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Additional Information

One-Day-Ahead Electricity Demand Forecasting in Holidays using discrete-interval moving seasonalities

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Abstract

Transmission System Operators provide forecasts of electricity demand to the electricity system. The producers and sellers use this information to establish the next day production units planning and prices. The results obtained are very accurate. However, they have a great deal with special events forecasting. Special events produce anomalous load conditions, and the models used to provide predictions must react properly against these situations. In this article, a new forecasting method based on multiple seasonal Holt-Winters modelling including discrete-interval moving seasonalities is applied to the Spanish hourly electricity demand to predict holidays with a 24-hour prediction horizon. It allows the model to integrate the anomalous load within the model. The main results show how the new proposal outperforms regular methods and reduces the forecasting error from 9.5% to under 5% during holidays.

Keywords: time series, forecasting, electricity demand, anomalous load

1. Introduction

The responsibility of the electricity production plan and schedule in western countries relies on the transmission system operators (TSO). They have to provide accurate forecasts of the next-day and next-hour electricity demand to the electricity system, known as short-term load forecasting (STLF). The rest of the system entities –producers, sellers and resellers – use this information to obtain an accurate production plan, that translates to realistic prices of energy. The overproduction must be avoided, and the underproduction is completed with high-costs operations. Time series tools-based forecasts made by the TSO's are very accurate. In Red Eléctrica de España (REE), the Spanish TSO, a forecast´s error under 2% is reported [1]. However, while anomalous load occurs, the daily forecasting is not so accurate. The forecast error in these conditions rises to values that could easily exceed 10%. These situations are commonly known as special events. They can be produced due to extreme weather conditions change, but most commonly due to the calendar effect: national holidays. The models try to reproduce a pattern, but in case a special event occurs, models have to react properly and model them. There is a strong appeal for forecasters to improve the forecasts during the special days.

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Thus, TSOs create complex models where those circumstances are taken into account. Those models can reach several hundred parameters to solve the calendar problem.

This article proposes a new model to one-day-ahead forecast Spanish hourly electricity demand, focused on national holidays and bridges – a source of anomalous loads –. The model uses Multiple Seasonal Holt-Winters models with Discrete-Interval Moving Seasonalities (nHWT-DIMS). This model considers special loads as a part of the model itself. All special days are modelled as a new discrete seasonality, completely integrated, with only one new parameter. Thus, the model is still simple but also improves the forecasting results. The forecasting error during special events sinks under 5% using only a few parameters and the global forecasting error remains under 2%. The new model is doubly validated. First, a benchmark against other well-known forecasting methods is carried out using the Spanish hourly electricity demand. Next, a comparison with other previous validated results obtained by specific methods for anomalous demand [2] for French half-hourly electricity demand is made.

This paper is organized as follows: in section 2 the standing literature regarding this topic is reviewed; section 3 presents the nHWT-DIMS for holidays methodology; in section 4 the nHWT-DIMS methodology applied to the case study is presented; section 5 summarizes the results obtained and in section 6 the conclusions reached are shown.

2. Related work

Artificial intelligence techniques have been widely used to predict electricity demand in recent years. The methods using Artificial Neural Network (ANN) and state techniques used to deal with the problem of holidays are based on treating holidays as weekends, as applied in [3–5]. Other techniques include dummy variables as regressors for holidays. An alternative technique to consider holidays is to divide into the predictions for the proportional curve and the daily extremum of electricity demand [6]. Fallah et al. [7] analyze the methodology while using ANN (similar-pattern, variable selection, hierarchical forecasting and weather station selection), concluding that a hybrid methodology would result in the most effective. Support Vector Regressions [8] and fuzzy logic [9] are also commonly used, in which the holidays are integrated as a part of the regression inputs. Recently, there is a special interest in Spatio-temporal techniques such as the use of Wavelets for forecasting. Zhang et al. [10] use models based on Wavelet Neural Networks in which the optimization is carried out using Fly Fruit optimization algorithms (FOA). Pattern-similarity based forecasting [11–13] clusters the time series to select patterns to provide new forecasts, including holidays. However, artificial intelligence techniques need a large amount of data to train and the number of holidays or long weekends is too low to achieve good results in addition to other drawbacks such as very poorly interpretable models or a high number of hyper-parameters.

Traditionally, statistical techniques developed to manage the special events try to adapt the series with external elements. Tarsitano and Amerise [14] apply use dummy variables for public holidays, local public holidays, Easter, days near holidays and part-time holidays while forecasting short-term electricity demand in some regions of Italy, confirming the most significant variables influencing hourly load are calendar dummy variables. This methodology has some drawbacks: the parameters accompany a series of dummy variables to modify the original model and adapt it to each special situation. López et al. [3] started a classification of the special events that can be applied to models. They found that there are more than 40 types of different special days. The Spanish' TSO included this classification in their STLF methodology [15]. However, the occurrences of the same conditions are sporadic and, therefore, the training for these models is complicated and insufficient. On other occasions, it is simply chosen to replace special days with other similar days, with the consequent loss of information [16]. Pros and cons of these approaches to deal with public holidays such as removing them from the data set, treating them as weekends or introducing separate holiday dummies using German electricity load are analyzed in [17].

Within the statistical techniques, the application to exponential smoothing methods and Holt-Winters methods in order to forecast anomalous demand has usually been done by modifying the original series. In particular, special events are replaced in the time series by a normal day, thus filtering the series. García-Díaz and Trull generalized Holt-Winters models and included new initialization methods. The authors stated double seasonal models offer more precise forecasts than those of three seasonalities applying to the hourly electricity demand in Spain when using the raw time series with no filtering [18]. This situation occurs because it is necessary to rework the series and eliminate the effects produced by the special days to optimize the use of the three seasons. Therefore, modifying the original time series has some disadvantages that affect the performance of these methods. In recent studies, it has been identified that although parameters are mostly stable, the main focus of instability is the calendar effect [19].

In the last years, other approaches based on exponential smoothing and Holt-Winters methods have been provided for the management of special events, in which the use of variables external to the model is avoided and is managed with internal variables. Bermudez [20] considered the inclusion of covariates within the exponential smoothing model itself, considering value 1 for normal days, and different values according to the expected weight for each type of special event. Göb et al. [21] also introduced covariates within the model that allow modelling both the effect of temperature and special days. Arora and Taylor [2] used rule-based methods to establish a model without the use of exogenous variables. In this methodology, the period and length of the seasonality are modified in a selected way, so that the demand for a similar day is used to feed the model and make forecasts. Trull et al. [22] presented a Holt-Winters model, with discrete-interval moving seasonalities (DIMS) included in the model and used it to improve predictions during an Easter period. This period in Spain is characterized by having a duration of 120 hours (from Holy Thursday to Easter Monday), with similar and repetitive behavior each year, despite occurring on different dates. This technique has been recently used to forecast irregular seasonal situations [23].

Regarding which variables to use to obtain electricity demand forecasts in the short term, multivariate methods use meteorological variables such as air temperature and humidity [24,25]. Spanish' TSO includes weather variables in its own model [15]. However, there are two reasons not to use them on these exponential smoothing and Holt-Winters models. The first, as indicated by Bunn [26], for short-term demand, the temperature varies slightly and is related to sunlight, which the time series itself reflects in addition to the model includes a 24-hour seasonality that addresses these variations. Only sudden extreme changes could have a big influence on the model. A second drawback is that aggregated demand in Spain is almost insensitive to temperatures as it almost is related to firms' consumption [27,28]. Other variables that could appear to be related, such as electricity prices, are not influential, since the electricity market in Spain is a regulated market and demand is mainly related to the country's activity [29,30]. However, the calendar has an important effect on their forecasts. Time factors such as special load days, usually related to the calendar, cannot be modelled. Many articles proposing new methods to forecast electricity demand commonly omitted the forecasts during special days.

This article introduces DIMS for forecasting electricity demand on holidays and bridges in order to make these particular days part of the model itself through a new discrete seasonality but with a single parameter. The approach is completely different: public holidays generally occur between 5 and 8 times a year, and it is not the general rule that they repeat more than 4 consecutive times. These holidays occur at different times of the year, and therefore their behavior is not easily generalizable. In an even more complex way, in a year between 3 and five bridges can occur, without being related to the previous year. Of course, on practically random days for the time series. As a consequence of this complexity, the use of DIMS to make forecasts on these dates can be complex, but in this article, it can be seen how the robustness of the methods used allows making really accurate forecasts in a simple way, with a minimum number of parameters.

3. Methodology

3.1. nHWT model

The Holt-Winters models initially introduced in the '60s by P.R. Winters [31] included a single seasonality in the equations. The model remained unmodified until double [32] and triple seasonal [33] methods were introduced by Taylor. García-Díaz and Trull [18] generalized to multiple seasonalities, namely nHWT models shown in (1-4).

$$L_t = \alpha\left(\frac{X_t}{\prod_{i=1}^{n_s} I_{t-s_i}^{(i)}}\right) + (1-\alpha)(L_{t-1} + T_{t-1}) \quad (1)$$

$$T_t = \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1} \quad (2)$$

$$I_t^{(i)} = \delta^{(i)}\left(\frac{X_t}{L_t \prod_{j=1, j\neq i}^{n_s} I_{t-s_j}^{(j)}}\right) + \left(1-\delta^{(i)}\right)I_{t-s_i}^{(i)} \quad (3)$$

$$\hat{X}_{t+k} = (L_t + kT_t)\prod_{i=1}^{n_s} I_{t-s_i+k}^{(i)} + \varphi_{AR}^k \varepsilon_t \quad (4)$$

where α, γ and $\delta^{(i)}$ are the smoothing parameters for the level (L_t), trend (T_t) and seasonalities ($I_t^{(i)}$) with cycle length s_i. X_t are the observed data and $\hat{X}_{t+k}$ are the k-ahead forecasts, through a forecasting equation $\hat{X}_{t+k}$ that use the information contained in the smoothing equations to provide forecasts. φ_{AR}^k stands for the parameter of an adjustment using the first-order autocorrelation error (ε_t).

3.2. nHWT-DIMS model for holidays

The main source of instability in the nHWT models is the effect produced by special events, especially the calendar effect [19]. The model tries to reproduce the previous behaviour of the time series and cannot deal with the anomalous load occurring during the special event, as these models behave robust against variations [19]. Trull et al. [22] include discrete seasonalities related to special events, which only apply when the event occurs and can be smoothed as other general seasonality (DIMS). The general nHWT-DIMS models are described in Appendix A. The nHWT-DIMS model for the holidays, bridges and Easter holidays are shown in Equations (5-12).

$$L_t = \alpha \left(\frac{X_t}{I_{t-24}^{(24)} I_{t-168}^{(168)} D_{t_{Easter}^* - s_{Easter}^*}^{(Easter)} D_{t_{hol}^* - s_{hol}^*}^{(hol)} D_{t_{brg}^* - s_{brg}^*}^{(brg)}} \right) + (1-\alpha)(L_{t-1} + T_{t-1}) \quad (5)$$

$$T_t = \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1} \quad (6)$$

$$I_t^{(24)} = \delta^{(24)} \left(\frac{X_t}{L_t I_{t-168}^{(168)} D_{t_{Easter}^* - s_{Easter}^*}^{(Easter)} D_{t_{hol}^* - s_{hol}^*}^{(hol)} D_{t_{brg}^* - s_{brg}^*}^{(brg)}} \right) + \left(1 - \delta^{(24)}\right) I_{t-24}^{(24)} \quad (7)$$

$$I_t^{(168)} = \delta^{(168)} \left(\frac{X_t}{L_t I_{t-24}^{(24)} D_{t_{Easter}^* - s_{Easter}^*}^{(Easter)} D_{t_{hol}^* - s_{hol}^*}^{(hol)} D_{t_{brg}^* - s_{brg}^*}^{(brg)}} \right) + \left(1 - \delta^{(168)}\right) I_{t-168}^{(168)} \quad (8)$$

$$D_{t_h^*}^{(Easter)} = \delta_D^{(Easter)} \left(\frac{X_t}{L_t I_{t-24}^{(24)} I_{t-168}^{(168)} D_{t_{hol}^* - s_{hol}^*}^{(hol)} D_{t_{brg}^* - s_{brg}^*}^{(brg)}} \right) + \left(1 - \delta_D^{(Easter)}\right) D_{t_{Easter}^* - s_{Easter}^*}^{(Easter)} \quad (9)$$

$$D_{t_h^*}^{(hol)} = \delta_D^{(hol)} \left(\frac{X_t}{L_t I_{t-24}^{(24)} I_{t-168}^{(168)} D_{t_{hEaster}^* - s_{Easter}^*}^{(Easter)} D_{t_{brg}^* - s_{brg}^*}^{(brg)}} \right) + \left(1 - \delta_D^{(hol)}\right) D_{t_{hol}^* - s_{hol}^*}^{(hol)} \quad (10)$$

$$D_{t_h^*}^{(brg)} = \delta_D^{(brg)} \left(\frac{X_t}{L_t I_{t-24}^{(24)} I_{t-168}^{(168)} D_{t_{hEaster}^* - s_{Easter}^*}^{(Easter)} D_{t_{hol}^* - s_{hol}^*}^{(hol)}} \right) + \left(1 - \delta_D^{(brg)}\right) D_{t_{brg}^* - s_{brg}^*}^{(brg)} \quad (11)$$

$$\hat{X}_{t+k} = (L_t + kT_t) I_{t-24+k}^{(24)} I_{t-168+k}^{(168)} D_{t_{hEaster}^* - s_{Easter}^* + k}^{(Easter)} D_{t_{hol}^* - s_{hol}^* + k}^{(hol)} D_{t_{brg}^* - s_{brg}^* + k}^{(brg)} + \varphi_{AR}^k \varepsilon_t \quad (12)$$

The model includes 2 regular seasonalities: $I_t^{(24)}$ of length 24 hours and $I_t^{(168)}$ of length 168 hours with smoothing parameters $\delta^{(24)}$ and $\delta^{(168)}$. For the holidays three DIMS are included: $D_{t_h^*}^{(Easter)}$ managing Easter holidays with smoothing parameter $\delta_D^{(Easter)}$ of length 120 hours, $D_{t_h^*}^{(hol)}$ for national holidays and smoothing parameter $\delta_D^{(hol)}$ and finally $D_{t_h^*}^{(brg)}$ for bridges and smoothing parameter $\delta_D^{(brg)}$, having both a seasonal length of 24 hours.

Figure 1 shows the procedure to follow to make forecasts with the nHWT-DIMS models. The diagram is organized into 3 work areas. Area 1 includes the data acquisition and the calendar associated with the data period. With this information, the model can be determined, as described in area 2. It is necessary to determine the trend and seasonality method to use, as well as the number of seasonality and their cycle length. Subsequently, the DIMS are defined, determining how many types of DIMS are to be used, and determining their period length. From there, the appearances in the time series and recursion are located. With all this information, the parameters of the model are established.

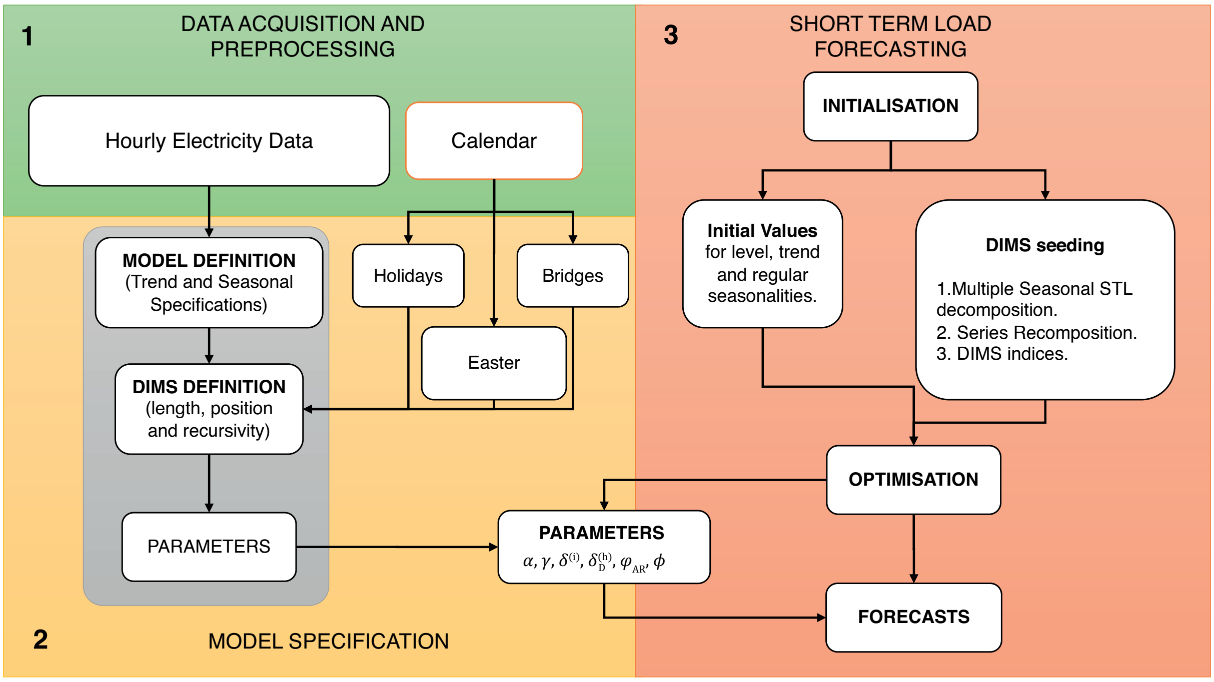

Figure 1. Flow chart of the forecasting process using the nHWT-DIMS models.

Area 3 describes the process for using the nHWT-DIMS models. The first step is initialization. Holt-Winters models are recursive; thus, they need initial values to be able to feed the model [34,35]. They are obtained in a two-step process: initially, the regular initial values are obtained, and the initial values of the DIMS. The procedure is as follows: firstly, a multiple Seasonal and Trend decomposition using Loess (STL) [36] is applied to the series, from which three components are obtained: trend, seasonalities and remainder. The remainder is removed from the series, cleaning it from the anomalous load. Then, the seasonal indices of the DIMS are obtained by weighting the original series over the recomposed one, thus the anomalous load effect is captured by DIMS. Finally, an optimization algorithm is used to fit the model to the observed data and obtain the parameter values. From this point on, forecasts are made.

4. Case study: hourly electricity demand in Spain during holidays

To demonstrate the improvement made by the new model, this methodology is applied to the time series of hourly electricity demand in Spain. The data series has been obtained from REE through its website (http://www.ree.es). The data includes the hourly electricity demand in Spain in the period from 1/01/2008 to 1/01/2018. The data set has been split into two subsets, the first one including from 1/1/2008 until 23:00 hours on 12/31/2016 and another from 1/1/2017 until 23:00 hours on 1/1/2018. The first subset is used to obtain the parameters and adjust the model, while the second subset is used to validate the results.

A sample of the data is shown in **Figure 2**. A short period of 10 weeks has been chosen to show, in order to better appreciate the demand series. In the representation, you can see how August 15 (marked in red), a holiday in Spain, has a unique and anomalous behaviour compared to the rest of the series.

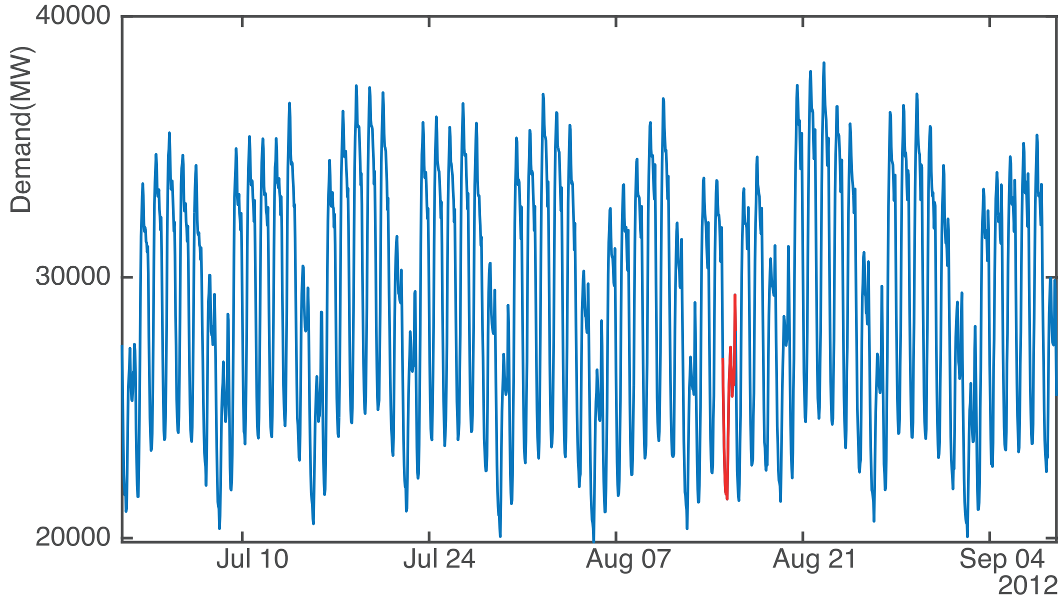

Figure 2. Hourly electricity demand in Spain.

During this period, Easter took place in the weeks listed in **Table 1**. It starts with Palm Sunday and ends with Resurrection Sunday. Throughout the ten years used, it can be observed how the position of Easter in the year is completely variable.

Year	**2008**	**2009**	**2010**	**2011**	**2012**
Palm Sunday	16 March	5 April	28 March	17 April	1 April
Resurrection Sunday	23 March	12 April	4 April	24 April	8 April
Year	**2013**	**2014**	**2015**	**2016**	**2017**
Palm Sunday	24 March	13 April	19 March	20 March	9 April
Resurrection Sunday	31 March	20 April	5 April	27 March	16 April

Table 1. General Easter Week period in Spain.

Figure 3 shows the distribution of the hourly electricity demand in Spain during this period. It is shown 8 days, from Monday before Easter till Easter Monday. The central line stands for the median, while the greyed area stands for the 2-sigma dispersion. It is remarkable the high variability of the data, even though 10 years have been used to build the graph. Easter officially starts in Spain on Holy Friday and ends on Resurrection Sunday, but many regions start on Holy Thursday and end on Easter Monday. In some regions, Easter Monday also depends on the year. The strategy for modelling Easter is to use DIMS with 120 hours length [22].

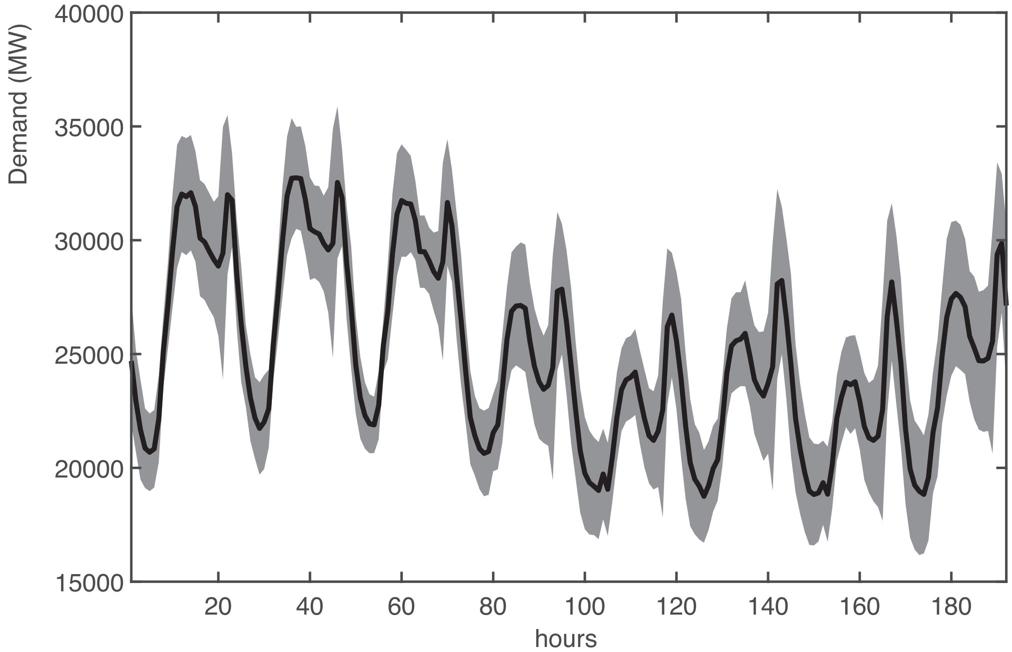

Figure 3. Distribution of the hourly electricity demand in Spain during Easter. Data from year 2008 to 2017.

The holidays considered in this article are gathered in **Table 2**, where crosses indicate occurrence, while blanks indicate the holidays took place during the weekend and has been removed from the study. The shaded part indicates the dates on which the forecasts were made for validation purposes.

	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
1 January	x	x	x			x	x	x	x		x
6 January		x	x	x	x		x	x	x	x	
1 May	x	x			x	x	x	x		x	
15 August	x			x	x	x	x		x	x	
12 October		x	x	x	x			x	x	x	
1 November			x	x	x	x			x	x	
6 December			x	x	x	x			x	x	
8 December	x	x	x	x			x	x	x	x	
25 December	x	x			x	x	x	x		x	

Table 2. Holidays during the period 2008-2018.

In order to describe the behavior of the electricity load during a holiday, **Figure 4** has been included. This figure collects the median load during the holidays between 2008 and 2017, split by the day of the week. The black line shows the median of the electricity demand during a working day (no distinction was needed, as all days behave similarly). The load during holidays have been graphed in different colors, depending on the day of the week. No weekend has been included since the influence of a holiday is weak. It can be stated that, independently of the day of the week, holidays in Spain behave similarly, decreasing the load consumption in the same ratio.

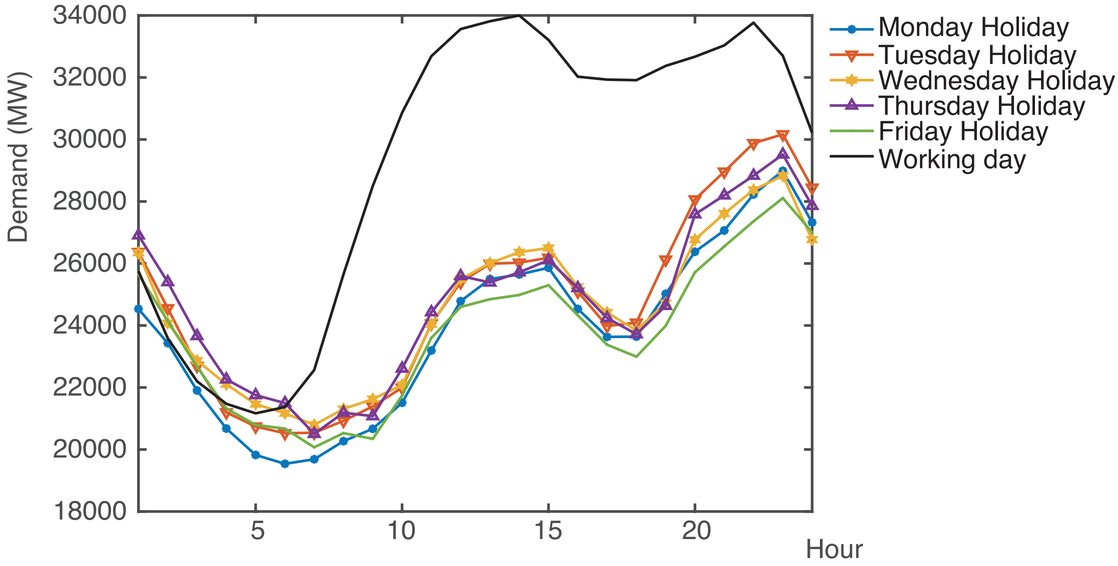

Figure 4. Hourly electricity demand in a working day compared to holidays, depending on the day of week.

The electricity load in different holidays shows a similar pattern as shown in **Figure 5**. This figure shows the median of the holidays grouped by type. It can be seen how the demand in different holidays has essentially similar behaviour, albeit at different levels. This situation happens because the holidays occur at different times of the year, where energy consumption is highly dependent on climatological characteristics. The Holt-Winters model uses the trend (if it is a trending model) and the level to adjust for these variations.

As a result of both figures together, it can be determined that the electrical demand on holidays follows an anomalous pattern, which is cyclical and which repeats on all holidays, but different from the daily pattern. This situation allows the use of discrete interval mobile seasonality, 24-hour in size, and which begins at 00:00 on the holiday and ends at 23:00 on the holiday.

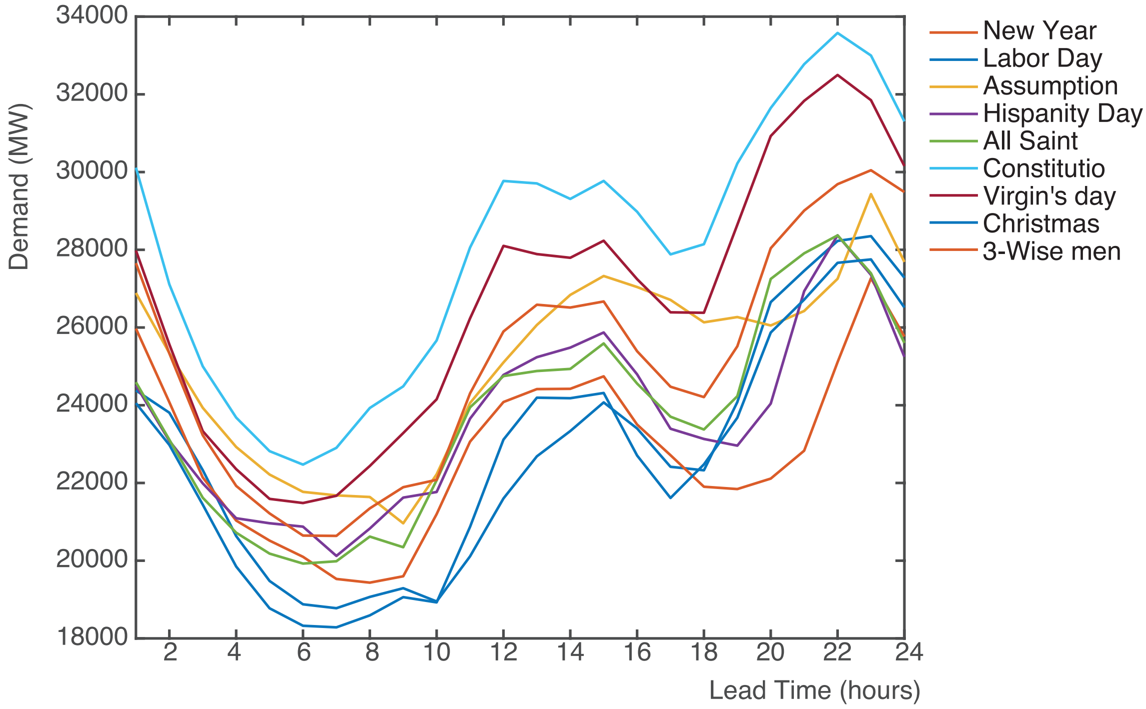

Figure 5. Median of hourly electricity load during the holidays.

Electricity demand during the days of bridges is presented very unevenly. **Table 3** enumerates the bridges in the period considered. Each row shows the bridges that occurred during the year in the row. Figure **6** presents a representation of the median electricity demand during bridges depending on the day of the week. An analysis of the graph does not conclude on the possibility of establishing a specific seasonality for the bridge days. The great variability that is shown may be associated with specific occurrences and their dependence on holidays to which it is associated. This distinction has been considered in previous works, and therefore, it is also considered.

2008	02-may	26-dec	31-dec		
2009	02-jan	05-jan	07-dec	31-dec	
2010	11-oct	07-dec	31-dec		
2011	07-jan	31-oct	05-dec	07-dec	09-dec
2012	30-apr	02-nov	07-dec	24-dec	31-dec
2013	16-aug	31-dec			
2014	02-may	26-dec	31-dec		
2015	02-jan	05-jan	07-dec	31-dec	
2016	31-oct	05-dec	07-dec	09-dec	
2017	14-aug	13-oct	07-dec		

Table 3. Bridges during the period considered.

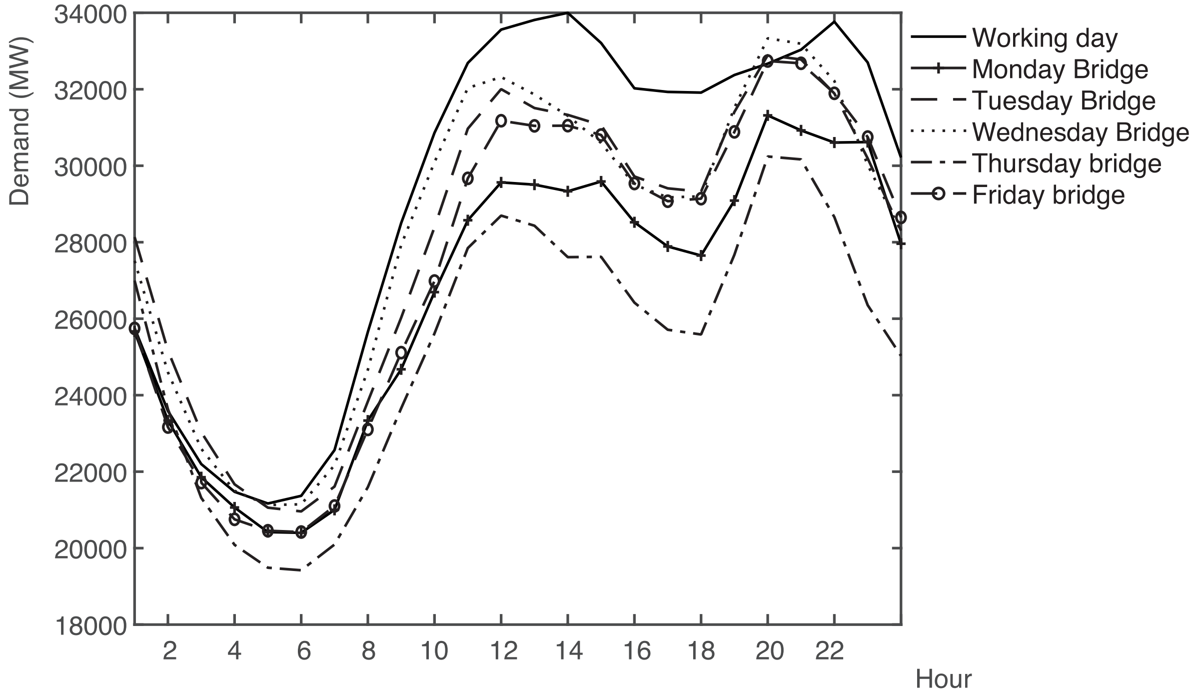

Figure 6. Median of the hourly electricity demand during bridges.

Modeling discrete seasonalities requires an analysis of the behavior of demand in the discrete intervals where it applies. **Figure 7** shows the distribution of demand on holidays in the data set (a), and on bridge days (b). Different behavior is observed for each case, but in both, there is a limited variability that throws possibilities on the use of a discrete seasonality to model this cyclical pattern.

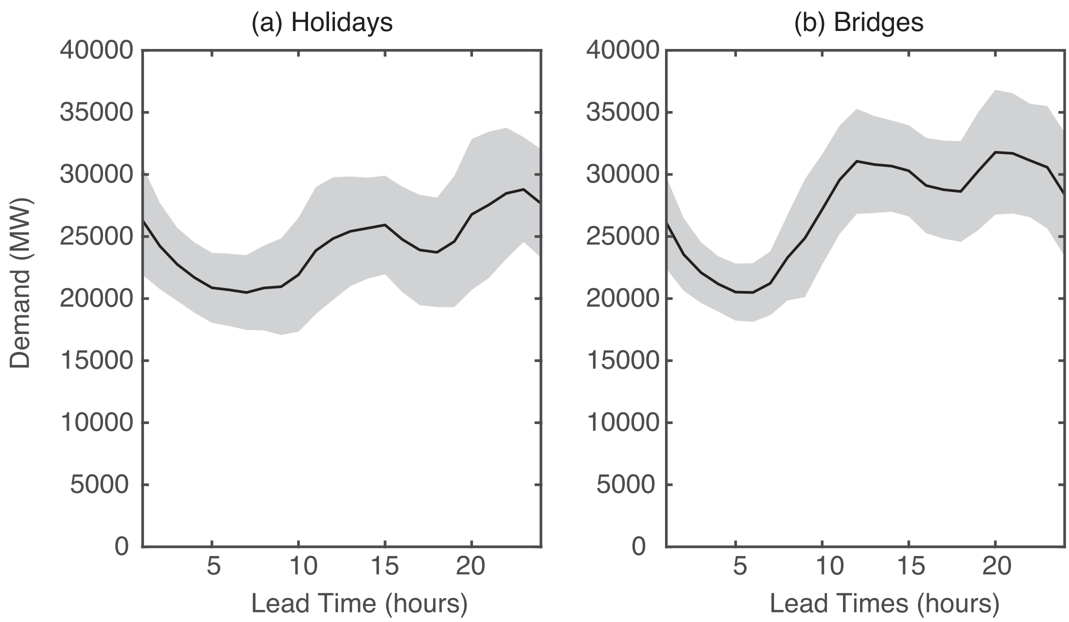

Figure 7. Distribution of the hourly electricity demand in: (a) the holidays and (b) bridges.

As indicated in **Figure 1**, the model is defined with Model and DIMS definitions. The major input data here is the DIMS specification. It is necessary to determine for each of them the length of the discrete seasonality, as well as the position in which it appears in the time series and the recursion associated with each appearance.

Figure 8 shows an example of how to determine the recursion and position of the DIMS in the time series. The regular series is shown in black, while public holidays and bridge days are shown in red and blue. The vertical lines help to identify the beginning and end of each discrete seasonality, which in both cases is 24 hours. In the same way, it would be carried out for Easter, where the established length is 120 hours. Recursion is the distance between an occurrence and the one immediately before it. Thus, for December 8, 2016, the recursion is 48 hours (the previous appearance is December 6). The recursion for December 6 goes back to November 1, which involves 840 hours. In the same way, it is calculated for bridges, although with fewer appearances.

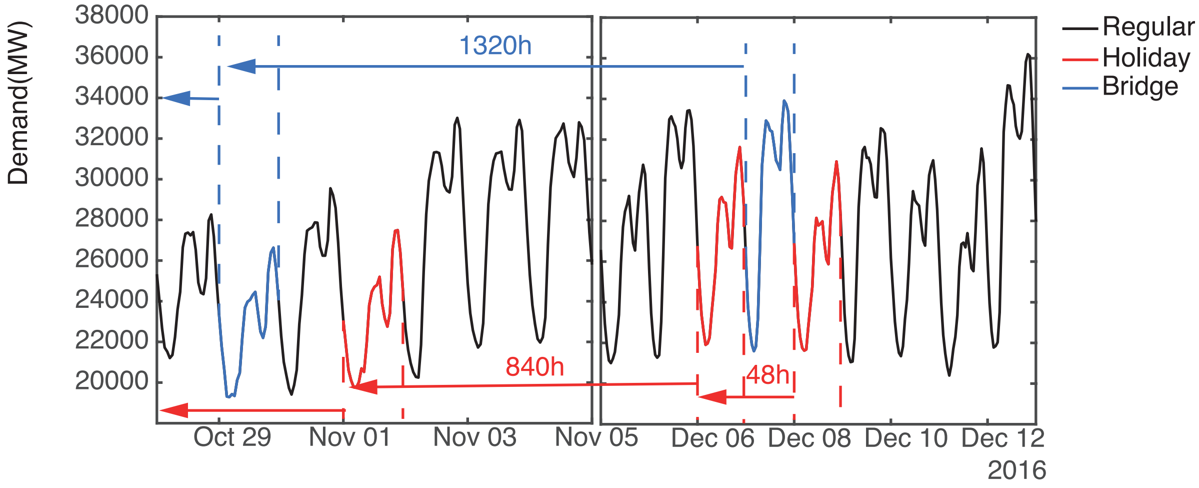

Figure 8. Example of determining the position and recursion of DIMS.

Finally, Table 4 shows the values of the DIMS used and the recursion in hours at each occurrence. The first occurrence has no recursion (January 1, 2008, and May 2, 2008). Easter DIMS has a length of 120 hours while holidays and bridges have a length of 24 hours.

			Easter					
Nr	**Date**	**Recurs.**	**Nr**	**Date**	**Recurs.**	**Nr**	**Date**	**Recurs.**
1	16-Mar-2008	---	5	01-Apr-2012	8280	9	20-Mar-2016	8448
2	05-Apr-2009	9120	6	24_mar-2013	8448	10	09-Apr-2017	9120
3	28-Mar-2010	8448	7	13-Apr-2014	9120			
4	17-Apr-2011	9120	8	29-Mar-2015	8280			
			Holidays					
Nr	**Date**	**Recurs.**	**Nr**	**Date**	**Recurs.**	**Nr**	**Date**	**Recurs.**
1	01-Jan-2008	---	22	06-Dec-2011	840	43	01-Jan-2015	168
2	01-May-08	2904	23	08-Dec-2011	48	44	06-Jan-2015	120
3	15-Aug-2008	2544	24	06-Jan-2012	696	45	01-May-15	2760

4	08-Dec-2008	2760	25	01-May-12	2784	46	12-Oct-15	3936
5	25-Dec-2008	408	26	15-Aug-2012	2544	47	08-Dec-2015	1368
6	01-Jan-2009	168	27	12-Oct-12	1392	48	25-Dec-2015	408
7	06-Jan-2009	120	28	01-Nov-12	480	49	01-Jan-2016	168
8	01-May-09	2760	29	06-Dec-2012	840	50	06-Jan-2016	120
9	12-Oct-09	3936	30	25-Dec-2012	456	51	15-Aug-2016	5328
10	08-Dec-2009	1368	31	01-Jan-2013	168	52	12-Oct-16	1392
11	25-Dec-2009	408	32	01-May-13	2880	53	01-Nov-16	480
12	01-Jan-2010	168	33	15-Aug-2013	2544	54	06-Dec-2016	840
13	06-Jan-2010	120	34	01-Nov-13	1872	55	08-Dec-2016	48
14	12-Oct-10	6696	35	06-Dec-2013	840	56	06-Jan-2017	696
15	01-Nov-10	480	36	25-Dec-2013	456	57	01-May-17	2760
16	06-Dec-2010	840	37	01-Jan-2014	168	58	15-Aug-2017	2544
17	08-Dec-2010	48	38	06-Jan-2014	120	59	12-Oct-17	1392
18	06-Jan-2011	696	39	01-May-14	2760	60	01-Nov-17	480
19	15-Aug-2011	5304	40	15-Aug-2014	2544	61	06-Dec-2017	840
20	12-Oct-11	1392	41	08-Dec-2014	2760	62	08-Dec-2017	48
21	01-Nov-11	480	42	25-Dec-2014	408	63	25-Dec-2017	408

Bridges

Nr	**Date**	**Recurs.**	**Nr**	**Date**	**Recurs.**	**Nr**	**Date**	**Recurs.**
1	02-May-08	---	13	05-Dec-2011	840	25	31-Dec-2014	120
2	26-Dec-2008	5712	14	07-Dec-2011	48	26	02-Jan-2015	48
3	31-Dec-2008	120	15	09-Dec-2011	48	27	05-Jan-2015	72
4	02-Jan-2009	48	16	30-Apr-2012	3432	28	07-Dec-2015	8064
5	05-Jan-2009	72	17	02-Nov-12	4464	29	31-Dec-2015	576
6	07-Dec-2009	8064	18	07-Dec-2012	840	30	31-Oct-16	7320
7	31-Dec-2009	576	19	24-Dec-2012	408	31	05-Dec-2016	840
8	11-Oct-10	6816	20	31-Dec-2012	168	32	07-Dec-2016	48
9	07-Dec-2010	1368	21	16-Aug-2013	5472	33	09-Dec-2016	48
10	31-Dec-2010	576	22	31-Dec-2013	3288	34	14-Aug-2017	5952
11	07-Jan-2011	168	23	02-May-14	2928	35	13-Oct-17	1440
12	31-Oct-11	7128	24	26-Dec-2014	5712	36	07-Dec-2017	1320

Table 4. List of DIMS positions and recursivities (in hours) for holidays and bridges.

The optimization of the parameters is obtained by minimizing the 1-hour ahead forecast error through an error measurement indicator. The optimization uses the root mean squared error (RMSE), as shown in Eq. (9). The Nelder-Mead's simplex-based minimization algorithm [37] in the MATLAB® platform was chosen to carry out this optimization process. Alternatives were also considered, such as FOA, as used in [38,39], but didn't outperform the results.

$$RMSE = \sqrt{\sum \frac{(\hat{X}_t - X_t)^2}{N}} \tag{13}$$

5. Results

Results applied to the Spanish hourly electricity demand to forecast holidays and bridges with 24-hour forecast horizon are reported in this Section. In particular, an analysis of the forecasts obtained by the nHWT with DIMS method is carried out in Section 5.1. Section 5.2 compares the performance of the proposed model with well-known forecasting methods. Finally, a comparison with other previous validated results obtained by specific methods for anomalous demand [2] for French half-hourly electricity demand is made in Section 5.3.

5.1 Analysis of results

All the HWT-DIMS models described in Table A.1 in Appendix A were included in the experimentation. After obtaining their parameters, forecasts have been made for the days indicated in the gray part of **Table 2** and **Table 3**. The measurement of the forecasting accuracy has been performed using the Mean Average Percentage Error (MAPE) defined in Eq. (10).

$$MAPE(\%) = 100 \times \frac{1}{N} \sum \frac{|\hat{X}_t - X_t|}{X_t} \tag{14}$$

Among the possible forecasting strategies that can be used, two versions have been chosen: the first, where the data from the adjustment set are used to obtain the parameters, and with them projections are made for every day of the year, checking the accuracy of the predictions (named Method 1); the second, the adjustment period has been extended to the moment immediately prior to the forecast, where the parameters have been readjusted, and the same projections have been made (named Method 2). Among all the proposed models, only the $AMC_{24,168,Easter,Hol,Brg}$ and $NMC_{24,168,Easter,Hol,Brg}$ models are shown. The results obtained with other models are within the same order. These are chosen because they are the most commonly used for the use of nHWT models. The results are shown in the **Table 5**. In this table, the 24 hours ahead MAPE is shown for the days listed in the first column, including all models 2 seasonalities and DIMS for Easter, holidays and bridges. The information is split into Method 1 and Method 2.

	Method 1		**Method 2**	
Forecast	**AMC**	**NMC**	**AMC**	**NMC**
6 January 2017	2.39	2.05	2.14	2.05
1 May 2017	5.81	5.60	6.03	5.62
15 August 2017	4.30	4.72	4.51	4.57
12 October 2017	1.74	2.36	1.69	2.39
1 November 2017	1.26	1.30	1.25	1.32
6 December 2017	4.06	3.75	3.98	3.81
8 December 2017	3.52	3.76	3.61	3.73
25 December 2017	10.49	9.86	10.61	9.86
1 January 2018	4.26	4.78	4.27	4.78
14 August 2017	3.57	4.36	3.54	4.35
13 October 2017	2.82	2.99	2.68	3.01

7 December 2017	4.49	3.29	4.23	3.32
Mean	4.06	4.07	4.05	4.07

Table 5. 24-hours ahead MAPE (%) of the forecasts produced for the holidays and bridges.

The results show that there is no great difference between the two methodologies. Therefore, the results offered by method 1 are those used, as they turn out to be more accurate. There is also no great difference between the different models, the differences being on the order of less than 1%. This time, the model that does not include a trend turns out to be more accurate in the predictions than the one that has a trend.

The 24-hour MAPE values are below 5% when the regular method is not capable of inferring predictions whose MAPE is below 10%. Thus, the model DIMS proposed to forecast holidays, Easter and bridges show a good performance in terms of accuracy.

It is necessary to pay attention to the forecasts made for December 25. After an analysis of the time series, it can be concluded that the demand during the previous weeks to December 25, in particular on December 6 and 8, was much higher and irregular than in previous years, which has an influence on the computation of the discrete seasonality associated with holidays ($D_{t_h^*}^{(hol)}$). Thus, these two days can be considered outliers and the DIMS for just these two days can be removed in order to obtain a more robust model. The effect is thus minimized, and forecasts are improved. Taking into account this consideration, a reduction of the MAPE around 2.5% can be obtained for the forecast of Christmas day. However, the prediction error made on December 25 will remain high compared to other holidays and bridges due to demand was unexpectedly high on that day compared to demand on the same day in nearby years. The results are shown in **Table 5**.

To observe the behavior of the predictions, **Figure 9** is shown. This figure shows the evolution of the MAPE according to the forecast horizon, both for holidays and for bridges, compared to working days.

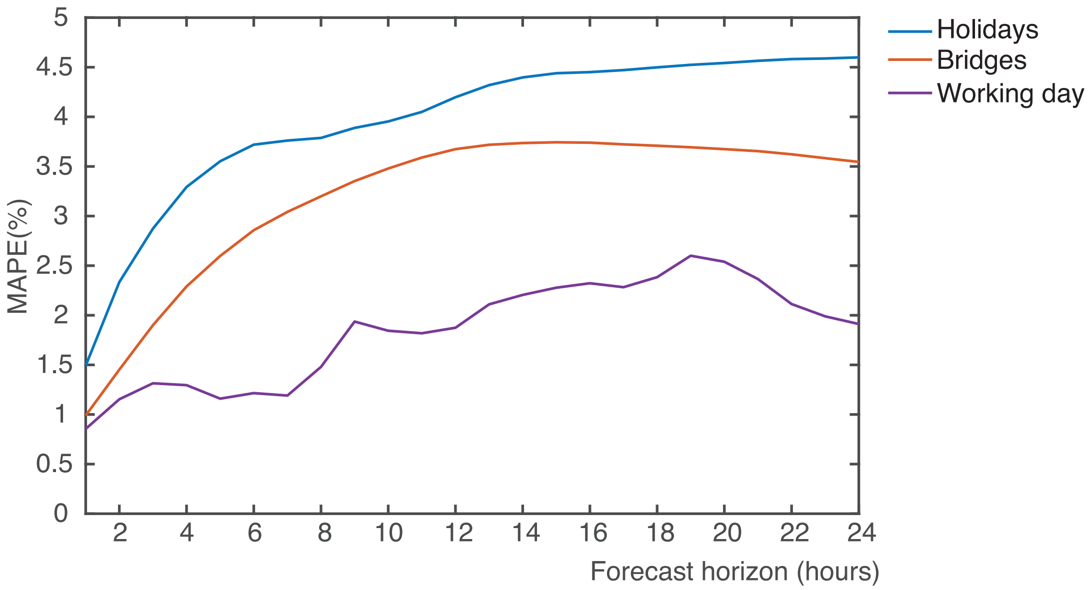

Figure 9. Forecasting MAPE for Holidays and Bridges compared to working days.

The comparison allows us to see how the appearance of abnormal demand days has a challenge for the models. Demand on working days moves in precision values around 2% of MAPE, while holidays around 4.5%. An intermediate case is the bridge days. At the modeling level there is no distinction between bridge days and holidays, but the fact is those bridge days, despite being much less, behave more regularly than holidays. Therefore, although predictions during bridges show a better performance, with lower MAPE and thus better predictions, the main improvement is done with holidays forecasts that denote a large decrease in MAPE.

5.2 Comparison with benchmark methods

After verifying the effectiveness of nHWT models with DIMS for forecasting demand on special days, a comparison is made with other methods commonly used for electrical demand. These methods have been ANN, Bagged Regression Trees (BRT), Exponential Smoothing with Box Cox transformation and ARMA modeling of the residuals (BATS) and the Trigonometric Seasonal version of this last (TBATS). The results are summarized in **Table 6**.

Among the most common models of Machine Learning, the use of ANN stands out for its good performance [40]. Similarly, BRTs are in wide use [41]. In this case, MATLAB software is used to make forecasts with these two methods. The variables used have been demand, holidays and historical demand. Climatological variables have not been used. With these data, the models have been trained, and forecasts have been made on the days of analysis. In the case of the state spaces, both BATS and TBATS, the R *forecast* [42] library has been used.

	ANN	BRT	nHWT	BATS	TBATS	nHWT DIMS
6/1/17	15.35	15.24	14.28	6.19	6.29	3.83
1/5/17	21.17	14.56	20.50	10.15	10.59	4.69
15/8/17	12.86	6.18	8.98	5.94	5.86	2.84
12/10/17	15.82	16.09	15.27	6.72	6.52	3.22
1/11/17	15.61	16.40	16.84	6.45	6.17	2.06
6/12/17	13.13	11.95	12.47	4.42	4.84	4.97
8/12/17	8.36	7.96	7.47	2.76	3.31	2.15
25/12/17	32.55	25.57	23.22	8.40	8.88	6.08
1/1/18	13.83	9.96	23.69	10.87	11.13	4.26
14/8/17	9.60	5.47	2.70	13.86	14.42	4.45
13/10/17	6.06	5.45	5.03	7.22	6.66	3.13
7/12/17	5.48	3.28	1.09	24.52	24.75	4.84
mean	14.15	11.51	12.63	8.96	9.12	3.87

Table 6. Daily MAPE(%) for the models tested.

To carry out the comparison, forecasts have been made out of the sample of electricity demand on the days indicated in the first column (both holidays and bridges), and it has been compared with the real value of the demand on those days. It can be seen that the different methods tackle the problem from different points of view. As a result, artificial intelligence methods turn out to offer the worst results, even though the BRTs have made very good predictions on bridge days. However, the nHWT method with DIMS offers the best results, with an average daily MAPE of around 4%.

5.3 Comparison with RB-SARMA

Finally, in order to compare the nHWT with DIMS with a specific forecasting method for anomalous demand, the Rule-Based SARMA (RB-SARMA) method proposed by Arora and Taylor in [2] has been selected. The RB-SARMA is a specific method for calculating demand on abnormal load days, such as holidays and bridges using France's electricity demand. This article is a reference point to make a comparison of the effectiveness of the forecasts obtained by the proposed method. The prediction made for the average hourly electricity demand in France has been taken as a reference. In this case, the same data set has been used, and the same forecasts have been made. Several combinations of models were tested, but the most accurate one has been $NMC_{48,336,Holidays,Bridges}$, which is a non-trend model with multiplicative seasonality.

The objective is not to improve the results obtained in that article, as it would imply a deeper work, but simply to use them as a reference to observe if the forecasts given by the proposed model are relevant, obtaining the results shown in **Figure 10**. The reality is that our expectations have been exceeded, and the predictions made are comparable with those of that article, which makes us validate the process previously carried out with the Spanish time series. On average, the MAPE for these days using RB-SARMA has been 4.3% while nHWT-DIMS has been 3.7%.

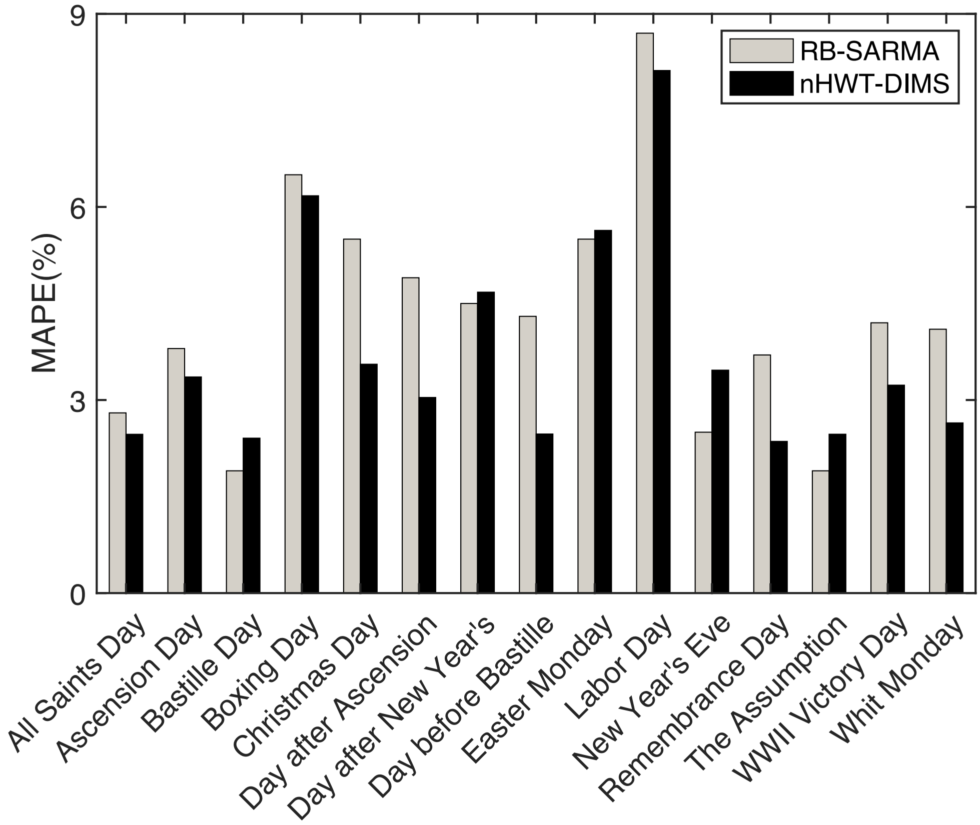

Figure 10. 48-hours ahead MAPE comparison between the RB-SARMA from [2] with nHWT $NMC_{48,336,Hol,Brg}$.

6. Conclusions

In this article, the work presented in [22] is taken up where discrete seasonality was used to model special events, such as Easter in Spain. The advantage of using DIMS is that they are seasonalities that are integrated within the model, without being external modifiers of the series. Here the same concept is used to model holidays and bridge days, using nHWT-DIMS models with 2 regular seasons, and with 3 discrete seasons. These methodologies have been applied to the hourly electricity demand in Spain to predict holiday and bridge days in the period of the year 2017.

The results show that the use of DIMS allows to greatly improve the results obtained with the regular model. Although the regular model is highly capable of forecasting on regular days (those that do not suffer variations due to special events), they are not as efficient when a special event occurs. The use of DIMS allows nHWT models to reduce the MAPE of the forecasts by almost half.

To test the usefulness of the model, it has been compared with other common forecasting models, models based on machine learning and state spaces. The comparison shows that the results obtained improve those of any of the other models. Furthermore, and in order to be able to validate the process, a comparison between a reference article and the nHWT-DIMS models using the French half-hourly electricity demand has been carried out. The results show that the proposed models offer competitive forecasts and it can thus be validated the DIMS to predict anomalous days such as holidays and bridges.

This methodology has been applied to STLF but could be easily adapted to higher term forecasts. The only limitations of the methods are: The events to forecast must have appeared in the past at least once, and it is needed to check when two special events could overlap and use only one event. This methodology is limited to those events that have already occurred in the past and that follow a repetitive behavior.

The proposed method improves the results obtained by other methodologies by adding DIMS for special events, also taking advantage of the prediction reliability of the nHWT models. Its application can be extended to other special events, as long as they have occurred in the past: strikes, sporting events, etc.

Recently, Moral-Carcedo and Pérez-García [28] have studied the variation in electricity demand according to daily sunlight. This concept has not been included in the model, although in future works it may be included as a special fact.

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Abbreviations

$AMC_{24,168,Easter,Hol,Brg}$	nHWT-DIMS model with additive trend, multiplicative seasonalities of 24 and 168 hours and DIMS for Easter, Holidays and Bridges
ANN	Artificial neural networks
ARMA	Autoregressive moving average
BATS	Exponential smoothing state space model with Box–Cox transformation, ARMA errors, trend and seasonal components
BRT	Bagged regression trees
FOA	Fly Fruit opmization algorithm
DIMS	Discrete interval moving seasonalities
MAPE	Mean absolut percentage error
nHWT	Multiple seasonal Holt–Winters
nHWT–DIMS	Multiple seasonal Holt–Winters with discrete interval moving seasonalities
$NMC_{24,168,Easter,Hol,Brg}$	nHWT-DIMS model with no trend, multiplicative seasonalities of 24 and 168 hours and DIMS for Easter, Holidays and Bridges
RB-SARMA	Rule-Based Seasonal ARMA process
REE	Red Eléctrica de España (Spanish' TSO)
RMSE	Root of mean squared error
STL	Seasonal–trend decomposition procedure using Loess
STLF	Short-term load forecasting

TBATS	Exponential smoothing state space model with Box–Cox transformation, ARMA errors, trend and trigonometric seasonal components
TSO	Transmission System Operator

Appendix A

The nHWT-DIMS general model comprises of some smoothing equations: Level L_t, trend T_t, as many seasonal indices $I_t^{(i)}$ as the number of seasonal patterns considered (n_s). The new model includes as many discrete seasonal indices $D_{t_h^*}^{(h)}$ as special events considered (n_{DIMS}). Superscripts i and h are used to name each seasonality and discrete seasonality respectively. The general model is shown in equations (A.1-A.5).

$$L_t = \alpha \left(\frac{X_t}{\prod_{i=1}^{n_s} I_{t-s_i}^{(i)} \prod_{h=1}^{n_{DIMS}} D_{t_h^*-s_h^*}^{(h)}} \right) + (1-\alpha)(L_{t-1} + T_{t-1}) \quad \text{(A.1)}$$

$$T_t = \gamma(L_t - L_{t-1}) + (1-\gamma)T_{t-1} \quad \text{(A.2)}$$

$$I_t^{(i)} = \delta^{(i)} \left(\frac{X_t}{L_t \prod_{j=1, j\neq i}^{n_s} I_{t-s_j}^{(j)} \prod_{h=1}^{n_{DIMS}} D_{t_h^*-s_h^*}^{(h)}} \right) + \left(1-\delta^{(i)}\right) I_{t-s_i}^{(i)} \quad \text{(A.3)}$$

$$D_{t_h^*}^{(h)} = \delta_D^{(h)} \left(\frac{X_t}{L_t \prod_{i=1}^{n_s} I_{t-s_i}^{(i)} \prod_{m=1, m\neq h}^{n_{DIMS}} D_{t_m^*-s_m^*}^{(m)}} \right) + \left(1-\delta_D^{(h)}\right) D_{t_h^*-s_h^*}^{(h)} \quad \text{(A.4)}$$

$$\hat{X}_{t+k} = (L_t + kT_t) \prod_{i=1}^{n_s} I_{t-s_i+k}^{(i)} \prod_{h=1}^{n_{DIMS}} D_{t_h^*-s_h^*+k}^{(h)} + \varphi_{AR}^k \varepsilon_t \quad \text{(A.4)}$$

where α, γ, $\delta^{(i)}$ and $\delta_D^{(h)}$ are the smoothing parameters for the level, trend, seasonalities and DIMS. Seasonalities have cycle length s_i whereas DIMS have s_h. X_t are the observed data and $\hat{X}_{t+k}$ are the k-ahead forecasts, through a forecasting equation $\hat{X}_{t+k}$ that use the information contained in the smoothing equations to provide forecasts. Time is expressed as t, but for the discrete seasonalities, it has been defined $t_h{}^*$. This variable reflects the time t in which DIMS h must be included, otherwise it does not apply. The recurrence term s_h^* indicates the time distance to the previous occurrence of the DIMS h. Both values s_h^* and $t_h{}^*$ are variable and must be calculated on every appearance of the DIMS. φ_{AR}^k stands for the parameter of an adjustment using the first-order autocorrelation error (ε_t).

The calculation of seasonal indices can be done in several ways, such as a multiplicative ratio, which is the case shown in equations (A.1-A.5) or using an additive composition. Furthermore, depending on the method used to combine the equations, models with additive or multiplicative tendency can be built. To identify the method used in the model, three letters are used, the first to refer to the type of trend used (N: no trend, A: additive, M: multiplicative, dA: damped additive and DM: damped multiplicative); a second letter to indicate the type of seasonality (N:

no seasonality, A: additive and M: multiplicative); finally a third letter (C) to indicate if an adjustment using the first order autoregressive error has been applied or (L) to indicate that there is no adjustment. Next, the seasonality used is indicated by subscripts indicating the length of every seasonal period considered, and the DIMS used with identifying names. As an example, a model $AMC_{24,168,Hol}$ stands for the model with additive trend, multiplicative seasonality and two seasonalities: intraday and intraweek with 24 and 168 hours of length, respectively, and DIMS for holidays.

All possible models resulting from the aforementioned combination is shown in Table A.1, where the multiple seasonal Holt–Winters with DIMS formulae are shown. They are organized according to the trend (rows) and seasonal method (columns). Here, S_t stands for the level equation. T_t stands for an additive trend smoothing equation, whereas R_t stands for a multiplicative one, with α and β as smoothing parameters. $I_t^{(i)}$ are the seasonal indices with smoothing parameter $\delta^{(i)}$. The superscript (i) stands for the seasonality. There are as many indices of length s_i as seasonalities considered. $D_{t_h^*}^{(h)}$ is the DIMS index *(h)*, with a smoothing parameter $\delta_D^{(h)}$. In this case, t_h^* is the region in the time series where $D_{t_h^*}^{(h)}$ is defined. The DIMS length is indicated by s_h, but the recursivity is indicated by s_h^*, that must be updated on every new appearance of the DIMS. X_t are the observed values and $\hat{X}_t(k)$ are the k-ahead forecasted values. φ_{AR} is the factor of the adjustment with the first autocorrelation error (ε_t).

	Additive Trend
Additive Seasonality	$\begin{gathered} S_t = \alpha\left(X_t - \sum_i I_{t-s_i}^{(i)} - \sum_h D_{t_h^*-s_h^*}^{(h)}\right) + (1-\alpha)(S_{t-1}+T_{t-1}) \\ T_t = \gamma(S_t - S_{t-1}) + (1-\gamma)T_{t-1} \\ I_t^{(i)} = \delta^{(i)}\left(X_t - S_t - \sum_{j\neq i} I_{t-s_j}^{(j)} - \sum_h D_{t_h^*-s_h^*}^{(h)}\right) + (1-\delta^{(i)})I_{t-s_i}^{(i)} \\ D_{t_h^*}^{(h)} = \delta_D^{(h)}\left(X_t - S_t - \sum_j I_{t-s_j}^{(j)} - \sum_{m\neq h} D_{t_m^*-s_m^*}^{(m)}\right) + (1-\delta_D^{(h)})D_{t_h^*-s_h^*}^{(h)} \\ \hat{X}_t(k) = (S_t + k\,T_t) + \sum_i I_{t-s_i+k}^{(i)} + \sum_h D_{t_h^*-s_h^*+k}^{(h)} + \varphi_{AR}^k \varepsilon_t \end{gathered}$
	Multiplicative Trend

$$S_t = \alpha\left(X_t - \sum_i I^{(i)}_{t-s_i} - \sum_h D^{(h)}_{t^*_h - s^*_h}\right) + (1-\alpha)(S_{t-1}R_{t-1})$$

$$R_t = \gamma(S_t/S_{t-1}) + (1-\gamma)R_{t-1}$$

$$I^{(i)}_t = \delta^{(i)}\left(X_t - S_t - \sum_{j\neq i} I^{(j)}_{t-s_j} - \sum_h D^{(h)}_{t^*_h - s^*_h}\right) + (1-\delta^{(i)})I^{(i)}_{t-s_i}$$

$$D^{(h)}_{t^*_h} = \delta^{(h)}_D\left(X_t - S_t - \sum_j I^{(j)}_{t-s_j} - \sum_{m\neq h} D^{(h)}_{t^*_h - s^*_h}\right) + (1-\delta^{(h)}_D)D^{(h)}_{t^*_h - s^*_h}$$

$$\hat{X}_t(k) = S_t R^k_t + \sum_i I^{(i)}_{t-s_i+k} + \sum_h D^{(h)}_{t^*_h - s^*_h + k} + \varphi^k_{AR}\varepsilon_t$$

Table A.1. Multiple seasonal methods with DIMS equations.

Multiplicative Seasonality	Additive Trend
	$$S_t = \alpha\left(X_t - \sum_i I^{(i)}_{t-s_i} - \sum_h D^{(h)}_{t^*_h - s^*_h}\right) + (1-\alpha)(S_{t-1}R_{t-1})$$ $$R_t = \gamma(S_t/S_{t-1}) + (1-\gamma)R_{t-1}$$ $$I^{(i)}_t = \delta^{(i)}\left(X_t - S_t - \sum_{j\neq i} I^{(j)}_{t-s_j} - \sum_h D^{(h)}_{t^*_h - s^*_h}\right) + (1-\delta^{(i)})I^{(i)}_{t-s_i}$$ $$D^{(h)}_{t^*_h} = \delta^{(h)}_D\left(X_t - S_t - \sum_j I^{(j)}_{t-s_j} - \sum_{m\neq h} D^{(h)}_{t^*_h - s^*_h}\right) + (1-\delta^{(h)}_D)D^{(h)}_{t^*_h - s^*_h}$$ $$\hat{X}_t(k) = S_t R^k_t + \sum_i I^{(i)}_{t-s_i+k} + \sum_h D^{(h)}_{t^*_h - s^*_h + k} + \varphi^k_{AR}\varepsilon_t$$
	Multiplicative Trend
	$$S_t = \frac{\alpha X_t}{\prod_i I^{(i)}_{t-s_i} \prod_h D^{(h)}_{t^*_h - s^*_h}} + (1-\alpha)\left(S_{t-1}R^{\phi}_{t-1}\right)$$ $$R_t = \gamma(S_t/S_{t-1}) + (1-\gamma)R^{\phi}_{t-1}$$ $$I^{(i)}_t = \delta^{(i)}\frac{X_t}{S_t \prod_{j\neq i} I^{(j)}_{t-s_j} \prod_h D^{(h)}_{t^*_h - s^*_h}} + (1-\delta^{(i)})I^{(i)}_{t-s_i}$$ $$D^{(h)}_{t^*_h} = \delta^{(h)}_D\frac{X_t}{S_t \prod_j I^{(j)}_{t-s_j} \prod_{m\neq h} D^{(m)}_{t^*_m - s^*_m}} + (1-\delta^{(h)}_D)D^{(h)}_{t^*_h - s^*_h}$$ $$\hat{X}_t(k) = \left(S_t R_t^{\sum_{j=1}^k \phi^j}\right)\prod_i I^{(i)}_{t-s_i+k} \prod_h D^{(h)}_{t^*_h - s^*_h} + \varphi^k_{AR}\varepsilon_t$$

Table A.1 (cont). Multiple seasonal methods with DIMS equations

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