quad u(0) = \varphi(x) \in L_2(\Gamma),$$

the statement for the function $u(1)$ follows. The same statement, given the ratio

$$\Lambda u(k) = -\frac{1}{\tau}u(k) + f_\tau(k) + \frac{1}{\tau}u(k-1), \quad u(0) = \varphi(x) \in L_2(\Gamma),$$

remains true for $k = 2, 3, \ldots, M$. Below, when obtaining an a priori estimate for the functions $u(k)$, the boundary τ_0 for the change τ will be specified. The theorem is proved.

3.3. Sustainability of the Control Solutions of the Transfer Processes

The stability of mathematical models of the transfer process (controlling the decision process) with continuously and discretely changing time is considered below. Note that the term "process control solution" in applied problems of practical economics means fixing (selecting) the initial data of the problem in accordance with the current economic need and the capabilities of the economic entity.

3.3.1. Sustainability of Solutions with Continuously Changing Time

We will conduct a mathematically based study of the stability parameters of the found solution of system (4), which describes the dynamics of the logistics process, formalized in the form of mass transfer of goods and cargo through the network. The transport network, in turn, is represented as a mathematical graph.

Assume that $0 \le b(x) \le \beta$ for $x \in \Gamma$, which guarantees the positivity of the eigenvalues λ_i, $i \ge 1$. In the cylinder $\Gamma_\infty = \Gamma_0 \times (0, \infty)$, consider the system (4) defined above.

Let us introduce the notation: $\Gamma_{t_0,t} = \Gamma_0 \times (t_0, t)$, $\partial\Gamma_{t_0,t} = \partial\Gamma \times (t_0, t)$, $(0 < t_0 < t < \infty)$, $\Gamma_{t_0,\infty} = \Gamma_0 \times (t_0, \infty)$, $\partial\Gamma_{t_0,\infty} = \partial\Gamma \times (t_0, \infty)$; obviously $\Gamma_{t_0,t} \subset \Gamma_t$.

By the assumption given above, $f(x,t) \in CL_{2,1}(\Gamma_T)$ ($CL_{2,1}(\Gamma_T)$ is denoted by the space of functions continuous over the variable t and the norm of the space $L_{2,1}(\Gamma_T)$), and:

$$\int_t^{t+1} \| f(\cdot, \varsigma) \|^2_{L_2(\Gamma)} \, d\varsigma \le A$$

($A > 0$ is a fixed constant) for an arbitrary $t \ge 0$.

Let the weak solution for (4) be $\overline{y}(x,t) \in V^{1,0}(a, \Gamma_{t_0,\infty})$, which is a generalized solution of system (4) in $\Gamma_{t_0,\infty}$. In this case, we formulate the initial and boundary conditions as follows:

$$y\Big|_{t=t_0} = \overline{\varphi}(x), \quad x \in \Gamma, \quad y\Big|_{x \in \partial\Gamma_{t_0,\infty}} = 0.$$

We introduce the relation $y(x,t) \in V^{1,0}(a, \Gamma_{t_0,\infty})$ valid for the weak solution of system (4) in the $\Gamma_{t_0,\infty}$ graph under consideration. At the same time, we also formulate the initial and boundary conditions as follows:

$$y\Big|_{t=t_0} = \varphi(x), \quad x \in \Gamma, \quad y\Big|_{x \in \partial\Gamma_{t_0,\infty}} = 0$$

The weak solution $\overline{y}\,(x,t)$ for (4) is denoted by the term 'unperturbed'. On the other hand, for the solution $y\,(x,t)$, we introduce the definition of 'perturbed'. The previously obtained relations for the generalized solution for the system (4), (5) determine the dependencies $\overline{y}(x,t)$, $y(x,t)$ in $\Gamma_{t_0,\infty}$ corresponding to the conditions. They also make it possible to determine $V^{1,0}(a, \Gamma_{t_0,\infty})$ at $f(x,t) \in CL_{2,1}(\Gamma_\infty)$.

We define the stability of its weak solution for (4), (5) in the same way as the Lyapunov stability for the problem under consideration.

Definition 3. *We assume that the unperturbed weak solution $\overline{y}(x,t)$ for equations (4) is stable in the case when for any $t_0 > 0$ and $\varepsilon > 0$ there exists $\delta(t_0, \varepsilon) > 0$ such that if $\| \varphi - \overline{\varphi} \|_{L_2(\Gamma)} < \delta(t_0, \varepsilon)$, then done inequality $\| y\,(\cdot,t) - \overline{y}\,(\cdot,t) \|_{W^1(a,\Gamma)} < \varepsilon$ $t \ge t_0$. In this case, $y\,(x,t)$ is a perturbed weak solution for (4).*

It is also possible for the practical needs of management to introduce a definition of the conditions of uniform sustainability. This is necessary for long-term planning of logistics activities. In this case, the unperturbed state of system (4) in the domain $\Gamma_{t_0,\infty}$ under consideration will be determined completely by analogy with Definition 3.

Additionally, for practical applications in long-term planning, the asymptotic and exponential stability can be determined in the framework of (4). Note that the reformulation is permissible due to the linearity of the system (4).

We show that the unperturbed weak solution (state) of system (4) in Γ_T is stable if the inequality $0 < b(x) \le \beta$, $x \in \Gamma$ is satisfied.

This is a consequence of the linearity of system (4). Let $\theta(x,t) = y(x,t) - \overline{y}(x,t)$, which means $\theta(x,t) \in V^{1,0}(a, \Gamma_{t_0,\infty})$, and let $\theta(x,t)$ be a generalized solution of the problem for a homogeneous system (4) satisfying the initial and boundary conditions

$$\theta\Big|_{t=t_0} = \phi(x), \quad x \in \Gamma, \quad \theta\Big|_{x \in \partial\Gamma_{t_0,\infty}} = 0,$$

where $\phi(x) = \varphi(x) - \overline{\varphi}(x)$.

It follows that problem (4) is uniquely generalized and solvable. Its weak solution is representable as $\theta(x,t) = \sum_{i=1}^{\infty} \phi_i e^{-\lambda_i t} u_i(x)$, $\phi_i = (\phi, u_i)$ and is the limit of the weakly converging sequence $\{\theta^N\}_{N\geq 1}$ of its approximations:

$$\theta^N(x,t) = \sum_{i=1}^{N} \phi_i\, e^{-\lambda_i t} u_i(x).$$

In this case, the following inequality is performed simultaneously for all N:

$$\| \theta^N \|_{2,\Gamma_t} \leq \sum_{i=1}^{N} \phi_i^2\, e^{-2\lambda_i t}, N = 1, 2, \ldots$$

In this inequality, moving to the limit at $N \to \infty$, we obtain an upper bound, given $e^{-2\lambda_i t} < 1$ $(i = 1, 2, \ldots)$:

$$\| \theta \|_{2,\Gamma_t} \leq C^* \| \phi \|_{L_2(\Gamma)}$$

for all $t \in [t_0, \infty)$; C^* is a constant independent of t.

This means that:

$$\| \theta(\cdot,t) \|_{W^1(a,\Gamma)} \leq C^* \| \phi \|_{L_2(\Gamma)}$$

Fix $\varepsilon > 0$ and take $\delta = \frac{\varepsilon}{C^*}$, then by virtue of the inequality:

$$\| \phi \|_{L_2(\Gamma)} = \| \varphi - \overline{\varphi} \|_{L_2(a,\Gamma)} < \delta$$

we obtain the inequality $\| \theta(\cdot,t) \|_{W^1(a,\Gamma)} = \| y(\cdot,t) - \overline{y}(\cdot,t) \|_{W^1(a,\Gamma)} < \varepsilon$ for arbitrary $t > t_0$ and, hence, the stability of the unperturbed weak solution (state) of system (4) in Γ_T.

3.3.2. Sustainability of Solutions with Discretely Varying Time

First of all, we note that due to the representation (8) of the differential-difference analog of the continuous system (4), the discrete time analogue for the continuously changing time variable $t \in (0,T]$ is the set $\{t = k\tau, k = 0, 1, 2, \ldots, M\}$, where $\tau = T/M$. For the differential-difference system (8), (9), stability is defined as the continuous dependence of its solution on the initial data: the functions $f_\tau(k) \in L_2(\Gamma)$ $(k = 1, 2, \ldots, M)$ and $\varphi(x) \in L_2(\Gamma)$.

Definition 4. *A differential-difference system (4), (5) is called stable if the relation is valid for small:*

$$\| u(k) \|_{2,\Gamma} \leq C(\| \varphi \|_{2,\Gamma} + \| f_\tau(k) \|_{2,1,\Gamma})$$

for any $k = 1, 2, \ldots, M$; C is a positive constant independent of τ; $\| u(k) \|_{2,\Gamma}$ is the norm in the space $L_2(\Gamma)$; and the norm $\| f_\tau(k) \|_{2,1,\Gamma}$ is defined by the formula

$$\| f_\tau(k) \|_{2,1,\Gamma} = \tau \sum_{s=1}^{k} \| f_\tau(s) \|_{2,\Gamma}.$$

Theorem 3. *If the conditions $f(x,t) \in L_{2,1}(\Gamma_T)$ and $\varphi(x) \in L_2(\Gamma)$ are met, the differential-difference system (4), (5) is stable.*

Proof. From the equality:

$$u(k-1)^2 = (u(k) - \tau u(k)_t)^2 = u(k)^2 + \tau^2 u(k)_t^2 - 2\tau u(k) u(k)_t$$

the relation follows:

$$2\tau u(k) u(k)_t = u(k)^2 + \tau^2 (u(k)_t)^2 - u(k-1)^2. \quad (11)$$

Taking the relation (8) $v(x) = 2\tau u(k)$, and taking into account (9) as well as the lower bound a_* for $a(x)$, we obtain the inequality:

$$\begin{array}{l}\int\limits_{\Gamma} u(k)^2 dx - \int\limits_{\Gamma} u(k-1)^2 dx + \tau^2 \int\limits_{\Gamma} (u(k)_t)^2 dx + 2a_*\tau \int\limits_{\Gamma} (\frac{du(k)}{dx})^2 dx \le \\ \le -2\tau \int\limits_{\Gamma} b(x) u(k)^2 dx + 2\tau \int\limits_{\Gamma} f_\tau(k) u(k) dx,\end{array}$$

hence the inequality follows:

$$\begin{array}{l}\| u(k) \|_{2,\Gamma}^2 - \| u(k-1) \|_{2,\Gamma}^2 + \tau^2 \| u(k)_t \|_{2,\Gamma}^2 + 2a_*\tau \| \frac{du(k)}{dx} \|^2 \le \\ \le 2\beta\tau \| u(k) \|_{2,\Gamma}^2 + 2\tau \| f_\tau(k) \|_{2,\Gamma} \| u(k) \|_{2,\Gamma} .\end{array}$$

For $k = 1, 2, \ldots, M$ and $\rho = 2\beta$, we obtain:

$$\| u(k) \|_{2,\Gamma}^2 - \| u(k-1) \|_{2,\Gamma}^2 \le \rho\tau \| u(k) \|_{2,\Gamma}^2 + 2\tau \| f_\tau(k) \|_{2,\Gamma} \| u(k) \|_{2,\Gamma} . \quad (12)$$

For $\| u(k) \|_{2,\Gamma} + \| u(k-1) \|_{2,\Gamma} > 0$, given the inequality $\frac{\|u(k)\|_{2,\Gamma}}{\|u(k)\|_{2,\Gamma} + \|u(k-1)\|_{2,\Gamma}} \le 1$, we obtain at $\tau \le \tau_0 < \frac{1}{2\rho}$ the estimate

$$\| u(k) \|_{2,\Gamma} \le \frac{1}{1-\rho\tau} \| u(k-1) \|_{2,\Gamma} + \frac{2\tau}{1-\rho\tau} \| f_\tau(k) \|_{2,\Gamma}, \quad (13)$$

For $\| u(k) \|_{2,\Gamma} + \| u(k-1) \|_{2,\Gamma} = 0$, the relation (12) follows $0 \le \rho\tau \| u(k) \|_{2,\Gamma} + 2\tau \| f_\tau(k) \|_{2,\Gamma}$, i.e., the relation $\| u(k) \|_{2,\Gamma} \le \rho\tau \| u(k) \|_{2,\Gamma} - \| u(k-1) \|_{2,\Gamma} + 2\tau \| f_\tau(k) \|_{2,\Gamma}$ and hence the inequality (13). From the recurrent property of the estimate (13), we obtain:

$$\begin{array}{l}\| u(k) \|_{2,\Gamma} \le \frac{1}{1-\rho\tau} \| u(k-1) \|_{2,\Gamma} + \frac{2\tau}{1-\rho\tau} \| f_\tau(k) \|_{2,\Gamma} \le \\ \le \frac{1}{(1-\rho\tau)^k} \| u(0) \|_{2,\Gamma} + 2\tau \sum\limits_{s=1}^{k} \frac{1}{(1-\rho\tau)^{k-s+1}} \| f_\tau(s) \|_{2,\Gamma} \le \\ \le e^{2\rho T} (\| u(0) \|_{2,\Gamma} + 2 \| f_\tau(k) \|_{2,1,\Gamma}).\end{array} \quad (14)$$

So, the score in definition 4 is where $C = e^{2\rho T}$, and the theorem is proved.

Note 3. One can significantly refine the estimate (14) presented in the proof of the statement of Theorem 3.

For any $m = 1, 2, \ldots, M$:

$$\sum_{k=1}^{m} \| f_\tau(k) \|_{2,1,\Gamma} = \sum_{k=1}^{m} \left(\tau \sum_{s=1}^{k} \| f_\tau(s) \|_{2,\Gamma} \right) \le m\tau \sum_{s=1}^{m} \| f_\tau(s) \|_{2,\Gamma} = m \| f_\tau(m) \|_{2,1,\Gamma},$$

given the evaluation of Theorem 3, we obtain the inequality:

$$\sum_{k=1}^{m} \| u(k) \|_{2,\Gamma} \le m e^{2\rho T} (\| \varphi \|_{2,\Gamma} + 2 \| f_\tau(m) \|_{2,1,\Gamma}).$$

Summing up the inequalities

$$\begin{array}{l}\| u(k) \|_{2,\Gamma}^2 - \| u(k-1) \|_{2,\Gamma}^2 + \tau^2 \| u(k)_t \|_{2,\Gamma}^2 + 2a_*\tau \| \frac{du(k)}{dx} \|^2 \le \\ \le 2\beta\tau \| u(k) \|_{2,\Gamma}^2 + 2\tau \| f_\tau(k) \|_{2,\Gamma} \| u(k) \|_{2,\Gamma}\end{array}$$

for k from 1 to $m \leq M$ and using the estimates obtained above, we arrive at the inequality:

$$\begin{aligned}
&\| u(m) \|_{2,\Gamma}^2 + 2a_*\tau \sum_{k=1}^{m} \| \tfrac{du(k)}{dx} \|^2 + \tau^2 \sum_{k=1}^{m} \| u(k)_t \|_{2,\Gamma}^2 \leq \\
&\leq \sum_{k=1}^{m} \Big(\beta\tau \| u(k) \|_{2,\Gamma}^2 + 2\tau \| f_\tau(k) \|_{2,\Gamma} \| u(k) \|_{2,\Gamma} \Big) \leq \\
&\leq \sum_{k=1}^{m} \| u(k) \|_{2,\Gamma} \sum_{k=1}^{m} (\beta\tau \| u(k) \|_{2,\Gamma} + 2\tau \| f_\tau(k) \|_{2,\Gamma}) \leq \\
&\leq me^{2\rho T} \Big\{ (3m\beta T e^{2\rho T} + 1) \| \varphi \|_{2,\Gamma}^2 + (6m\beta T e^{2\rho T} + 5) \| f_\tau(m) \|_{2,1,\Gamma}^2 \Big\}.
\end{aligned}$$

Hence, to the inequality:

$$\| u(m) \|_{2,\Gamma}^2 + 2a_*\tau \sum_{k=1}^{m} \| \frac{du(k)}{dx} \|^2 \leq C(\| \varphi \|_{2,\Gamma}^2 + \| f_\tau(m) \|_{2,1,\Gamma}^2), \tag{15}$$

where the constant C depends only on a_*, β, and T.

3.4. Connection of Transport Models with Continuously and Discretely Changing Time

Inequalities (14) and (15) make it possible to justify the applicability of the Rote method and to establish a connection between the initial boundary value problem (4), (5) and the differential-difference system (8), (9).

Let $u_M(x,t) = u(k),\ t \in ((k-)1\tau, k\tau]$, $k = 1, 2, \ldots, M$. Obviously, $u_M(x,t)$ belongs to ${W_0}^{1,0}(a, \Gamma_T)$ and the estimate (15) is valid for it, as well as the estimate

$$\| u_M \|_{2,\Gamma_T} + \| \frac{\partial u_M}{\partial x} \|_{2,\Gamma_T} \leq C^*, \tag{16}$$

with a constant C^*, independent of τ, through $\| . \|_{2,\Gamma_T}$—the norm in $L_2(\Gamma_T)$.

Let $f_M(x,t) = f_\tau(x,k)$, $t \in ((k-1)\tau, k\tau]$, $k = 1, 2, \ldots, M$ and $M \to \infty$. By virtue of (16) of $\{u_M(x,t)\}$, you can select the sequence $\{\widetilde{u}_M(x,t)\}$, which weakly converges in the norm of the space $W_2^{1,0}(\Gamma_T)$ to $u(x,t) \in {W_0}^{1,0}(a, \Gamma_T)$.

We show that $u(x,t)$ defines a weak solution to the initial boundary value problem (4), (5); in other words, the identity (6) is valid for $u(x,t)$. We check the validity of this identity for functions $\eta(x,t) \in C^1(\Gamma_{T+\tau})$ that satisfy the matching conditions for nodes $\xi \in J(\Gamma)$ for any $t \in (0,T)$ and the conditions $\eta|_{\partial\Gamma_T} = 0$, $\eta|_{t\in[T,T+\tau]} = 0$.

By $\eta(x,t)$, we define the function $\eta(k) = \eta(x, k\tau)$, $k = 1, 2, \ldots, M$, and $\eta(k)_{t'} = \frac{1}{\tau}[\eta(k+1) - \eta(k)]$.

As for $u(k)$, the piecewise continuous functions $\eta_M(x,t)$, $\frac{\partial \eta_M(x,t)}{\partial x}$, and $\frac{\partial \eta_M(x,t)}{\partial t}$ are defined with respect to $\eta(k)$.

Obviously, $\eta_M(x,t)$, $\frac{\partial \eta_M(x,t)}{\partial x}$, $\frac{\partial \eta_M(x,t)}{\partial t}$ converge uniformly on $\overline{\Gamma}_T$ to respectively $\eta(x,t)$, $\frac{\partial \eta(x,t)}{\partial x}$, $\frac{\partial \eta(x,t)}{\partial t}$, and $\eta_M(x,t) = 0$, $t \in [T, T+\tau]$.

Let $\nu(x) = \tau\eta(k)$ in (10) and summing up the obtained identities with respect k and from 1 to M, we arrive at the relation

$$\begin{aligned}
&-\int_{\Gamma_T} u_M(x,t) \tfrac{\partial \eta_M(x,t)}{\partial t} dxdt + \ell_T(u_M, \eta_M) - \int_{\Gamma} \varphi(x)\eta(1)dx = \\
&= \int_{\Gamma_T} f(x,t)\eta_M(x,t)dxdt.
\end{aligned} \tag{17}$$

From the obtained relation (17) at $M \to \infty$, let us pass to the limit with respect to the sequence $\{\widetilde{u}_M(x,t)\}$ and obtain identity (6), which means that $u(x,t)$ (the function $u(x,t) \in W_0^{1,0}(a, \Gamma_T)$) is a weak solution (state) of system (4) and (5). By virtue of the uniqueness of the solution $u(x,t)$ to problem (4) and (5) on the basic fundamentals and estimate (16), the sequence $\{u_M(x,t)\}$ converges weakly to $u(x,t)$. The theorem is proved.

Note 4. The results can be used to analyze evolutionary and differential-difference systems with boundary conditions of the second and third types. For this, it is sufficient to set the second or third boundary condition instead of the Dirichlet boundary condition for the boundary value problem (7).

3.5. On Computational Aspects of Transport Models with Continuously and Discretely Varying Time

Mathematical models (4), (5) and (8), (9) allow for effective approximation by difference schemes on a two-dimensional grid $\Gamma_T^{h,\tau} = \{(ih, j\tau),\ ih \in \gamma,\ j\tau \in [0,T]\}$ and a one-dimensional grid $\Gamma^h = \{ih\}$ (the points of splitting the edges $\gamma \in \Gamma$ and the time interval $[0,T]$ are numbered by indices i, j) with approximation errors equal to $O(h,\tau)$ and $O(h)$, respectively. In this case, the error in the approximation of systems (4) and (8) on the edges is equal to $O(h^2+\tau)$ and $O(h^2)$ and at the nodes is equal to $O(h)$ in both cases; the approximation of boundary conditions (5) and (9) is exact. Numerical analysis of test examples according to schemes for systems (4), (5) and (8), (9) showed almost the same result in the accuracy of calculations, but preference should be given to calculation schemes (8), (9); due to the one-dimensionality of Γ^h, the number of arithmetic operations is several orders of magnitude fewer. The restrictions on the use of the presented results are determined by the linearity property of the differential operators of systems (4), (5) and (8), (9).

4. Discussion

Digital logistics is based on some platform-based business models, so the issues of the implementation of digital business models for sustainable development could be a topic for future research. Companies' information about their socio-environmental actions is important as well as both the rankings of corporate social responsibility [8] and the degree of similarity in international brand valuation rankings that apply to the IT sector [9].

The digital transformation of logistics networks has been driven by different factors, including the approaches of such giant companies as Apple, Alibaba, Facebook, and Google [10] to developing the platform's ecosystem [11] as well as new business models on the basis of the digital twin concept, which was initially implemented in the manufacturing industry [12]. Researchers take into account some different aspects of the digital logistics networks [13–16] and the features of digital ecosystems [17,18] as well.

The competitiveness of all types of commercial activities using the services of logistics operators, including the concepts of 3PL and 4PL outsourcing, depends on the quality of algorithms for processing data arrays on the movement of goods (taking into account sustainable human resource management [19]). The reader can consider a graph as a prototype of a logistics network. The authors propose the use of the formalisms of initial boundary value problems or differential-difference systems to construct a mathematical model of transfer problems in the class of summable functions, since such systems and the class of functions used help to more accurately take into account the key property of the transferred continuous or discrete media–multiphase media, which involves a lack of continuity in the functions describing the quantitative characteristics of the transferred media. Modern mathematical methods and approaches developed by the authors provide the necessary mathematical tools not only to describe the process on linear network fragments but to obtain adequate mathematical relationships describing the process at the junction nodes of linear network fragments. Additionally, the authors applied the integral functional of the quality of the logistics network [20–24]. The article investigates the boundaries of the stability of an evolutionary parabolic system and a differential-difference system with distributed parameters on a selected graph [25–29]. A computational complexity analysis to demonstrate the computational performance of the proposed solution methodologies could be a topic for future research because of the limitation on the size of the article.

5. Conclusions

The modern transnational business structure is based on a complex multi-level and constantly evolving transport network. In the middle of the 20th century, simple models for finding optimal routes were developed using linear programming methods. The need to work under conditions of uncertainty and in a dynamic market environment requires us to consider the fundamental difference between discrete and quasi-continuous transportation.

Currently, the theoretical fundamentals of logistics comprise a complex of different methods and models for managing logistics networks. The activity of logistics operators is based on predictive planning and operates with a stream of data on the condition of the cargo and its location in real time. Projecting the movement of material flows onto the logistics network and choosing the optimal supply chains require the creation of effective digital logistics platforms. The core of these systems is formed by an algorithm implemented on mathematical models and the theory of making optimal decisions based on the criteria of economic efficiency. Real logistic activity takes place under the influence of numerous random disturbances. For practical business tasks, it is critically important to analyze the sustainability of the proposed solutions, which not only provides an assessment of investment risks but also allows us to switch to predictive indicators of advanced planning.

In this study, the development of classical approaches to the study of the sustainability of solutions obtained by using mathematical modeling based on systems of differential equations is carried out. The complex topology of logistics networks within which supply chains are built makes it necessary to apply deeper methods in the study of the dynamics of logistics flows. Based on the real conditions of logistics activities, it was assumed that this functionality is not always continuous in terms of movement and time parameters. The complexity of solving the problems of forming effective supply chains has increased under the influence of the trend of the consolidation of business into network structures. The application of the results obtained in this work is not limited to the analysis of the sustainability of solutions. Their use is relevant in assessing investment risks, the sustainability of the business model of outsourcing logistics operators, as well as for gaining competitive advantages.

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