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Additional Information

An experimental and one-dimensional modeling analysis of turbulent gas ejection in pre-chamber engines

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Abstract

Experimental results from a study on the evolution of gas jets ejected through the orifices of a pre-chamber in a heavy-duty optical engine are presented. The work examines conditions without fuel inside the main-chamber, which helps to describe the dynamics of the ejected gas jets without the interference of subsequent combustion in the main-chamber. Experimental diagnostics consist of high-speed visible intensified imaging and low-speed infrared imaging. Additionally a one-dimensional gas jet model is used to characterize the spatial distribution of the ejected flow, including parameters such as tip penetration, which are then validated based on experimental results. Different stages in the ejection of pre-chamber jets are identified, with chemical activity restricted to a maximum distance of 5 to 10 orifice diameters downstream of the orifice as indicated by the recorded visible radiation. Sensitivity of cycle-to-cycle variations in pre-chamber jet development to the air-to-fuel ratio in the pre-chamber observed in the experiments is in most part attributed to the variations in the timing of combustion initiation in the pre-chamber. The influence of the ejection flow on the penetration of the gas jet on a cycle-to-cycle basis is presented using the one-dimensional model. The one-dimensional model also indicates that the local flow exhibits highest sensitivity to operating conditions during the start of ejection until the timing when maximum flow is attained. Differences that exist during the decreasing mass-flow ejection time-period tend to smear out in part due to the transient slowdown of the ejection process.

Keywords: Pre-chamber spark ignition, 1-D gas jet model, cycle-to-cycle variability, infrared imaging, high-speed visualization

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1 1. Introduction, motivation and objectives

In the search for higher thermal efficiency and lower fuel consumption in internal combustion en-2 gines, pre-chamber spark-ignition system is leading a new chapter in the improvement of spark-ignition 3 engines' performance. The considerable increase in scientific publications related to this technology 4 over the past few years demonstrates the appreciable growth in both scientific and commercial inter-5 ests in pre-chamber spark-ignition systems. Many researchers have documented the main advantages 6 of pre-chamber spark-ignition systems for use in high-power stationary power plants [1, 2] and in trans-7 port applications [3, 4, 5, 6] through several engine experiments. Under suitable operating conditions, 8 pre-chamber spark-ignition guarantees extremely low cycle-to-cycle variation (CCV) and an accelerated 9 combustion process [7], which minimizes knocking due to end-gas auto-ignition compared to conven-10 tional spark-ignition systems [8, 9, 10, 11], and thus allows for an increase in the compression ratio that 11 can further improve engine thermal efficiency [12]. In addition, its active version, with a dedicated fuel 12 injector inside the pre-chamber, allows to increase the dilution limit to extremely lean mixtures ($\lambda \sim$ 13 2.5), while reducing nitrogen oxides (NO_x) emissions to near zero levels [7, 13]. 14

Despite the general benefits of this ignition system being well established, the fundamental aspects of turbulent jet-based ignition are not fully understood, which hinders a complete concept optimization that in turn limits its market penetration [14]. For example, currently, there is no consensus on the combustion regimes that are encountered during the ejection of the gases from the pre-chamber at engine-relevant conditions involving ultra-lean mixtures.

To shed some light on this aspect, several researchers have utilized different visualization tech-20 niques applied to simplified devices such as divided constant-volume chambers and rapid compression 21 machines (RCM). In a pioneering work [15] the visualization of pre-chamber ignition process in a 22 divided chamber bomb through high-speed schlieren imaging and OH* radiation was performed. Dif-23 ferent ignition patterns were identified when modifying the nozzle diameter and the equivalence ratio 24 in the main chamber. More recently, several experiments in a similar device using simultaneous high-25 speed schlieren [16] and OH* chemiluminescence imaging [17] were carried out to describe the jet 26 penetration and ignition process. A relationship between the Damköhler number and the combustion 27 regime in the main-chamber was established using semi-empirical correlations. 28

The temporal evolution of the ejected pre-chamber jets was related to the ignition kernel development within the pre-chamber in [18, 19] using experimental visualizations and numerical simulation in a simplified pre-chamber apparatus at ambient pressure. The measurements showed the influence of the jet dynamic structure and mixture composition on the main-chamber ignition process. In particular, the authors emphasized on the importance of jet penetration length, jet duration, and reaction zone presence in the performance of this system.

The lack of fundamental analyses of the characteristics of premixed turbulent jets under enginerelevant conditions motivated other research works based on RCM optical measurements. In [20] and [21], the influence of nozzle diameter and equivalence ratio on jet morphology and ignition location in a radially mounted single hole pre-chamber was studied. Similarly, pre-chamber spark-ignition systems were compared in [22] with conventional systems using both high-speed OH* imaging and integrated UV emissions using a photomultiplier in an axially assembled four-nozzle pre-chamber that allowed for visualization of interaction between the reacting jets.

Despite some attempts involving studies based on real engine architecture to characterize the im-42 pact of the pre-chamber spark-ignition systems on the engine cycle-to-cycle variability, the root causes 43 of this phenomenon have not been analyzed in detail due to facility limitations [23]. The complexity 44 of applying visualization techniques in internal combustion engines due to the limited optical access 45 increases significantly when considering a small dead volume (pre-chamber) connected by small ori-46 fices. Indeed, in small engines, it is practically impossible even to include a pressure transducer inside 47 the pre-chamber. In this sense, there is an important dearth of knowledge related to the fundamentals 48 of cycle-to-cycle variability in pre-chamber spark-ignited engines. Only the work in [24] delves into 49 one of the possible root causes of CCV. They studied the asymmetries in the jet dynamics and structure 50 of the pre-chamber jets (jet-to-jet variation) using numerical methods. Jet-to-jet variations were at-51 tributed to the asymmetric formation of the initial spark kernel inside the pre-chamber, which resulted 52 in asymmetrical distribution of the turbulent jets actuated from six different orifices. 53

In the context of numerical simulations, a large number of investigations have been performed in parallel to the previously discussed experiments. Starting from simplistic simulations based on unsteady Reynolds Averaged Navier-Stokes (URANS) formulation [25, 26, 27, 28] to more sophisticated ones such as Large-Eddy Simulations (LES) [29, 19, 30] and even Direct Numerical Simulations (DNS) [31, 32, 33], multiple researchers have addressed different aspects of the pre-chamber combustion

process including the filling process, local flow distribution, flame quenching through the nozzle and 59 the associated composition field. However, it is not easy to establish a direct and quantitative con-60 nection based upon such highly sophisticated calculation tools between pre-chamber combustion and 61 the resulting characteristics of the ejected gas jet, which is essentially central for the prediction of the 62 combustion process in the main-chamber. The development of simplified models accounting for the 63 fundamental physics of pre-chamber gas jet ejection and the associated combustion process is an area 64 where contributions are rather scarce. Few examples are available, such as the one in [18], where a 65 two-zone model was developed to predict the time evolution of conditions inside the pre-chamber up 66 to the nozzle. However, no link to the ejected gas jet structure was established. In terms of simplified 67 tools, one-dimensional (1-D) models have been successfully utilized for analysis of diesel-type sprays, 68 thereby coupling low computational costs and high prediction capabilities. Especially when boundary 69 conditions are highly controlled, they can deliver an accurate prediction of tip penetration and overall 70 mixing behavior [34, 35, 36]. Most of the assumptions used to develop these models are based on 71 turbulent gas jet theory [37]. For example, one of the core simplifications is that the radial spread of 72 axial momentum and mass fraction of the injected stream are self-similar, which reduces the flow to a 73 quasi-steady 1-D problem. Therefore, pre-chamber gas jets are good candidates for the application of 74 these diesel-type spray models. To the best of the authors' knowledge, no direct application of such an 75 approach has been reported in the literature. The availability of such tools with low computational cost 76 would help to bridge the gap between the pre-chamber combustion process and the associated gas jet 77 ejection, which would eventually contribute to a more efficient optimization of this concept for engine 78 applications [23, 38, 39]. 79

In the present work, pre-chamber spark-ignited engine experiments are conducted in a single-80 cylinder, heavy-duty, optical engine to characterize the development of turbulent jets and to discuss 81 the role of the ejected flow in the subsequent ignition of the main-chamber mixture. To this end, fuel is 82 only injected in the pre-chamber i.e., no main-chamber fueling is performed, so that the development 83 of the turbulent pre-chamber gas jets into an air ambient can be analyzed without the interference of 84 combustion in the main-chamber. Although some of the previously cited investigations have applied 85 this method with pre-chamber jets ejecting into the main-chamber without any fuel, they only use it 86 as a reference case for the later analysis of a realistic fueling case in the engine, without performing 87 a detailed analysis of the jet dynamics under realistic engine conditions. Cycle-to-cycle and jet-to-jet 88

variations will be the predominant focus of this work. In this study, experimental analysis is supported using a 1-D jet model, which is validated and applied for evaluation of cycle-to-cycle variability in tip penetration. Boundary conditions for the 1-D jet model in terms of mass-flow rate at the nozzle and in-cylinder conditions are estimated based on closed-cycle engine models, including both pre-chamber and main chamber. Both experimental and modeling efforts contribute to the understanding of the relevant phenomena during the pre-chamber ejection process.

95 2. Experimental methods

96 2.1. Optical engine and operating conditions

Experiments are performed in a single-cylinder, heavy-duty optical engine. Visualization is carried
out through a Bowditch-type piston with an open, right-cylindrical bowl fitted with a flat fused silica
piston-crown window (Figure 1). The major specifications of the engine are summarized in Table 1,
with further details about the facility available elsewhere [40].

Engine base type	Cummins N-14, DI diesel
Displacement [L]	2.34
Bore x Stroke [cm]	13.97 x 15.24
Base compression ratio [-]	11.2
Combustion chamber	Quiescent, direct injection
Bowl Width x Depth [cm]	9.78 x 1.55
Swirl ratio [-]	0.5

Table 1: Major specifications of the single-cylinder optical engine.

Although originally developed as a heavy-duty, optically accessible, single-cylinder diesel engine, it has been suitably modified to operate as a gas engine. The engine is fitted with a pre-chamber sparkignition module located centrally in the cylinder. The pre-chamber has a volume of 4.66 ml, with 8 equally spaced, 1.6 mm diameter orifices machined with an included angle of 130°. The pre-chamber tip protrudes 10.6 mm below the fire deck. The pre-chamber houses a Rimfire Z1 spark plug and a Bosch HDEV5 GDI injector with 6 unequally spaced, 0.17 mm diameter orifices (Figure 1). A synthetic mixture comprising 95% CH_4 , 4% C_2H_6 and 1% C_3H_8 by volume is used as a surrogate for natural gas.

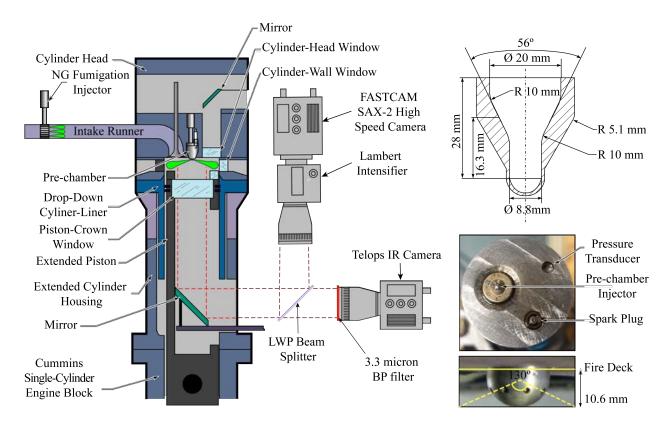


Figure 1: Schematic layout of the (a) optical engine and imaging setup (b) internal geometry of the prechamber (c) pre-chamber spark ignition system and its components as installed in the optical engine.

According to the objective of the study, no fuel is injected at the intake manifold, so that the ejection 108 of pre-chamber jets into an air-ambient can be investigated. The engine is operated at 1200 rpm with 109 constant intake conditions (105 kPa and $41^{\circ}C$) such that nominal bulk air conditions of 19 bar and 110 730 K are reached at a spark timing of 343 CAD (crank angle degree), which is maintained constant. 111 A sweep of air-fuel ratio (λ) in the pre-chamber (Table 2) has been performed by varying the start and 112 duration of pre-chamber injector energization, so that a constant timing of 336.6 CAD for the end of 113 solenoid energization (6.4 CAD before spark timing) is maintained. Injection pressure is kept constant 114 at 100 bar throughout the study. Air-fuel ratio values in the table have been calculated by considering 115 the air mass contained in the pre-chamber at spark timing and the total injected fuel mass, since no 116 fuel leakages into the main-chamber can be expected before combustion-induced gas ejection. 117

The optical engine is operated in a 9 : 1 skip-fire mode, i.e. nine motored cycles precede each fired cycle, which minimizes the amount of residual gases present inside the pre-chamber and the mainchamber clearance volume. This allows for a fundamental study governing the ejection of pre-chamber gas jets avoiding any residual gas effects. Each experimental test run consists of 30 fire cycles after the engine is motored for 60 seconds at constant speed.

Table 2: Operating conditions as defined by a λ sweep in the pre-chamber along with the corresponding injected fuel mass.

λ	Fuel mass
	[mg]
1.65	1.56
1.50	1.72
1.25	2.06
1.07	2.41
0.94	2.76
0.83	3.10
0.75	3.45

123 **2.2.** Experimental diagnostics

Diagnostics include time-resolved pressure measurements both in the pre-chamber and the mainchamber along with imaging of broadband luminosity in the visible and infrared region. Figure 1 shows the schematic layout of the optical engine fitted with the pre-chamber spark ignition system along with the imaging setups.

Main-chamber pressure P_{MC} and pre-chamber pressure P_{PC} are measured using an AVL QC34D piezoelectric pressure transducer and an uncooled KISTLER PiezoStar piezoelectric pressure transducer, respectively. Both pressure traces are recorded every quarter crank angle degree. For every fired cycle during which images are acquired, the pressure difference between pre-chamber and main-chamber $(\Delta P = P_{PC} - P_{MC})$ is used extensively for analysis. Furthermore, an apparent heat release rate is calculated based on the whole instantaneous in-cylinder volume, i.e. pre-chamber plus main-chamber [41]. This curve will be used as an input to the modeling approach described later.

Radiation from the main-chamber is imaged through the piston window using a beam splitter arrangement for a two-camera system: Broadband visible (VIS) radiation is imaged using a Photron FASTCAM SAX-2 high-speed camera equipped with a Lambert Hi-CATT high-speed intensifier with a S-20 photocathode. This setup records time-resolved images at a 0.5 *CAD* resolution with a projected pixel size of 7.5 *pix/mm*. The camera effective exposure time is setup by the intensifier gain, which is 50 µs. Due to the absence of soot in this type of combustion, VIS radiation primarily corresponds to chemiluminescence of products of the pre-chamber combustion being ejected into the main-chamber, or of those species recombining to final products in the main-chamber.

• Infrared (IR) radiation is imaged using a Telops Hyp4 camera equipped with a Spectrogon bandpass filter centered at 3.3 μm with a 215 nm full width at half-maximum. Due to slow acquisition speed of this camera, it can only acquire only one image per cycle, with a projected pixel size of 4.5 pix/mm. The camera exposure time is 15 μs . IR radiation is emitted due to the C -H vibrational stretching of hot unreacted fuel at elevated in-cylinder temperatures caused by compression and/or combustion, or due to the thermal radiation by the hot combustion products emitting VIS radiation.

Due to the difference in the temporal resolution between the two cameras, the experimental method-151 ology has consisted of recording information from 30 fired cycles, for each of which a single IR image, 152 a VIS image sequence and pre-chamber and main-chamber pressure traces are acquired. Image acqui-153 sition timing has been suitably adjusted such that every single IR image is simultaneous with one of the 154 VIS images from the 30 acquired sequences. For that purpose, the engine shaft synchronization system 155 has been programmed to send the trigger pulses to both cameras at the specified camera acquisition 156 CAD timings. Figure 2 shows an example of the acquired information for one fired cycle: A single 157 IR image from that particular cycle, 5 selected VIS snapshots from the recorded VIS sequence along 158 with the corresponding ΔP signal and the penetration of the pre-chamber gas jets derived from both 159 IR/VIS images. The latter information is obtained by processing the corresponding IR/VIS images using 160 typical Diesel spray processing algorithms as detailed in [35]. Detected contours (colored outlines in 161 Figure 2) have been overlaid onto the images to improve visualization. Maximum axial extent of the 162 IR/VIS radiation measured from the orifice exit will be referred to as IR penetration and VIS penetration 163 respectively. Only IR images show the maximum axial-extent (penetration) of the gas jet tip clearly, 164

while VIS images only show a radiation zone that is mostly limited to the near orifice region. Hence, the wording *gas jet penetration* is only appropriate for the IR-derived information. However, confusion will be avoided by using the corresponding IR/VIS acronym.

168 **3. Modelling approach**

As discussed earlier in the introduction, the aim of this work is to validate a modeling approach that speeds up the design process by predicting gas jet penetration, which has been found to be a governing parameter in pre-chamber combustion. This methodology combines two main tools, a 0-D engine model and a 1-D jet model. Compared to CFD approaches, the main advantage of combining these tools is the fast computation time, which leads to a reduction in the associated computation costs while ensuring a reasonable agreement with experimental results.

A schematic layout of the overall modelling workflow is presented in Figure 3. Inputs to the 0-D engine model are pressure and temperature at inlet valve closing (IVC), spark timing, injected fuel mass and apparent heat release rate (AHRR). Momentum flux and mass flow rate through the holes are

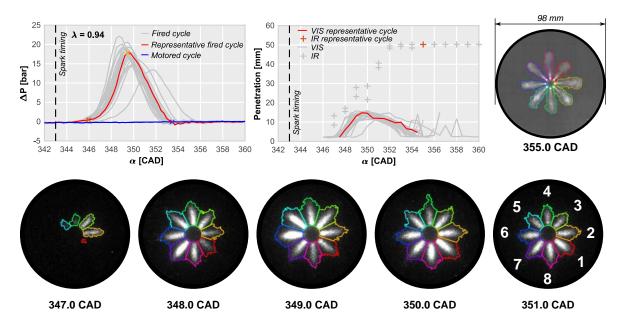


Figure 2: Sample of acquired and processed information from a single fired cycle for $\lambda = 0.93$. Top row: ΔP (left), IR- and VIS-penetration (middle) and IR image (right)), simultaneous with the VIS image acquired at 349 CAD. Bottom row: Sequence of five VIS images. Overlaid colour lines show detected contours from the image processing algorithm on the corresponding IR/VIS images.

obtained along with the thermodynamic conditions in the main-chamber from the engine modelling part. These results serve as the main inputs for the 1-D jet model, which predicts jet penetration for a single ejected gas jet (S). This modeling workflow can be used on a fully predictive basis, as proposed in [42] by using reasonable assumptions for the AHRR in the pre-chamber. However, the overarching goal of this work is the validation of the whole simulation workflow against experimental data. To this end, experimental AHRR is used as input, while the final predicted pre-chamber gas jet tip penetration profile is compared with the corresponding experimental values.



Figure 3: Schematic layout of the modeling workflow coupling a 0-D engine model with the 1-D gas jet model.

185 **3.1.** O-D Engine model

The engine model is built up within the framework of a commercial 1-D engine modelling software 186 (GT-Power) following the methodology from a previous work [23]. However, in the present case the 187 approach has been simplified to consider only the closed-volume part of the engine cycle, where pre-188 chamber gas jet ejection and combustion occur. A submodel is implemented for the main-chamber and 189 the pre-chamber, where they are both treated as engine cylinders connected through the pre-chamber 190 orifices with the pre-chamber considered as a non-moving piston. This model reproduces the pressure 191 evolution in both chambers along with the gas exchange between them based on the imposed heat-192 release rate profile. 193

Heat release profiles can be imposed either only on the pre-chamber or on the main-chamber, or 194 on both of them. The experimental AHRR includes the combined effect of both chemical heat release 195 and heat transfer in the whole main- and pre-chamber volume. However, because of the absence of 196 fuel in the main chamber, the chemical part of AHRR should only be occurring in the pre-chamber. 197 For the simulation part, the chemical part is the one to be imposed only in the pre-chamber. The 198 reconstruction of this chemical part starting from the experimental one is performed in a simplified 199 way, namely the experimental AHRR trace is properly scaled to deliver the fuel energy content in the 200 pre-chamber (fuel mass times lower heating value). Other predictive approaches for the heat-release 201

rate are also feasible, but since the goal of this work is focused on the validation of the workflow, this
quasi-diagnostics approach applied here is considered reasonable for our purposes, i.e., a quantitative
description of the gas jet flow using the 1-D jet model.

Simulations are carried out into two different steps. The first one consists of a motored cycle, compressing air starting from IVC until the spark timing as set in the experiments. This is followed by a second simulation starting from the start of spark until the end of the cycle. In the second simulation, the fuel-air mass in the pre-chamber is initialized at pressure and temperatures values obtained at the end of the previous simulation. This defines the initial conditions on which the apparent heat-release rate profile is imposed to complete the remaining portion of the closed cycle.

Gas within the pre-chamber is modeled as a perfectly homogeneous mixture with a composition that evolves with time as combustion progresses in a single-zone fashion. Hence, the ejected pre-chamber flow is a mixture of air, fuel and burned products (CO_2 , H_2O and N_2), which is indeed a simplification of reality as confirmed by the experimental results. The composition of the burned products is set by considering complete stoichometric combustion. The eight pre-chamber orifices are simulated using a combination of hole and pipe templates. The ejection velocity, mass flow rate and momentum flux from one of the eight orifices is then later used as input for the 1-D gas jet model.

The engine model is calibrated using the experimental data by varying the heat transfer and orifice discharge coefficients until there is reasonable agreement between the simulated and experimental pressure profiles in both the pre-chamber and main-chamber. An example of the calibration results for $\lambda = 0.94$, i.e., experimental (blue) and simulated (orange) pressure profiles is shown in Figure 4. The light blue area corresponds to the standard deviation in the experimental pressure profiles. Validation results indicate that the 0-D engine model is able to accurately reproduce the experimental pressure trends, thereby making it suitable for our analysis.

225 **3.2.** 1-D jet model

As discussed earlier in the introduction, one of the main objectives of this work is to validate this modeling approach for the prediction of ejected gas jet penetration. For this purpose, an existing 1-D jet model [36, 43, 44, 45] is adapted for this application. Although this 1-D jet model has been compared in the past with CFD RANS simulations of gas jets [36, 43], most of its applications have been Diesel-like sprays. A detailed description of this model can be found in [36, 43] so only the modifications carried out to simulate the gas jets for the pre-chamber configuration are described here:

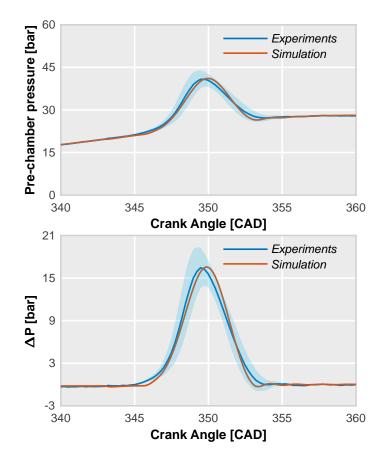


Figure 4: Left: Experimental and simulated pre-chamber pressure profile for $\lambda = 0.94$. Right: Pressure difference (ΔP) between the pre-chamber and main-chamber for the same operating condition.

- An inert configuration is analyzed in this work, i.e., the model describes the turbulent mixing of
 hot ejected pre-chamber gases with the air ambient in the main-chamber. Although VIS images
 show some radiation in the vicinity of the pre-chamber nozzle, most of the ejected gas jet volume
 shows no evidence of chemical activity further downstream of this region.
- The pre-chamber gas jets are modeled as a gas stream injected through the orifices from the pre chamber into the main-chamber filled with air. Hence, an ideal gas equation of state is used, and
 a low Mach approach is assumed in terms of compressibility.

Flow inputs to the 1-D jet model are the time evolution of mass and momentum fluxes through
 the pre-chamber orifice, which are outputs obtained from the previously described 0-D engine
 simulations. As opposed to the typical flat top-hat injection profiles used for diesel-like sprays,
 evolution of such quantities for the pre-chamber configuration is found to be highly transient as
 shown later.

- While in Diesel-like sprays a cold flow is injected in a hot environment, in this case a stream 244 of a high-temperature fluid (around 1500 K based on 0-D engine simulations) from the pre-245 chamber is ejected into the main-chamber with the ambient at a relatively lower temperature 246 $(700 \ K)$. Temperature and mixture composition at the pre-chamber orifice exit are assumed 247 to time-independent (constant) for the sake of simplicity. Since the focus of this work is on 248 penetration predictions, the most critical parameters are the nozzle momentum flux and ambient 249 density, which govern the jet dynamics. In the most simplified scenario, a non-reacting gas jet could be considered as a constant-density flow [46]; thus, the role of orifice temperature and 251 mixture composition can be effectively ignored. 252
- Boundary conditions of the main-chamber into which the pre-chamber gas is ejected consists of a
 time varying density and temperature profile that are also obtained from 0-D engine simulations.
- One of the critical parameters for the application of the 1-D jet model is the radial cone angle,
 which is often used as a fitting parameter for the experimental tip-penetration data [35]. Here,
 a constant cone angle of 25° is used. Modelled start of ejection has been shifted for all cases to
 accommodate for the initial part of ejection process, which is highly challenging to capture in 1-D
 models due to the uncertainties during the first instants of flow ejection.
- Although interaction between the pre-chamber gas jets and the piston-bowl wall are to be expected in experiments, the 1-D gas jet model only accounts for free-jet propagation.

Figure 5 shows the predicted ΔP and the momentum flux at the orifice as derived from the modelling approach as an example of the coupling between the 0-D engine model and the 1-D gas jet one. Mainchamber density is also included, as both momentum flux at the orifice and density of the ambient into which ejection occurs govern the tip penetration of the pre-chamber gas jets, similar to Diesel sprays [43]. Density in the main chamber during the main ejection period is roughly around $11.0-11.5 kg/m^3$

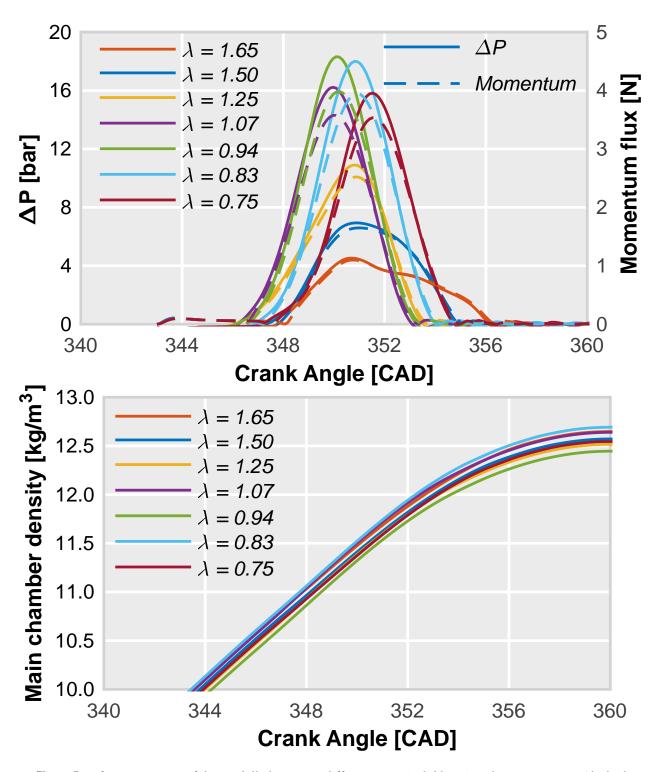


Figure 5: Left: Comparison of the modelled pressure difference ΔP (solid lines) and jet momentum (dashed lines) for various λ . Right: Evolution of main-chamber density for various λ .

for all conditions, as engine intake conditions are constant in this study. On the other hand, ΔP is an 267 indicator of the thermal state in the pre-chamber as a result of heat release and it is also the governing 268 parameter for the ejection velocity (and hence momentum) through the pre-chamber holes. Results will 269 evidence the strong relationship between both variables, and hence between pre-chamber combustion 270 and gas jet ejection. Momentum delivery starts when pre-chamber pressure increases as a result of 271 combustion to a value high enough to overcome the cylinder pressure and establish a flow through the 272 orifice. The expected proportionality in the time evolution between both variables (ΔP and momentum 273 flux) is observed, which also suggests the prominent role of pre-chamber pressure in gas jet ejection, 274 even though compressibility effects might play a role. In terms of λ sensitivity, the highest values of 275 peak ΔP are achieved under fuel-rich conditions ($\lambda = 0.94$), where pre-chamber combustion is fastest, 276 with peak ΔP values decreasing when moving away from these conditions, specially for the leanest 277 278 cases.

4. Analysis of pre-chamber combustion cycle-to-cycle variation

280 4.1. Experimental assessment of cycle-to-cycle variation

One of the most prominent features observed in this study has been the cycle-to-cycle variability in 281 the experiments. Figure 6 shows results that exemplify this phenomenon for $\lambda = 1.50, 0.94, 0.75$. The 282 results are from 30 individual fired cycles along with a representative cycle (highlighted in red) identified 283 based on the selection criteria discussed below. Both pressure difference ΔP between pre-chamber and 284 the main-chamber along with jet tip penetrations from both IR and VIS images are included. High 285 cycle-to-cycle variability is noticeable in all three metrics for both the lean ($\lambda = 1.5$) and rich cases 286 $(\lambda = 0.75)$, while this is not so apparent for the near stoichiometric case ($\lambda = 0.94$). In terms of 287 pressure difference, variations influence both the timing of the initial rise, as well as the values of peak 288 ΔP attained. Variation in timing is also observable in the VIS-based penetration results with similar 289 peak values. As a single IR image is acquired every fired cycle, the cycle to cycle variation cannot be 290 described based upon IR images. 291

To normalize this timing variability, a so-called 'Start of Ejection' (SOE_{*p*}) is defined as the crankangle position where the pressure increase (ΔP) exceeds 0.5 *bar*. Figure 7 presents the same results of Figure 6 but on a SOE_{*p*}-referenced crankangle basis. From a qualitative perspective, it is clear that the variation in ΔP decreases significantly especially for the rich case on this modified SOE_{*p*}-based

time referencing. Therefore, it is reasonable to infer that the timing of initial pressure rise within the 296 pre-chamber is most likely to be a major source of the observed cycle-to-cycle variability. However, the 297 overall shape of ΔP evolution, which dictates the apparent heat-release rate, remains similar. Using 298 the new SOE_p -based time-reference also decreases the variations in both IR and VIS-based penetration. 299 Even though cycle-to-cycle variability is lowest for stoichiometric conditions, some IR-based penetration 300 measurements (350 CAD) are consistently off-trend compared to the adjacent timings. However, when 301 plotted against a SOE_p -based crank angle scale, they tend to follow a single consistent trend. Similar 302 to ΔP , all the VIS-based penetration traces are now in phase, with peak penetration values occurring 303 close to the maximum ΔP timing. This minimization in cycle-to-cycle variation of ΔP and IR/VIS-based 304 penetration evolution is very evident for the rich case. For the lean case, although the SOE_p-based crank 305 angle reference removes the variation in IR/VIS-based penetration for the most part, some spread in 306 the trends is still noticeable. Thus, the cycle-to-cycle variation observed in pre-chamber events for 307 stoichiometric and rich conditions is mainly related to the fluctuations in the start timing of the ejection 308 event. Otherwise, this remains repeatable in terms of both ΔP and IR/VIS-based penetration. 309

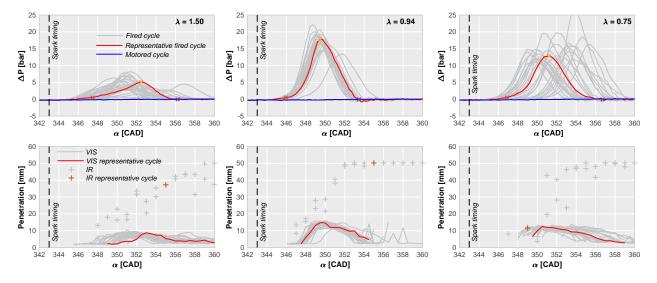


Figure 6: Pressure difference ΔP between pre-chamber and main-chamber (top) and jet tip penetration (bottom) based on IR (single markers) and VIS (lines) images. $\lambda = 1.50$ (left), 0.94 (middle) and 0.75 (right). Results are from 30 individual fired cycles (gray lines and markers) along with a representative cycle (highlighted in red) identified based on the selection criteria discussed in Section 4.1. Vertical dashed line corresponds to the timing of the spark.

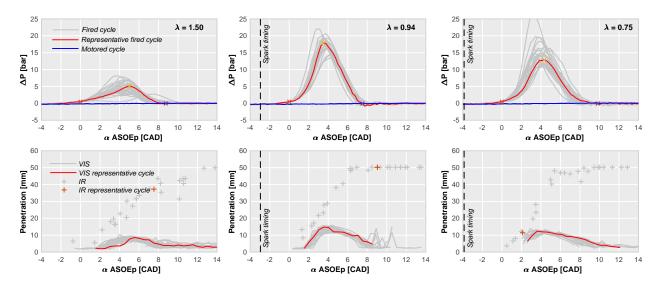


Figure 7: Pressure difference ΔP between pre-chamber and main-chamber (top) and jet tip penetration (bottom) based on IR (single markers) and VIS (lines) images. $\lambda = 1.50$ (left), 0.94 (middle) and 0.75 (right). Crank angle values are referenced to the start of ejection SOE_p defined based on ΔP exceeding 0.5 bar. Results are from 30 individual fired cycles (gray lines and markers) along with a representative cycle (highlighted in red) identified based on the selection criteria discussed in Section 4.1. Vertical dashed line corresponds to the timing of the spark.

The earlier analysis revealed a strong link between the pressure difference and the timing of pre-310 chamber gas jet evolution on a cycle-to-cycle basis. To shed further light on this one-way dependence, 311 Figure 8 compares the evolution of the start of ejection as derived from the pressure difference (SOE_p) 312 to that from VIS images, now defined as the timing of VIS penetration exceeding 5 mm (SOE_{VIS}). The 313 dependence of start of ejection (SOE_p and SOE_{VIS}) on λ is quite similar for both cases, with the start 314 of ejection reaching minimum values around stoichiometry and increasing with rich and lean mixtures. 315 Another characteristic crank angle position is also plotted in Figure 8, namely the timing of maximum 316 pressure difference $\alpha_{\Delta P_{max}}$, which also exhibits a similar dependency with λ . Differences between these 317 characteristic timings are almost constant, namely a 2.5° CA difference between the two start of ejection 318 definitions and 1.0° CA difference between the start of ejection (SOE_{VIS}) and $\alpha_{\Delta P_{max}}$. This confirms 319 a stable time sequence of pre-chamber events independent of λ : starting with the pressure rise in the 320 pre-chamber, followed by the appearance of visible light caused by the ejection of active species and 321 products of combustion from the pre-chamber, leading to maximum pressure difference between the 322 pre-chamber and the main-chamber beyond which the combustion begins to recede. 323

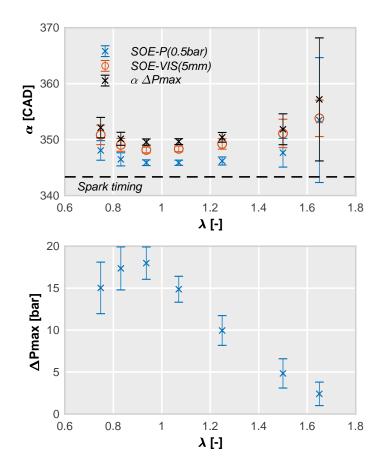


Figure 8: Start of ejection based on ΔP exceeding 0.5 bar (SOE_p) and VIS-based penetration exceeding 5 mm (SOE_{VIS}) and the timing of maximum ΔP (top) and maximum ΔP values (bottom) for various λ . Markers indicate the sample-averaged value with error bars corresponding to one standard deviation.

The peak pressure difference (ΔP_{max}) shown in Figure 8 can be considered as an indicator of the 324 maximum heat release rate inside the pre-chamber during the combustion process. Compared to the 325 timing evolution, the dependence of ΔP_{max} on λ appears to be reversed, with ΔP_{max} reaching highest 326 levels at slightly rich conditions, consistent with laminar flame speeds peaking at slightly rich condi-327 tions. The magnitude of error bars on the start of ejection timing also indicate that the variation in 328 combustion timing becomes significant at mixture compositions that deviate from stoichiometry. For 329 ΔP_{max} , the absolute variation becomes more pronounced on the rich side, while the relative variation 330 (i.e. the size of the error bar compared to average value) is especially large for lean mixtures. The main 331 conclusion is that the combustion timing in the pre-chamber, the peak pressure difference (ΔP_{max}) and 332 the subsequent ejection processes are strongly dependent on λ , with fastest and most stable combustion 333 happening around stoichiometry. 334

Figure 9 shows the ΔP profile for individual fired cycles for the same three λ values (0.94, 1.5 335 and 0.75) as analyzed previously with four cycles highlighted: the ensemble-averaged cycle, the single 336 cycles with the maximum and minimum peak pressure difference (ΔP_{max}) and the most representative 337 cycle in the sample [47]. The representative cycle is defined as the individual cycle that most resembles 338 the average of the samples both in terms of combustion timing and peak pressure difference. In practice, 339 it is chosen as the one cycle that minimizes the merit function f in equation (1) amongst all cycles in the 340 sample. This merit function for a given jth cycle takes into account the maximum pressure difference 341 (ΔP_{max}) and three characteristic crank angle timings, namely the crank angle of maximum pressure 342 difference ($\alpha_{\Delta P_{max}}$), the start of combustion (α_{SoC}) and the end of combustion (α_{EoC}) as defined based 343 on the apparent heat-release rate. These variables for every firing cycle are compared against the 344 sample-averaged value of all cycles (denoted by the overbar in equation (1)). 345

$$f^{j} = \frac{|\Delta P_{max}^{j} - \overline{\Delta P}_{max}|}{\overline{\Delta P}_{max}} + \frac{|\alpha_{\Delta P_{max}}^{j} - \overline{\alpha}_{\Delta P_{max}}|}{\overline{\alpha}_{\Delta P_{max}}} + \frac{|\alpha_{SoC}^{j} - \overline{\alpha}_{SoC}|}{\overline{\alpha}_{SoC}} + \frac{|\alpha_{EoC}^{j} - \overline{\alpha}_{EoC}|}{\overline{\alpha}_{EoC}}$$
(1)

Though the results in Figure 9 indicate that the representative cycle is similar to the mean cycle 346 under stoichiometric conditions, where cycle-to-cycle variations remain low, the differences become 347 more evident at lean and rich mixtures. It is precisely this increased spread (dispersion) in the profiles 348 that results in an ensemble-averaged cycle that exhibits a much broader combustion duration with lower 349 ΔP_{max} values than most cycles in the sample and thereby is not considered representative of the typical 350 combustion evolution. For example, the ΔP_{max} value for the ensemble-averaged cycle for a rich mixture 351 $(\lambda = 0.75)$ is lower than the cycle with minimum ΔP_{max} values. Thus, the representative cycle based 352 on the function of merit described in equation (1) is more appropriate than ensemble averaging to get a 353 realistic evolution of combustion. Corresponding ΔP profiles and VIS- and IR-based penetration values 354 for this representative cycle have been also highlighted in Figures 6 and 7. 355

356 4.2. 1-D modelling analysis of scattering

Since 1-D models usually rely on ensemble-averaged cycle results, they cannot take into account the effect of cycle-to-cycle variability intrinsically. To overcome this caveat, an indirect evaluation of jet tip penetration variability is obtained by running the model with different input values. As described earlier in Section 3.1, the experimental heat release rate is an input parameter to the modeling workflow, which is calculated based on the pressure evolution in the pre-chamber and main-chamber.

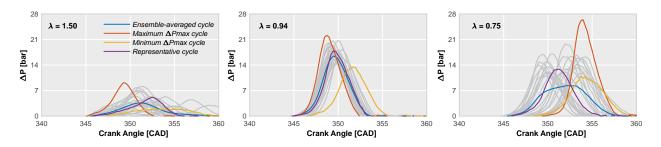


Figure 9: Experimental pressure difference ΔP for individual cycles and $\lambda = 0.94$, 1.5 and 0.75. Four cycles have been plotted in a different colour, namely the sample mean case, those with the maximum and minimum peak of ΔP and the representative cycle, which has been obtained as described in the text.

Our previous analysis has shown that a range of pressure traces ranging from a maximum to a minimum pressure difference between the pre-chamber and main-chamber are obtained. Aside from the ensemble-averaged mean cycle, an experimental cycle, chosen based on a merit function is considered as a good representation of a typical cycle. This section evaluates the variability of the predicted jet tip penetration by feeding these different cycles into the modeling workflow as varying inputs.

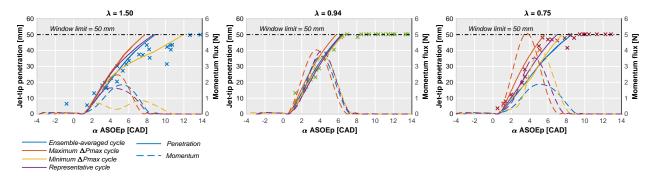


Figure 10: Comparison of jet tip penetration values calculated based on IR images (single markers) with 1-D gas jet simulation results (colored lines). Simulated jet momentum profiles for the four different input cycles (dashed lines) are also shown. Crank angle values are referenced to the start of ejection SOE_p based on ΔP threshold exceeding 0.5 bar.

Figure 10 compares the simulated jet tip penetration for each of the four characteristic cycles highlighted in Figure 9 with the experimental IR-based penetration values, which indicate the tip of the ejected pre-chamber jets. Input momentum fluxes are also included to bridge the link to orifice conditions. The time scale in Figure 10 is referenced to the start of ejection (SOE_p) , as this has proven to be a more meaningful way of analyzing the experimental results to shed light on the average behavior of the gas jet independent of its cyclic variability, as discussed in the previous sections. However, it should be noted that this start of ejection (SOE_p) time-referencing removes the timing variations in combustion and only considers intensity differences in combustion development for comparison, but the limited temporal resolution of the IR images do not allow for resolving this information on a cycle-to-cycle basis anyways.

The spread (dispersion) in the simulated jet tip penetration values among the different cycles is 377 similar to the observed experimental trend, i.e., the overall prediction is more accurate for stoichiomet-378 ric and rich cases. The spread is minimum for the stoichiometric condition and especially high for the 379 lean case. If this variation is quantified in terms of jet tip penetration reaching 50 mm (the limit of the 380 visualization window), it is clear that for the stoichiometric case all cycles reach this value of jet tip 381 penetration within 0.5 CAD of each other. However, this interval widens to 3.0 CAD and 4.0 CAD for 382 the rich and lean cases respectively. In addition to this spread, the predicted jet tip penetration shows 383 good agreement with the experimental values for the ensemble-averaged cycle only under stoichiomet-384 ric conditions. For the rich and lean conditions, as the pressure difference (and as a consequence the 385 input heat-release rate) evolution of the ensemble-averaged cycle is much flatter and wider than most 386 of the cycles (c.f. Figure 9), there is a reduced momentum flux leading to less accurate jet tip penetra-387 tion prediction results. Again under extreme conditions such as the rich case ($\lambda = 0.75$), the predicted 388 jet tip penetration for the ensemble-averaged cycle is slower than that of the minimum ΔP_{max} cycle, 389 which is in good agreement with the analysis presented in Figure 9. 390

On the other hand, the predicted jet tip penetration evolution for the representative case is in good agreement with the IR-based penetration values for stoichiometric and rich conditions, with the predicted values typically falling between cycles with minimum and maximum ΔP_{max} . However, for the lean case, the agreement is not so good with experimental cycle-to-cycle variations strongly indicating that accurate predictions on a single-cycle basis become increasing difficult due to combustion instability at such lean conditions.

397 4.3. Hole-to-hole variation

In the previous analysis, the image-based start of ejection values have been averaged over all eight of the pre-chamber orifices, however, high-speed VIS images can also be used to discern the jet-to-jet dispersion between pre-chamber jets emanating from different orifices within a single cycle. Low speed acquisition prevents from using IR images for this purposes. Figure 11 compares the time taken to reach the 5 mm VIS penetration threshold (SOE_{VIS}) for each individual orifice to the orifice-averaged

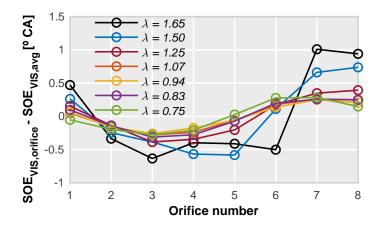
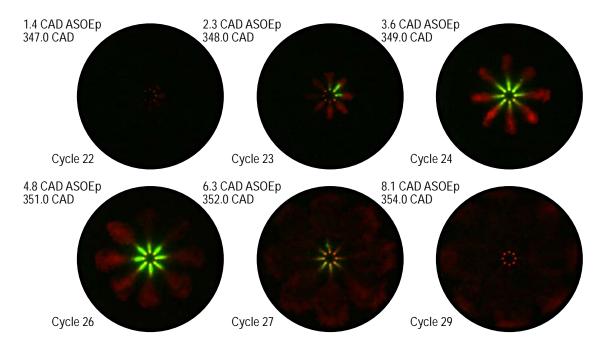


Figure 11: Average deviation of the orifice-resolved start of ejection based on VIS images compared to the sample average one.

SOE_{VIS} value (both ensemble-averaged over 30 cycles). Negative and positive values for this deviation 403 parameter indicate earlier and later start of ejection respectively for a particular orifice when compared 404 to the orifice-averaged value. In general Figure 11 shows that a clear gas jet ejection pattern exists 405 in azimuthal direction around the pre-chamber axis, with orifices #2 through #5 ejecting earlier than 406 the average (based on images such as those in Figure 2, orifice #2 corresponds to the 3 o'clock loca-407 tion and the subsequent ones are sequentially numbered in anti-clockwise direction). This sequence is 408 independent of λ and is most likely caused by the asymmetry existing inside the pre-chamber due to 409 the location of the spark plug and the injector, which results in a preferential direction for pre-chamber 410 flame propagation towards the faster orifices (#2 - #5). The maximum and minimum dispersion of this 411 deviation parameter around the zero value for a given λ indicate the strength of this variation, with 412 results consistently closer to zero and maximum observable dispersion under stoichiometric and lean 413 conditions respectively. Such jet-to-jet dispersion in the pre-chamber gas jet ejection is discussed only 414 in [24], where it is mainly attributed to the asymmetries (non-uniformities) in formation of the ini-415 tial spark-kernel inside the pre-chamber. Some of the potential factors that contribute to cycle-to-cycle 416 and jet-to-jet dispersion in pre-chamber gas jet ejection are spark kernel repeatability, fuel-air mixture 417 stratification and / or turbulence inside the pre-chamber. However, since no optical access into the 418 pre-chamber is currently available, one cannot fully discern the true reasons for this behavior using the 419 current engine configuration. 420



421 **5.** Analysis of events during pre-chamber gas jet ejection

Figure 12: Composite snapshots consisting of simultaneous IR images (red) overlaid on VIS images (green) showing the sequence of events in pre-chamber gas jet ejection for $\lambda = 0.94$. Acquisition CAD (both in absolute terms and after start of ejection ($ASOE_p$) based on ΔP threshold exceeding 0.5 bar) and acquisition cycle number are indicated on the to-left and bottom-left corner of each image.

Figure 12 shows a representative sequence of composite snapshots for $\lambda = 0.94$, created by super-422 imposing simultaneously acquired IR (employing a red color map) and VIS (employing a green color 423 map) images. This allows to specifically distinguish between regions of burnt gases (stronger IR emis-424 sion from combustion products) and active chemical reactions (stronger chemiluminescence emission 425 from combustion intermediates) respectively, while describing the overall spatial and temporal evolu-426 tion of the pre-chamber jets. Due to the selected color scheme, regions of overlap between IR and VIS 427 activity appears in shades of yellow. Acquisition CAD for the images is referenced to both absolute 428 and after start of ejection (ASOE_{*p*} defined based on ΔP) based timing based on the individual cycle. 429 The start of ejection based timing reference results in a steadily increasing IR penetration and a fairly 430 cycle-independent ΔP , which helps minimize the effect of cycle-to-cycle variations on the analysis to 431 some extent. Furthermore, the selected operating condition ($\lambda = 0.94$) corresponds to that with the 432 lowest variation as described earlier. 433

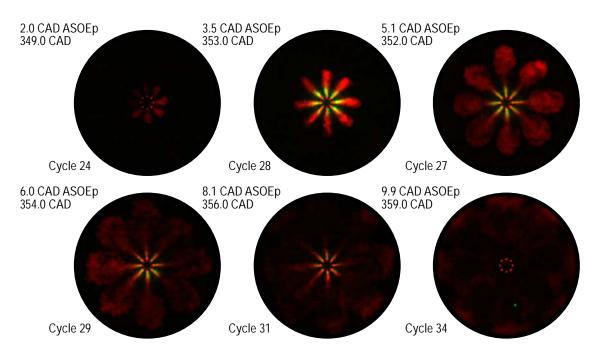


Figure 13: Composite snapshots consisting of simultaneous IR images (red) overlaid on VIS images (green) showing the sequence of events in pre-chamber gas jet ejection for $\lambda = 0.75$. Acquisition CAD (both in absolute terms and after start of ejection ($ASOE_p$) based on ΔP threshold exceeding 0.5 bar) and acquisition cycle number are indicated on the to-left and bottom-left corner of each image.

- ⁴³⁴ Different events can be observed based upon these images:
- Ejection of fresh mixture: The first composite snapshot at 1.4 *CAD* ASOE_p shows a preliminary
 ejection of mass from the pre-chamber. Since only IR signal is observable without any signs of
 broadband chemiluminescence (only shades of red with no shades of green or yellow), it is most
 likely that this stems from fresh fuel-air mixture being ejected from the pre-chamber before the
 premixed flame-front reaches the orifice.
- Start of ejection of active products: The second snapshot at 2.3 *CAD* ASOE_p (roughly coinciding with VIS-based start of ejection) shows the first appearance of broadband chemiluminescence
 i.e., pre-chamber gas jet ejection is discernible from VIS images by means of radiation (greenish regions) in the near-nozzle region, which suggests pre-chamber combustion products reaching
 the corresponding orifice. As described earlier, there is a clear asymmetry in the initial VIS penetration, with the 3 o'clock and the adjacent jet in the anti-clockwise direction (#2 and #3 respectively) already penetrating until roughly 10 *mm*, while no luminosity is observable in other

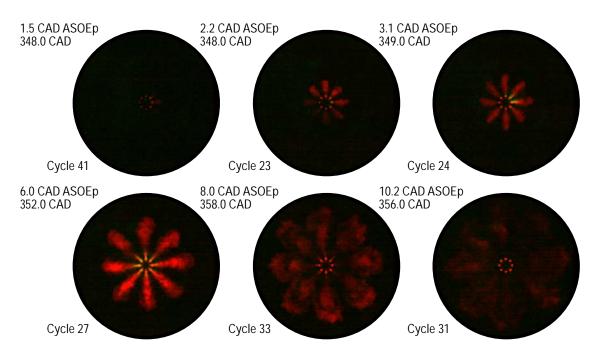


Figure 14: Composite snapshots consisting of simultaneous IR images (red) overlaid on VIS images (green) showing the sequence of events in pre-chamber gas jet ejection for $\lambda = 1.50$. Acquisition CAD (both in absolute terms and after start of ejection ($ASOE_p$) based on ΔP threshold exceeding 0.5 bar) and acquisition cycle number are indicated on the to-left and bottom-left corner of each image.

- orifices. The IR signal extends further away from the pre-chamber nozzles indicating that the initially ejected fuel-air mixture reaches deep into the main-chamber. The corresponding IR-based
 jet tip penetration is significantly longer than the VIS-based value, with a much more symmetrical
 layout. The observed symmetry in the IR imaging confirms that the IR jets initially correspond to
 fresh unreacted fuel-air mixture being forced through the pre-chamber orifices, which is mainly
 governed by the pressure difference P, thereby resulting in minimal jet-to-jet variation,
- Peak pressure increase: The third composite snapshot at 3.6 *CAD* ASOE_p corresponds to the timing of maximum VIS penetration and roughly maximum ΔP_{max} between the pre-chamber and main-chamber. The overall appearance of the pre-chamber jets is fairly similar to the previous snapshot, but with IR signal penetration progressing further into the main-chamber. The main difference between the two snapshots is in the near-orifice region, where VIS luminosity is almost symmetrical extending from the eight pre-chamber orifices. The axial extent of the VIS region

at this instant is around 11 mm and is consistent for all pre-chamber orifices. This potentially
 indicates that conditions upstream of the orifices are similar, i.e., the premixed flame-front has
 most likely consumed all the available fuel-air mixture in pre-chamber.

- Piston window impingement: The fourth composite snapshot at 4.8 CAD ASOE_p corresponds to 462 around 1 CAD after maximum pressure difference (ΔP_{max}) for that particular cycle is attained. 463 This indicates that the combustion in the pre-chamber is receding, i.e., heat-release cannot com-464 pensate for the ejection of gas and heat transfer to the surroundings leading to decreasing ΔP . 465 Again, a similar overall pre-chamber gas jet structure is observable based on the IR and VIS im-466 ages, with a very intense VIS radiation. However, the IR radiation at the jet tip appears wider 467 when compared to the earlier snapshots. As no IR luminosity change is observable in this region, 468 no additional heat-release is to be expected, i.e., there is no evidence of the initially unreacted 469 fuel-air mixture igniting and undergoing combustion. Hence, this widening of the pre-chamber 470 gas jet head cannot be due to combustion, as in a diesel-jet after ignition [44]. Instead, this radial 471 widening of the jet tip appears to be consistent with the gas jet impinging on the piston window, 472 which is confirmed by comparing the IR-based jet tip penetration with the position of the bowl 473 window at that timing. 474
- End of ejection: The last two composite snapshots at 6.3 and 8.1 CAD ASOE_p correspond to later 475 stages when the pressure difference ΔP drops back to zero. The intensity of gas jet radiation 476 in both VIS and IR is significantly lower than the earlier snapshots. This is especially evident in 477 the VIS region, where radiation primarily stems from chemically reacting products being ejected 478 through the nozzle orifices, which are depleted by the end of pre-chamber combustion process. 479 VIS radiation eventually disappears, while the IR signal extends further in the form a wall gas jet 480 along the piston window towards the cylinder wall. The jet structure essentially vanishes in the 481 last snapshot, where the injected mass appears to be spread across the entire main-chamber. In 482 this particular instant, the radiation at the exit of the pre-chamber orifice is clearly visible, which 483 is most likely caused by thermal radiation emanating from the hot mass remaining within the 484 pre-chamber and / or hot surfaces inside the pre-chamber. 485
- Figure 13 and 14 show similar sequence of composite snapshots for a rich ($\lambda = 0.75$) and lean ($\lambda =$ 1.50) condition respectively. The rich case appears fairly similar to the stoichiometric one discussed earlier in terms of the sequence of events, observed gas jet radiation structure and spatial evolution

of the pre-chamber gas jets during the ejection process. Note that in the rich case, it is reasonable to 489 expect some chemical activity extending in the downstream direction, due to the excess fuel in the pre-490 chamber possibly being oxidized by the air in the main-chamber. However, the IR images do not show 491 any indication of strong radiation at the jet tip, which would possibly indicate local heat release and 492 subsequent temperature rise. The 1-D gas jet model will be used to show that as the gas jet penetrates 493 further downstream of the nozzle, the increasing air entrainment strongly dilutes the ejected mass to 494 mixtures that are too lean to support combustion. This suggests that ambient entrainment plays a 495 leading role in the evolution of the ejected combustion products. 496

As for the lean case, the sequence of events remains consistent but with much lower VIS radiation levels due to the less intense heat release associated with lean combustion. The ejected combustion products from a lean mixture will be significantly different in composition when compared to the other two cases. Furthermore, the lower pressure difference results in lower gas jet momentum at the nozzle leading to a shorter penetration of the gas jet, as modeling will confirm.

502 5.1. 1-D Modeling analysis of the gas jet ejection process

Figure 15 summarizes 1-D modeling results of jet tip penetration for all operating conditions in 503 this study. Only 1-D model predictions based upon the representative cycle are shown with predicted 504 jet tip penetration values compared to experimental IR- and VIS-based results. The IR-based jet tip 505 penetration values are plotted for all 30 cycles, while only the representative cycle is included for the 506 VIS-based information. Starting with the leanest case, the trends clearly show that IR-based jet tip 507 penetration becomes faster with richer pre-chamber conditions down to $\lambda = 0.94$. The remaining rich 508 cases show very similar jet tip penetration results though the peak momentum slightly decreases, which 509 is in good agreement with the measured peak ΔP values shown in Figure 8. 510

The agreement of the predicted 1-D modeling based jet tip penetration values with the IR-based 511 results is good starting from $\lambda = 1.25$ and into the richer conditions. For the leanest of the conditions, 512 accuracy is just fair. An almost 1 CAD shift between the initial timing of the penetration based on 513 the modeling and experimental results is also observable with the three leanest cases. For these cases, 514 the spread in experimental data is evident, which indicates that combustion is more unstable not just 515 in timing but also in development, hence generating reliable modeling predictions becomes far more 516 difficult. For the other stoichiometric and rich conditions, the agreement between the modeling and 517 experimental results is quite remarkable. 518

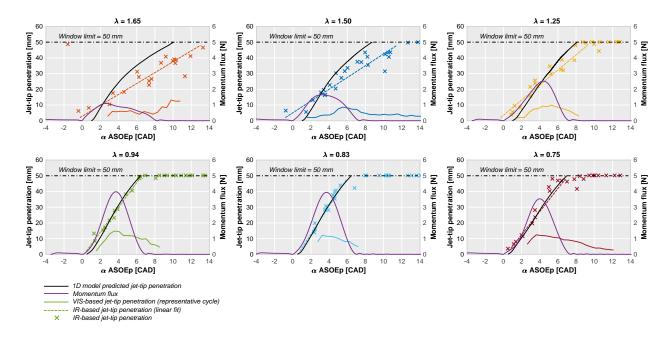


Figure 15: Comparison of jet tip penetration based on IR (single markers - all cycles, dotted line - linear fit) and VIS (colored line - only representative cycle) imaging with predicted jet tip penetration values (black line) using the input data from the representative cycle. Momentum flux used for the simulation is also included. Crank angle values are referenced to the start of ejection SOE_p based on ΔP exceeding 0.5 bar.

Figure 16 provides further insight into the evolution of flow dynamics along the gas jet axis during 519 the ejection process for $\lambda = 0.94$ using the representative engine cycle. Corresponding momentum 520 flux at the nozzle and the modeled gas jet tip penetration have been presented earlier in Figure 10. 521 Mass and momentum fluxes along the gas jet axial coordinate at different instances during the ejection 522 process are shown with colors signifying the crank angle timing. To facilitate analysis, plots are split into 523 two intervals corresponding to timings before and after the occurrence of maximum momentum flux. 524 During the initial stage of the ejection process (left plots in Figure 16), both momentum flux and mass 525 flow increase with time as expected. A gas jet can be considered as a set of momentum flux parcels that 526 travel downstream incorporating ('entraining') ambient gas, which is quantified in terms of the mass 527 flux. At any given instant, the momentum flux decreases with the axial distance as the parcels with the 528 highest momentum are the latest injected ones, which are much closer to the orifice. Such parcels tend 529 to push the ones ahead thereby transferring momentum towards the jet tip. On the other hand, mass 530 flux at any given instant is always increasing with axial distance as the entrainment of ambient mass 531

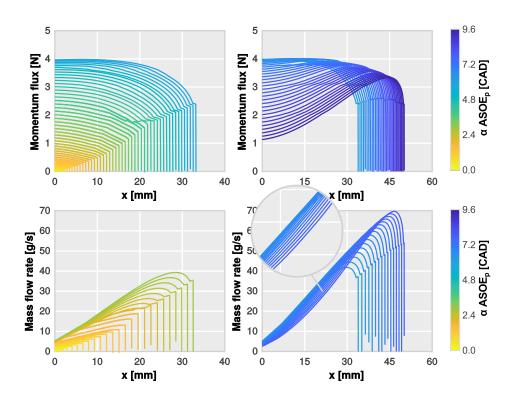


Figure 16: Spatial distribution of momentum (top) and mass fluxes (bottom) along the axial coordinate of the gas jet at different instances during pre-chamber gas jet ejection process for $\lambda = 0.94$ using the representative cycle. Colored contours correspond to different timings indicated by CAD values after SOE_p as shown by the color bar. Plots on the left correspond to timings between start of ejection and the occurrence of maximum momentum flux, while plots on the right correspond to subsequent timings until the end of ejection.

from the main-chamber starts at the nozzle and increases with the parcel timeline, i.e. along the axial
distance. This means that the ejected pre-chamber gas jet flow becomes increasingly diluted with axial
distance.

Consistent with the previous description of the gas jet, the later ejected parcels (right plots in Figure 16) have lower momentum than the ones ejected earlier. Hence, starting from the earlier slightly decreasing axial distribution, the shape of the momentum flux progressively turns into an increasing function. This means that the newly injected parcels do not help in pushing the jet tip anymore. As for the mass flux, the axially-increasing trend remains the same during the second phase, as ambient mass is steadily being incorporated into the spray due to the momentum flux. However, mass flux values at a given axial position steadily decrease with time, although the temporal decrease rate is much lower than that of the momentum flux. This evolution is similar to the end of injection event in diesel-like sprays, investigated in detail in [34]. Owing to the decreasing momentum flux, progress in jet tip penetration in the second phase is roughly only 15 *mm* compared to the initial increase of more than 30 *mm* during the first phase of the pre-chamber gas jet ejection process.

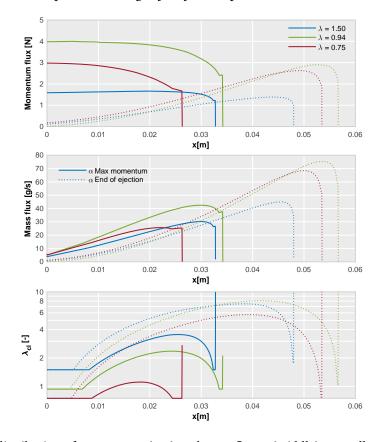


Figure 17: Spatial distribution of momentum (top) and mass fluxes (middle), as well as air-excess ratio λ_{cl} (bottom) along the axial coordinate of the gas jet for $\lambda = 1.55$, 0.94 and 0.75 using the representative cycle. Timings correspond to the instance of maximum momentum flux (solid line) and the end of ejection (dashed line).

Figure 17 compares the magnitudes of mass and momentum fluxes for three operating conditions, namely $\lambda = 1.55,0.94$ and 0.75 at two characteristic timings, namely during the occurrence of peak momentum flux at the orifice (last timing on Figure 16, left) and at the end of ejection (last timing on Figure 16, right). The previously observed trends for momentum and mass flux distribution along the spray axis remain relatively similar for all three operating conditions at both timings. During the maximum momentum flux timing at the orifice, the results clearly indicate higher local momentum and mass fluxes for the stoichiometric condition compared to the other conditions, due to the more rapid pre-chamber combustion development. A higher momentum flux leads to more intense entrainment of ambient air and hence increased mass flow.

Entrainment effects are also estimated in terms of a third variable shown on the plot, namely the 555 air-fuel ratio λ_{cl} along the gas jet axis. This will be the richest location within the jet cross-section at 556 each axial position of the gas jet, as the fuel mass fraction profiles are assumed to follow a Gaussian 557 distribution in the 1-D model. In the near-orifice region, the local value of λ_{cl} correlates to the cor-558 responding pre-chamber λ value, and increases after remaining constant for a short distance. Beyond 559 this region, the stoichiometric pre-chamber condition exhibits the most prominent increase in λ_{cl} up to 560 around 15 mm, indicating faster entrainment; despite this, it never reaches λ_{cl} values associated with 561 the leanest condition. Furthermore, none of the three shown λ_{cl} evolution intersect with each other. 562 The decreasing trend observed at the tip of the jet is an artifact of the 1D simplification of the jet tip 563 zone, which is specially evident for highly transient injection velocity, as already shown in [36]. 564

At the end of ejection timing, results show that the spray tip propagates with larger momentum for 565 the stoichiometric condition compared to the other two. However, distribution of mass and momentum 566 fluxes remain fairly similar for all three conditions starting from the orifice exit up to around 30 mm 567 downstream. This indicates that the second phase of gas jet ejection (following the occurrence of peak 568 momentum flux) results in a very similar flow state near the vicinity of the pre-chamber orifice at the 569 end of the ejection process. In other words, the initial ramp-up period of ejection until the occurrence 570 of maximum momentum flux is the most dependent on pre-chamber combustion characteristics. For 571 scenarios with fuel-air mixtures present in the main-chamber, the subsequent combustion development 572 is most likely to be influenced by this initial ejection phase, as the ramp-down phase of ejection collapses 573 to very similar flow distributions, almost independent of the operating conditions. 574

Information derived from the 1-D model analysis also provides a rough quantification for the dilution of the ejected pre-chamber gases. As seen from the experimental images, there is no evidence of chemical activity in regions situated further downstream of the maximum VIS penetration length (8 to 15 *mm* depending on λ). According to the estimations of the 1-D model, the richest locations within the gas jet at the timing of maximum momentum flux will all be below a local $\lambda = 2$ for distances longer than 15 *mm*, except for the stoichiometric case. At the end of ejection timing, the plot shows all conditions being much leaner than the $\lambda = 2$. Keeping in mind that most of the ejected mass will be leaner than the values shown on the spray axis, the resulting mixture is most likely too lean to react,
which is in agreement with the absence of chemical activity in the experimental images downstream of
the maximum VIS penetration.

585 6. Summary and conclusions

This work reports an experimental study of pre-chamber gas jets ejected into the main-chamber (air ambient) in a heavy-duty, single-cylinder optical engine. This pre-chamber only fueling strategy eliminates the effect of ignition and subsequent combustion development in the main-chamber on prechamber gas jet flow dynamics. Different objectives have been achieved, namely the detailed description of the near-orifice hot jets structure by means of high-speed visible imaging, the quantification of the tip penetration by means of low speed infrarred imaging, a detailed analysis of cycle-to-cycle scattering as well as the validation of a 1-D gas jet model to describe the ejected gas flow dynamics.

⁵⁹³ The main conclusions of this study are summarized as follows:

- Cycle-to-cycle variation in the ejection process is quantified in detail in terms of both pressure
 difference and jet-penetration. As expected, stable combustion is achieved around stoichiometric
 conditions due to rapid heat-release.
- Although cycle-to-cycle variations are linked to maximum burning rate and thereby to the peak 597 pressure in the pre-chamber, most of the observed fluctuations tend to disappear when refer-598 encing the time evolution to the start of ejection, which is defined based on a threshold pres-599 sure difference of 0.5 bar or a visible radiation penetration threshold of 5 mm. Thus, most of 600 the fluctuations are related to the start of combustion in the pre-chamber. Jet-to-jet variations 601 shows a repeatable ignition sequence with some orifices ejecting earlier than others do. This most 602 likely stems from the differences in the dynamics of premixed flame-front propagation within the 603 pre-chamber. This is most likely due to the intense turbulence induced stratification of the pre-604 chamber charge caused by the fuel injection event and the stochastic nature of ignition event, but 605 further investigation is warranted to confirm this hypothesis. 606
- Combustion evolution is described using visible and infrared images. Ejection of unreacted fresh
 fuel-air mixture into the main-chamber is detected from infrared images initially due to the fa vorable pressure drop across the orifice. Visible radiation is detected only after a delay (typically

in the order of 2.5 *CAD*) from the start of ejection as defined in terms of pressure difference (ΔP). This chemiluminescence activity is limited exclusively to the vicinity of the orifice, while downstream of this region only infrared radiation is observable. No indication of chemical activity is detectable in the infrared-only region, which is most likely due to the ultra-lean mixture composition (no fueling of the main-chamber).

- A 1-D spray model is adapted to predict tip-penetration of ejected pre-chamber gas jets. Flow
 conditions at the orifice (mass and momentum fluxes) are obtained using a 0-D analysis of the
 pre-chamber and main chamber. Time-varying ambient conditions in the pre-chamber are also
 obtained from the 0-D analysis.
- Variation in the tip-penetration is quantified based on 1-D model predictions with different input 619 data derived from corresponding experimental runs. Conditions of sample mean pressure rise 620 along with extreme values (maximum and minimum) of pressure increase are evaluated along 621 with a representative cycle, selected based on a merit function. The representative cycle case pro-622 vides most accurate predictions of tip-penetration, while the sample mean case performs poorly 623 especially in conditions with significant cycle-to-cycle variation. Hence, the proposed 1-D com-624 putational tool can be used for pre-chamber gas jet tip-penetration predictions with reasonable 625 certainty. 626
- Analysis of the 1-D jet model predictions suggests that the initial ejection process, i.e., until the point of maximum momentum flux at the nozzle, is highly dependent on the air-fuel ratio λ in the pre-chamber. Hence, the local flow and most likely the associated combustion development for fuelled main-chamber scenario will be highly dependent on this early-ejection phase. Flow evolution at later timings becomes relatively independent of λ in the pre-chamber, with a characteristic fast slow-down due to the steeply decreasing ejection velocities.

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644 References

- [1] S. Heyne, M. Meier, B. Imbert, D. Favrat, Experimental investigation of prechamber autoignition in a natural gas engine
 for cogeneration, Fuel 88 (3) (2009) 547 552. doi:https://doi.org/10.1016/j.fuel.2008.09.032.
- [2] A. Jamrozik, Lean combustion by a pre-chamber charge stratification in a stationary spark ignited engine, Journal of
 Mechanical Science and Technology 29 (5) (2015) 2269–2278. doi:https://doi.org/10.1007/s12206-015-0145-7.
- [3] J. Benajes, R. Novella, J. Gomez-Soriano, I. Barbery, C. Libert, F. Rampanarivo, M. Dabiri, Computational assessment to wards understanding the energy conversion and combustion process of lean mixtures in passive pre-chamber ignited en gines, Applied Thermal Engineering 178 (2020) 115501. doi:https://doi.org/10.1016/j.applthermaleng.2020.
 115501.
- [4] P. Hlaing, M. Echeverri Marquez, E. Singh, F. Almatrafi, E. Cenker, M. Ben Houidi, B. Johansson, Effect of pre-chamber
 enrichment on lean burn pre-chamber spark ignition combustion concept with a narrow-throat geometry, WCX SAE
 World Congress Experience 2020-01-0825 (2020). doi:10.4271/2020-01-0825.
- [5] A. Shah, Improving the efficiency of gas engines using pre-chamber ignition, Ph.D. thesis, Lund University (2015).
- [6] E. Toulson, H. J. Schock, W. P. Attard, A review of pre-chamber initiated jet ignition combustion systems, SAE 2010
 Powertrains Fuels & Lubricants Meeting 2010-01-2263 (2010). doi:https://doi.org/10.4271/2010-01-2263.
- [7] A. Shah, P. Tunestal, B. Johansson, Effect of relative mixture strength on performance of divided chamber 'avalanche
 activated combustion' ignition technique in a heavy duty natural gas engine, SAE 2014 World Congress & Exhibition
 2014-01-1327 (2014). doi:https://doi.org/10.4271/2014-01-1327.
- [8] W. P. Attard, M. Bassett, P. Parsons, H. Blaxill, A new combustion system achieving high drive cycle fuel econ omy improvements in a modern vehicle powertrain, SAE 2011 World Congress & Exhibition 2011-01-0664 (2011).
 doi:https://doi.org/10.4271/2011-01-0664.
- [9] J. Pan, Z. Hu, H. Wei, M. Pan, X. Liang, G. Shu, L. Zhou, Understanding strong knocking mechanism through high strength optical rapid compression machines, Combustion and Flame 202 (2019) 1–15. doi:https://doi.org/10.
 1016/j.combustflame.2019.01.004.
- [10] C. Chen, P. Pal, M. Ameen, D. Feng, H. Wei, Large-eddy simulation study on cycle-to-cycle variation of knocking com bustion in a spark-ignition engine, Applied Energy 261 (2020) 114447. doi:https://doi.org/10.1016/j.apenergy.
 2019.114447.

- [11] A. Broatch, R. Novella, J. García-Tíscar, J. Gomez-Soriano, P. Pal, Investigation of the effects of turbulence modeling
 on the prediction of compression-ignition combustion unsteadiness, International Journal of Engine Research (2021)
 1468087421990478.
- [12] W. P. Attard, H. Blaxill, A gasoline fueled pre-chamber jet ignition combustion system at unthrottled conditions, SAE
 International Journal of Engines 5 (2) (2012) 315–329. doi:https://doi.org/10.4271/2012-01-0386.
- [13] C. E. C. Alvarez, G. E. Couto, V. R. Roso, A. B. Thiriet, R. M. Valle, A review of prechamber ignition systems as lean
 combustion technology for si engines, Applied Thermal Engineering 128 (2018) 107 120. doi:https://doi.org/
 10.1016/j.applthermaleng.2017.08.118.
- [14] Natural gas vehicle research workshop, U.S. Department of Energy Vehicle Technologies Office, 2017.
- [15] S. Yamaguchi, N. Ohiwa, T. Hasegawa, Ignition and burning process in a divided chamber bomb, Combustion and Flame
 59 (2) (1985) 177 187. doi:https://doi.org/10.1016/0010-2180(85)90023-9.
- [16] S. Biswas, S. Tanvir, H. Wang, L. Qiao, On ignition mechanisms of premixed ch4/air and h2/air using a hot turbulent jet
 generated by pre-chamber combustion, Applied Thermal Engineering 106 (2016) 925 937. doi:https://doi.org/
 10.1016/j.applthermaleng.2016.06.070.
- [17] S. Biswas, L. Qiao, Ignition of ultra-lean premixed hydrogen/air by an impinging hot jet, Applied energy 228 (2018)
 954–964. doi:https://doi.org/10.1016/j.apenergy.2018.06.102.
- [18] E. Mastorakos, P. Allison, A. Giusti, P. De Oliveira, S. Benekos, Y. Wright, C. Frouzakis, K. Boulouchos, Fundamental aspects of jet ignition for natural gas engines, SAE International Journal of Engines 5 (10) (2017) 2429–2438. doi: https://doi.org/10.4271/2017-24-0097.
- [19] P. Allison, M. de Oliveira, A. Giusti, E. Mastorakos, Pre-chamber ignition mechanism: Experiments and simulations on
 turbulent jet flame structure, Fuel 230 (2018) 274–281. doi:10.1016/j.fuel.2018.05.005.
- [20] G. Gentz, B. Thelen, P. Litke, J. Hoke, E. Toulson, Combustion visualization, performance, and cfd modeling of a pre chamber turbulent jet ignition system in a rapid compression machine, SAE International Journal of Engines 8 (2)
 (2015) 538–546. doi:https://doi.org/10.4271/2015-01-0779.
- [21] M. Gholamisheeri, B. C. Thelen, G. R. Gentz, I. S. Wichman, E. Toulson, Rapid compression machine study of a premixed,
 variable inlet density and flow rate, confined turbulent jet, Combustion and Flame 169 (2016) 321 332. doi:https:
 //doi.org/10.1016/j.combustflame.2016.05.001.
- [22] S. Schlatter, B. Schneider, Y. M. Wright, K. Boulouchos, Comparative study of ignition systems for lean burn gas engines
 in an optically accessible rapid compression expansion machine, 11th International Conference on Engines & Vehicles
 2013-24-0112 (2013). doi:https://doi.org/10.4271/2013-24-0112.
- [23] J. Benajes, R. Novella, J. Gomez-Soriano, P. Martinez-Hernandiz, C. Libert, M. Dabiri, Evaluation of the passive pre chamber ignition concept for future high compression ratio turbocharged spark-ignition engines, Applied Energy 248
 (2019) 576 588. doi:https://doi.org/10.1016/j.apenergy.2019.04.131.
- [24] X. Li, W. Zhang, Z. Huang, D. Ju, L. Huang, M. Feng, X. Lu, Z. Huang, Pre-chamber turbulent jet ignition of methane/air
 mixtures with multiple orifices in a large bore constant volume chamber: effect of air-fuel equivalence ratio and pre mixed pressure, Frontiers in Energy 13 (3) (2019) 483–493. doi:10.1007/s11708-019-0631-1.

- [25] A. Shah, P. Tunestål, B. Johansson, Cfd simulations of pre-chamber jets' mixing characteristics in a heavy duty natural
 gas engine, JSAE/SAE 2015 International Powertrains, Fuels & Lubricants Meeting 2015-01-1890 (2015). doi:https:
 //doi.org/10.4271/2015-01-1890.
- [26] G. Kammel, F. Mair, J. Zelenka, M. Lackner, A. Wimmer, G. Kogler, E. Bärow, Simulation based predesign and experimental validation of a prechamber ignited hpdi gas combustion conceptdoi:https://doi.org/10.4271/2019-01-0259.
- [27] G. Xu, Y. M. Wright, M. Schiliro, K. Boulouchos, Characterization of combustion in a gas engine ignited using a small
 un-scavenged pre-chamber, International Journal of Engine Research 21 (7) 1085–1106. doi:https://doi.org/10.
 1177/1468087418798918.
- [28] B. Korb, K. Kuppa, H. D. Nguyen, F. Dinkelacker, G. Wachtmeister, Experimental and numerical investigations of charge motion and combustion in lean-burn natural gas engines, Combustion and Flame 212 (2020) 309–322. doi:https://doi.org/10.1016/j.combustflame.2019.11.005.
- [29] A. Validi, H. Schock, F. Jaberi, Turbulent jet ignition assisted combustion in a rapid compression machine, Combustion
 and Flame 186 (2017) 65–82. doi:https://doi.org/10.1016/j.combustflame.2017.07.032.
- [30] Q. Malé, G. Staffelbach, O. Vermorel, A. Misdariis, F. Ravet, T. Poinsot, Large eddy simulation of pre-chamber ignition in an internal combustion engine, Flow, Turbulence and Combustion 103 (2) (2019) 465–483. doi:https://doi.org/ 10.1007/s10494-019-00026-y.
- [31] A. Validi, F. Jaberi, Numerical study of turbulent jet ignition in a lean premixed configuration, Flow, Turbulence and
 Combustion 100 (1) (2018) 197–224. doi:https://doi.org/10.1007/s10494-017-9837-7.
- [32] F. Qin, A. Shah, Z.-w. Huang, L.-n. Peng, P. Tunestal, X.-S. Bai, Detailed numerical simulation of transient mixing and
 combustion of premixed methane/air mixtures in a pre-chamber/main-chamber system relevant to internal combustion
 engines, Combustion and Flame 188 (2018) 357–366. doi:https://doi.org/10.1016/j.combustflame.2017.10.
 006.
- [33] S. Benekos, C. E. Frouzakis, G. K. Giannakopoulos, M. Bolla, Y. M. Wright, K. Boulouchos, Prechamber ignition: An
 exploratory 2-d dns study of the effects of initial temperature and main chamber composition, Combustion and Flame
 215 (2020) 10–27. doi:https://doi.org/10.1016/j.combustflame.2020.01.014.
- [34] M. P. B. Musculus, K. Kattke, Entrainment waves in diesel jets, SAE International Journal of Engines 2 (11) 1170–1193.
 doi:10.4271/2009-01-1355.
- [35] L. M. Pickett, J. Manin, C. L. Genzale, D. L. Siebers, M. P. B. Musculus, C. A. Idicheria, Relationship Between Diesel
 Fuel Spray Vapor Penetration/Dispersion and Local Fuel Mixture Fraction, SAE International Journal of Engines (1)
 764–799. doi:10.4271/2011-01-0686.
- [36] J. V. Pastor, J. J. López, J. M. García, J. M. Pastor, A 1d model for the description of mixing-controlled inert diesel sprays,
 Fuel 87 (13) (2008) 2871 2885. doi:https://doi.org/10.1016/j.fuel.2008.04.017.
- [37] G. N. Abramovich, The theory of turbulent jets, The MIT Press Classics, 1963.
- [38] K. Bardis, P. Kyrtatos, G. Xu, C. Barro, Y. M. Wright, K. Boulouchos, Development and validation of a novel quasi dimensional combustion model for un-scavenged prechamber gas engines with numerical simulations and engine experiments, International Journal of Engine Research (2020) 1468087420951338.
- [39] J. Benajes, R. Novella, J. Gomez-Soriano, I. Barbery, C. Libert, Advantages of hydrogen addition in a passive pre-chamber
 ignited si engine for passenger car applications, International Journal of Energy Research.

- [40] J. O'Connor, M. P. B. Musculus, Effects of exhaust gas recirculation and load on soot in a heavy-duty optical diesel engine
 with close-coupled post injections for high-efficiency combustion phasing, International Journal of Engine Research (4)
 421–443. doi:10.1177/1468087413488767.
- [41] J. B. Heywood, et al., Internal combustion engine fundamentals, McGraw-Hill New York, 1988.
- [42] R. Novella, J. Gomez-Soriano, P. Martinez-Hernandiz, C. Libert, F. Rampanarivo, Improving the performance of the
 passive pre-chamber ignition concept for spark-ignition engines fueled with natural gas, Fuel 290 (2021) 119971.
 doi:https://doi.org/10.1016/j.fuel.2020.119971.
- [43] J. Desantes, J. Pastor, J. García-Oliver, J. Pastor, A 1d model for the description of mixing-controlled reacting diesel sprays, Combustion and Flame 156 (1) (2009) 234 249. doi:https://doi.org/10.1016/j.combustflame.2008.
 10.008.
- [44] J. Desantes, J. García-Oliver, T. Xuan, W. Vera-Tudela, A study on tip penetration velocity and radial expansion of reacting
 diesel sprays with different fuels, Fuel 207 (2017) 323 335. doi:https://doi.org/10.1016/j.fuel.2017.06.108.
- [45] J. V. Pastor, J. M. Garcia-Oliver, J. M. Pastor, W. Vera-Tudela, One-dimensional diesel spray modeling of multicomponent
 fuels, Atomization and Sprays 25 (6) (2015). doi:10.1615/AtomizSpr.2014010370.
- [46] J. Desantes, R. Payri, F. Salvador, A. Gil, Development and validation of a theoretical model for diesel spray penetration,
 Fuel 85 (2006) 910–917. doi:10.1016/j.fuel.2005.10.023.
- [47] P. Pal, C. Kolodziej, S. Choi, S. Som, A. Broatch, J. Gomez-Soriano, Y. Wu, T. Lu, Y. C. See, Development of a virtual cfr engine model for knocking combustion analysis, SAE International Journal of Engines 11 (6) (2018) 1069–1082.
 doi:https://doi.org/10.4271/2018-01-0187.