# Optimising 2-parameter Lambert Conformal Conic projections for ground-to-grid distortions

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**Abstract.** A Lambert Conformal Conic (LCC) projection with two true-scale parallels of latitudes $\varphi_l$ and $\varphi_u$ can be recast in a LCC projection with one standard parallel of latitude $\varphi_0$ and scale $k_0$, having the practical advantage that the same type of definition can be used for the two conformal projections universally used: LCC and Transverse Mercator (TM). While equations giving $\varphi_0$ and $k_0$ in terms of $\varphi_l$ and $\varphi_u$ can be found in the literature, inverse relationships are not readily found. They are derived in the present paper. These may be necessary in views of the planned future definition of the United States State Plane Coordinate System (SPCS) 2022 for the users of particular mapping software requiring to specify the two latitude values instead of the central latitude and central scale. While map projection parameters are customary selected to minimise ellipsoid-to-grid distortions for a region, in some cases it could be more convenient to study and minimise ground-to-grid distortions. Also bearing in mind the design of SPCS 2022, we discuss the advantages and disadvantages of working with each type of distortion definition.

**Keywords**: Map projections; distortion; Lambert Conformal Conic (LCC); State Plane Coordinate System (SPCS).

## Introduction

A Lambert Conformal Conic (LCC) projection is usually defined to preserve scale in two parallels usually referred to as the lower true-scale parallel $\varphi_l$ and the upper true-scale parallel $\varphi_u$.

A LCC projection defined by its pair of true-scale parallels $\varphi_l$ and $\varphi_u$ can be alternatively defined by the latitude of its central parallel $\varphi_0$ and its corresponding scale factor $k_0$ (Fig. 1).

*Figure 1 here*

This fact is true for the LCC projection due to its conformal character but not for other widely used projections such as Albers Conic Projection (which is equal-area but not conformal), and it is acknowledged in the literature (e.g. Snyder 1987, p. 105) although many surveyors and cartographers may be unaware of it, as exemplified in the illuminating webinar by Dennis (2018a). By contrast a TM can be defined by means of $\varphi_0$ and $k_0$ but cannot be alternatively defined by two true-scale meridians (of longitudes $\lambda_1$ and $\lambda_2$) since the true-scale lines do not exactly follow any meridian.

As Snyder (1987) mentions, the choice of standard parallels $\varphi_l$ and $\varphi_u$ in a LCC projection has the effect of reducing the scale $k_0$ of the central parallel $\varphi_0$ by an amount which cannot be expressed simply in exact form. The formulas to obtain both the latitude of the central parallel and its scale from the true-scale parallel latitudes, i.e. $\varphi_0$ and $k_0$ from $\varphi_l$ and $\varphi_u$ can be found in the literature (e.g. Dennis 2019, p. 31), but this is not the case of the reverse formulas, i.e. $\varphi_l$ and $\varphi_u$ from $\varphi_0$ and $k_0$, which some surveyors and cartographers may need for their mapping software once the SPCS 2022 is defined by means of $\varphi_0$ and $k_0$ both for TM and LCC projections (Dennis 2018b). In the following section both types of formulas are given for the sake of completion.

## 2LCC to 1LCC plus scale

### *Direct formulation*

The expressions to obtain $\varphi_o$ and $k_o$ in terms of $\varphi_l$ and $\varphi_u$ can be derived using some formulas in Snyder (1987). Given a latitude $\varphi$ on an ellipsoid whose first eccentricity is $e$, the corresponding quantities $m$ and $t$ can be obtained as

$$m = \frac{\cos\varphi}{\sqrt{1 - e^2\sin^2\varphi}} \tag{1}$$

$$t = \frac{\tan\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)}{q} \tag{2}$$

being

$$q = \left(\frac{1 - e\sin\varphi}{1 + e\sin\varphi}\right)^{e/2} \tag{3}$$

We will use subscripts $0$, $l$ and $u$ to denote the corresponding values for these equations for $\varphi_o$ (still unknown), $\varphi_l$ and $\varphi_u$ latitudes, that is: $m_0$, $t_0$ and $q_0$, $m_l$, $t_l$ and $q_l$, and $m_u$, $t_u$ and $q_u$, respectively.

Now we give an easy demonstration to quickly find simple expressions for $\varphi_o$ and $k_o$ in terms of $\varphi_l$ and $\varphi_u$.

The scale factor for a parallel of latitude $\varphi$ can be given (Snyder 1987, p. 108) as

$$k = \frac{m_l t^n}{m {t_l}^n}$$

(4)

with

$$n = \frac{\ln m_l - \ln m_u}{\ln t_l - \ln t_u}$$

(5)

This is true, in particular, for the parallel of latitude $\varphi_o$ (still unknown)

$$k_0 = \frac{m_l {t_0}^n}{m_0 {t_l}^n}$$

(6)

Now, an LCC projection with two standard parallels $\varphi_l$ and $\varphi_u$ is exactly the same as an LCC projection with one standard parallel $\varphi_o$ plus a scale factor $k_0$. As Snyder (1987, p.108) mentions, in the case of only one standard parallel (of latitude $\varphi_o$) Eq. (5) becomes indeterminate and it has to be replaced by

$$n = \sin\varphi_0$$

(7)

As said, both LCC projections, one defined by two standard parallels and the other with one standard parallel plus a scale factor, give the same results exactly. This is true not only for the final coordinates but also for the intermediate $n$ value having a direct influence on both $x$ and $y$ coordinates (we refer the reader again to Snyder 1987, p.107-108, for inspection of the entire formulation). Therefore, we can obtain the central latitude as

$$\varphi_0 = \text{asin}\,(n) \tag{8}$$

using the $n$ obtained in Eq. (5).

Eq. (8), first, and then Eq. (6) solve the desired problem of obtaining $\varphi_o$ and $k_o$ in terms of $\varphi_l$ and $\varphi_u$. These equations use the intermediate variable definitions given in Eqs. (1), (2), (3) and (5).

Some users may prefer, however, compact expressions that do not require the computation of intermediate variables. Such explicit expressions in terms of only $\varphi_l$, $\varphi_u$ and the ellipsoid's eccentricity $e$ can be obtained for $\varphi_o$ and $k_o$ by introducing the definitions of all intermediate variables in Eqs. (8) and (6), resulting in

$$\varphi_0 = asin\left(\frac{\ln\dfrac{cos\varphi_l}{\sqrt{1-e^2sin^2\varphi_l}} - \ln\dfrac{cos\varphi_u}{\sqrt{1-e^2sin^2\varphi_u}}}{\ln\dfrac{\tan\left(\dfrac{\pi}{4}-\dfrac{\varphi_l}{2}\right)}{\left(\dfrac{1-e\sin\varphi_l}{1+e\sin\varphi_l}\right)^{e/2}} - \ln\dfrac{\tan\left(\dfrac{\pi}{4}-\dfrac{\varphi_u}{2}\right)}{\left(\dfrac{1-e\sin\varphi_u}{1+e\sin\varphi_u}\right)^{e/2}}}\right) \tag{9}$$

$$k_0 = \frac{\frac{cos\varphi_l}{\sqrt{1-e^2sin^2\varphi_l}}}{\frac{cos\varphi_0}{\sqrt{1-e^2sin^2\varphi_0}}}\left(\frac{\tan\left(\frac{\pi}{4}-\frac{\varphi_0}{2}\right)\left(\frac{1-e\sin\varphi_l}{1+e\sin\varphi_l}\right)^{e/2}}{\tan\left(\frac{\pi}{4}-\frac{\varphi_l}{2}\right)\left(\frac{1-e\sin\varphi_0}{1+e\sin\varphi_0}\right)^{e/2}}\right)^{\sin\varphi_0}$$

(10)

These are equivalent and similar in complexity to the ones given in Dennis (2019, p.31).

### *Inverse formulation*

The problem now is to obtain $\varphi_l$ and $\varphi_u$ in terms of $\varphi_0$ and $k_0$.

From Eq. (6) we obtain

$$t_l = \left(\frac{m_l}{m_0k_0}\right)^{1/n} t_0$$

(11)

Equating the right-hand side of this equation with the right-hand side of Eq. (2), now to be used with latitude $\varphi_l$, and operating to extract $\varphi_l$ from the numerator of the latter we can obtain

$$\varphi_l = \frac{\pi}{2} - 2\,\mathrm{atan}\left[\left(\frac{m_l}{m_0k_0}\right)^{1/n} t_0q_l\right]$$

(12)

An iterative calculation can now be proposed. Before, we want to give two critical suggestions, however:

- The starting value $\varphi_l$ to be used for the computation of $m_l$ and $q_l$ on the right-hand side of Eq. (12) has to be sufficiently close to the final solution. Since for the General Conic projection with spherical earth of radius of curvature $R$ (Fig. 2) we have

*Figure 2 here*

$$\cos\alpha = \frac{d}{R} = k_0 \tag{13}$$

that is $\alpha = \operatorname{acos} k_0$, and $\varphi_l = \varphi_0 - \alpha$, $\varphi_u = \varphi_0 + \alpha$, one can use as an initial suitable value $\varphi_l = \varphi_0 - \operatorname{acos} k_0$

- Being the convergence of the process relatively slow, one may want to improve it with the help of Aitken's method (e.g. Burden and Faires 2011).

The algorithm can be summarized in:

Step 1. Obtain an initial value $\varphi_{l1}$ by means of $\varphi_{l1} = \varphi_0 - \operatorname{acos} k_0$. In this iteration we define $\varphi'_{l1} = \varphi_{l1}$

Step 2. Use this approximate value to compute $m_{l1}$ and $q_{l1}$ and introduce them in the right-hand side of Eq. (12) to obtain an improved value $\varphi_{l2}$. In this iteration we define $\varphi'_{l2} = \varphi_{l2}$

Step 3a. Use this value to compute $m_{l2}$ and $q_{l2}$ and introduce them in the right-hand side of Eq. (12) to obtain $\varphi_{l3}$

Step 3b. Use values $\varphi'_{l1}$, $\varphi'_{l2}$ and $\varphi_{l3}$ to obtain an improved value $\varphi'_{l3}$ by means of

$$\varphi'_{l3} = \varphi_{l3} - \frac{(\varphi_{l3} - \varphi'_{l2})^2}{\varphi_{l3} - 2\varphi'_{l2} + \varphi'_{l1}} \tag{14}$$

Return to Step 3a and iterate. That is, use this value to compute $m_{l3}$ and $q_{l3}$ and introduce them in the right-hand side of Eq. (12) to obtain $\varphi_{l4}$ and then by Eq. (14) with $\varphi'_{l2}$, $\varphi'_{l3}$ and $\varphi_{l4}$ (instead of $\varphi'_{l1}$, $\varphi'_{l2}$ and $\varphi_{l3}$, respectively) obtain $\varphi'_{l4}$, and so on. The iterations may be stopped when the differences between successive latitudes are negligible.

In the following table an example of application is given.

*Table 1 here*

Similarly, the upper true-scale parallel latitude $\varphi_u$ can be obtained by using the initial approximation $\varphi_{u1} = \varphi_0 + \operatorname{acos} k_0$ in Step 1 and introducing upper latitudes instead of lower latitudes in Eqs. (12) and (14).

Using these formulas, the defining parameters for the LCC projections for the different zones of the SPCS 83 have been computed and are given in the Appendix; not only the lower and upper standard parallel latitudes $\varphi_l$ and $\varphi_u$, as usual, but also the latitude and scale of the central parallel, $\varphi_0$ and $k_0$.

## Ellipsoid-to-grid and ground-to-grid distortions

Given a point and an infinitesimally small surrounding region on the ellipsoid surface there can be defined a scale distortion factor $k$ which brings the original distance $ds$ on the ellipsoid to the resulting distance $ds'$ on the grid

$$k = \frac{ds'}{ds} \tag{15}$$

Prior to the projection onto the grid, the surveyor has to reduce every measured distance from the ground ($d_g$) to the ellipsoid surface ($d_e$). This can be done by means of the computation for every particular distance of the correction to the chord, followed by the chord-to-normal-section correction (Meyer 2010, p.123) which can be assumed coincident in length with the geodesic line, or, alternatively, with an "approximate method [that] serves most surveyors and engineers well" (Stem 1990, p. 49) which uses an elevation factor – encapsulating these corrections for a mean elevation and mean geoid height – to reduce from ground to ellipsoid (Dennis 2019, Meyer 2010, Stem 1990):

$$d_e = \left(\frac{R}{R + H + N}\right) d_g = k_e d_g \tag{16}$$

where

$$k_e = \frac{R}{R + H + N} \tag{17}$$

is the elevation factor in terms of mean elevation $H$ (orthometric height), mean geoid height $N$ and a mean radius of curvature $R$ (say the geometric mean of the principal radii of curvature).

This correction of distances from the ground to the ellipsoid, be it rigorous or approximate as in Eq. (16), has often been left out of the process of customization of a map projection except for the definition of low distortion projections and special cases in the history of usage of the SPCS, e.g. the Montana Department of Transportation use SPCS scaled to ground to minimize linear distortions in projects with tight tolerances (Dennis 2018c).

For these latter cases a combined scale and elevation factor $k'$ can be used

$$k' = k \times k_e \tag{18}$$

The next section analyzes the advantages and disadvantages of designing a map projection for minimizing the scale factor distortion Eq. (15), i.e. the ellipsoid-to-grid distortion, versus the design minimizing the combined factor Eq. (18), i.e. the ground-to-grid distortion. Then a tool is presented and made available to the reader for evaluating and optimizing map projections for user-defined areas of interest in terms of both types of distortion, and some examples are commented on.

## *Pros and cons*

First of all, one must acknowledge that the ground-to-grid distortion factor, Eq. (18), is approximate due to the fact of the elevation factor being an approximate correction itself. This is so because of the approximation in reducing to the ellipsoid by means of a mean elevation and a mean geoid height, first, and, second, because of the use of a mean radius of curvature, when this radius changes not only with location but also with the bearing of the line.

It is clear that the smaller the area of use and the smoother the relief the better the results obtained by the ground-to-grid factor. But how small and how smooth? How can the degree of approximation be known for a particular case? As Dracup (1974) already pointed out, "The simplest approach is to examine the scale factors at the extremities and the range of elevations and compare these values with those computed for the center of the system and at the mean elevation of the project".

As clear advantages in favor of the ground-to-grid distortion factor, we must first mention its ease of use. It also avoids a quite common oversight: users may (more than often!) forget the fact that field measurements have to be conveniently reduced to the ellipsoid before using the projection scale factor. Finally, the reader should never forget that no ground-to-grid transformation can be made conformal, that is, no ground-to-grid transformation exists for which linear distortion is the same in every direction from a point. Among other things this means that a rigorous ground-to-grid "factor", not the one in Eq. (18) which is only an approximation, would not be a single value for a point on the earth's surface but rather a varying quantity depending on the orientation of the line of interest. Even worse, we could not always expect smooth variations along a point's neighborhood (or in other words, expect the rigorous ground-to-grid distortion to be represented by an analytic function) inasmuch as the earth relief is not smooth nor possible to be represented by an analytic function (just think of the earth's relief and the occasional discontinuities in some of its slopes).

### *Practical computation of ground-to-grid distortions*

For the general evaluation of ground-to-grid distortions in the present paper, we propose to use the ETOPO1 1-arc-minute global relief model (Amante and Eakins 2009) to obtain elevations along with the EGM2008 earth gravitational model (Pavlis *et al*. 2012) to obtain geoid heights so that the ellipsoid height of every point given by its latitude and longitude coordinates can be computed.

We will use a new release of the *TestGrids* software for Evaluating and Optimizing Map Projections which was presented in Baselga (2019) and makes use of Fibonacci lattices for efficiently sampling distortion functions over the geographic domain of interest. The latest release of this tool, Fig. 3, permits to choose between evaluation and optimization of distortions from ellipsoid-to-grid or distortions from ground-to-grid. The application also computes automatically both types of possible defining parameters for LCC projections (standard parallels $\varphi_l$ and $\varphi_u$, as well as the equivalent central parallel and scale, $\varphi_0$ and $k_0$). The application can be downloaded from the author's personal web page at http://personales.upv.es/serbamo/TestGrids_r2/index.htm.

*Figure 3 here*

## Example of application

We analyze here the SPCS 83 Colorado Central, which uses a LCC projection with lower standard parallel $\varphi_l$ = 38º27' = 38.45º and upper standard parallel $\varphi_u$ = 39º45' = 39.75º (Dennis 2018c). It is also known as projection EPSG:3501 in the International Association of Oil & Gas Producers (OGP) database (EPSG 2019), which has become a standard for the definition of coordinate reference systems. The corresponding geographic area, denoted as EPSG:2183, is limited by parallels 38.14ºN and 40.09ºN and meridians 109.06ºW and 102.04ºW.

Using these defining values we compute by using *TestGrids* the distortion statistics for the ellipsoid-to-grid and the ground-to-grid transformations, using 10 million points for the sampling Fibonacci lattice. The results are shown in the first columns of Table 2 (for ellipsoid-to-grid) and Table 3 (for ground-to-grid). In each case we have also optimized the latitudes of the standard parallels to minimize the resulting typical distortion (see second columns of Tables 2 and 3) or the extreme distortion (third columns of Tables 2 and 3).

*Table 2 here*

*Table 3 here*

In Table 2, ellipsoid-to-grid transformation, we can see that the defining latitudes for the standard parallels are already quite well optimized both in the sense of minimum typical distortion as well as minimum extreme distortions. In fact, these latitudes $\varphi_u$ = 39.75º and $\varphi_l$ = 38.45º represent a compromise between those yielding the minimum average distortion $\varphi_u$ = 39.6776º and $\varphi_l$ = 38.5523º and the ones resulting in the minimum extreme distortions $\varphi_u$ = 39.8054º and $\varphi_l$ = 38.4280º also in terms of the respective statistics for typical, average, maximum and minimum distortions.

The ground-to-grid transformation is analyzed in Table 3. We can see that the typical distortion amounts to 366 ppm, that is, more than 3 cm for a distance of 100 m, which recalls that the surveyor should normally not forget the reductions from field distance measurements to grid values. As a result of the significant differences in relief for the area of use of the projection, the modification of the defining values of the standard parallels do not improve much the situation: the typical distortion for the entire area cannot be decreased below 306 ppm, and the minimum existing distortion cannot be better than -611 ppm. The values of the standard parallels, or equivalently for the central parallel and its scale, optimizing typical and extreme distortions are relatively similar and considerably far from the current defining values.

## Conclusions

Apart from simple relationships for obtaining the defining parameters $\varphi_0$ and $k_0$ that permit to recast a 2 standard parallel LCC projection in a one standard parallel LCC plus scale projection, a handy formulation for solving the inverse problem ($\varphi_l$ and $\varphi_u$ in terms of $\varphi_0$ and $k_0$) has being obtained.

The differences in performance evaluation and optimization of projections in terms of ground-to-grid and ellipsoid-to-grid distortions were also discussed. A tool for general use in a user-defined arbitrary region was prepared for evaluation and optimization in terms of both types of scale distortion. It is available for the readers. Examples of application were finally provided.

All these ideas and methods may play an important role in the forthcoming definition of the new SPCS2022.

## Appendix. LCC projection defining parameters for SPCS 83

*Table A1 here*

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## Tables

**Table 1.** Latitudes of the lower true-scale parallel $\varphi'_l$ in successive iterations for $\varphi_0$ = 42° and $k_0$ = 0.99995 (GRS80 ellipsoid).

| Iteration No. | $\varphi_l$ | $\varphi'_l$ |
|---|---|---|
| 1 | 41.427039817518° | 41.427039817518° |
| 2 | 41.427018236228° | 41.427018236228° |
| 3 | 41.426996894787° | 41.425097964315° |
| 4 | 41.425098000955° | 41.425098000955° |
| 5 | 41.425098037186° | 41.425101255574° |
| 6 | 41.425101255511° | 41.425101255511° |
| 7 | 41.425101255448° | 41.425101249927° |
| 8 | 41.425101249927° | 41.425101249927° |

**Table 2.** Statistics for ellipsoid-to-grid distortions in Colorado Central SPCS 83 (EPSG:3501).

| | Current definition | *Optimized for min. typical distortion* | *Optimized for min. extreme distortion* |
|---|---|---|---|
| $\varphi_u$ | 39.75° | 39.6776° | 39.8054° |
| $\varphi_l$ | 38.45° | 38.5523° | 38.4280° |
| $\varphi_0$ | 39.1010° | 39.1158° | 39.1178° |
| $k_0$ | 0.999936 | 0.999952 | 0.999928 |
| Typical distortion (ppm) | 46 | 43 | 49 |
| Average distortion (ppm) | -16 | 0 | -24 |
| Maximum distortion (ppm) | 85 | 96 | 72 |
| Minimum distortion (ppm) | -64 | -48 | -72 |

**Table 3.** Statistics for ground-to-grid distortions in Colorado Central SPCS 83 (EPSG:3501).

| | Current definition | *Optimized for min. typical distortion* | *Optimized for min. extreme distortion* |
|---|---|---|---|
| $\varphi_u$ | 39.75° | 39.1150° | 39.1150° |
| $\varphi_l$ | 38.45° | 39.1148° | 38.9411° |
| $\varphi_0$ | 39.1010° | 39.1149° | 39.0280° |
| $k_0$ | 0.999936 | 1.000000 | 0.999999 |
| Typical distortion (ppm) | 366 | 306 | 307 |
| Average distortion (ppm) | -345 | -281 | -281 |
| Maximum distortion (ppm) | 0 | 0 | 7 |
| Minimum distortion (ppm) | -678 | -615 | -611 |

**Table A1.** Defining parameters for LCC projection for the different zones of the SPCS 83: ($\varphi_l$, $\varphi_u$) are the lower and upper standard parallel latitudes, respectively, ($k_0$, $\varphi_0$) are the scale and latitude of the central parallel, respectively.

| Zone abbrev | Zone code | $\varphi_l$ | $\varphi_u$ | $k_0$ | $\varphi_0$ |
|---|---|---|---|---|---|
| Alaska (AK): SPCS 83 | | | | | |
| AK10 | 5010 | 51°50'N | 53°50'N | 0.999848060 | 52°50'13.95273"N |
| Arkansas (AR): SPCS 83 | | | | | |
| AR N | 301 | 34°56'N | 36°14'N | 0.999935935 | 35°35'03.22204"N |
| AR S | 302 | 33°18'N | 34°46'N | 0.999918470 | 34°02'03.873752"N |
| California (CA): SPCS 83 | | | | | |
| CA 1 | 401 | 40°00'N | 41°40'N | 0.999894637 | 40°50'6.38205"N |
| CA 2 | 402 | 38°20'N | 39°50'N | 0.999914673 | 39°05'04.862119"N |
| CA 3 | 403 | 37°04'N | 38°26'N | 0.999929179 | 37°45'03.849971"N |
| CA 4 | 404 | 36°00'N | 37°15'N | 0.999940762 | 36°37'33.09351"N |
| CA 5 | 405 | 34°02'N | 35°28'N | 0.999922127 | 34°45'03.799131"N |
| CA 6 | 406 | 32°47'N | 33°53'N | 0.999954142 | 33°20'02.122601"N |
| Colorado (CO): SPCS 83 | | | | | |
| CO N | 501 | 39°43'N | 40°47'N | 0.999956846 | 40°15'02.561233"N |
| CO C | 502 | 38°27'N | 39°45'N | 0.999935910 | 39°06'03.654042"N |
| CO S | 503 | 37°14'N | 38°26'N | 0.999945398 | 37°50'02.976973"N |
| Connecticut (CT): SPCS 83 | | | | | |
| CT | 600 | 41°12'N | 41°52'N | 0.999983140 | 41°32'01.046165"N |
| Florida (FL): SPCS 83 | | | | | |
| FL N | 903 | 29°35'N | 30°45'N | 0.999948433 | 30°10'02.112795"N |
| Iowa (IA): SPCS 83 | | | | | |
| IA N | 1401 | 42°04'N | 43°16'N | 0.999945368 | 42°40'03.525435"N |
| IA S | 1402 | 40°37'N | 41°47'N | 0.999948370 | 41°12'03.167141"N |
| Kansas (KS): SPCS 83 | | | | | |
| KS N | 1501 | 38°43'N | 39°47'N | 0.999956851 | 39°15'02.47301"N |
| KS S | 1502 | 37°16'N | 38°34'N | 0.999935918 | 37°55'03.504219"N |
| Kentucky (KY): SPCS 83 | | | | | |
| KY1Z | 1600 | 37°05'N | 38°40'N | 0.999904942 | 37°52'35.19065"N |
| KY N | 1601 | 37°58'N | 38°58'N | 0.999962080 | 38°28'02.114288"N |
| KY S | 1602 | 36°44'N | 37°56'N | 0.999945402 | 37°20'02.924351"N |
| Louisiana (LA): SPCS 83 | | | | | |
| LA N | 1701 | 31°10'N | 32°40'N | 0.999914741 | 31°55'03.740121"N |
| LA S | 1702 | 29°18'N | 30°42'N | 0.999925745 | 30°00'03.022354"N |
| LASH | 1703 | 26°10'N | 27°50'N | 0.999894794 | 27°00'03.784619"N |
| Maryland (MD): SPCS 83 | | | | | |
| MD | 1900 | 38°18'N | 39°27'N | 0.999949848 | 38°52'32.83682"N |
| Massachusetts (MA): SPCS 83 | | | | | |

| Zone abbrev | Zone code | $\varphi_l$ | $\varphi_u$ | $k_0$ | $\varphi_0$ |
|---|---|---|---|---|---|
| MA M | 2001 | 41°43'N | 42°41'N | 0.999964550 | 42°12'02.251034"N |
| MA I | 2002 | 41°17'N | 41°29'N | 0.999998483 | 41°23'00.09366394"N |
| Michigan (MI): SPCS 83 | | | | | |
| MI N | 2111 | 45°29'N | 47°05'N | 0.999902834 | 46°17'07.100223"N |
| MI C | 2112 | 44°11'N | 45°42'N | 0.999912706 | 44°56'36.09153"N |
| MI S | 2113 | 42°06'N | 43°40'N | 0.999906878 | 42°53'06.054488"N |
| Minnesota (MN): SPCS 83 | | | | | |
| MN N | 2201 | 47°02'N | 48°38'N | 0.999902817 | 47°50'07.490779"N |
| MN C | 2202 | 45°37'N | 47°03'N | 0.999922023 | 46°20'05.707721"N |
| MN S | 2203 | 43°47'N | 45°13'N | 0.999922040 | 44°30'05.35829"N |
| Montana (MT): SPCS 83 | | | | | |
| MT | 2500 | 45°00'N | 49°00'N | 0.999392636 | 47°00'45.52353"N |
| Nebraska (NE): SPCS 83 | | | | | |
| NE | 2600 | 40°00'N | 43°00'N | 0.999658595 | 41°30'21.1692"N |
| New York (NY): SPCS 83 | | | | | |
| NY L | 3104 | 40°40'N | 41°02'N | 0.999994900 | 40°51'00.3090314"N |
| North Carolina (NC): SPCS 83 | | | | | |
| NC | 3200 | 34°20'N | 36°10'N | 0.999872592 | 35°15'06.330961"N |
| North Dakota (ND): SPCS 83 | | | | | |
| ND N | 3301 | 47°26'N | 48°44'N | 0.999935842 | 48°05'04.98783"N |
| ND S | 3302 | 46°11'N | 47°29'N | 0.999935852 | 46°50'04.776813"N |
| Ohio (OH): SPCS 83 | | | | | |
| OH N | 3401 | 40°26'N | 41°42'N | 0.999939140 | 41°04'03.716122"N |
| OH S | 3402 | 38°44'N | 40°02'N | 0.999935908 | 39°23'03.690642"N |
| Oklahoma (OK): SPCS 83 | | | | | |
| OK N | 3501 | 35°34'N | 36°46'N | 0.999945409 | 36°10'02.804168"N |
| OK S | 3502 | 33°56'N | 35°14'N | 0.999935942 | 34°35'03.105994"N |
| Oregon (OR): SPCS 83 | | | | | |
| OR N | 3601 | 44°20'N | 46°00'N | 0.999894583 | 45°10'07.413463"N |
| OR S | 3602 | 42°20'N | 44°00'N | 0.999894608 | 43°10'06.919559"N |
| Pennsylvania (PA): SPCS 83 | | | | | |
| PA N | 3701 | 40°53'N | 41°57'N | 0.999956840 | 41°25'02.667447"N |
| PA S | 3702 | 39°56'N | 40°58'N | 0.999959500 | 40°27'02.420495"N |
| South Carolina (SC): SPCS 83 | | | | | |
| SC | 3900 | 32°30'N | 34°50'N | 0.999793657 | 33°40'09.672498"N |
| South Dakota (SD): SPCS 83 | | | | | |
| SD N | 4001 | 44°25'N | 45°41'N | 0.999939112 | 45°03'04.264566"N |
| SD S | 4002 | 42°50'N | 44°24'N | 0.999906870 | 43°37'06.209699"N |
| Tennessee (TN): SPCS 83 | | | | | |
| TN | 4100 | 35°15'N | 36°25'N | 0.999948401 | 35°50'02.618685"N |
| Texas (TX): SPCS 83 | | | | | |

| Zone abbrev | Zone code | $\varphi_l$ | $\varphi_u$ | $k_0$ | $\varphi_0$ |
|---|---|---|---|---|---|
| TX N | 4201 | 34°39'N | 36°11'N | 0.999910876 | 35°25'04.455416"N |
| TXNC | 4202 | 32°08'N | 33°58'N | 0.999872623 | 33°03'05.833995"N |
| TX C | 4203 | 30°07'N | 31°53'N | 0.999881744 | 31°00'05.007016"N |
| TXSC | 4204 | 28°23'N | 30°17'N | 0.999863244 | 29°20'05.41983"N |
| TX S | 4205 | 26°10'N | 27°50'N | 0.999894794 | 27°00'03.784619"N |
| Utah (UT): SPCS 83 | | | | | |
| UT N | 4301 | 40°43'N | 41°47'N | 0.999956841 | 41°15'02.652047"N |
| UT C | 4302 | 39°01'N | 40°39'N | 0.999898821 | 39°50'05.918907"N |
| UT S | 4303 | 37°13'N | 38°21'N | 0.999951297 | 37°47'02.650647"N |
| Virginia (VA): SPCS 83 | | | | | |
| VA N | 4501 | 38°02'N | 39°12'N | 0.999948385 | 38°37'02.893135"N |
| VA S | 4502 | 36°46'N | 37°58'N | 0.999945401 | 37°22'02.927838"N |
| Washington (WA): SPCS 83 | | | | | |
| WA N | 4601 | 47°30'N | 48°44'N | 0.999942253 | 48°7'04.494517"N |
| WA S | 4602 | 45°50'N | 47°20'N | 0.999914598 | 46°35'06.305232"N |
| West Virginia (WV): SPCS 83 | | | | | |
| WV N | 4701 | 39°00'N | 40°15'N | 0.999940741 | 39°37'33.44126"N |
| WV S | 4702 | 37°29'N | 38°53'N | 0.999925678 | 38°11'04.102788"N |
| Wisconsin (WI): SPCS 83 | | | | | |
| WI N | 4801 | 45°34'N | 46°46'N | 0.999945345 | 46°10'03.977587"N |
| WI C | 4802 | 44°15'N | 45°30'N | 0.999940705 | 44°52'34.12811"N |
| WI S | 4803 | 42°44'N | 44°04'N | 0.999932547 | 43°24'04.464095"N |
| Puerto Rico and U.S. Virgin Islands (PR and VI): SPCS 83 | | | | | |
| PRVI | 5200 | 18°02'N | 18°26'N | 0.999993944 | 18°14'00.1413267"N |

## Figure captions

Fig. 1. True-scale parallels $\varphi_l$ and $\varphi_u$, central parallel $\varphi_0$ and corresponding scale factor $k_0$ in an LCC projection.

Fig. 2. General Conic projection with spherical earth.

Fig. 3. TestGrids release 2.0 layout.