ANALYSIS OF THE INFLUENCE OF THE BALLAST TRACK IN THE DYNAMIC BEHAVIOUR OF SINGLE-TRACK RAILWAY BRIDGES OF DIFFERENT TYPOLOGIES

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Abstract: *Short-to-medium span simply-supported (SS) railway bridges are prone to experience high levels of vertical acceleration due to train passage. The necessity of predicting accurately their dynamic behaviour for design, safety and maintenance reasons, requires a deep understanding of the train induced vibrations in these structures. A key factor of this phenomenon is the influence exerted by the ballast track on their dynamic response. This paper provides a detailed sensitivity analysis over a single-track bridge catalogue covering lengths of interest from 10 to 25 m considering two different typologies, (i) girder-deck bridges and (ii) slab-deck bridges. The effect of the vertical flexibility of elastic bearings is also analysed. A 2D Finite-Element (FE) track-bridge interaction model is implemented with the aim to evaluate the influence of the track parameters on the modal properties of the bridges and the dynamic response under train passages. The results obtained reveal the influence of the ballast shear stiffness and damping in the dynamic behaviour of the structures, especially in the case of the shortest girder bridges.*

1 INTRODUCTION

In a context of an increasing demand of personal and freight mobility around the world, railway systems have experienced a sustained development that projects them as a reliable and sustainable way of transportation for the time to come. For this reason, dynamic effects on railway bridges are considered of major interest and concern for scientists and engineers, especially since the appearance of High Speed (HS) [1]. In this regard, short-to-medium span (10 – 25 m) SS railway bridges are particularly prone to experience an excessive level of vertical acceleration at the deck during train passage, due to its usually associated low mass and structural damping, especially at resonance [2]. This could cause discomfort for the passengers, flaws in the ballast layer, a rise in the maintenance service cost of the track and an increased risk of derailment in the worst-case scenario. Train induced vibrations in railway bridges is a rather complex interaction problem, which is affected by several factors. Apart from the mechanical and geometrical properties of the bridge and the characteristics of the train, interaction mechanisms regarding the vehicle, the track and the soil may also affect the response of the structure, which are currently under investigation [3]. In addition, the computational cost of including these mechanisms is considerable, thus, simplified models that usually disregard them are commonly used in engineering consultancies. This work is dedicated to the investigation of the effect exerted by the ballast track on the vertical dynamic response of SS

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railway bridges. To this aim, the influence of the main track parameters on the bridge modal properties and on the dynamic response due to train passage is evaluated. With this purpose, a 2D FE track-bridge interaction model is implemented, where the track is represented using a three-layer discrete model, based on the work by Zhai et al. [4]. The model is employed to perform a sensitivity analysis over a bridge catalogue covering bridges of two different deck typologies and for a selected range of lengths of interest from 10 to 25 m. In sections 2 and 3, the bridge catalogue is presented, and the numerical model is described. In section 4, the results of dynamic analyses under train passage are included. Finally, in section 5, the main conclusions are summarized.

2 BRIDGE CATALOGUE

The catalogue contemplates single-track railway bridges of span lengths that range from 10 to 25 m in 5 m intervals. For each length, two common deck typologies are considered: (i) pre-stressed concrete girder decks; and (ii) voided or solid concrete slabs, or pre-stressed filler beams encased in a concrete pseudo-slab. As for the vertical support of the decks, infinitely rigid supports and elastic supports accounting for the vertical flexibility of neoprene bearings are differentiated.

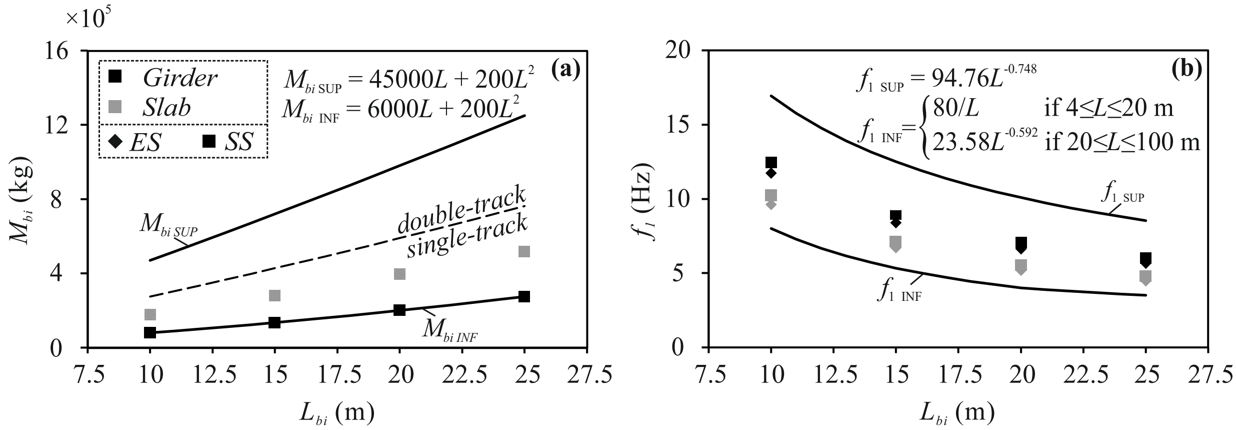

Figure 1: (a) Mass per span and (b) fundamental frequency of the bridges under study.

The main characteristics regarding the mass and the fundamental frequency of the bridges are calculated according to the work presented by Doménech et al. [5], where an ensemble of existing bridges of the considered typologies was studied. Fig. 1 shows for the 16 bridges of the catalogue the total mass per bridge span and the fundamental frequency. For the girder decks, the mass of the reported single-track existing bridges approaches the inferior limit. Additionally, this corresponds to the worst-case scenario for the vertical acceleration criterion. The fundamental frequency is selected as 50% of the difference between the Eurocode 1 (EC1) simplified method limits for each length [6]. For the slab decks, the mass value is selected as 25% of the difference between the upper and the lower limits for each length. This corresponds to an average value for the mass of existing single-track slab bridges. For the fundamental frequency, the same criterion is applied, and the frequency is calculated as 25% of the difference between the limits for each length. In addition, an elastically-supported (ES) version for each bridge is also defined admitting that the ratio κ between the bridge bending stiffness and the vertical stiffness of the bearings is approximately equal to 0.05, which leads to a reduction of the fundamental frequency of 3-4% with respect to the SS case [7], as indicated in Eq. 1. In this equation, $E_{bi}I_{ybi}$ stands for cross-section flexural stiffness of each section, $\bar{K}_{bi,dyn}$ for the vertical dynamic stiffness of the elastic bearings and L_{bi} for the span length.

$$\kappa = \frac{E_{bi}\, I_{ybi}\, \pi^3}{\bar{K}^n_{bi,dyn}\, L^3_{bi}} \approx 0.05 \tag{1}$$

The mechanical properties of the bridges of the catalogue are shown in Tables 1 and 2, where the data is expressed per bridge span. From left to right, the columns show the information relative to the span length, L_{bi}, fundamental frequency, f_1, total mass, M_{bi}, cross-section flexural stiffness of the span section, $E_{bi}I_{ybi}$, and the vertical dynamic stiffness of the elastic bearings $\bar{K}_{bi,dyn}$, respectively. The last column stands for the identification code for each bridge, which contains the typology, the type of support and the span length (e.g. GD-ES-10 stands for girder-deck bridge, elastically-supported with 10 m of span length).

$L_{bi}[m]$	$f_1[Hz]$	$M_{bi}[t]$	$E_{bi}I_{ybi}[MN/m^2]$	$\bar{K}_{bi,dyn}[MN/m]$	ID
10	12.46	80.0	$3.56{\cdot}10^3$	∞	GD-SS-10
	11.72	80.0	$3.18{\cdot}10^3$	$3.12{\cdot}10^3$	GD-ES-10
15	8.92	135.0	$1.06{\cdot}10^4$	∞	GD-SS-15
	8.39	135.0	$9.63{\cdot}10^3$	$2.70{\cdot}10^3$	GD-ES-15
20	7.04	200.0	$2.41{\cdot}10^4$	∞	GD-SS-20
	6.62	200.0	$2.20{\cdot}10^4$	$2.49{\cdot}10^3$	GD-ES-20
25	6.02	275.0	$4.93{\cdot}10^4$	∞	GD-SS-25
	5.66	275.0	$4.51{\cdot}10^4$	$2.50{\cdot}10^3$	GD-ES-25

Table 1: Mechanical properties of the girder bridges.

$L_{bi}[m]$	$f_1[Hz]$	$M_{bi}[t]$	$E_{bi}I_{ybi}[MN/m^2]$	$\bar{K}_{bi,dyn}[MN/m]$	ID
10	10.22	177.5	$6.63{\cdot}10^3$	∞	SD-SS-10
	9.62	177.5	$6.06{\cdot}10^3$	$4.67{\cdot}10^3$	SD-ES-10
15	7.12	281.3	$1.66{\cdot}10^4$	∞	SD-SS-15
	6.70	281.3	$1.53{\cdot}10^4$	$3.59{\cdot}10^3$	SD-ES-15
20	5.52	395.0	$3.32{\cdot}10^4$	∞	SD-SS-20
	5.19	395.0	$3.05{\cdot}10^4$	$3.03{\cdot}10^3$	SD-ES-20
25	4.76	518.8	$6.41{\cdot}10^4$	∞	SD-SS-25
	4.48	518.8	$5.95{\cdot}10^4$	$2.96{\cdot}10^3$	SD-ES-25

Table 2: Mechanical properties of the slab bridges.

3 TRACK-BRIDGE INTERACTION MODEL

For the subsequent analysis, the discrete FE 2D track-bridge interaction model shown in Fig. 2 is implemented. A three-layer discrete model for the track is configured, based on that proposed by Zhai et al. [4], which couples a series of elastically or simply-supported bridge spans. The track admits Ahlbeck hypothesis, so it can be assumed that the load transmitted from each sleeper to the ballast has a cone distribution. In the proposed model, the rail is represented with a Bernoulli-Euler (B-E) beam, where E_r, I_{yr}, and m_r stand for the rail Young Modulus, cross-section moment of inertia with respect to the Y axis and linear mass, respectively. Below, the vertical damping and stiffness of the rail pads (C_p, K_p), of the mobilized ballast (C_b, K_b) and of the subgrade (C_f, K_f) are included at the sleepers locations. The continuity and coupling effect of the interlocking ballast granules is also considered in the model by means of spring-damper elements (C_w, K_w) that link relative vertical displacements between adjacent ballast masses. Then, M_{sl} and M_b stand for the mass of each sleeper and the vibrating ballast mass under each support, respectively. Damping and stiffness on the bridge deck (C_f^b, K_f^b) are set to 0 and $100 \cdot K_f$, respectively, assuming that the ballast rests directly on the bridge deck. The longitudinal interaction between the rails and the deck through the ballast layer is disregarded in a first approach given the high flexural stiffness of the bridges. As shown in Fig. 2, rail and track parameters are multiplied by a factor of two, as only one rail is explicitly included in the model. The bridge is represented by means of N_{sp} simply or elastically-supported B-E beams representing each span of the bridge. In the present paper, N_{sp} is set to a value of 2, as two identical spans are considered for each bridge. The vertical stiffness of the neoprene bearings is introduced by the constant equivalent vertical stiffness $\bar{K}_{bi,dyn}$ at each end section of the i-th bridge span. The parameters L_{bi}, E_{bi}, I_{bi} and m_{bi} stand for the length, Young Modulus, cross-section moment of inertia with respect to the Y axis and linear mass of the i-th bridge span, respectively. Due to the presence of the continuous ballast track, a weak interaction takes place between successive spans. In the simulations, a track length of $L_{r,prev} = 20$ m is included before and after the bridge, which is considered sufficient according to previous publications [8], corresponding to 33.3 times the sleeper distances. The rail is discretized into two beam elements between consecutive sleepers, and so are the bridge beams.

The train excitation is represented by means of a constant moving load model, which implies that vehicle-structure interaction effects are neglected. In this sense, it is intended to isolate the effect of the track components affecting the dynamic behaviour of the bridges to investigate their influence separately. For the track parameters, an important dispersion has been found among different publications. Based on a review presented by the authors in [9], the values selected are shown in Table 3, expressed per rail seat. M_b, K_b and K_f are calculated with the equations given in [4]. Data from the European [10] and Spanish Standards [12], and from [11] are adopted for the rail, rail pads and sleepers properties. In the case of the ballast shear stiffness and damping, the authors have found that most of the times these parameters are not considered in track models. In the few cases where included, the majority of them adopted those proposed in [4]. For this reason, in this work, these same values are employed, and its influence is investigated. The model is implemented in ANSYS. For the computation of the bridges response under passing trains (see section 4), mass, stiffness and damping matrices are exported to MATLAB, and the equations of motion of the full model are integrated in the time domain applying the Newmark-beta constant acceleration algorithm. The time step for the numerical integration is set as the minimum between 1/50 times the smaller period of interest and 1/20 times the load travelling time between two consecutive sleepers.

4 SENSITIVITY ANALYSIS: MODAL PROPERTIES AND VERTICAL ACCELERATION

This section presents the results for the sensitivity analysis regarding the influence of the track properties on the dynamic behaviour of the bridges. The authors have found that the only parameters that affect significantly the modal properties of the bridges at low frequencies are the ballast shear stiffness and damping (K_w, C_w). In this sense, Zhai et al. [4] pointed out their great influence on the dynamic behaviour of the track too. Thus, in what follows, individual variations of these track parameters are considered to evaluate how this impacts the modal properties and the vertical acceleration on the bridge deck under train passages. It is also intended to determine what bridges are the most affected by these variations.

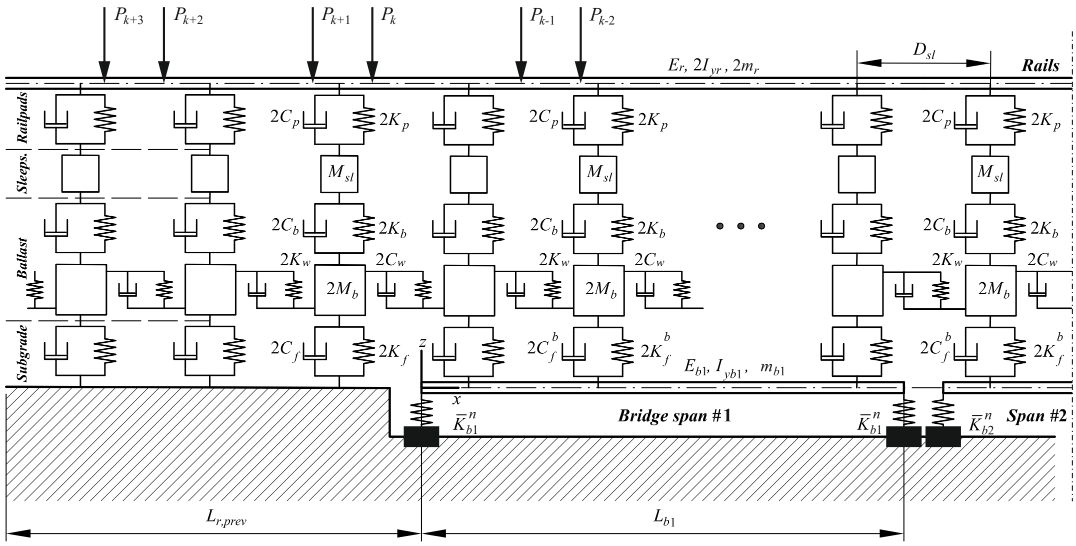

Figure 2: Track-bridge interaction model.

4.1 Influence of K_w on the bridge modal parameters

In this section the influence of the ballast shear stiffness on the bridge frequencies is evaluated. To this aim, the first, third and fifth longitudinal bending modal frequencies are calculated under individual variations of K_w. Fig. 3 shows the results for all the bridges in the catalogue, grouped per bridge length. Each plot shows the variation in the natural frequency f_i for $i =$ 1, 3, 5 when factors [0.0, 0.5, 1.0, 1.5, 2.0] multiply the nominal value of K_w (Table 3) with respect to the nominal case. From the results obtained, the following is observed:

- Natural frequencies increase with K_w. Bridges with shorter spans in a certain typology are more affected with the variation of this parameter.
- The fundamental frequency f_1 corresponding to the first longitudinal bending mode is significantly more affected than higher frequencies. The effect of K_w reduces with the frequency number.
- Regarding the typology, girder bridges, with lower longitudinal bending stiffness, are affected to a higher extent than slab bridges.
- As per the bridge supports, bridges on elastic supports are slightly more affected by K_w variations than rigidly supported bridges. Nevertheless, the difference is not significant, especially for modes higher than the fundamental one.

These results are consistent in all the considered bridges. From the sensitivity analysis it is concluded that regarding the modal parameters, short-span elastically-supported girder bridges are the most sensitive ones to the value of K_w. On this matter, the maximum variations for the frequency obtained for the first, third and fifth modes are 20%, 6% and 3%, respectively, for the shortest bridge considered (GD-ES-10), and 10%, 3% and 1.5% for the longest one (GD-ES-25).

4.2 Influence of K_w and C_w on the deck vertical acceleration due to train passage

The influence of K_w and C_w on the vertical acceleration at the bridge deck under train passages is investigated in this section. To this aim, several dynamic analyses are carried out on

Notation	Parameter	Value	Unit	Reference
E_r	Rail UIC 60 elastic modulus	$2.100 \cdot 10^{11}$	Pa	[10]
I_{yr}	Rail UIC 60 moment of inertia	$3038.3 \cdot 10^{-8}$	m^4	[10]
m_r	Rail UIC 60 mass per unit of length	60.21	kg/m	[10]
K_p	Rail pad vertical stiffness	$1.000 \cdot 10^8$	N/m	[11]
C_p	Rail pad damping	$7.500 \cdot 10^4$	Ns/m	[4]
M_{sl}	Sleeper mass	300	kg	[12]
D_{sl}	Sleeper distance	0.600	m	[12]
l_e	Half sleeper effective supporting length	0.950	m	[4]
l_b	Sleeper width	0.300	m	[12]
α	Ballast stress distribution angle	35	$\circ$	[4]
h_b	Ballast thickness	0.300	m	[12]
ρ_b	Ballast density	1800	kg/m^3	[4]
M_b	Ballast vibrating mass	317.910	kg	[4]
E_b	Ballast elastic modulus	$1.100 \cdot 10^8$	Pa	[4]
K_b	Ballast vertical stiffness	$1.933 \cdot 10^8$	N/m	[4]
C_b	Ballast damping	$5.880 \cdot 10^4$	Ns/m	[4]
E_f	Subgrade K_{30} modulus	$9.000 \cdot 10^7$	Pa/m	[4]
K_f	Subgrade vertical stiffness	$7.399 \cdot 10^7$	N/m	[4]
C_f	Subgrade damping	$3.115 \cdot 10^4$	Ns/m	[4]
K_w	Ballast shear stiffness	$7.840 \cdot 10^7$	N/m	[4]
C_w	Ballast shear damping	$8.000 \cdot 10^4$	Ns/m	[4]

Table 3: Bridge-track interaction model parameters, per rail seat.

the GD-ES-10 bridge under the circulation of HSLM-A1 Universal Train presented in the EC1. Only this bridge is selected for the sake of conciseness and for being the most influenced one by the ballast shear stiffness and damping properties. The acceleration response is calculated for the HSLM-A1 train in the range of velocities [40, 117] m/s (e.g. [144, 420] km/h) every 1 m/s at a quarter, mid-span and three quarters of both spans. A 3rd order Chebyshev filter is applied to the response in order to filter contributions below 1 Hz and above 60 Hz. Then, maximum response envelopes are obtained for each speed. The following individual variations of the track parameters are imposed: $[0.0, 0.5, 1.0, 1.5, 2.0] \cdot K_w$ and $[0.5, 1.0, 1.5, 2.0] \cdot C_w$. Also, Rayleigh damping is assumed according to EC1 for pre-stressed concrete bridges as 1.7% for the GD-ES-10 bridge. This ratio is applied on the first and fifth natural frequencies.

In Fig. 4 (a-b), an envelope of the maximum acceleration response at the bridge deck is represented at the most critical section which corresponds to the center of the second span. The maximum acceleration level is not relevant as an unrealistically high design velocity is considered in order to capture low order and clear resonances of the bridge. Also, and in order to visualize how the variation of K_w and C_w affects the bridge response in different situations, the acceleration time-history at the same section is represented for three different velocities. In this way, the analysis is started with the second resonance speed of the first mode (e.g. $j = 2$, $n = 1$ in Eq. 2, according to [13]), which is equals to 380 km/h (see Fig. 4 (c-d)).

$$V^r_{nj} = \frac{d_k}{j\,T_n} = \frac{d_k\ \omega_n}{2\pi j} \tag{2}$$

In the previous equation, d_k stands for the characteristic distance of the HSLM-A1 train (18 m), T_n is the n-th natural period of the bridge and j the resonant order. Following that, the

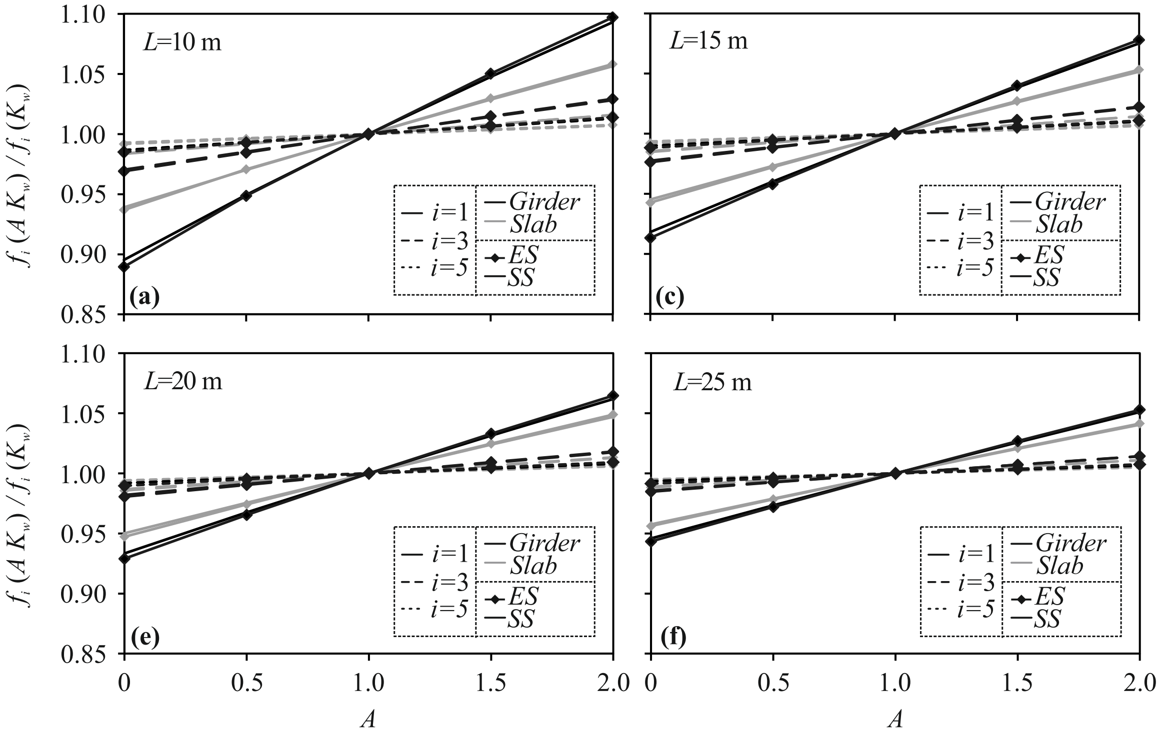

Figure 3: Influence of the variation of K_w in f_1, f_3 and f_5 with respect to the frequency in the nominal case.

response is computed at 324 km/h, far both from resonance and from cancellation of resonance (see Fig. 4 (e-f)). Finally, it is determined for a speed near a cancellation of resonance condition, given by Eq. 3, in agreement with [13]:

$$\left(\frac{L_{bi}}{d_k}\right)^c_{nji} = \left(\frac{\lambda_n}{n\pi}\right)^2 \frac{n}{2jK^c_{ni}}, \qquad n, j, i \geq 1 \tag{3}$$

In this way, when the relation L_{bi}/d_k between the length of each span and the characteristic distance of the train approaches the i-th cancellation ratio given by Eq. 3, the cancellation of the resonance is produced, and the vibration level gets significantly attenuated. For the case of the GD-ES-10 bridge associated to the circulation of the HSLM-A1 train, the third resonance speed of the first mode, equal to 253 km/h, approaches the first $(L_{bi}/d_k)^c_{nji}$ theoretical condition of cancellation for this resonance (e.g. $j = 3$, $n = 1$, $i = 1$, respectively), although it is not coincident (the difference is approximately 15%). Nevertheless, the phenomenon is visible, leading to a quite reduced resonant peak. These results are shown in Fig. 4 (g-h). In summary, the subsequent observations can be made:

- An increase in K_w leads to a rise in the resonant velocities, in the same proportion that the resonant frequency is modified by this parameter (in this particular case, neglecting or doubling K_w entails variations of -17.4% to +9.3% of the resonant velocity for the nominal case). This affects similarly different order resonances.

- For the range of K_w values considered, resonance at a certain speed may or may not take place depending on K_w (see Fig. 4(c-e)).

- Regarding the effect of the ballast shear damping, it is only relevant at resonance, leading to a pronounced reduction of the acceleration response. In this particular case, if C_w is doubled with respect to its nominal value, the vertical acceleration reduces by a 26%. The effect of this parameter on the second resonant peak ($V = 380$ km/h) is much higher than the effect on the third one ($V = 253$ km/h). Nevertheless, this last peak is close to cancellation and no conclusions can be extracted in this regard.

- Finally, for the resonance speed approaching the cancellation conditions, a very significant attenuation of the acceleration level is observed with a small influence of the track parameters.

5 CONCLUSIONS

The longitudinal coupling effect exerted by the continuity of the ballasted track in single-track railway bridges composed by several isostatic consecutive spans is evaluated in this work. Specifically, the influence of the ballast shear stiffness and damping in the modal parameters and vertical acceleration under train passages is investigated. In the first place, a bridge catalogue considering short-to-medium span lengths and two common bridge deck typologies has been prepared. Then, a sensitivity analysis has been performed by means of a 2D FE track-bridge interaction model. Individual variations of the track parameters have been imposed in order to study their influence on the dynamic behaviour of the bridges. The main conclusions for this work are summarized as follows:

- In the discrete track model presented, the ballast shear stiffness and damping are the parameters that affect the most the bridge response in the frequency range of interest. The influence of the remaining parameters is negligible compared to these two.

- Regarding the modal parameters of the bridges, K_w exerts a notable influence on them, which is stronger in shorter bridges. When it comes to the typology, girder-deck bridges are the most affected due to their initially lower bending stiffness. The correlation with the flexibility of elastic supports is minor.

- With respect to the vertical acceleration level caused by the passage of a train, it is found that the effect of K_w and C_w is significant, especially at resonance. In particular, an increment of K_w leads to an important rise in resonant velocity, while an increment of C_w results into a reduction of the resonant acceleration amplitude. The effect of C_w far from resonance is negligible. These results are consistent, since, higher K_w values lead to an increase on the natural frequencies, especially of the fundamental one and in the case of short flexible structures.

- Future investigations are required in order to understand completely the influence of these shear parameters. It is also needed to find clear ways to determine their value, since their influence on the dynamic behaviour of railway bridges is significant and the information about it found in the literature is scarce. Experimentally appraised values for these parameters could be quite useful in the case of using discrete track models, which is a reasonable solution permitting solving the dynamic equations of motion in the time domain performing a full analysis in a reasonable amount of time.

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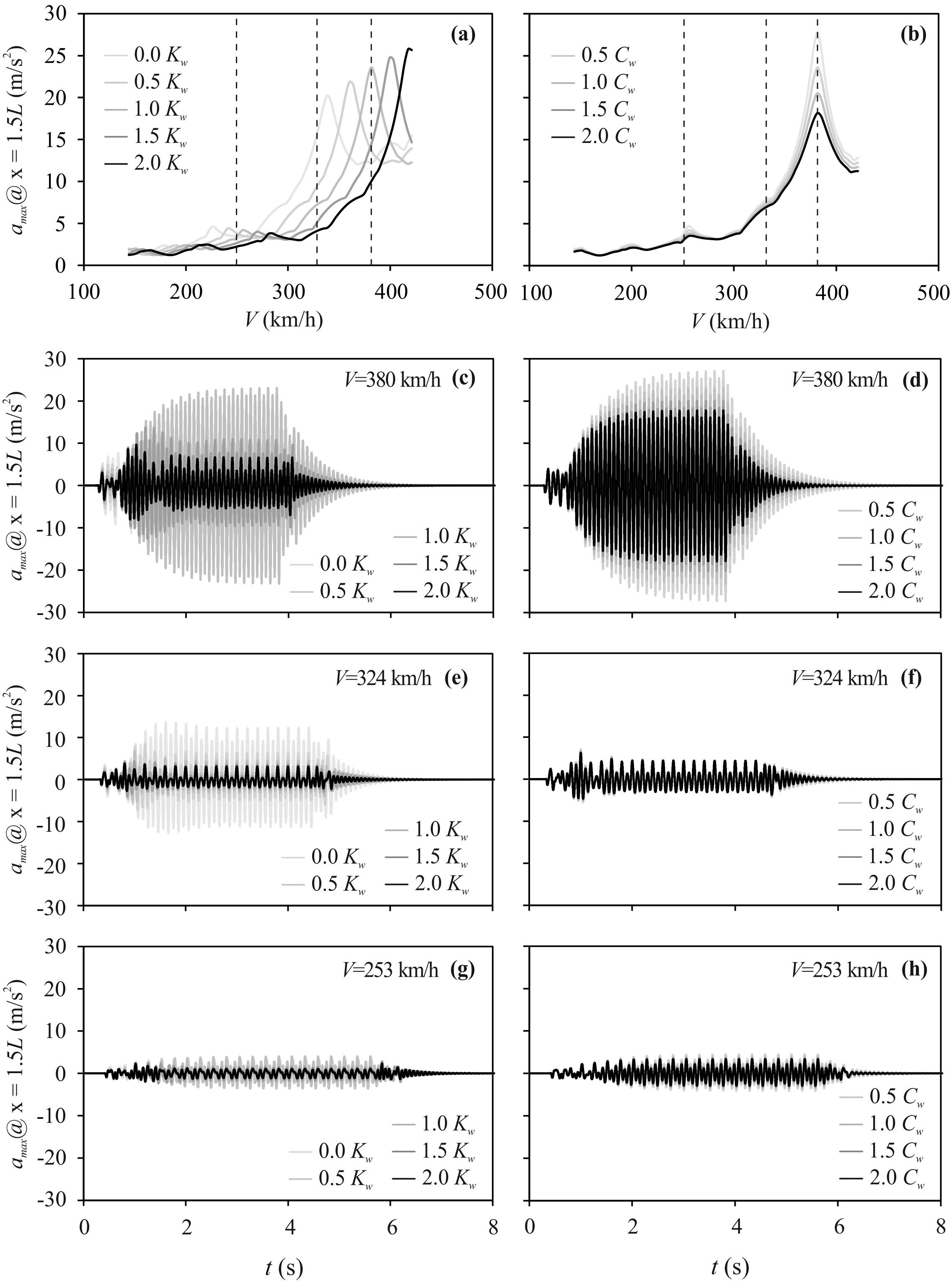

Figure 4: GD-ES-10 bridge. Maximum acceleration response for each velocity (a-b), and acceleration time-history at different speeds (c-h).

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