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A capacitated lot-sizing model with sequence-dependent setups, parallel machines and bi-part injection moulding

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ABSTRACT

This paper aims to propose a model for solving a capacitated lot-sizing problem with sequence-dependent setups (CLSD) and parallel machines in a bi-part injection moulding (BPIM) context that consists of injecting two different parts or products into the same mould in separate injection cavities. These two parts require the same sequence order and available capacity at the same instant, but generate different part inventories. The addressed problem should be considered a multi-machine CLSD-BPIM one. We provide a mixed integer linear programming (MILP) model for it. The proposed model is based on a real-world case study from a second-tier supplier of the automotive sector. In sequence setup, inventory and resource setup costs terms, the benefits of the proposed multi-machine CLSD-BPIM model are validated by comparing it with a single-machine CLSD-BPIM model that replicates the current planning scenario carried out in the company under study. Additionally, random instances based on the real problem have been generated and tested. The computational results exhibit optimal solutions for the small instances with an average of 0.2-second solution time, the medium instances present average gaps of 0.47% with 2.6 h and large instances present average gaps of 4% with 6 h solution time.

1. Introduction

The plastic industry transforms petrochemical inputs into finished goods for the final market or components for other transformation industries, mainly for the automotive, food-processing and building sectors. To convert raw materials into finished goods or, for instance, automotive parts, the plastic industry uses different transformation processes, and injection moulding is one of the most important ones. Here technological evolution has provided new injection moulding machines

List of acronyms: CLSD, Capacitated lot-sizing problem with sequence-dependent setups; BPIM, Bi-part injection moulding; MILP, Mixed integer linear programming; DLSP, Discrete lot-sizing and scheduling problem; CSLP, Continuous setup lot-sizing problem; PLSP, Proportional lot-sizing and scheduling problem; GLSP, General lot-sizing and scheduling problem; TSP, Travelling salesman problem; SME, Small- and medium-sized enterprise; PPAP, Production part approval process; MRP, Material requirements planning; MPS, Master production schedule.

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and equipment to enable better performance in terms of higher speed, accuracy, flexibility and resourcefulness, among others. Accordingly, production management and research need to provide novel optimisation models and solution approaches to improve injection moulding processes through better production scheduling programmes. Such motivation led us to present our work after considering the needs of an automotive injection moulding plant to optimise its production scheduling processes.

The formulations for simultaneous lot-sizing and scheduling problems can be classified mainly into two groups according to the maximum number of setups per period: small-bucket models (the planning horizon is decomposed into relatively small intervals or "microperiods" within which one item can be produced at the most) and large-bucket models (multiple items can be produced during each period or "macroperiod") [1]. According to [2], models based on microperiods include: the discrete lot-sizing and scheduling problem (DLSP), the continuous setup lot-sizing problem (CSLP) and the proportional lot-sizing and scheduling problem (PLSP). Macroperiods are used in the formulations of the general lot-sizing and scheduling problem (GLSP), and the capacitated lot-sizing problem with sequence-dependent setups (CLSD). In CLSD models [3], the sequence of lots is described by numbering the products produced during the period, based on a tour of a travelling salesman problem (TSP). Gupta and Magnusson [4] develop a single-machine CLSD model by introducing new variables and a heuristic solution for a sandpaper company. Almeder and Almada-Lobo [5] add new constraints to the formulation of [4] to avoid disconnected subtours.

The first attempt to optimise the CSLP with parallel machines for multiple items in an injection moulding context is provided by Dastidar and Nagi [6], who propose an integer programming formulation to minimise inventory, backorder and setup costs, and a solution approach based on a two-phases work centre-based decomposition scheme. Recently, Masoud and Mason [7] extended this model by simultaneously optimising transportation decisions, while incorporating crane assignment decisions into the model. They also developed a hybrid simulated annealing algorithm to solve the proposed problem based on the automotive industry. Ibarra-Rojas et al. [8] formulate an integer quadratically constrained programming model to address the problem from a part-mould-machine manufacturing perspective, in which limited moulds are set up on different machines. The formulation is also decomposed into two subproblems: one for lot-sizing and another for verifying feasible schedules. Rios-Solis et al. [9] report a decomposition approach based on mathematical programming and heuristics to solve a lot-sizing and scheduling problem in a plastic injection production environment where pieces are produced using moulds to form finished products. Previously, single-machine scheduling problems [10,11] and multi-machine scheduling problems [12] have been addressed in an injection moulding context by heuristic and metaheuristic procedures [13].

Based on the previous works developed by Gupta and Magnusson[4] and Almada-Lobo [14], we address the optimisation of a CLSD problem with parallel injection moulding machines producing similar parts that belong to the right- and left-hand side of a car. In this context, injection moulds have two cavities for right and left parts, which are managed as the same part in the CLSD. We previously addressed this problem, but in a single machine context for a bi-part injection moulding (BPIM) capacitated lot-sizing problem (CLSP) [15]. This problem can also arise in shoe companies that inject plastic soles, whose requirements and schedules are generally considered by pairs instead of units [16] . However, to the best of our knowledge, no contribution is available on capacitated lot-sizing and scheduling on parallel machines in a BPIM context.

This paper aims to propose and evaluate a mixed integer linear programming (MILP) model for the simultaneous lot-sizing and scheduling problem in parallel machines with BPIM that accounts for different part inventories that are independently managed due to quality, transportation, manufacturing or holding issues, and avoids order post-processing. The main contributions of this paper are to: (i) propose a new multi-machine CLSD-BPIM optimisation model; (ii) develop three different sized datasets (small, medium, and large) based on real data from a second-tier supplier that produces plastic parts for the automotive sector; (iii) propose, validate and compare two different scenarios to carry out computational experiments considering the proposed monolithic approach (parallel-machine scheduling) and an alternative decomposition approach through a single-machine optimisation model executed serially (single-machine scheduling) based on current real world heuristic practices. Specifically, the studied company uses a manual spreadsheet procedure based on heuristics for production scheduling that iteratively sequences the parts needed on each single-machine. This process is extremely time-consuming (about 13 h) as requires much replanning and data updating.

The remainder of this paper is organised as follows: Section 2 briefly presents a literature overview on models for simultaneous lot-sizing and scheduling with parallel machines. Section 3 describes the planning process and other technical features of the case study. Section 4 illustrates a CLSD with parallel machines and bi-part injection moulding, and formulates it as a multi-machine CLSD-BPIM model. Section 5 reports all the data used for testing, some computational experiments related to instance sizes, and a comparison to a decomposition approach with a single-machine optimisation model. Section 6 offers some final conclusions and identifies further research lines.

2. Literature review

Here we present a brief overview of multi-machine CLSD models (Table 1). Cheng and Sin [17] provide an exhaustive review of relevant contributions to parallel machine scheduling problems addressed in different contexts. Drexl and Kimms [18] review models CLSP, CSLP, DLSP, GLSP and PLSP for a single-stage multiple product system. CLSP is an early large-bucket model that determines lot sizes, but not the sequence of lots [2]. Karimi et al. [19] focus their literature review on single-level CLSP models and solution approaches. In this context, Diaz-Madroñero et al. [15] propose a MILP model for the single machine CLSP with sequence-dependent setup costs to adapt it to a second-tier supplier in the automotive sector con-

Table 1
Literature overview of recent multi-machine CLSD models

Authors	Research problem	Model's characteristics	Industrial Application	Modelling and solution approach
Almeder and Almada-Lobo [5]	CLSD GLSP	Parallel machines, backordering, two types of scarce resources, tool sequence-dependent setup times and costs, continuous lot sizes	Wafer testing in the semiconductor industry	MILP-commercial solver
James and Almada-Lobo [33]	CLSD	Single and parallel machines comparative, no backorder, sequence-dependent setup times and costs, continuous maximum lot sizes	-	MILP-heuristics
Camargo et al.[34]	CLSD	Parallel machines, secondary resources, two-stage, bill of materials, continuous lot sizes	Process industry	MILP-commercial solver
Ferreira et al. [1]	CLSD GLSP	Parallel machines, backordering, synchronised two-stage production, small and large buckets, asymmetric TSP constraints, bill of materials, continuous between minimum and maximum lot sizes	Soft drink bottling plant	MILP-commercial solver
Ruiz-Torres et al. [35]	CLSD	Parallel machines, backordering, variable demand, predefined lot size, single-level of production, setup times are product-specific, campaigns are considered for scheduling	Pharmaceutical industry	MILP-heuristics
Amorim et al. [36]	CLSD	Parallel machines, no backordering, complex setup structure, continuous lot sizes	Consumer goods industry	MILP-commercial solver
Xiao et al. [37]	CLSD	Parallel machines, sequence-dependent setup times, time windows, machine eligibility, preference constraints, continuous lot sizes	Semiconductor industry	MILP-heuristics
Xiao et al. [38]	CLSD	Parallel machines, sequence-dependent setup times, time windows, machine eligibility, preference constraints, continuous lot sizes	Semiconductor industry	MILP- heuristics
Tempelmeier and Copil [39]	CLSD	Parallel machines, sequence-dependent setups, simultaneous scheduling of a common setup resource, perishability and quarantine issues, time windows, multiple setups per product and period, continuous lot sizes	Food company	MILP-heuristics
De Armas and Laguna [40]	CLSD	Parallel machines, shared resources, stochastic production rates	Pipe insulation industry	MILP-heuristics

templating raw material requirements and inventories, inventory coverages, holding space limitations, overtime costs and scheduling plans. Dastidar and Nagi [6] classify research works into lot-sizing and scheduling models (basic, time period, multi-machine models) and solution methods (optimisation and heuristic approaches). Quadt and Kuhn [20] present a literature review on problems that extend the standard CLSP formulation by incorporating: backordering, multi-level product structure, overtime, setup carryover and crossover [21], sequence-dependent setup costs and times [22] and parallel machines. MILP combined with heuristic algorithms is the most widely used modelling and solution approach for CLSP models. Buschkühl et al. [23] present a review of four decades of research into different modelling approaches of dynamic lot-sizing with capacity constraints, specifically optimisation problems and algorithmic solution approaches, which they classify into five groups: mathematical programming heuristics, lagrangian heuristics, decomposition and aggregation heuristics, metaheuristics and problem-specific Greedy heuristics. Of them, mathematical programing and the use of metaheuristics are identified as a vivid flourishing research field. Clark et al. [24] identify industrial extensions and research opportunities in lot-sizing and scheduling. Hot topics include perishability, synchronisation of resources, non-triangular setups, delivery time windows, multiple stages with parallel machines, supply chain and reverse logistics aspects. However, research into the effect of using real-life instances and the integration of algorithms with interactive decision support systems is lacking. There are other variants of the classic lot sizing and scheduling problems, such as multi-stage production [25] and deterioration issues [26], among others. Readers are referred to other reviews by Quadt and Kuhn [27], Robinson et al. [28], Guimaraes et al. [29] and Copil et al. [2], which establish the body of knowledge of simultaneous lot-sizing and scheduling, and show the relevance of this research field. Regarding methodological approaches, we refer readers to Ball [30], Della-Croce et al. [31] and Ghirardi and Amerio [32].

Regarding CLSD models, Copil et al. [2] present a detailed review of 31 works from 1996 to 2015, of which 12 models consider parallel machines, and exact or MILP-based solution approaches are mainly used. Table 1 provides an overview of recent works on multi-machine CLSD models to show the addressed research problems, the main characteristics of the proposed model, the industrial application, and the modelling and solution approach.

Our proposal considers these CLSD foundations and extends them by characterising and modelling the multi-machine CLSD-BPIM, which contemplates parallel machines, raw material requirements (bill of materials) and inventories, inventory coverages (or safety lead times), holding space limitations, demand backorders, discrete multiple lot sizes, sequence-dependent setups, setup times and costs, and bi-part injection mouldings constraints. The industrial application is based

Table 2
Notation for the CLSD-BPIM model

Set of indices	
I	Set of products (parts and raw materials) (indexed by i: $i = 1, \ldots, \|I\| \cup j$: $j = 1, \ldots, \|J\|$)
J	Set of parts in the bill of materials (indexed by i: $i = 1, \ldots, \|I\| \cap j$: $j = 1, \ldots, \|J\|$)
R	Set of resources (indexed by r: $r = 1, \ldots, \|R\|$)
T	Set of time periods (indexed by t: $t = 1, \ldots, \|T\|$)
Subsets of indices	
$Isr(i,r)$	Set of primary parts i (sequenced parts) that may be produced on machine or resource r
$Ins(i)$	Set of secondary parts (non-sequenced bi-parts), where $Ins(i) \in J$
$Js(j)$	Set of primary parts (sequenced parts), where $Js(j) \in J$
$To(t)$	Set of time periods with total overtime production, where $To(t) \in T$
Data	
B_{i0}	Initial backorder of product $i \in J$
bip_i	Bi-part product $i \in Ins(i)$
bom_{ji}	Bill of materials, a unit of part j is produced with a quantity of product i
c_{tr}	Total time units of available capacity during time period t on resource r
cb_{it}	Backorder cost of product i during time period t
ci_{it}	Inventory cost of product i during time period t
co_{tr}	Overtime production costs during time period $t \in To(t)$ on resource r
cov_{it}	Number of time periods for the inventory coverage of product $i \in J$ during time period t
cs_{itr}	Moulding setup cost for product $i \in Isr(i,r)$ during time period t on resource r
$cseq_{ijtr}$	Sequence setup cost for product $i \in Isr(i,r)$ to product $j \in Js(j)$ during time period t for r
cv_{it}	Penalisation cost for a soft inventory coverage constraint for product i during t
d_{it}	Demand of product i during time period t
INV_{i0}	Initial inventory of product i
$INVMAX_{it}$	Maximum inventory units for product i during time period t
lot_{ir}	Production lot size of product $i \in Isr(i,r)$ on resource r
M	Big number
N_r	Total number of parts $i \in Isr(i,r)$ to be sequenced on resource r
SR_{it}	Scheduled receptions of product i during time period t
tl_i	Lead time to supply a unit of product i
tp_{ir}	Time required to produce a unit of product $i \in Isr(i,r)$ on resource r
ts_{ijr}	Setup time for moulding exchange for product $i \in Isr(i,r)$ to product $j \in Js(j)$ on resource r
Decision variables	
α_{itr}	1 if product $i \in Isr(i,r)$ is produced first during time period t on resource r, 0 otherwise
β_{itr}	1 if product $i \in Isr(i,r)$ is produced last during time period t on resource r, 0 otherwise
δ_{tr}	0 if exactly one product is produced during time period t on resource r, an unrestricted non-negative number otherwise
γ_{itr}	1 if a machine is setup for product $i \in Isr(i,r)$ at the end of time period t on resource r, 0 otherwise
ω_{tr}	Strictly positive when at least one product is produced during time period t on resource r, 0 otherwise
B_{it}	Backorder of product $i \in J$ during time period t
H_{it}	Auxiliary variable to generate a soft inventory constraint for product $i \in J$ during time period t
INV_{it}	Inventory level of product i at the end of time period t
k_{itr}	Number of lots to produce of $i \in Isr(i,r)$ during time period t on resource r
Q_{it}	Amount of product i to order to suppliers during time period t
S_{ijtr}	1 if a setup occurs for product $i \in Isr(i,r)$ to product $j \in Js(j)$ and $i \neq j$ during time period t on resource r, 0 otherwise
V_{itr}	Auxiliary continuous variable to eliminate disconnected subtours of product $i \in Isr(i,r)$ during time period t on resource r
X_{itr}	Amount, which is k_{itr} times the required production lot size, of primary parts $i \in Isr(i,r)$ to produce during time period t on resource r
Xs_{itr}	Amount of primary and secondary parts $i \in J$ to produce during time period t on resource r
Y_{itr}	1 if product $i \in Isr(i,r)$ is produced during time period t on resource r, 0 otherwise

on an automotive second-tier supplier. Here we adopt a MILP modelling approach and a solution procedure based on the standard algorithm embedded in a commercial solver (Gurobi, details in Section 5.1).

3. The case study

The case study refers to a small- and medium-sized enterprise (SME) that manufactures small parts, which are finished goods, for the automotive industry by thermoplastics bi-injection moulding. It is a second-tier supplier for exterior and interior plastic automotive technical parts that supplies worldwide. The production facilities of the second-tier supplier house 18 injection moulding machines with the following technologies: co-injection machines; 2K machines with rotative plates or two-shot injection moulding, which consists of processing two different polymers (or two different colours of one polymer) in an end product by one-injection moulding process that enables several functions to be integrated into an injection moulded product; heat and cool systems; machines controlled by a central server (quality reports, data monitoring, etc.); artificial vision and a painting line subdivision (small parts). The company also has in-house engineering capabilities, such as tool engineering for the products it manufactures (tools and control measurements), automation engineering, metrology, production part approval process (PPAP) and full PPAP reports. The company specialises in energy-saving plastics auxiliary equipment, including plastics dryers, powerguard dryers, plastics vacuum-conveying equipment and systems, plastics blending, plastics feeding and vacuum-conveying controls.

By taking 6 months, 7 days and the daily demand data sent by the automotive assembler to the first-tier suppliers as the main input data, they calculate their car components production plans and parts requirements planning, which are the input for the demand plan for the second-tier supplier under study. From the capacities and current inventory levels, the second-tier supplier calculates the parts injection plans and the material requirements planning (MRP) to optimise manufacturing and logistics assets.

In this specific case study, simultaneous lot-sizing and scheduling appears mainly conditioned, on the one hand, by the plant's technological features with high setup costs, long product-sequence dependent times and costs related to mould changing and cleaning tasks. Additionally, the BPIM for the left- and right-hand side car parts makes the management of the lot-sizing and scheduling plans more complex. On the other hand, the automotive sector's strict demand fullfilment requirements imply very high backorder costs and using inventory coverage constraints, which involves safety lead times.

The second-tier supplier under study produces about 314 different automotive parts, with 80 manufactured every week on average in a single stage operation. Parallel machines are considered independent because they can process assigned parts with differents costs and production speeds. Demand is deterministic. The plastic plant works 5 days a week with three working shifts (24 h) with normal production, and 1 extra day with two working shifts (16 h) consisting of a whole overtime period of production, whenever necessary. An unitary overtime cost is included for these extra production time periods.

Inventory coverage and backordering for parts are also considered. Presently, high-volume batches are usually produced in 24-hour (1-day) runs. Some pairs of the right- and left-hand side car parts use the same injection mould at the same instant to produce the same amount of both parts per time period. Nevertheless, independent inventories are managed for these right- and left-hand side parts that are subject to product quality, transportation, manufacturing and/or holding problems, which complicate sequence management, but avoid manual order post-processing.

The problem also contemplates the quantities to order and the inventories for raw materials. It can also be assumed that:

- The plastic injection moulding process is performed in a single stage that corresponds to the operation of transforming plastic raw material into a finished automotive part.
- The bill of materials is characterised by a single-level product structure, i.e. each automotive part has one predecessor or petrochemical raw material at the most.
- Lot sizes of a fixed value (1 or others) can be considered.
- Setup times and costs are sequence-dependent on the injection moulding machines due to mould changes or cleaning tasks.
- Storage costs are considered for both parts and raw materials. Penalisation in terms of additional costs is also included for not fulfilling inventory coverages.

4. Model formulation

The multi-machine CLSD-BPIM proposal considers the initial single-machine BPIM-CLSP model by Díaz-Madroñero et al. [15], based on the former works of Gupta and Magnusson [4] and Almada-Lobo et al. [14], but extends them by considering: a bill of materials where parts, which are composed of raw materials, are sequenced (Table 2); part and raw material inventories and inventory coverages; holding space limitations; demand backorders; overtime costs; the simultaneous manufacturing of two parts in the same moulds; setups per time period; lot-size requirements. Thus, parts are produced and sequenced and raw materials are purchased to outside suppliers according to demand and inventory levels. This new multi-machine CLSD-BPIM model is extended to manage simultaneous lot-sizing and scheduling problems with parallel machines with no limits of setups per period. It is important to highlight that in the BPIM context, two parts are produced at the same instant using the same mould and machine capacity. Therefore, only one part should be considered sequenced and time-consuming for normal and overtime production, and this part is known as the primary part. Here, secondary parts are associated with primary parts, and every time a primary part is sequenced, a secondary part is also produced in the same injection mould and stored. Thus, primary and secondary parts are created as a mathematical artifice in order to model the real BPIM sequencing process. All the resources considered are machines. Thus, multiple machines are used to meet the demand of multiple parts over multiple time periods. We consider a set of products including parts and raw materials. We use the same set for parts and raw materials in order to provide a more general model, diminish the number of decision variables and simplify the further structure of the model implementation as proposed by [41,42]. The notation of the MILP model for the CLSD-BPIM problem is shown in Table 2.

The objective function minimises total costs, which include sequence-dependent setup costs, inventory costs, overtime costs, penalisation costs for not fulfilling inventory coverages, backorder costs and moulding setup costs. Here it is important to highlight that overtime costs are established according to time periods, i.e. there will be time periods with total overtime production (weekends, etc.) and others when only normal production is available with null overtime costs. In addition, cost parameters have been defined on a time-period basis to provide more flexibility in defining the model according to the application context.

$$Min\ z = \sum_{i\in Isr}\sum_{j\in Js}\sum_{t\in T}\sum_{r\in R} cseq_{ijtr}\cdot S_{ijtr+}\sum_{i\in I}\sum_{t\in T} ci_{it}\cdot INV_{it} + \sum_{i\in Isr}\sum_{t\in To}\sum_{r\in R} co_{tr}\cdot X_{itr} + \sum_{i\in J}\sum_{t\in T} cv_{it}\cdot H_{it} + \sum_{i\in J}\sum_{t\in T} cb_{it}\cdot B_{it}$$
$$+ \sum_{i\in Isr}\sum_{t\in T}\sum_{r\in R} cs_{itr}\cdot Y_{itr} \quad (1)$$

Subject to:

4.1. Inventory balance equations

The inventory balance equations for parts (2) and raw materials (3) guarantee the appropriate values for inventories, quantities to produce or supply, and the backorders for each time period t.

$$INV_{i,t-1} - B_{i,t-1} + B_{it} + \sum_{r} Xs_{itr} - INV_{it} = d_{it} \qquad \forall i\in J, \forall t\geq 1 \quad (2)$$

$$INV_{i,t-1} + SR_{i,t} + Q_{it-tl_i} - INV_{it} = \sum_{j}\sum_{r} bom_{ji}\,\boldsymbol{.}\,Xs_{j,t,r} \qquad \forall i\in I:\ i\notin J, \forall t\geq 1 \quad (3)$$

4.2. Inventory coverage constraints

Constraint (4) limits the inventory levels for each product according to the available space for inventory holding. There are hard constraints, which must necessarily be satisfied, and other soft constraints, which prevent undesirable solutions. Accordingly, constraint (5) is a soft constraint with a penalty cost in the objective function for the inventory coverage requirement of parts. The function of penalty costs is to ensure that part inventory levels at the end of each time period come as close as possible to the sum of the demands during the following time periods according to the inventory coverage.

$$INV_{it} \leq INVMAX_{it} \qquad \forall i\in I,\ \forall t\in T \quad (4)$$

$$INV_{it} + H_{it} \geq \sum_{t+1}^{t+cov_{it}} d_{it} \qquad \forall i\in J,\quad \forall t\ \in T \quad (5)$$

4.3. Capacity and setup changes constraints

Constraint (6) guarantees that, when $X_{itr} > 0$, binary decision variable Y_{itr} is set to 1 when product i is sequenced in the allowed machine r. Constraint (7) determines the total required capacity per time period t and machine r to ensure that the total production and setup times are less than or equal the available capacity per time period t and machine r. Here, additional capacity constraints related to common manpower resources [39], overtimes [15] or maximum number of setup changes per period [43,44], among others, can be easily included if required by the addressed problem. Additionally, if required, sequence dependent setup costs and moulding setup costs could be aggregated in a unique setup cost. Here, we model both of them in a separate way in order to provide a reference of the proportion of the cost associated to the sequence, i.e. mould changing, cleaning tasks and/or machine raw materials feeding tasks. Thus, the moulding setup cost is the common part of the mould change cost, which is an incurred cost regardless of the sequence; and the sequence dependent setup cost is the variable part of the mould change cost, which depends on the sequence.

$$X_{itr} \leq M\,\boldsymbol{.}\,Y_{itr} \qquad \forall (i,r)\in Isr(i,r),\ \forall t\in T \quad (6)$$

$$\sum_{i\in Is} tp_{ir}\cdot X_{itr} + \sum_{i\in Is}\sum_{j\in Js} ts_{ijr}\cdot S_{ijtr} \ \leq c_{tr} \qquad \forall t\in T, \forall\ \ \in R \quad (7)$$

4.3. Sequence constraints

Based on Gupta and Magnusson [4], Constraints (8)–(21) determine the optimal sequence for the multi-machine CLSD-BPIM problem. In Constraint (8), ω_{tr} is established as 1 if any part is produced during time period t on machine r. Constraints (9) and (14) imply that when more than one part is produced during a single time period, the same part cannot be produced both first and last during that given time period. Constraints (10) and (11) ensure that α_{itr} and β_{itr} are 1 for exactly one part during a given time period if production occurs during that time period. Constraints (12) and (13) ensure that α_{itr} and β_{itr} are set at 0 when no parts are produced. If only one part is produced during a time period, then values α_{itr} and β_{itr} must equal 1 for that part and 0 for all the other parts. This also forces δ_{tr} to be 0 in (14). However, if more than one part is produced during a time period, δ_{tr} becomes positive by (9). In this case, Constraint (14) ensures that if the α_{itr} value for a part is 1, then the corresponding β_{itr} value is 0, and *vice versa*. Constraint (15) establishes that only one part can be set up at the end of each time period. Constraints (16) and (17) apply whenever more than one part is produced during a single time period. They force at least one S_{ijtr}'s to be 1 per part i, except when this part is either the first or the last part in the sequence. Constraint (18) forces a setup during production-free time periods when the machine's setup state at the end of the time period is not the same as the setup status at the end of the following time period. Constraints (19) and (20) count the setups between the time periods during which the machine is not idle. Setup carryovers are contemplated through binary variables α_{itr}, β_{itr} and γ_{itr} [4]. Constraint (21) eliminates sequence subtours according to Almada-Lobo et al. [14]. It is noteworthy that sequencing in this model begins with the requirement of a part, while the proposal of Gupta and Magnusson [4] requires the definition of the initial part to be sequenced.

$$Y_{itr} \le \omega_{tr} \qquad \forall (i,r) \in Isr(i,r),\ \forall t \in T \tag{8}$$

$$\sum_{i \in Isr} Y_{itr} - 1 \le (N_r - 1) \cdot \delta_{tr} \qquad \forall t \in T,\ \forall r \in R \tag{9}$$

$$\omega_{tr} \le \sum_{i \in Isr} \alpha_{itr} \le 1 \qquad \forall t \in T,\ \forall r \in R \tag{10}$$

$$\omega_{tr} \le \sum_{i \in Isr} \beta_{itr} \le 1 \qquad \forall t \in T,\ \forall r \in R \tag{11}$$

$$\alpha_{itr} \le Y_{itr} \qquad \forall (i,r) \in Isr(i,r),\ \forall t \in T \tag{12}$$

$$\beta_{itr} \le Y_{itr} \qquad \forall (i,r) \in Isr(i,r),\ \forall t \in T \tag{13}$$

$$\alpha_{itr} + \beta_{itr} \le 2 - \delta_{tr} \qquad \forall (i,r) \in Isr(i,r),\ \forall t \in T \tag{14}$$

$$\sum_{i \in Isr} \gamma_{itr} = 1 \qquad \forall t \in T,\ \forall r \in R \tag{15}$$

$$\sum_{j \in Js} S_{jirt} \ge Y_{itr} - \alpha_{itr} \qquad \forall (i,r) \in Isr(i,r), \forall t \in T \tag{16}$$

$$\sum_{j \in Js} S_{ijtr} \ge Y_{itr} - \beta_{itr} \qquad \forall (i,r) \in Isr(i,r), \forall t \in T \tag{17}$$

$$S_{ijtr} \ge \gamma_{i,t-1,\ r} + \gamma_{jtr} - \omega_{tr} - 1 \qquad \forall (i,r) \in Isr(i,r), \forall j \in Js(j),\quad i \ne j, \forall t \in T \tag{18}$$

$$S_{jitr} \ge \alpha_{itr} + \gamma_{j,t-1,\ r} - 1 \qquad \forall (i,r) \in Isr(i,r), \forall j \in Js(j),\ i \ne j, \forall t \in T \tag{19}$$

$$S_{ijtr} \ge \beta_{itr} + \gamma_{jtr} - 1 \qquad \forall (i,r) \in Isr(i,r), \forall j \in Js(j),\ i \ne j, \forall t \in T \tag{20}$$

$$V_{jtr} \ge V_{itr} + N_r \cdot S_{ijtr} - (N_r - 1) - N_r \cdot \gamma_{i,t-1,\ r} \qquad \forall (i,r) \in Isr(i,\ r), \forall j \in Js(j),\ i \ne j, \forall t \in T \tag{21}$$

$$0 \le \omega_{tr} \le 1 \qquad \forall t \in T,\ \forall r \in R \tag{22}$$

4.4. BPIM and lot size constraints

Constraints (23) and (24) allow the produced quantities of the sequenced and primary parts, X_{itr}, to be copied and stored in the associated non-sequenced or secondary parts, Xs_{itr}, where bip_i refers to the corresponding primary part to be sequenced for secondary parts (see Table 5). Constraint (25) considers the number of established lots per parts. Thus, we include this constraint to guarantee X_{itr} is multiple of lot_{ir}. Here we can replace X_{itr} with $k_{itr} \bullet lot_{ir}$ along the model, but we maintain both decision variables to gain a better understanding of the results, and even of the model formulation.

$$Xs_{itr} = X_{itr} \qquad \forall (i,r) \in Isr(i,r),\ \forall t \in T \tag{23}$$

$$Xs_{itr} = X_{bip_i,t,r} \qquad \forall i \in Ins(i),\ \forall t \in T,\ \forall r \in R \tag{24}$$

$$X_{itr} = k_{itr} \cdot lot_{ir} \qquad \forall (i,r) \in Isr(i,r), \forall t \in T \tag{25}$$

$$Y_{itr}, S_{ijtr}, \alpha_{itr}, \beta_{itr}, \gamma_{itr} \in \{0, 1\} \qquad \forall (i, r) \in Isr(i, r), \forall j \in Js(j), \forall\ t \in T \tag{26}$$

$$INV_{it} \in \mathbb{N} \qquad \forall i \in I,\ \forall t \in T \tag{27}$$

$$B_{it}, H_{it} \in \mathbb{N} \qquad \forall i \in J, \forall t \in T \tag{28}$$

$$Xs_{itr} \in \mathbb{N} \qquad \forall i \in J, \forall t \in T,\ \forall r \in R \tag{29}$$

$$\omega_{tr}, \delta_{tr} \in R \qquad \forall t \in T,\ \forall r \in R \tag{30}$$

$$X_{itr}, k_{itr}, V_{itr} \in \mathbb{N} \qquad \forall (i, r) \in Isr(i, r),\ \forall t \in T \tag{31}$$

$$Q_{it} \in \mathbb{N} \qquad \forall i \in I : i \notin J,\ \forall t \in T \tag{32}$$

5. Computational experiments

We perform computational experiments in three different sized datasets that reflect the distinct characteristics of the real-world case study. In the next sections, the results are summarised with two features: (i) small, medium and large instances are tested by the monolithic approach (multi-machine CLSD-BPIM); (ii) a comparative test with a multi-machine decomposition problem in single-machine subproblems is run. We aim to investigate, on the one hand, the impact of the size of datasets on the computational results to test our proposal's applicability to optimise industrial problems in a reasonable time. On the other hand, we compare the results provided by the monolithic approach (multi-machine) with those obtained from applying the single-machine model by decomposing the multi-machine problem into several single-machine subproblems through an optimisation model for single machine CLSD-BPIM.

From the case study characteristics, for all the datasets, backorder costs, cb_{it}, are considered to be equal to M (99999) for all parts and time periods to penalise backorders, but to allow the model's feasibility in all possible cases. A representative planning horizon with 14 time periods for an operative sequencing problem is considered for the medium and large dataset sizes. A 3-day inventory coverage, cov_{it}, is also contemplated for all parts and time periods. Similarly, the penalisation costs for not fulfilling inventory coverages are set at M. A lot-for-lot technique is followed through a fix lot size, lot_{ir}, of 1 for all parts and machines. The moulding setup cost, cs_{itr}, is considered to be 40 for all parts, time periods and machines. The selection of these values derives from the addressed real-world problem. All the costs referred to throughout this paper are in euros.

5.1. Monolithic approach

We solve the multi-machine CLSD-BPIM monolithic approach (1)-(32) for small-, medium- and large-sized datasets. The small-sized dataset (S0) is considered mainly a short planning horizon of three time periods (for instance, 3 days) where a few parts have to be sequenced, which would be the minimum data size required to solve the addressed problem. Medium (M0) and large (L0) datasets consider a representative sequencing problem with a typical 2-week or 14-day planning horizon, where the master production schedule (MPS) is frozen with a medium and large number of parts to be sequenced, respectively. The large dataset represents the data size from the real-world company under study.

Table 3 provides the characteristics of the different datasets tested, including capacity utilisation, referred to the percentage of the machine time spent, and computational efficiency in terms of the number of constraints, variables, integer decision variables and the solution time using the optimisation solver Gurobi 9.1.0 under two Intel Xeon 2.00 GHz processors and 16 GB RAM memory conditions. The optimality gap value (the difference between the upper and lower bounds) for each solution is also provided.

Table 3 presents the solution times from about 0.41 seconds for a small dataset comprised of six parts, two raw materials, two machines, three time periods and 82% capacity utilisation when considering the first two periods of normal available capacity; 3 h for a medium and more realistic dataset size comprised of 30 parts, 10 raw materials, 10 machines, 14 time periods and 71% capacity utilisation, with optimality gaps between 0 and 0.98% for the small and the medium dataset sizes, respectively. Additionally, a more realistic scenario contemplates a larger dataset size with 80 parts, 40 raw materials, 20 machines, 14 time periods and 76% capacity utilisation, with 4.25%, 3.88% and 1.88% solution gap and a computational time of 3, 4.13 and 8.5 h, respectively. Therefore, we state that the monolithic approach offers reasonable computational efficiencies according to the required solution gap. Next all the details of the experiments are shown through the input and output data of the three experimented instances S0, M0 and L0. Finally, the average results are presented for the 12 more generated instances.

Table 3
Characteristics of the S0, M0 and L0 datasets and computational efficiency

		Dataset size Small (S0)	Medium (M0)		Large (L0)	
Number of parts		6	30	80		
Number of raw materials		2	10	40		
Number of machines		2	10	20		
Number of time periods		3	14	14		
Capacity utilisation		82%	71%	76%		
	Constraints	783	307,734	4,082,414		
Computational efficiency	Variables	375	79,254	904,414		
	Integer	216	70,000	860,720		
	Solution time	0.41 sec	3 hours	3 h	4.13 h	8.5 h
	Gap %	0	0.98	4.25	3.88	1.88

Table 4
Part demand of the S0 dataset

d_{it} (units)	t=1	t=2	t=3
***i*=1**	98	0	120
***i*=2**	98	0	120
***i*=3**	70	0	120
***i*=4**	70	0	120
***i*=5**	30	50	0
***i*=6**	0	100	0

Table 5
Inventory costs, maximum inventory capacity and bi-part products of the S0 data set for all time periods

i	ci_{it} (€)	$INVMAX_{it}$ *(units)*	bip_i *(part number)*
1	0.7225	29,160	0
2	0.7225	29,160	1
3	0.7185	28,800	0
4	0.7185	28,800	3
5	0.3193	65,520	0
6	0.5093	42,480	0
7	2.9	99,999	0
8	3.3	99,999	0

5.1.1. Input data

In the small-sized dataset, we consider 2 machines. There are 8 products, 6 parts and 2 raw materials; among the 6 parts there are 2 pairs of bi-parts (primary and secondary parts). Part 2 is a secondary part associated with primary part 1, and part 4 is a secondary part associated with primary part 3. This gives four parts (1, 3, 5 and 6) to be considered in the sequence constraints. Each part can be processed in the assigned machine for the corresponding production times. The initial inventory for all the parts and raw materials is assumed to be 1 to avoid backorders during the first time period. Initial backorders are considered null for all parts. We consider a 3-day planning horizon. Thus, time periods 1 and 2 last 24 h (3 shifts) of normal available capacity, while time period 3 lasts 16 h (2 shifts) of possible overtime, with an extra cost of 100. Stock coverage is set as 1 day of demand. The additional input data for this small-sized dataset are provided in Table 4, Table 5, Table 6, Table 7 and Table 8.

We consider 10 machines in the medium-sized dataset, M0. There are 40 products, 30 parts and 10 raw materials; among the 30 parts there are 10 pairs of bi-parts (primary and secondary parts). Therefore, we contemplate 10 primary parts in the sequence constraints. It is important to highlight that each part can be produced only on some defined machines. The initial inventory for all the parts and raw materials is assumed to be 1 to avoid backorders during the first time period. Initial backorders are considered null for all parts. We consider a 14-day planning horizon. Thus, time periods last 24 h (3 shifts) of normal available capacity, except for time periods 6 and 12, with 16 h (2 shifts) of possible overtime, with an extra cost of 100, and periods 7 and 14 with null available capacity. Stock coverage is set as 3 days of demand. The additional input data for the M0 dataset are available (see Supplementary material).

For the large-sized dataset, L0, we consider 20 machines. There are 120 products, 80 parts and 40 raw materials; among the 80 parts there are 27 pairs of bi-parts (primary and secondary parts). Therefore, we consider 27 primary parts in the sequence constraints. The initial inventory for all the parts and raw materials is assumed to be 100 to avoid backorders during the first time period. Initial backorders are considered null for all parts. We consider a 14-day planning horizon. Thus, time periods last 24 h (3 shifts) of normal available capacity, except for time periods 6 and 12 with 16 h (2 shifts) of

Table 6
Time required to produce a unit of part in each machine/resource of the S0 dataset

$i \in J$	R	tp_{ir} (hours)
1	1	0.1818
1	2	0.1667
2	1	0.1818
2	2	0.1667
3	1	0.1219
3	2	0.1099
4	1	0.1219
4	2	0.1099
5	1	0.1370
5	2	0.1449
6	1	0.1
6	2	0.0917

Table 7
Bill of materials of the S0 dataset

bom_{ji}	i=7	i=8
j=1	1	-
j=2	1	-
j=3	1	-
j=4	1	-
j=5	-	1
j=6	-	1

Table 8
Sequence setup times and costs of the S0 dataset for all time periods and machines

i	j	r	ts_{ijtr} (hours)	$Cseq_{ijtr}$ (€)
1	1	1	0	9999
3	1	1	1.2558	13.8140
5	1	1	1.2558	13.8140
6	1	1	1.2558	13.8140
1	3	1	1.1036	12.1392
3	3	1	0	9999
5	3	1	1.1036	12.1392
6	3	1	1.1036	12.1392
1	5	1	1.0079	11.0866
3	5	1	1.0079	11.0866
5	5	1	0	9999
6	5	1	1.0079	11.0866
1	6	1	0.5284	5.8127
3	6	1	0.5284	5.8127
5	6	1	0.5284	5.8127
6	6	1	0	9999
1	1	2	0	9999
3	1	2	0.8881	9.7687
5	1	2	0.8881	9.7687
6	1	2	0.8881	9.7687
1	3	2	0.5100	5.6096
3	3	2	0	9999
5	3	2	0.5100	5.6096
6	3	2	0.5100	5.6096
1	5	2	1.4893	16.3820
3	5	2	1.4893	16.3820
5	5	2	0	9999
6	5	2	1.4893	16.3820
1	6	2	0.6313	6.9441
3	6	2	0.6313	6.9441
5	6	2	0.6313	6.9441
6	6	2	0	9999

Table 9
Output data of the S0 data set

i	t	r	Xs_{it}						
(units)		i	j	T	r	S_{ijtr}			
1	1	1	2		5	6	1	1	1
2	1	1	2		6	1	1	1	1
5	1	1	79		1	3	1	2	1
6	1	1	99						
1	1	2	95						
2	1	2	95						
3	1	2	69						
4	1	2	69						
1	2	1	120						
2	2	1	120						
3	2	2	120						
4	2	2	120						

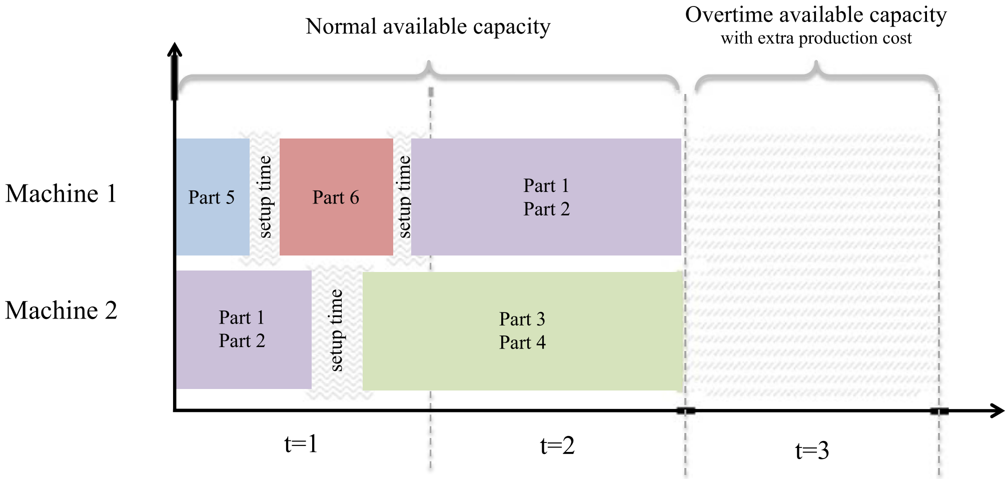

Fig. 1. Graphic output data of the S0 dataset

possible overtime, with an extra cost of 100, and periods 7 and 14 with null available capacity. Stock coverage is set as 3 days of demand. The additional input data for the L0 dataset are provided (see Supplementary material).

5.1.2. Output data

Table 9 and Figure 1 provide the S0 dataset with the quantities of i to be produced during time period t on resource r, Xs_{it}. The setups required per period and machine, S_{ijtr}, are also shown. Detailed cost information is provided in Table 10.

Thus, the S0 input data considers 3 time periods of production, the first two time periods are characterized by being normal production, while the third time period is used for overtime production (with an extra production cost). The S0 output data results on producing only in time periods 1 and 2, which indicates that the proposed model moves forward the production in order not to incur with overtime costs and also satisfying the inventory coverage constraint.

From the previous output data, we state that the multi-machine CLSD-BPIM model is capable of optimising the lot size of the produced parts at the constrained resources and at the same time that a machine sequence is provided when considering the existence of primary (to sequence) and secondary parts (associated with primary parts) in the injection mould context. The additional output data for the M0 dataset are also provided (see Supplementary material) and for the L0 data at (see Supplementary material).

5.2. Decomposition approach

This section validates and compares the previous results to a multi-machine problem decomposition approach into single-machine subproblems. Currently, the company under study follows a heuristic procedure based on single-machine lot sizing and scheduling for all machines sequentially. Here we propose a comparison between the monolithic approach, i.e. using a unique model for part sequencing in parallel in all the machines at the same time, and the decomposition approach, which serially sequences parts in each machine separately with an optimisation single machine CLSD-BPIM, which

Table 10
Comparative cost information of the S0, M0 and L0 datasets

S0 dataset Monolithic approach	Decomposition approach
Sequence-dependent setup cost = €25.24	Sequence-dependent setup cost = €165.39
Inventory cost = €412.73	Inventory cost = €20,664.33
Overtime cost = 0	Overtime cost = 0
Not fulfilling costs in inventory coverage = 0	Not fulfilling costs in inventory coverage = 0
Resource setup cost = €280	Resource setup cost = €320
Backorder cost = 0	Backorder cost = 0
Total cost = €717.97	**Total cost =€1,637.67**
Running time = 0.41 seconds	**Running time = 0.3 seconds**
M0 dataset	
Monolithic approach	**Decomposition approach**
Sequence-dependent setup cost = €469.46	Sequence-dependent setup cost = €895.68
Inventory cost = €26,293.66	Inventory cost = €32,720.51
Overtime cost = 0	Overtime cost =€ 5,793.31
Not fulfilling costs in inventory coverage = 0	Not fulfilling costs in inventory coverage = 0
Resource setup cost = €3,720	Resource setup cost = €4,800
Backorder cost = 0	Backorder cost = €13,199,868
Total cost = €30,483.12	**No solution without backorders was found**
Running time = 3 hours	**Running times = 7.18 hours**
L0 dataset	
Monolithic approach	**Decomposition approach**
Sequence-dependent setup cost = €1,267.70	Sequence-dependent setup cost = €2,391.17
Inventory cost = €128,743.92	Inventory cost = €70,125.92
Overtime cost = €176.59	Overtime cost = €19,267.07
Not fulfilling costs in inventory coverage = 0	Not fulfilling costs in inventory coverage = 0
Resource setup cost = €4,680	Resource setup cost = €12,040
Backorder cost = 0	Backorder cost = €307,296,924.83
Total cost = €134,868.21	**No solution without backorders was found**
Running time = 8.5 hours	**Running time = 10.52 hours**

represents the current company's heuristic procedure. Figure 2 presents the decomposition approach used to validate the multi-machine CLSD-BPIM.

The decomposition method is an iterative process that allows the single machine CLSD-BPIM to be computed in each machine separately to obtain lot-sizing and scheduling programmes of all the parts on all machines until the full capacity of all the machines is used or there are no backlogs ($B_{it}=0$). The decomposition method was designed according to the heuristic-based spreadsheet used by planners of the second-tier supplier to calculate the lot-sizing and sequencing plan. Here, however, an optimisation model for the single machine CLSD-BPIM model decomposition is utilised. In line with this, the decomposition method has as many iterations (k) as the maximum number of machines to be scheduled (*max. r*). The decomposition approach starts the first iteration ($k=1$) by calculating the lot-sizing and scheduling programme for machine r=1. The input of the first iteration includes the initial demand (d_{it}) of all the parts, the initial inventory (INV_{i0}) of all the products, and the parts produced prior to iteration k ($fpp_{it}{}^{k}$). In the first iteration $fpp_{it}{}^{k=1}=0$ as no parts have already been produced. The output of computing the single machine CLSD-BPIM in iteration k (machine r) is the amount of i to be produced during time period t on resource r ($Xs_{itr}{}^{k}$). The following iteration, $k=k+1$, will start only if there are backorders of parts during time period t and there are still machines to schedule parts ($k\leq max.k$). Accordingly, the new machine $r+1$ to be scheduled considers the initial demand (d_{it}) of all the parts, the initial inventory (INV_{i0}) of all the products, the parts produced prior to iteration $k=k+1$ ($fpp_{it}{}^{k}$), which coincides with the sum of the number of parts to produce during time period t on resource

r-1 obtained in the previous scheduled machines. Accordingly, $fpp_{it}^{k}=\sum_{k'=1}^{k=k-1}\sum_{r'=1}^{r=r-1}Xs_{itr'}^{k'}$. The iteration process stops when there are no backorders, and all the machines are scheduled. Thus, the machines' capacity is fulfilled, and no more parts can be scheduled.

The input data used in the decomposition approach are the same as those in the monolithic approach. Output data are presented in relation to the S0, M0 and L0 datasets in costs terms (Table 10). The solution considered for the monolithic approach is the optimal one for the S0 dataset. The solution obtained in the M0 and L0 datasets is that which considers a time limit up to 3 h in the M0 dataset and up to 8.5 h in the L0 dataset.

Hence the comparison between the monolithic and decomposition approaches shows that, as expected, the monolithic ones perform considerably better in sequence setup, inventory, resource setup and backorder costs terms for the S0 (55%), M0 and L0 datasets despite of the longer computational times of the monolithic approach. As for the overtime costs generated by the decomposition approach, we can intuitively deduce that the iterative model on each machine attempts to avoid backorders, which have very high costs, by proposing overtime production. However, considering only one resource in each iteration involves both high overtime and backorder costs, similarly to a manual procedure in which planners must replan

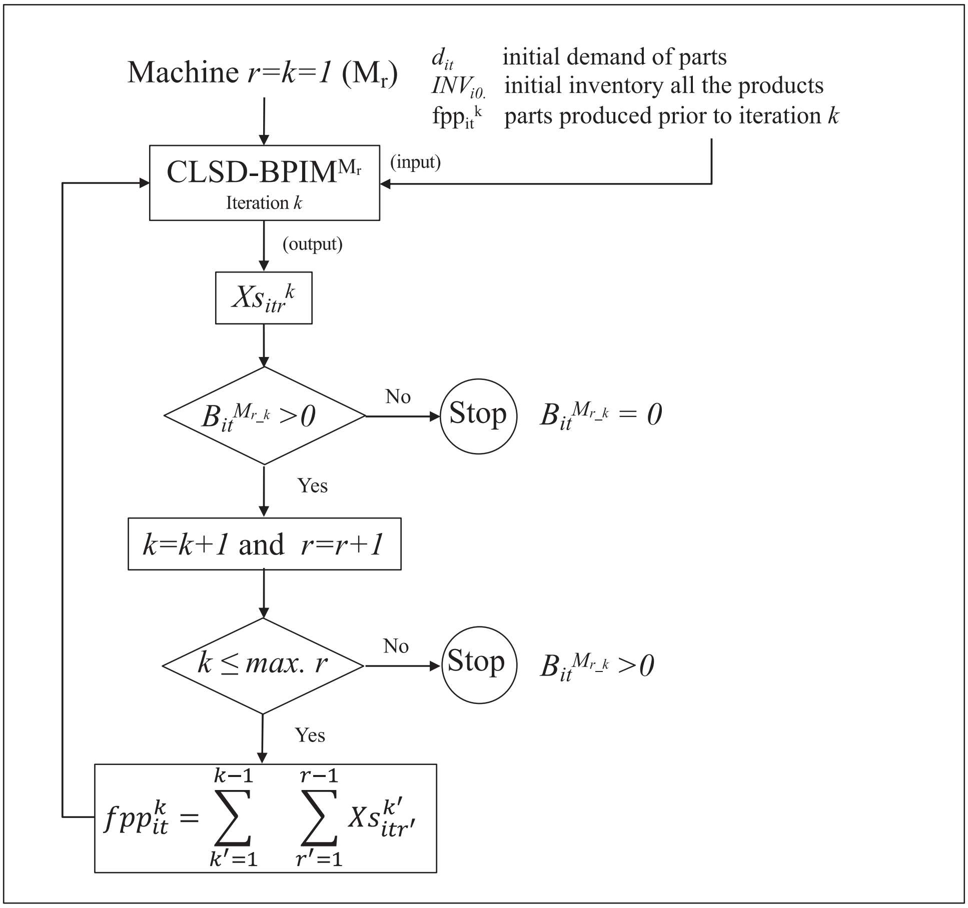

Fig. 2. Decomposition approach

Table 11
Average results per data size

Monolithic approach	Average results Small-sized datasets (S1, S2, S3, S4)	Medium-sized datasets (M1, M2, M3, M4)	Large-sized datasets (L1, L2, L3, L4)
Sequence-dependent setup cost (€)	4.08	29.16	210.67
Inventory cost (€)	722.88	19,385.03	58,788.12
Overtime cost (€)	0	0	1,323.56
Not fulfilling costs in inventory coverage (€)	0	0	0
Resource setup cost (€)	280	2,460	6,440
Backorder cost (€)	0	0	0
Solution time (hours)	0	2.49	5.94
Solution gap (%)	0	0.34%	4.04%

and update data plans to avoid these overtime and backorder costs. In this regard, it is important to highlight the reduction of the time spent by planners when following the monolithic approach instead of manually carrying out or following a simulation procedure to schedule all the parallel machines by taking into account the corresponding data updates after scheduling each machine.

Apart from the three representative datasets tested, additional random instances based on the real problem with different sizes –four small (S1, S2, S3, S4), four medium (M1, M2, M3, M4) and four large (L1, L2, L3, L4)– are generated to test the monolithic approach's computational performance and to present the average results for each size group of the 12 new experimented instances. Thus, a Python code to generate these random instances based on the real problem was developed: (see Supplementary material). Table 11 presents the average results for the 12 generated random instances based on the real problem.

Effectively, we have demonstrated that our proposal of a monolithic approach is performing better than the current procedure used in the company under study, which is simulated through the decomposition approach. Nowadays, the monolithic approach solution is computally limited by the size of the problem, mainly the number of time periods (t), number of machines (r) and number of products (i), although capacity utilisation along planning times is also crucial. Otherwise, if there are problems with a very big number of the indicated set of indices (i, j, r and t) or with overdemands, the proposed decomposition approach can be alternatively used.

6. Conclusion

We herein address for the first time a capacitated lot-sizing problem with sequence-dependent setups (CLSD) and parallel machines and bi-part injection moulding (BPIM). We considered a case study based on an automotive second-tier supplier that injects plastic automotive parts. Some pairs of these parts are similar and are injected into the same mould at the same instant for right- and left-hand sides of cars. Although the same amount of both parts is produced per time period, independent inventories are managed for each part given the appearance of part discards by quality, transportation, manufacturing and/or holding problems, which complicate sequence management, but avoid manual order post-processing. We herein propose a multi-machine CLSD-BPIM model based on MILP formulation to improve the automotive second-tier supplier's production scheduling.

The multi-machine CLSD-BPIM model was adopted based on its formulation in the previous works by: Gupta and Magnusson [4], who propose a single-machine CLSD; Almada-Lobo et al. [14], who improve Gupta and Magnusson's model [4]. Thus, the multi-machine CLSD-BPIM model minimises sequence-dependent setup costs, inventory costs, overtime costs, not fulfilling inventory coverage costs, resource setup costs and backorder costs. The main characteristics of the multi-machine CLSD-BPIM formulation include: bill of materials, safety inventory coverages, holding space limitations, backorders, discrete multiple lot sizes, sequence constraints, machine setup times and costs, bi-part injection mouldings constraints. An exact MILP-based solution approach was applied.

In order to test the model formulation and solution time, 15 different sized datasets (small, medium, large) based on a real-world case study were generated. After solving the 15 instances, reasonable computational times were obtained. Furthermore, the multi-machine CLSD-BPIM model was validated by comparing it to an alternative decomposition approach of the multi-machine problem into single-machine subproblems with similar production scheduling from the perspective of the studied plastic plant. The results demonstrate better performance in sequence setup, inventory, resource setup and backorder costs terms of the monolithic approach or multi-machine CLSD-BPIM model than the decomposition approach using a single-machine CLSD-BPIM model.

The managerial implications of this proposal may imply integrating the proposed model with the company's information system by previously testing and fixing the relation between the problem size and computational efficiency of the available solver and hardware equipment. The model can be used by CLSD planners of plastic bi-injection moulding companies, such as the automobile or shoe industry, among others. It would imply the optimisation and automation of CLSD planning. The main benefit of this model compared to others found in the literature is that it contemplates a simultaneous lot-sizing and scheduling problem in parallel machines with BPIM. The results recommend managers following a monolithic approach to schedule all parallel machines. Hence sequence setup, inventory, resource setup and backorder cost savings are expected, subject to problem size and computational efficiency limitations. Here a decomposition approach of the problem to address several multi-machine subproblems can be a reasonable procedure to face larger multi-machine CLSD-BPIM problems. In order to put it into practice, other modelling and solution alternatives can be addressed to use digital manufacturing platforms like C2NET, which has embedded make, source and delivery models and algorithms [45] and provides optimisation-as-a-service and ZDMP, which offer an open Industry 4.0 environment with zero-defect service applications and optimisation algorithms to reduce equipment changeovers and eliminate related errors [46].

We identified the following further research works while undertaking this work: (i) develop and test conditional constraints for modelling deliberate co-production [47] in the multi-machine CLSD-BPIM problem context; (ii) study and model the necessity of extending the bi-part moulding problem to a multi-part injection moulding problem; (iii) analyse the solution approaches for larger-sized datasets based on more case studies, where alternative solution methods should be studied, such as metaheuristic and/or matheuristic approaches even in the supercomputation and quantic contexts; (iv) contemplate the existence of common setup operators [39] in the multi-machine CLSD-BPIM model; (v) use and test alternative sequence constraints [48] in the multi-machine CLSD-BPIM; (vi) extend the CLSD-BPIM model to a multi-objective adaptive approach [45]; (vii) incorporate uncertain conditions of demand, supply or process into the model.

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Supplementary materials

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.apm.2021.07.028.

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